

# Syntactic Approaches for Natural Language Processing

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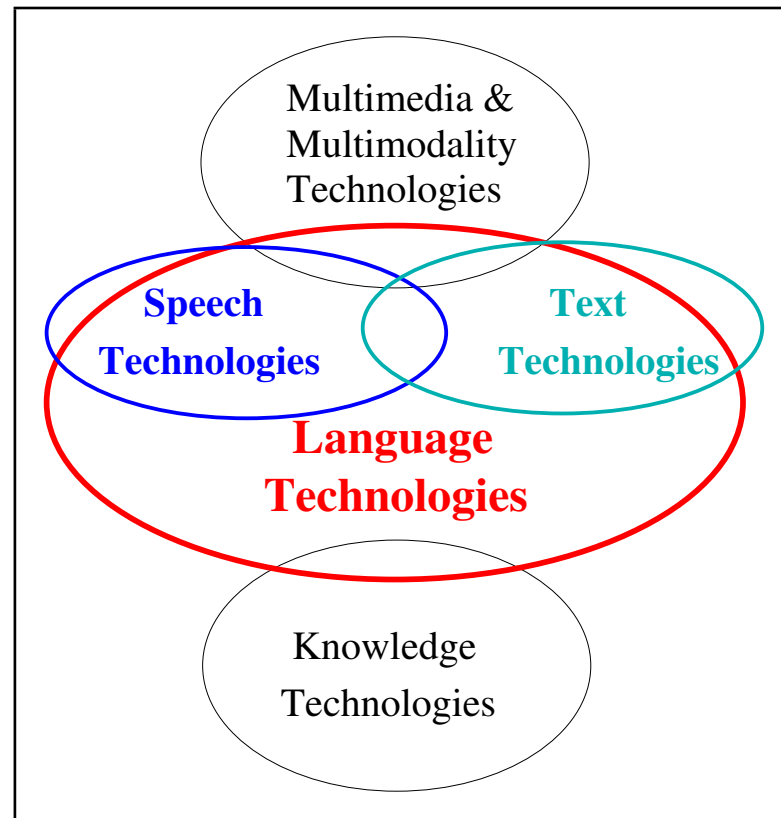
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# 1.1 INTRODUCTION

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“Computational Linguistics” deals with the most difficult communication process:  
**Natural Language**



# 1.1 INTRODUCTION

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## Goal:

To develop systems that are able to process, to understand and, to produce Natural Language

## Motivation:

- Natural Language is the main way to represent and to transfer human knowledge
- There exist lots of information and knowledge in Natural Language
- There exist a lot of potential users that need to communicate with computers in Natural Language

## Applications

- Systems for information extraction from text and speech

Examples:

information retrieval, information extraction, text categorization, ...

- Systems for speech/text to speech/text:

Examples:

machine translation, speech translation, speech recognition, ...

- Systems for communication with humans:

Examples:

dialog systems, query systems, ...

## Probabilistic approach

- **Interpretation** by using the probabilistic decision rule  
[ to generate a desired interpretation (output) ]
- **Modeling** the human perception with Statistical Decision techniques and Formal Language theory  
[ to define the statistical dependence between observations (input) and interpretation (output) ]
- **Learning** knowledge from examples  
[ to learn the model parameters from training examples ]

## Main goals of the lecture

- **To introduce syntactic approaches to deal with difficult problems related to Natural Language**
- To study fundamentals related to Computational Linguistics
- To learn basic techniques that are necessary to develop robust systems that are able to understand text data



# 1.1 INTRODUCTION

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## Applications

- Automatic Speech Recognition
- Machine Translation
- Dialog Systems
- Automatic Summarization
- Text Classification
- Information Retrieval
- . . .

## Abstract tasks

- *Language Modeling*
- *Part of Speech Tagging*
- *Parsing*
- Lexical Disambiguation
- Semantic Analysis
- Discourse Analysis
- . . .

## Knowledge levels in Natural Language:

- Morphology: word structure
- Syntax:
  - word category — *Part of Speech tagging*
  - sentence structure — *Parsing, Language Modeling*
- Semantics:
  - word semantics
  - sentence semantics
- Pragmatics: use of the language, cultural issues, environment
- Discourse: dialog structure

## 1.2 HMM AND POS TAGGING

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**Part of Speech Tagging Problem:** Given a set of PoS tags and a sentence, to assign a PoS tag to each word

Pierre/NNP Vinken/NNP ,/, 61/CD years/NNS old/JJ ,/, will/MD join/VB  
the/DT board/NN as/IN a/DT nonexecutive/JJ director/NN Nov./NNP 29/CD ./.

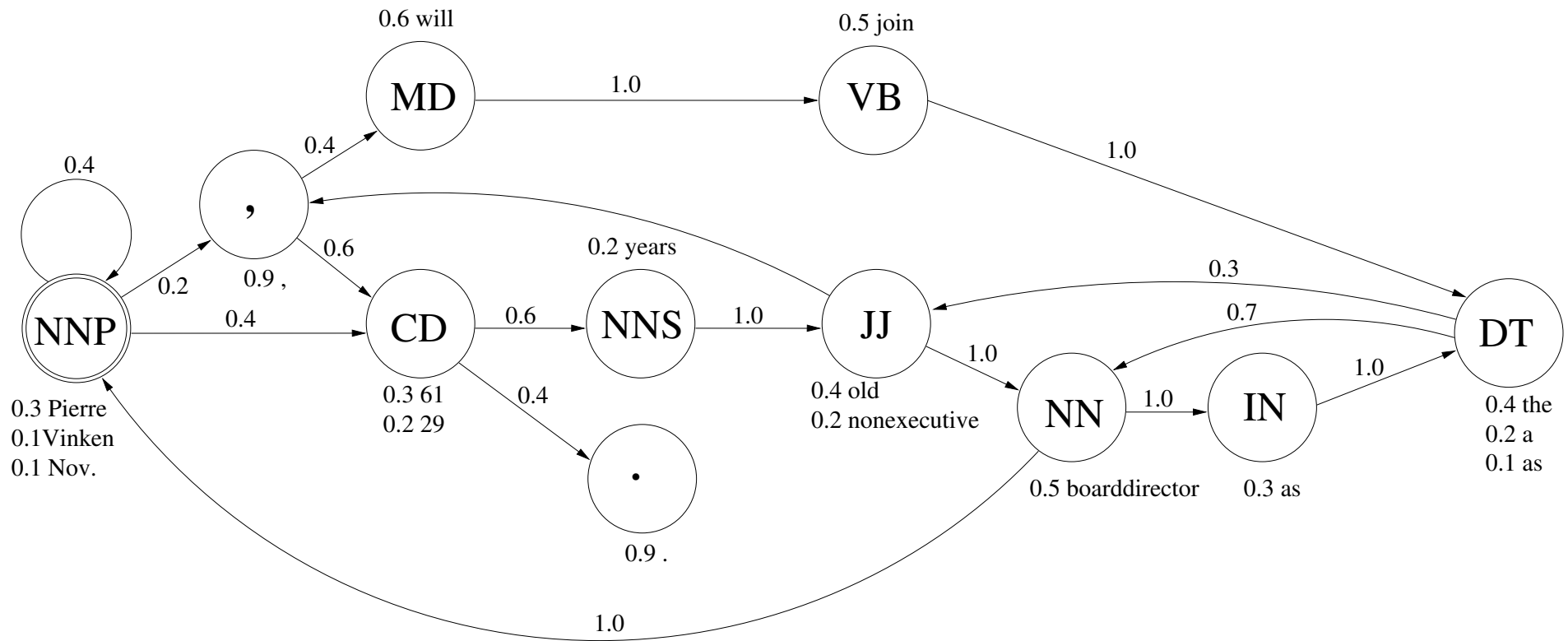
→ Problem is difficult because of ambiguity

Approaches:

- HMM
- Maximum Entropy
- SVM

# 1.2 HMM AND POS TAGGING

## HMM for PoS tagging: [Merialdo 94]



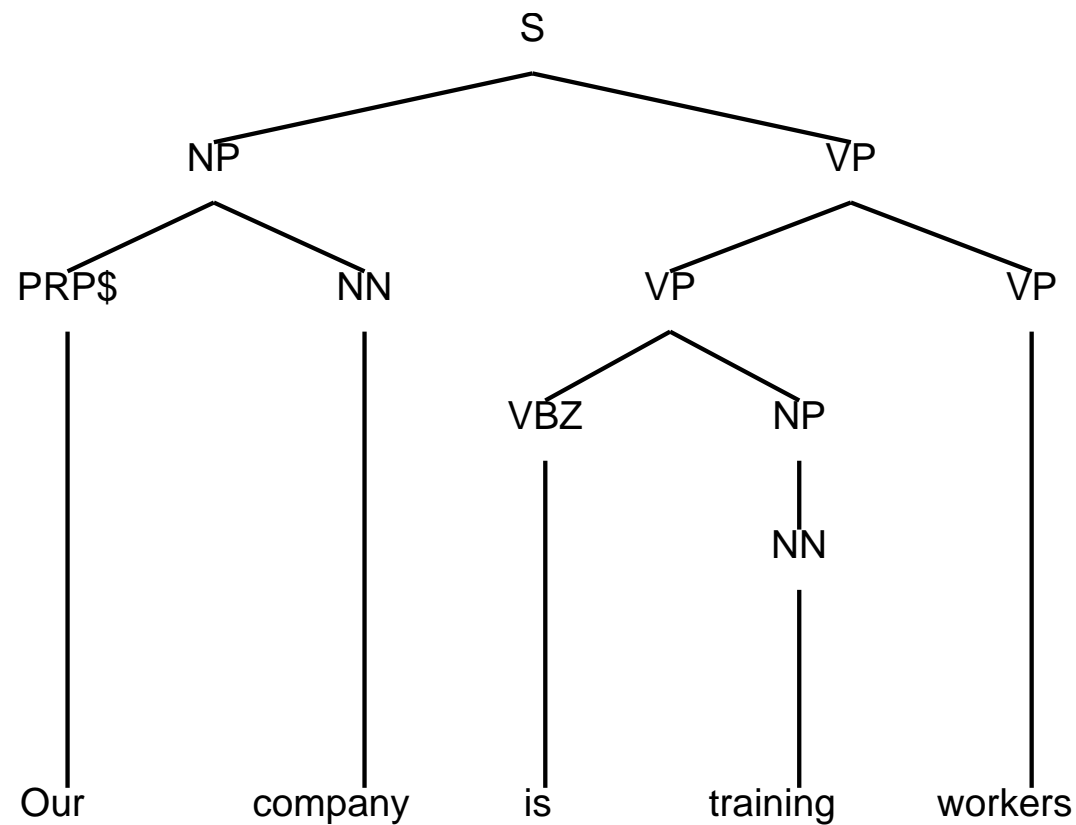
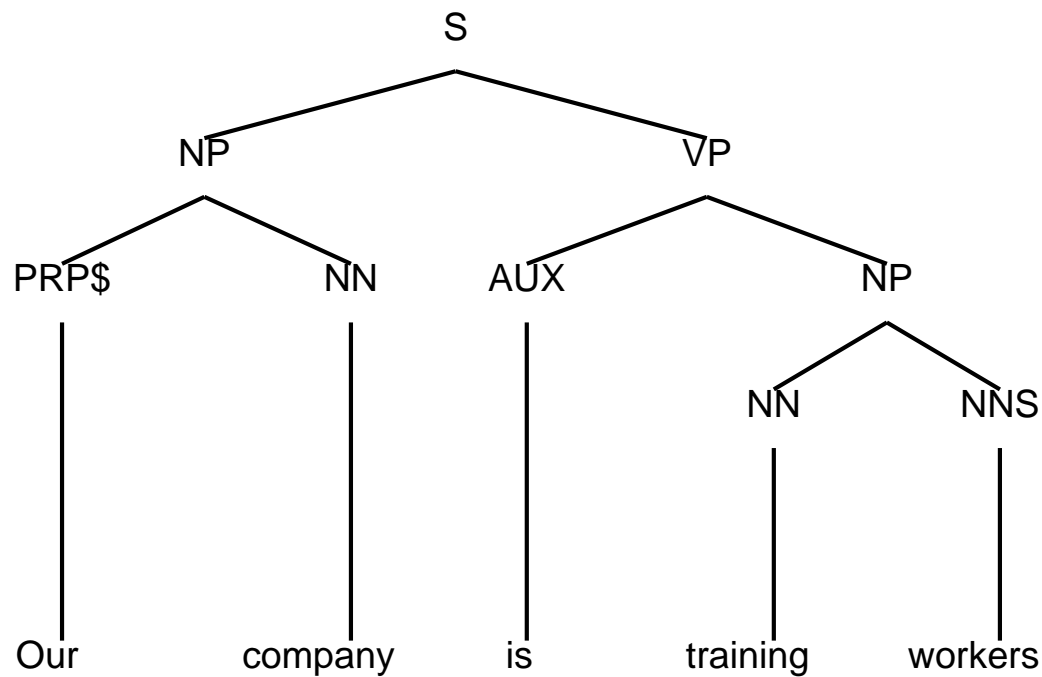
### Problems:

- Model learning
- Interpretation

# 1.3 PCFG AND PARSING

**Parsing Problem:** Given a sentence, to assign a parsing structure to the sentences

**Difficulties in Parsing: Ambiguity**



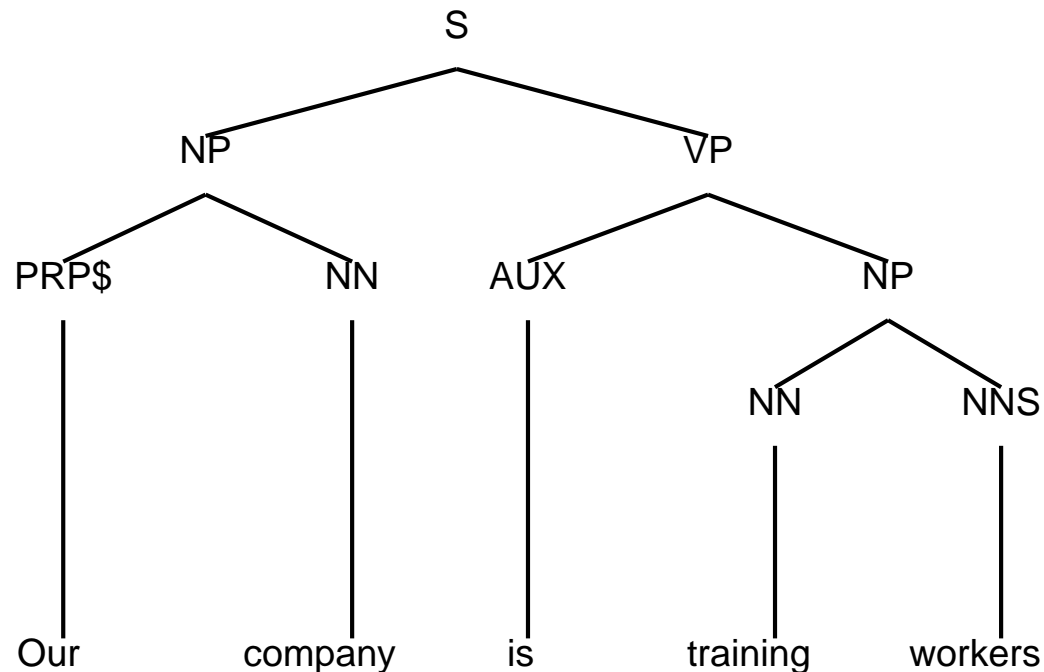
### Parsing with syntactic models: (Formal) grammar

S	→	NP VP	PRP\$	→	Our
NP	→	PRP\$ NN	NN	→	company
NP	→	NN NNS	AUX	→	is
NP	→	NN	NN	→	training
VP	→	AUX NP	NNS	→	workers
VP	→	VP VP			
VP	→	VBZ NP			

# 1.3 PCFG AND PARSING

## Parsing with syntactic models: (Formal) grammar

1.0	S	→	NP VP	1.0	PRP\$	→	Our
0.4	NP	→	PRP\$ NN	0.6	NN	→	company
0.3	NP	→	NN NNS	1.0	AUX	→	is
0.3	NP	→	NN	0.4	NN	→	training
0.5	VP	→	AUX NP	1.0	NNS	→	workers
0.3	VP	→	VP VP				
0.2	VP	→	VBZ NP				



### Recognition with noisy channel



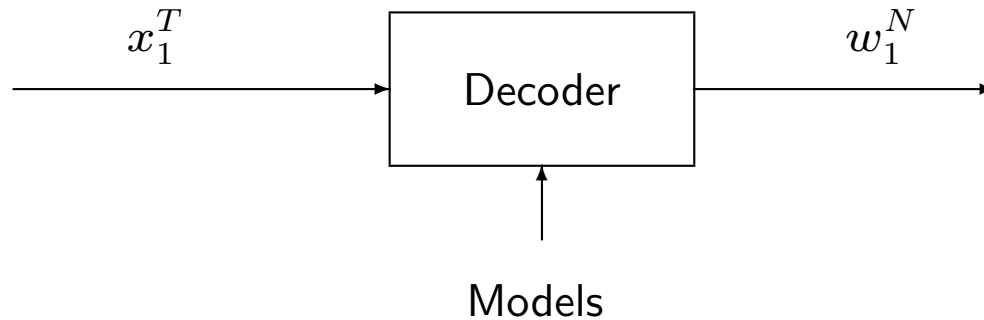
$$\hat{I} = \arg \max_I \Pr(I|O) = \arg \max_I \Pr(O|I) \Pr(I)$$

$\Pr(I)$ : language model probability

$\Pr(O|I)$ : channel probability



## Automatic Speech Recognition



$$\widehat{w_1^N} = \arg \max_{w_1^N} \Pr(w_1^N | x_1^T) = \arg \max_{w_1^N} \Pr(x_1^T | w_1^N) \Pr(w_1^N)$$

## Language Model

$$\Pr(w_1^N) = \Pr(w_1) \prod_{n=2}^N \Pr(w_n | w_1^{n-1})$$

## 1.4 PCFG FOR LANGUAGE MODELING

---

➤ **N-Gram models:** Restriction on the history length  $w_1^{n-1}$

$$\Pr(w_1^N) = \Pr(w_1) \prod_{n=2}^N \Pr(w_n | w_{n-k+1}^{n-1})$$

- ✗ don't capture long-term dependencies
- ✓ efficient to compute
- ✓ efficient methods to estimate the model parameters

➤ **Grammatical models:** No restriction on the history length  $w_1^{n-1}$

$$\Pr(w_1^N) = \Pr(w_1) \prod_{n=2}^N \Pr(w_n | w_1^{n-1})$$

- ✓ capture long-term dependencies
- ✗ expensive to compute
- ✗ efficient methods to estimate the model parameters, but expensive

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## 2.1 NOTATION AND DEFINITIONS

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### Hidden Markov Models: [Vidal 05a, Vidal 05b]

- Simple and compact models for representing regular relations
- Formal framework well understood
- Natural Language is no regular (but almost)
- Adequate representation of short-term syntactic structures
- Adequate modeling of ambiguity

## 2.1 NOTATION AND DEFINITIONS

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### Example

- Primitives: alphabet

words, punctuation symbols, . . .

- Object representation: written sentences

“Pierre Vinken, 61 years old, will join the board as a nonexecutive director Nov. 29.”

- Pattern set

sentences

- Interpretation: PoS tag association

Pierre/NNP Vinken/NNP ,/, 61/CD years/NNS old/JJ ,/, will/MD  
join/VB the/DT board/NN as/IN a/DT nonexecutive/JJ director/NN  
Nov./NNP 29/CD ./.

## 2.1 NOTATION AND DEFINITIONS

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- An **alphabet**  $T$  is a finite set of symbols.
- A **string**  $x = a_1 \cdots a_n$  ( $a_i \in T; i : 1 \dots n$ ), is a finite sequence of symbols of  $T$ . The length of the string is noted by  $|x|$ . Let  $x$  and  $y$  be two strings,  $x, y \in T^*$ , then the **concatenation** of  $x$  and  $y$  is the string  $xy$ .  $|xy| = |x| + |y|$ .
- The **empty string**  $\epsilon$ , is the string with length equal to zero. For any string  $x$ ,  $x \in T^*$ :  
 $\epsilon x = x \epsilon = x$ .
- The **closure**  $T^*$  is the infinite and countable set of all strings with finite length composed with symbols of  $T$ ,  $\epsilon$  included. The **positive closure**  $T^+$  is defined as:  
 $T^+ = T^* - \{\epsilon\}$ .
- A **language**  $L$  is a set of strings composed with symbols of  $T$  ( $L \subseteq T^*$ ).

## 2.1 NOTATION AND DEFINITIONS

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A discrete *HMM* is defined as  $M = (Q, T, a, b, \pi, q_f)$ :

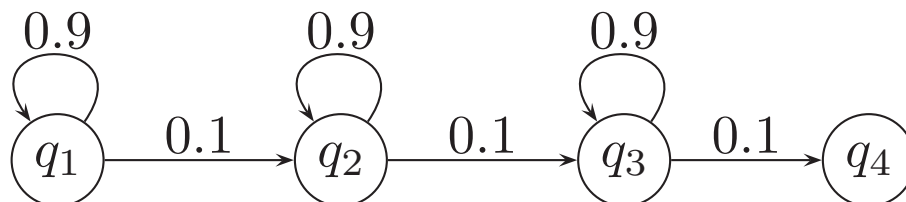
$$a : Q - \{q_f\} \times Q \rightarrow [0, 1]; \quad \forall q \in Q - \{q_f\}: \sum_{q' \in Q} a(q, q') = 1$$

$$b : Q - \{q_f\} \times T \rightarrow [0, 1]; \quad \forall q \in Q - \{q_f\}: \sum_{x \in T} b(q, x) = 1$$

$$\pi : Q \rightarrow [0, 1]; \quad \sum_{q \in Q} \pi(q) = 1$$

Example: Given  $T = \{a, b\}$ :

$$a \begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix} \quad a \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix} \quad a \begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix}$$

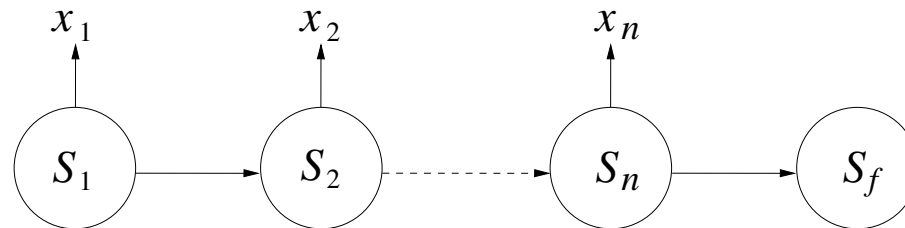


$$\pi(q_1) = 1.0$$

## 2.1 NOTATION AND DEFINITIONS

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Given  $x = x_1 \cdots x_n \in T^*$  and the HMM  $M$ :



$$b(s_1 = q_1, x_1)a(s_1, s_2)b(s_2, x_2) \dots a(s_{n-1}, s_n)b(s_n, x_n)a(s_n, q_f)$$

Let  $S = (s_1 = q_1, s_2, \dots, s_n, s_{n+1} = q_f)$  be a valid path through  $M$ . Then:

$$\Pr_M(S) = \prod_{i=1}^n a(s_i, s_{i+1}), \quad \text{and} \quad \Pr_M(x | S) = \prod_{i=1}^n b(s_i, x_i)$$

Let  $\mathcal{S}_M(x)$  be the set of all valid paths for  $x$ . Then:

$$\Pr_M(x) = \sum_{S \in \mathcal{S}_M(x)} \Pr_M(x | S) \Pr_M(S)$$



### Forward algorithm

– **Definition:**  $\alpha(i, q) = \Pr_M(x_1 \cdots x_i, q) \quad 1 \leq i \leq n + 1 \quad q \in Q \cup \{q_f\}$

– **Recursion:**  $\forall q \in Q$  with  $2 \leq i \leq n$

$$\alpha(i, q) = \left[ \sum_{q' \in Q} \alpha(i-1, q') a(q', q) \right] b(q, x_i)$$

$$\alpha(n+1, q_f) = \sum_{q' \in Q} \alpha(n, q') a(q', q_f)$$

– **Initialization:**  $\alpha(1, q) = \pi(q) b(q, x_1) \quad \forall q \in Q \cup \{q_f\}$

– **Result:**  $\Pr_M(x) = \alpha(n+1, q_f)$

## 2.2 FUNDAMENTAL ALGORITHMS

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### Forward algorithm: Example

	<i>a</i>	<i>b</i>	<i>b</i>	<i>a</i>
$q_1$	0.9	0.9 0.9 0.1		
$q_2$		0.9 0.1 0.9	0.081 0.1 0.9+ 0.081 0.9 0.9	
$q_3$			0.081 0.1 0.1	0.0729 0.1 0.9+ 0.00081 0.9 0.9
$q_4$				0.0072171 0.1

### Backward algorithm

– **Definition:**  $\beta(i, q) = \Pr_M(x_{i+1} \cdots x_n \mid q) \quad 1 \leq i \leq n+1 \quad q \in Q \cup \{q_f\}$

– **Recursion:**  $\forall q \in Q$  with  $1 \leq i \leq n-1$ :

$$\beta(i, q) = \sum_{q' \in Q} a(q, q') b(q', x_{i+1}) \beta(i+1, q')$$

– **Initialization:**  $\beta(n, q) = a(q, q_f) \beta(n+1, q_f) \quad \forall q \in Q.$   
 $\beta(n+1, q_f) = 1$

– **Result:**  $\Pr_M(x) = b(q_1, x_1) \beta(1, q_1)$

## 2.2 FUNDAMENTAL ALGORITHMS

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Let:

$$\widehat{S}_x = \max_{S \in \mathcal{S}_M(x)} \Pr_M(x | S) \Pr_M(S)$$

and:

$$\widehat{\Pr}_M(x) = \Pr_M(x, \widehat{S}_x)$$

### Viterbi algorithm

– **Definition:**  $\gamma(i, q) = \widehat{\Pr}_M(x_1 \cdots x_i, q) \quad 1 \leq i \leq n \quad q \in Q \cup \{q_f\}$

– **Recursion:**  $\forall q \in Q$  with  $2 \leq i \leq n$

$$\gamma(i, q) = \left[ \max_{q' \in Q} \gamma(i-1, q') a(q', q) \right] b(q, x_i)$$

$$\gamma(n+1, f) = \max_{q' \in Q} \gamma(n, q') a(q', q_f)$$

– **Initialization:**  $\gamma(1, q) = \pi(q) b(q, x_1) \quad \forall q \in Q \cup \{q_f\}$

– **Result:**  $\widehat{\Pr}_M(x) = \gamma(n+1, q_f)$

## 2.2 FUNDAMENTAL ALGORITHMS

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### Viterbi algorithm: Example

	<i>a</i>	<i>b</i>	<i>b</i>	<i>a</i>	
$q_1$	0.9	0.9 0.9 0.1			
$q_2$		0.9 0.1 0.9	0.081 0.1 0.9 , 0.081 0.9 0.9		
$q_3$			0.081 0.1 0.1	0.06561 0.1 0.9 , 0.00081 0.9 0.9	
$q_4$					0.0059049 0.1

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## 3.1 INTRODUCTION

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### ➤ Supervised methods

- Maximum likelihood estimation

$$\bar{a}(q, q') = \frac{C(q, q')}{C(q)}$$

- Annotated data is needed

### ➤ Non-supervised methods

- EM algorithms
- Problem: local optimum

## 3.1 INTRODUCTION

---

Let  $M$  be a HMM and  $\theta = (a, b, \pi)$ , and let  $\Omega = \{x_1, x_2, \dots, x_n\}$  be a training sample.

$$\hat{\theta} = \arg \max_{\theta} F_{\theta}(\Omega)$$

### ➤ Optimization method

➤ Growth transformations

### ➤ Optimization function

➤ **Maximum likelihood**

➤ Corrective training

➤ Maximum mutual information



## 3.1 INTRODUCTION

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### Theorem [Baum 72]

Let  $P(\Theta)$  be a homogeneous polynomial with non-negative coefficients. Let  $\theta = \{\theta_{ij}\}$  be a point in the domain  $D = \{\theta_{ij} \mid \theta_{ij} \geq 0; \sum_{j=1}^{q_i} \theta_{ij} = 1, i = 1, \dots, p; j = 1, \dots, q_i\}$ , and let  $Q(\theta)$  be a close transformation in  $D$ , that is defined as:

$$Q(\theta)_{ij} = \frac{\theta_{ij}(\partial P / \partial \Theta_{ij})_{\theta}}{\sum_{k=1}^{q_i} \theta_{ik}(\partial P / \partial \Theta_{ik})_{\theta}}$$

with the denominator different from zero. Then,  $P(Q(\theta)) > P(\theta)$  except if  $Q(\theta) = \theta$ .

```
input  $P(\Theta)$ 
 $\theta$  = initial values
repeat
    compute  $Q(\theta)$  using  $P(\Theta)$ 
     $\theta = Q(\theta)$ 
until convergence
output  $\theta$ 
```

### Optimization function

Given a sample  $\Omega$  and a model  $M$

$$\Pr_M(\Omega, \Delta_\Omega) = \prod_{x \in \Omega} \Pr_M(x, \Delta_M(x)),$$

such that:

- $\Delta_M(x) \subseteq \mathcal{S}_M(x)$
- $\Pr_M(x, \Delta_M(x)) = \sum_{S \in \Delta_M(x)} \Pr_M(x, S)$

- $\forall q, q' \in Q - \{q_f\}$  (See demonstration [Benedí 05])

$$\bar{a}(q, q') = \frac{\sum_{x \in \Omega} \frac{1}{\Pr_M(x, \Delta_M(x))} \sum_{S \in \Delta_M(x)} N((q, q'), S) \Pr_M(x, S)}{\sum_{x \in \Omega} \frac{1}{\Pr_M(x, \Delta_M(x))} \sum_{S \in \Delta_M(x)} N(q, S) \Pr_M(x, S)}$$

- $\forall q \in Q: \bar{a}(q, q_f)$
- $\forall q \in Q, \forall a \in t: \bar{b}(q, a)$

### Optimization function

$$\Pr_M(\Omega) = \prod_{x \in \Omega} \Pr_M(x),$$

### Baum-Welch algorithm

$$- \forall q, q' \in Q - \{q_f\}$$

$$\bar{a}(q, q') = \frac{\sum_{x \in \Omega} \frac{1}{\Pr_M(x)} \sum_{i=1}^{n-1} \alpha(i, q) a(q, q') b(q', x_{i+1}) \beta(i+1, q')}{\sum_{x \in \Omega} \frac{1}{\Pr_M(x)} \sum_{i=1}^n \alpha(i, q) \beta(i, q)}$$

$$- \forall q \in Q: \bar{a}(q, q_f)$$

$$- \forall q \in Q, \forall a \in t: \bar{b}(q, a)$$

$$- \forall q \in Q, \bar{\pi}(q)$$

Time complexity:  $O(|\Omega| |N| b)$

### Optimization function

$$\widehat{\text{Pr}}_M(\Omega) = \prod_{x \in \Omega} \widehat{\text{Pr}}_M(x),$$

### Viterbi algorithm

–  $\forall q, q' \in Q - \{q_f\}$

$$\bar{a}(q, q') = \frac{\sum_{x \in \Omega} N((q, q'), \hat{S}_x)}{\sum_{x \in \Omega} N(q, \hat{S}_x)}$$

–  $\forall q \in Q: \bar{a}(q, q_f)$

–  $\forall q \in Q, \forall a \in t: \bar{b}(q, a)$

–  $\forall q \in Q, \bar{\pi}(q)$

Time complexity:  $O(|\Omega| |N| b)$

## 3.3 VITERBI ALGORITHM

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1. Carrying out the maximization with  $M^{(i)}$ :  $\widehat{\mathcal{S}}_x^{(i)}$ ;

$$\widehat{\mathcal{S}}_x^{(i)} = \{\widehat{\mathcal{S}}_x^{(i)} : \widehat{\mathcal{S}}_x^{(i)} = \arg \max_{S \in \mathcal{S}_M(x)} \Pr_{M^{(i)}}(x, S)\}$$

2. Applying the transformation:  $M^{(i+1)}$ .

The function to be optimized is defined after step 1. This function is continuous and differentiable:

$$\prod_{x \in \Omega} \Pr_{M^{(i)}}(x, \widehat{\mathcal{S}}_x^{(i)}) \leq \prod_{x \in \Omega} \Pr_{M^{(i+1)}}(x, \widehat{\mathcal{S}}_x^{(i)}).$$

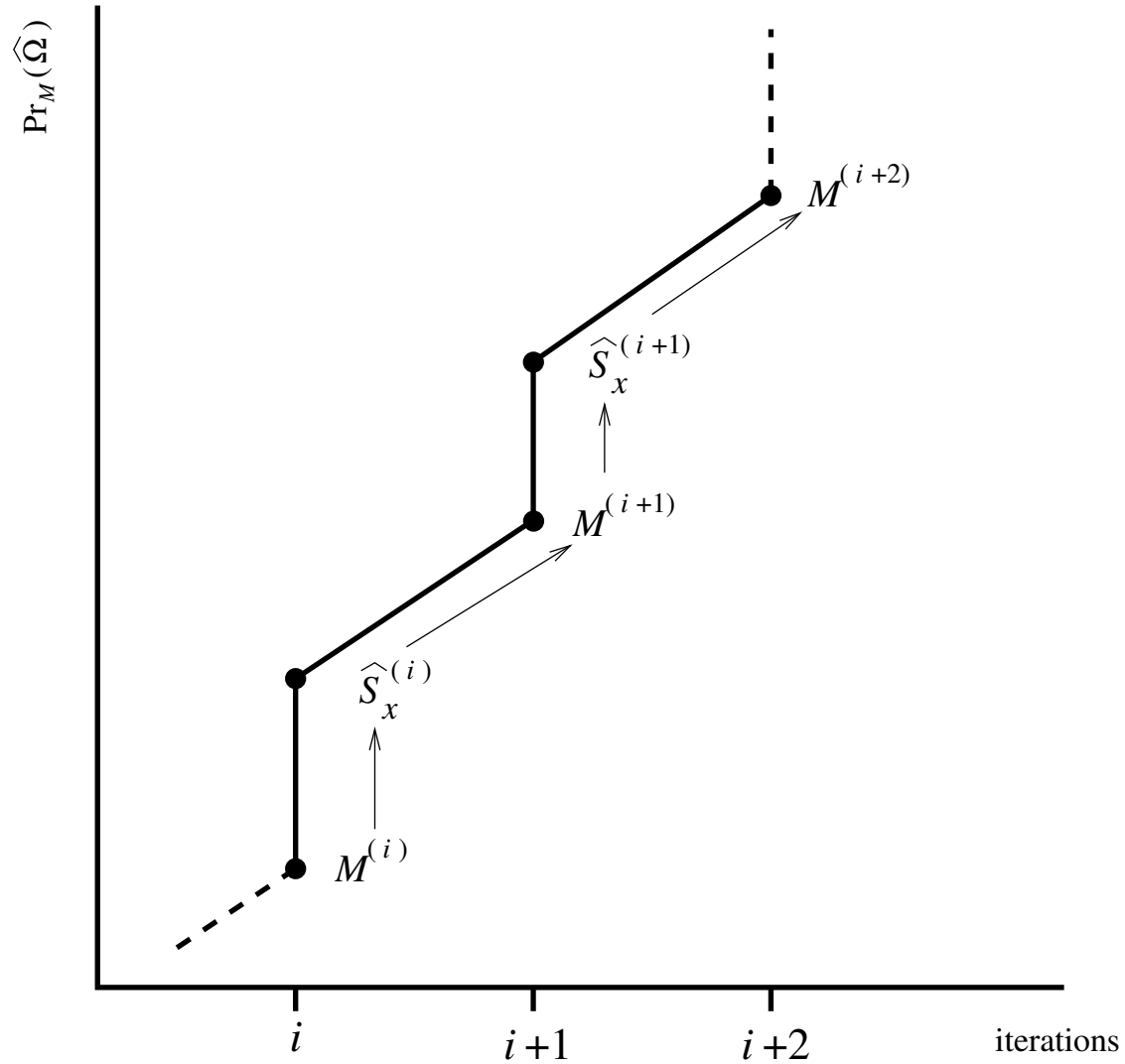
In the next step  $i + 1$ , the most probable sequence  $\widehat{\mathcal{S}}_x^{(i+1)}$  is computed for each string  $x$  with  $M^{(i+1)}$ , and therefore:

$$\Pr_{M^{(i+1)}}(x, \widehat{\mathcal{S}}_x^{(i)}) \leq \Pr_{M^{(i+1)}}(x, \widehat{\mathcal{S}}_x^{(i+1)}) \quad \forall x \in \Omega,$$

and hence

$$\prod_{x \in \Omega} \Pr_{M^{(i+1)}}(x, \widehat{\mathcal{S}}_x^{(i)}) \leq \prod_{x \in \Omega} \Pr_{M^{(i+1)}}(x, \widehat{\mathcal{S}}_x^{(i+1)}).$$

## 3.3 VITERBI ALGORITHM



## 3.4 USE OF HMM FOR POS TAGGING

---

**Problem:** Let  $W$  be a sentence and let  $\mathcal{C} = \{c_1, c_2, \dots, c_{|\mathcal{C}|}\}$  be a PoS tag set:

$$\begin{aligned}\hat{C} &= \arg \max_{C \in \mathcal{C}^{|\mathcal{W}|}} P(c_1 c_2 \dots c_{|\mathcal{W}|} \mid w_1 w_2 \dots w_{|\mathcal{W}|}) \\ &= \arg \max_{C \in \mathcal{C}^{|\mathcal{W}|}} P(c_1 c_2 \dots c_{|\mathcal{W}|}) P(w_1 w_2 \dots w_{|\mathcal{W}|} \mid c_1 c_2 \dots c_{|\mathcal{W}|})\end{aligned}$$

**Assumption:**

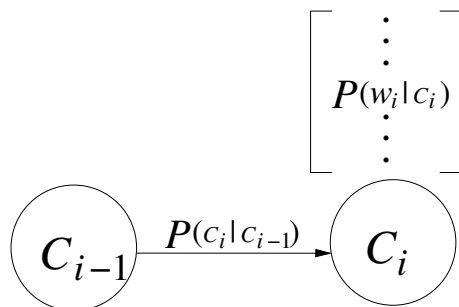
$$\begin{aligned}P(c_1 c_2 \dots c_{|\mathcal{W}|}) &\approx P(c_1) \prod_{i=2}^{|\mathcal{W}|} P(c_i \mid c_{i-1}) \\ P(w_1 w_2 \dots w_{|\mathcal{W}|} \mid c_1 c_2 \dots c_{|\mathcal{W}|}) &\approx \prod_{i=1}^{|\mathcal{W}|} P(w_i \mid c_i)\end{aligned}$$

## 3.4 USE OF HMM FOR POS TAGGING

---

Bigram approach:

$$\hat{C} = \arg \max_{C \in \mathcal{C}^{|W|}} P(c_1)P(w_1|c_1) \prod_{i=2}^{|W|} P(c_i|c_{i-1})P(w_i|c_i)$$



### Problems

- Labeling: Viterbi algorithm
- Parameter learning:
  - Non-supervised methods: Baum-Welch estimation.
  - Supervised methods:

$$P(c_i|c_{i-1}) = \frac{f(c_{i-1}c_i)}{f(c_{i-1})} \quad P(w_i|c_i) = \frac{f(w_i, c_i)}{f(c_i)}$$



## 3.4 USE OF HMM FOR POS TAGGING

### Example:

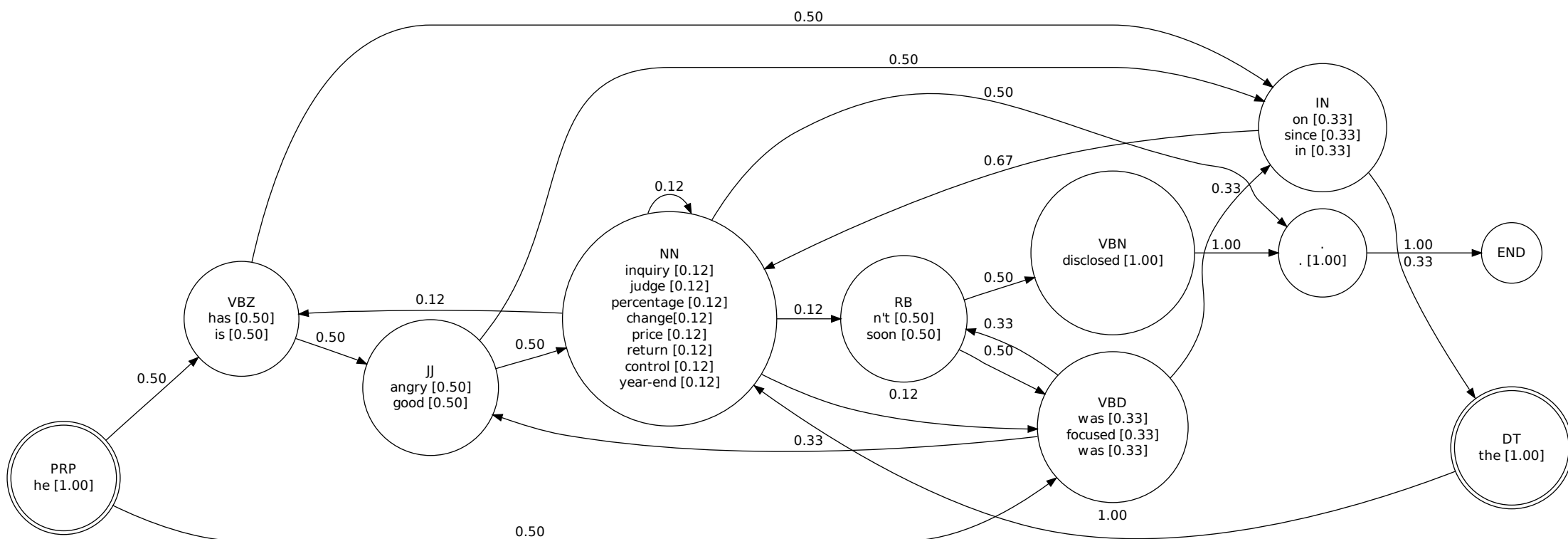
he/PRP has/VBZ good/JJ control/NN ./.

the/DT percentage/NN change/ NN is/VBZ since/IN year-end/ NN ./.

the/DT price/NN was/VBD n't/RB disclosed/VBN ./.

he/PRP becameVBD/ angry/JJ in/IN return/NN ./.

the/DT inquiry/NN soon/RB focused/VBD on/IN the/DT judge/NN ./.



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## 4.1 NOTATION AND DEFINITIONS

---

### Context-free grammar: [Aho,72]

- Simple and compact models for parsing
- Formal framework well understood
- Adequate representation of long-term syntactic structures
- Adequate modeling of ambiguity

## 4.1 NOTATION AND DEFINITIONS

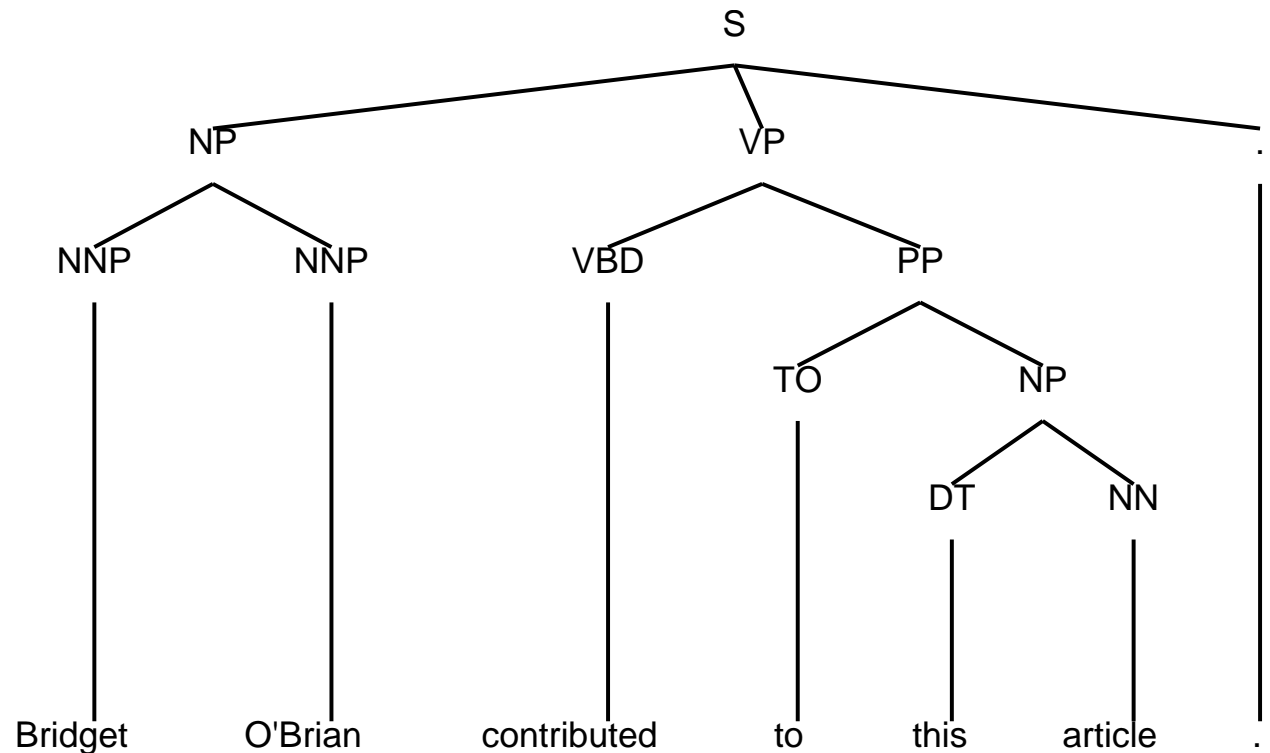
### Example

- Primitives: alphabet  
words, punctuation symbols, . . .
- Object representation: written sentences  
“Bridget O’Brian contributed to this article”

- Pattern set

sentences

- Interpretation: syntactic analysis



## 4.1 NOTATION AND DEFINITIONS

---

Similar definitions as in HMM:

- **Alphabet:**  $T$  is a finite set of symbols.
- **String:** a finite sequence of symbols of  $T$ .
- **Closure  $T^*$ :** the infinite and countable set of all strings with finite length composed with symbols of  $T$ ,  $\epsilon$  included.
- **Language:**  $L$  is a set of strings composed with symbols of  $T$  ( $L \subseteq T^*$ ).

## 4.1 NOTATION AND DEFINITIONS

---

➤ **Grammar:**  $G = (N, T, P, S)$

$$V = N \cup T; N \cap T = \emptyset; S \in N; (A \rightarrow \beta) \in P; \quad A \in N; \beta \in V^*$$

➤ **Derivation:**

$$\mu A \delta \Longrightarrow \mu \beta \delta \text{ iff } \exists (A \rightarrow \beta) \in P; \quad \mu, \delta \in V^*$$

➤ **Sentential Form:**

$$\alpha \in V^* \text{ is a } \textit{sentential form} \text{ of } G \text{ if } S \xRightarrow{*} \alpha$$

➤ **Language generated by  $G$ :**

$$L(G) = \{x \in T^* \mid S \xRightarrow{*} x\}$$

➤ **Grammar classification:**

- **Type 2:** *context free grammars*

$$A \rightarrow \beta$$

$$A \in N; \beta \in V^*$$

- **Type 3:** *regular grammars*

$$A \rightarrow aB, \quad A \rightarrow a$$

$$A, B \in N; a \in T$$

## 4.1 NOTATION AND DEFINITIONS

---

### Approaches

- Top-Down parsing
- Down-Top parsing

### Depending on time complexity

- Backtracking methods Exponential complexity
- Deterministic methods Linear complexity
  - Grammars: LL(1), SLR(1), LALR(1), LR(1), ...
- Tabular methods Cubic complexity
  - CKY algorithm**
  - Earley algorithm [Aho 72, Stolcke 95]

## 4.1 NOTATION AND DEFINITIONS

---

### ALGORITHM: Cocke-Kasami-Younger

**Input**  $G = (N, T, P, S)$  in CNF and  $\mathbf{x} = x_1 \dots x_n \in T^*$

**Output** Parsing table  $t[i, l]$  ( $1 \leq i, l \leq n$ )

$A \in t[i, l]$  if  $A \xrightarrow{*} x_{i+1} \dots x_l$

### METHOD

**for all**  $i : 0 \dots n - 1$  **do**

$t[i, i + 1] := t[i, i + 1] \cup \{A \mid (A \rightarrow b) \in P; b = x_{i+1}\}$

**for all**  $j : 2 \dots n$  **do**

**for all**  $i : 0 \dots n - j$  **do**

**for all**  $k : 1 \dots j - 1$  **do**

$t[i, i + j] := t[i, i + j] \cup \{A \mid (A \rightarrow BC) \in P;$

$B \in t[i, i + k]; C \in t[i + k, i + j]\}$

**if**  $S \in t[0, n]$  **then**  $x \in L(G)$  **else**  $x \notin L(G)$

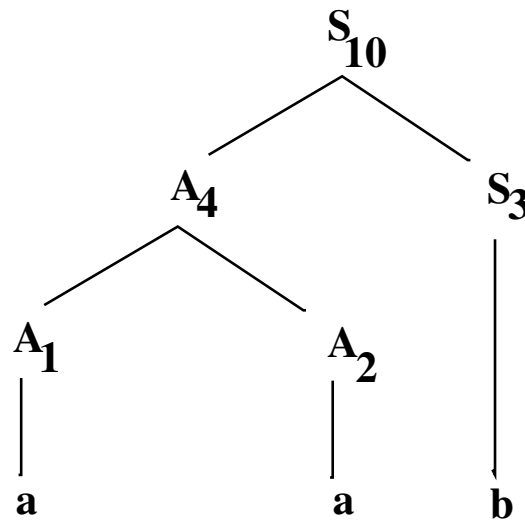
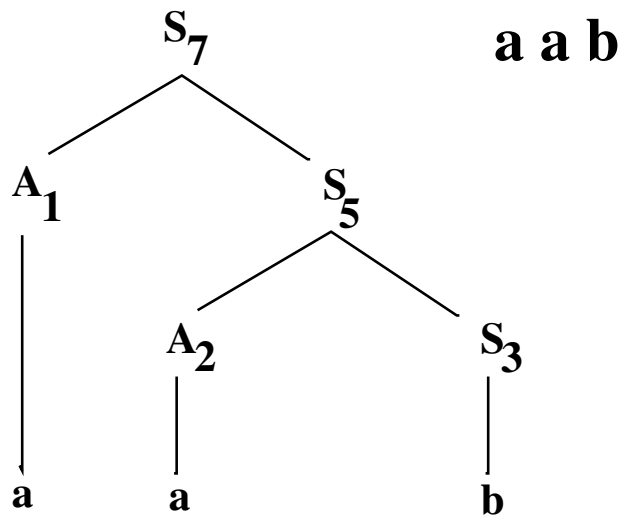
**END**



# 4.1 NOTATION AND DEFINITIONS

$S \rightarrow AS$   
 $S \rightarrow b$   
 $A \rightarrow AS$   
 $A \rightarrow AA$   
 $A \rightarrow a$

$l = 1$	$l = 2$	$l = 3$	
1: (A, 0, 0)	4: (A, 1, 2)	7: (S, 1, 5) 8: (A, 1, 5) 9: (A, 1, 6) 10: (S, 4, 3) 11: (A, 4, 3)	$i = 0$
	2: (A, 0, 0)	5: (S, 2, 3) 6: (A, 2, 3)	$i = 1$
		3: (S, 0, 0)	$i = 2$



## 4.1 NOTATION AND DEFINITIONS

---

Let  $x \in T^*$  and a stochastic model  $M$  characterized by a parameter vector  $\theta$ , we are interested in computing:  $p_\theta(x)$

**Stochastic language**  $(L, \phi)$  over  $T$  [Wetherell 80]:

- $L \subseteq T^*$  characteristic language
- $\phi : T^* \longrightarrow [0, 1]$  computable stochastic function:

- i)  $x \notin L \implies \phi(x) = 0 \quad \forall x \in T^*$
- ii)  $x \in L \implies 0 < \phi(x) \leq 1 \quad \forall x \in T^*$
- iii)  $\sum_{x \in L} \phi(x) = 1$

**Example** [Booth 73]

Given the alphabet  $T = \{a, b\}$ , the following language is defined:  $L = \{a^n b^n \mid n \geq 0\}$ , where  $\phi(x) = 0, \forall x \notin L$  and  $\phi(a^n b^n) = \frac{1}{en!}$

$$\sum_{x \in L} \phi(x) = \sum_{0 \leq n \leq \infty} \frac{1}{en!} = \frac{1}{e} \sum_{0 \leq n \leq \infty} \frac{1}{n!} = \frac{1}{e} e = 1$$

## 4.1 NOTATION AND DEFINITIONS

---

Probabilistic context-free grammar:  $G_s = (G, p)$

➤  $G = (N, T, P, S)$  characteristic grammar

➤  $p : P \rightarrow ]0, 1]$  probability of the rules.  $\forall A_i \in N$ :

$$\sum_{1 \leq j \leq n_i} p(A_i \rightarrow \alpha_j) = 1,$$

where  $n_i$  is the number of rules with  $A_i$  in the left side of the rules.

### Stochastic derivation for PCFG

Given a sequence of stochastic events:

$$S = \alpha_0 \xrightarrow{r_1} \alpha_1 \xrightarrow{r_2} \alpha_2 \cdots \alpha_{m-1} \xrightarrow{r_m} \alpha_m = x$$

the probability of  $x$  being generated by  $G_s = (G, p)$  from the rule sequence  $d_x = r_1, \dots, r_m$ , is:

$$\Pr_{G_s}(x, d_x) = p(r_1)p(r_2 \mid r_1) \cdots p(r_m \mid r_1 \cdots r_{m-1})$$

➤ **problem:** computation of the probabilities

➤ **restriction:**  $p(r_j \mid r_1 \cdots r_{j-1}) = p(r_j)$

$$\Pr_{G_s}(x, d_x) = \prod_{j=1 \cdots m} p(r_j)$$

## 4.1 NOTATION AND DEFINITIONS

---

Probability of a derivation  $d_x = r_1, \dots, r_m$

$$\Pr_{G_s}(x, d_x) = \prod_{j=1 \dots m} p(r_j) = \prod_{\forall (A \rightarrow \alpha) \in P} p(A \rightarrow \alpha)^{N(A \rightarrow \alpha, d_x)}$$

Probability of a string

$$\Pr_{G_s}(x) = \sum_{d_x \in D_x} \Pr_{G_s}(x, d_x)$$

Probability of the best derivation

$$\widehat{\Pr}_{G_s}(x) = \max_{d_x \in D_x} \Pr_{G_s}(x, d_x)$$

Probability of a string with a subset of derivations  $\Delta_x \subseteq D_x$

$$\Pr_{G_s}(x, \Delta_x) = \sum_{d_x \in \Delta_x} \Pr_{G_s}(x, d_x)$$

Language generated by a PCFG

$$L(G_s) = \{x \in L(G) \mid \Pr_{G_s}(x) > 0\}$$

### Consistent grammar

A PCFG  $G_s = (G, p)$  is consistent iff:

$$\sum_{x \in L(G)} \Pr_{G_s}(x) = 1$$

### Theorem [Booth 73]

There exist stochastic languages  $(L, \phi)$  that can not be generated by a stochastic grammar  $G_s = (G, p)$

**Dem. outline** Let  $L = \{a^n b^n \mid n \geq 0\}$  be a stochastic language:

$$\phi(a^n b^n) = \frac{1}{en!}$$

There is not any  $G_s$  such that  $\phi(x) = \Pr_{G_s}(x) \quad \forall x \in L$

## 4.3 CKY-BASED PARSING ALGORITHMS

---

### Inside algorithm for PCFG [Lari 90]

- Given  $x = x_1 \dots x_n \in T^*$  and  $A \in N$

$$e(A \langle i, l \rangle) = \Pr_{G_s}(A \xrightarrow{*} x_i \dots x_l)$$

- Compute  $\forall A \in N$ :

$$e(A \langle i, i \rangle) = p(A \rightarrow b) \delta(b, x_i) \quad 1 \leq i \leq n$$

$$e(A \langle i, j \rangle) = \sum_{B, C \in N} p(A \rightarrow BC) \sum_{k=i}^{j-1} e(B \langle i, k \rangle) e(C \langle k+1, j \rangle)$$

$$1 \leq i < j \leq n$$

- $\Pr_{G_s}(x) = e(S \langle 1, n \rangle)$
- Time complexity:  $O(|x|^3 |P|)$

## 4.3 CKY-BASED PARSING ALGORITHMS

---

### Inside algorithm for PCFG (bracketed version [Pereira 92])

➤ Bracketed sentence:

( ( ( Pierre Vinken ) , ( ( 61 years) old ) , ) ( will ( join ( the board ) ( as ( a nonexecutive director ) ) ( Nov. 29. ) ) ) .)

$$c(i, j) = \begin{cases} 1 & \text{if } (i, j) \text{ does not overlap any span in the sentence,} \\ 0 & \text{otherwise.} \end{cases}$$

➤ Compute  $\forall A \in N$ :

$$e(A \langle i, i \rangle) = p(A \rightarrow b) \delta(b, x_i) \quad 1 \leq i \leq n$$

$$e(A \langle i, j \rangle) = c(i, j) \sum_{B, C \in N} p(A \rightarrow BC) \sum_{k=i}^{j-1} e(B \langle i, k \rangle) e(C \langle k+1, j \rangle)$$
$$1 \leq i < j \leq n$$

➤ Linear if full bracketing



### Viterbi algorithm for PCFG [Ney 91]

- Given  $x = x_1 \dots x_n \in T^*$  and  $A \in N$

$$\widehat{e}(A \langle i, l \rangle) = \widehat{\text{Pr}}_{G_s}(A \xrightarrow{*} x_i \dots x_l)$$

- Compute  $\forall A \in N$ :

$$\widehat{e}(A \langle i, i \rangle) = p(A \rightarrow b) \delta(b, x_i) \quad 0 \leq i \leq n$$

$$\widehat{e}(A \langle i, j \rangle) = \max_{B, C \in N} p(A \rightarrow BC) \max_{k=i, \dots, j-1} \widehat{e}(B \langle i, k \rangle) \widehat{e}(C \langle k+1, j \rangle)$$

$$1 \leq i < j \leq n$$

- $\widehat{\text{Pr}}_{G_s}(x) = \widehat{e}(S \langle 1, n \rangle)$

- Time complexity:  $O(|x|^3 |P|)$  (Bracketed version: linear if full bracketing)

### Outside algorithm for PCFG

- Given  $x = x_1 \dots x_n \in T^*$  and  $A \in N$

$$f(A \langle i, l \rangle) = \Pr_{G_s}(S \xRightarrow{*} x_1 \dots x_{i-1} A x_{l+1} \dots x_n)$$

- Compute  $\forall A \in N$ :

$$f(A \langle 1, n \rangle) = \delta(A, S)$$

$$f(A \langle i, j \rangle) = \sum_{B, C \in N} p(B \rightarrow CA) \sum_{k=1}^{i-1} f(B \langle k, j \rangle) e(C \langle k, i-1 \rangle) \\ + \sum_{B, C \in N} p(B \rightarrow AC) \sum_{k=j+1}^n f(B \langle i, k \rangle) e(C \langle j+1, k \rangle)$$

$$1 \leq i \leq j \leq n$$

- $\Pr_{G_s}(x) = \sum_{A \in N} f(A \langle i, i \rangle) p(A \rightarrow x_i),$   $1 \leq i \leq n$

- Time complexity:  $O(|x|^3|P|)$  (Bracketed version: linear if full bracketing)

## 4.3 CKY-BASED PARSING ALGORITHMS

---

### Probability of an initial substring: LRI algorithm

$$T(A \Rightarrow B) = \sum_{\alpha} \Pr_{G_s}(A \xrightarrow{*} B\alpha)$$

$$T(A \Rightarrow BC) = p(A \rightarrow BC) + \sum_D T(A \Rightarrow D)p(D \rightarrow BC)$$

➤ Given  $x = x_1 \dots x_n \in T^*$  and  $A \in N$

$$e(A \ll i, l) = \Pr_{G_s}(A \xrightarrow{*} x_i \dots x_l \dots)$$

➤ Compute  $\forall A \in N$ :

$$e(A \ll i, i) = p(A \rightarrow x_i) + \sum_D T(A \Rightarrow D) p(D \rightarrow x_i) \quad 1 \leq i \leq n$$

$$e(A \ll i, j) = \sum_{B, C \in N} T(A \Rightarrow BC) \sum_{k=i}^{j-1} e(B \ll i, k) e(C \ll k+1, j) \quad 1 \leq i < j$$

➤  $\Pr_{G_e}(x_1 \dots x_k \dots) = e(S \ll 1, k)$

➤ Time complexity:  $O(|x|^3|P|)$

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## 5.1 INTRODUCTION

---

### ➤ Supervised methods

➤ Maximum likelihood estimation:

➤ Annotated data is needed (“treebank”)

$$\widehat{\text{Pr}}(A \rightarrow \alpha) = \frac{C(A \rightarrow \alpha)}{C(A)}$$

### ➤ Non-supervised methods

➤ EM algorithms

➤ Problem: local optimum

Let  $G_s$  a PCFG with parameters  $\theta$  and a sample  $\Omega = \{x_1, x_2, \dots, x_n\}$ .

$$\hat{\theta} = \arg \max_{\theta} F_{\theta}(\Omega)$$

➤ Optimization method: Growth transformations

➤ Optimization function: Maximum likelihood

## 5.1 INTRODUCTION

---

### Theorem [Baum 72]

Let  $P(\Theta)$  be a homogeneous polynomial with non-negative coefficients. Let  $\theta = \{\theta_{ij}\}$  be a point in the domain  $D = \{\theta_{ij} \mid \theta_{ij} \geq 0; \sum_{j=1}^{q_i} \theta_{ij} = 1, i = 1, \dots, p; j = 1, \dots, q_i\}$ , and let  $Q(\theta)$  be a close transformation in  $D$ , that is defined as:

$$Q(\theta)_{ij} = \frac{\theta_{ij}(\partial P / \partial \Theta_{ij})_{\theta}}{\sum_{k=1}^{q_i} \theta_{ik}(\partial P / \partial \Theta_{ik})_{\theta}}$$

with the denominator different from zero. Then,  $P(Q(\theta)) > P(\theta)$  except if  $Q(\theta) = \theta$ .

```
input  $P(\Theta)$ 
 $\theta$  = initial values
repeat
    compute  $Q(\theta)$  using  $P(\Theta)$ 
     $\theta = Q(\theta)$ 
until convergence
output  $\theta$ 
```

## 5.2 INSIDE-OUTSIDE ALGORITHM

---

Let a PCFG  $G_s$ , a sample  $\Omega$  and a set of derivations  $\Delta_x$  for each  $x \in \Omega$

$$\Pr_{G_s}(\Omega, \Delta_\Omega) = \prod_{x \in \Omega} \Pr_{G_s}(x, \Delta_x)$$

$\forall (A \rightarrow \alpha) \in P$  (See demonstration [Benedí 05])

$$\bar{p}(A \rightarrow \alpha) = \frac{\sum_{x \in \Omega} \frac{1}{\Pr_{G_s}(x, \Delta_x)} \sum_{\forall d_x \in \Delta_x} N(A \rightarrow \alpha, d_x) \Pr_{G_s}(x, d_x)}{\sum_{x \in \Omega} \frac{1}{\Pr_{G_s}(x, \Delta_x)} \sum_{\forall d_x \in \Delta_x} N(A, d_x) \Pr_{G_s}(x, d_x)}$$

## 5.2 INSIDE-OUTSIDE ALGORITHM

---

Optimization function ( $\Delta_x = D_x$ )

$$\Pr_{G_s}(\Omega) = \prod_{x \in \Omega} \Pr_{G_s}(x)$$

➤  $\forall(A \rightarrow BC) \in P; \forall(A \rightarrow b) \in P$  (See demonstration)

$$\bar{p}(A \rightarrow BC) =$$

$$\frac{\sum_{x \in \Omega} \frac{p(A \rightarrow BC)}{\Pr_{G_s}(x)} \sum_{i=0}^{n-j} \sum_{j=2}^n \sum_{k=1}^{j-1} f(A \langle i, i+j \rangle) e(B \langle i, i+k \rangle) e(C \langle i+k, i+j \rangle)}{\sum_{x \in \Omega} \frac{1}{\Pr_{G_s}(x)} \sum_{i=0}^{n-j} \sum_{j=1}^n f(A \langle i, i+j \rangle) e(A \langle i, i+j \rangle)}$$

$$\bar{p}(A \rightarrow b) = \frac{\sum_{x \in \Omega} \frac{1}{\Pr_{G_s}(x)} \sum_{i=0, b=x_i}^{n-1} f(A \langle i, i \rangle) p(A \rightarrow b)}{\sum_{x \in \Omega} \frac{1}{\Pr_{G_s}(x)} \sum_{i=0}^{n-j} \sum_{j=1}^n f(A \langle i, i+j \rangle) e(A \langle i, i+j \rangle)}$$

➤ Time complexity:  $O(|LT|^3|P|)$



## 5.3 VITERBI ALGORITHM

---

Optimization function ( $\Delta_x = \hat{d}_x$ ) [Benedí 05]

$$\Pr_{G_s}(\hat{\Omega}) = \prod_{x \in \Omega} \Pr_{G_s}(x, \hat{d}_x)$$

➤  $\forall (A \rightarrow \alpha) \in P$

$$\bar{p}(A \rightarrow \alpha) = \frac{\sum_{x \in \Omega} N(A \rightarrow \alpha, \hat{d}_x)}{\sum_{x \in \Omega} N(A, \hat{d}_x)}.$$

➤ Time complexity:  $O(|LT|^3|P|)$

## 5.4 PROBABILISTIC PROPERTIES OF THE ESTIMATED PCFG

---

**Theorem [Booth 73]** A PCFG is consistent if  $\rho(E) < 1$ , where  $\rho(E)$  is the spectral radius (absolute value of the largest eigenvalue) of matrix  $E$ .

**Probabilistic expectation matrix:**  $E = (e_{ij})$ , expected number of times that the non-terminal  $A_j$  is derived directly from  $A_i$ :

$$e_{ij} = \sum_{(A_i \rightarrow \alpha)} p(A_i \rightarrow \alpha) N(A_j, \alpha) \quad 1 \leq i, j \leq |N|$$

**Expectation matrix**

$$Q = \sum_{i=0}^{\infty} E^i. \quad \text{If } G_s \text{ is consistent, then the sum converges to: } Q = (I - E)^{-1}$$

**Theorem [Sánchez 97]** Let  $G_s = (G, p)$  be a PCFG and let  $\Omega$  be a sample from  $L(G)$ . If  $\overline{G}_s = (G, \overline{p})$  is a PCFG obtained from  $G_s$  when applying the previous growth transformation, the  $\overline{G}_s$  is consistent.

## 5.4 PROBABILISTIC PROPERTIES OF THE ESTIMATED PCFG

---

### Palindrome language

$$\{ ww^R \mid w \in \{a, b\}^+; R = \text{reverse string} \}$$

#### ➤ Original model

$$\begin{array}{llll} S \rightarrow AC & 0.4 & S \rightarrow BB & 0.1 & C \rightarrow SA & 1.0 & A \rightarrow a & 1.0 \\ S \rightarrow BD & 0.4 & S \rightarrow AA & 0.1 & D \rightarrow SB & 1.0 & B \rightarrow b & 1.0 \end{array}$$

#### ➤ Training set: 1000 strings

#### ➤ Initial model to be estimated

- 5 non-terminals and 2 terminals  $\Rightarrow$  130 rules
- Random probabilities attached to the rules

Algorithm	kld	Palindromes (%)	Non palindromes (%)
VS	6.00	1.9	98.1
IO	1.88	76.0	24.0

### Combination of N-Grams and PCFG for LM [Benedi 05]

$$\Pr(w) = \Pr(w_1 \dots w_n) = \prod_{k=1}^n \Pr(w_k | w_1 \dots w_{k-1})$$

$$\Pr(w) = \prod_{k=1}^n \Pr(w_k | w_{k-n+1} \dots w_{k-1})$$

$$\Pr(w_k | w_1 \dots w_{k-1}) = \alpha \Pr_N(w_k | w_{k-n+1} \dots w_{k-1}) + (1 - \alpha) \Pr_{M_s}(w_k | w_1 \dots w_{k-1})$$

⇒  $M_s$ : a PCFG  $G_c$  of categories (PoS tags) and a word-category distribution  $C_w$

$$\Pr_{G_c, C_w}(w_k | w_1 \dots w_{k-1})$$

### WSJ Experiments

➤ WSJ characteristics:

Data set	Directories	No. of senten.	No. of words
Training (full)	00-20	42,075	1,004,073
Training ( $\leq 50$ )	00-20	41,315 (98,2%)	959,390 (95,6%)
Tuning	21-22	3,371	80,156
Test	23-24	3,762	89,537

➤ Vocabulary (Training) 10,000 more frequent words

➤ 3-Gram model: (*linear discounting*)

○ *Tuning set perplexity*: 160.3;

○ *Test set perplexity*: 167.3;

## 5.5 USE OF PCFG FOR LM

---

### Test set perplexity

Model	Perplexity		% improvement
	Trigram	Interpolated	
[Chelba 00]	167.1	148.9	10.9
[Roark 01]	167.0	137.3	17.8
IOb	167.3	142.3	14.9

### WER

Model	Training Size	Vocabulary Size	LM Weight	WER
[Chelba 00]	20M	20K	16	13.0
[Roark 01]	1M	10K	15	15.1
Treebank trigram	1M	10K	5	16.6
No language model			0	16.8
Current model	1M	10K	6	16.0

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  - 3.3 Viterbi algorithm
  - 3.4 Use of HMM for PoS tagging
4. Preliminaries on PCFG
  - 4.1 Notation and definitions
  - 4.2 Basic probabilistic properties of syntactic models
  - 4.3 CKY-based parsing algorithms
5. Probabilistic estimation of PCFG
  - 5.1 Introduction
  - 5.2 Inside-Outside algorithm
  - 5.3 Viterbi algorithm
  - 5.4 Probabilistic properties of the estimated PCFG
  - 5.5 Use of PCFG for LM
6. **Advanced topics**
  - 6.1 On-line learning of syntactic models
  - 6.2 Active learning of syntactic models
  - 6.3 Interactive-predictive parsing: a framework for active learning

### Problem definition [Liang 09]:

- Probabilistic model:  $p(\mathbf{x}, \mathbf{z}; \theta)$

Input:  $\mathbf{x}$  (a sentence)

Hidden output:  $\mathbf{z}$  (a parse tree)

Parameters:  $\theta$  (rule probabilities)

- Given a set of unlabeled example  $\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(n)}$ , maximize the marginal log-likelihood:

$$l(\theta) = \sum_{i=1}^n \log p(\mathbf{x}^{(i)}; \theta)$$

- Evaluation of the trained model  $\hat{\theta}$ : accuracy

$$\text{true output } \mathbf{z}^{(i)} \leftrightarrow \arg \max_{\mathbf{z}} p(\mathbf{z} | \mathbf{x}^{(i)}; \theta)$$

- Training algorithm: **EM algorithm** [Dempster 77, Neal 98, Cappé 09]



## 6.1 ON-LINE LEARNING OF SYNTACTIC MODELS

### EM algorithm [Liang 09]:

#### Batch EM

```
 $\mu \leftarrow$  initialization  
for each iteration  $t = 1, \dots, T$ :  
   $\mu' \leftarrow 0$   
  for each example  $i = 1, \dots, n$ :  
     $s'_i \leftarrow \sum_{\mathbf{z}} p(\mathbf{z}|\mathbf{x}^{(i)}; \theta(\mu)) \phi(\mathbf{x}^{(i)}, \mathbf{z})$   
     $\mu' \leftarrow \mu' + s'_i$   
 $\mu \leftarrow \mu'$ 
```

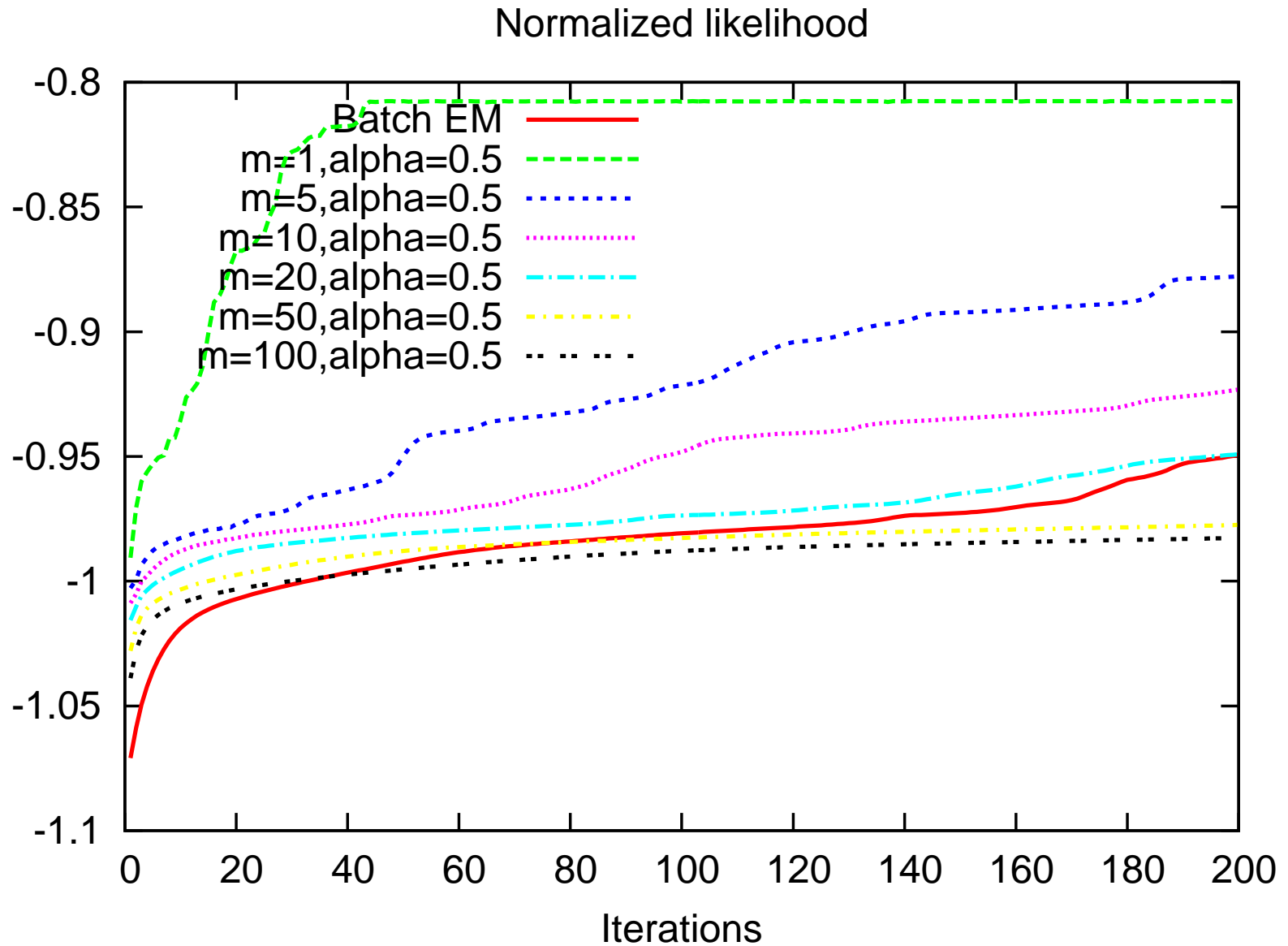
#### Stepwise EM

```
 $\mu \leftarrow$ ;  $k = 0$  initialization  
for each iteration  $t = 1, \dots, T$ :  
  for each example  $i = 1, \dots, n$  in  
  random order:  
     $s'_i \leftarrow \sum_{\mathbf{z}} p(\mathbf{z}|\mathbf{x}^{(i)}; \theta(\mu)) \phi(\mathbf{x}^{(i)}, \mathbf{z})$   
     $\mu \leftarrow (1 - \eta_k)\mu + \eta_k s'_i$   
     $k \leftarrow k + 1$ 
```

- $\phi(\mathbf{x}, \mathbf{z})$ : mapping from a labelled example  $(\mathbf{x}, \mathbf{z})$  to a vector of sufficient statistics  $(\mu)$
- $\theta(\mu)$ : maximum likelihood estimate
- Stepwise EM: convergence is guaranteed if  $\sum_{k=0}^{\infty} \eta_k = \infty$  and  $\sum_{k=0}^{\infty} \eta_k^2 < \infty$ 
  - $\eta_k = (k + 2)^{-\alpha}$  with  $0.5 < \alpha \leq 1$
  - Approach: take  $m$  examples at once

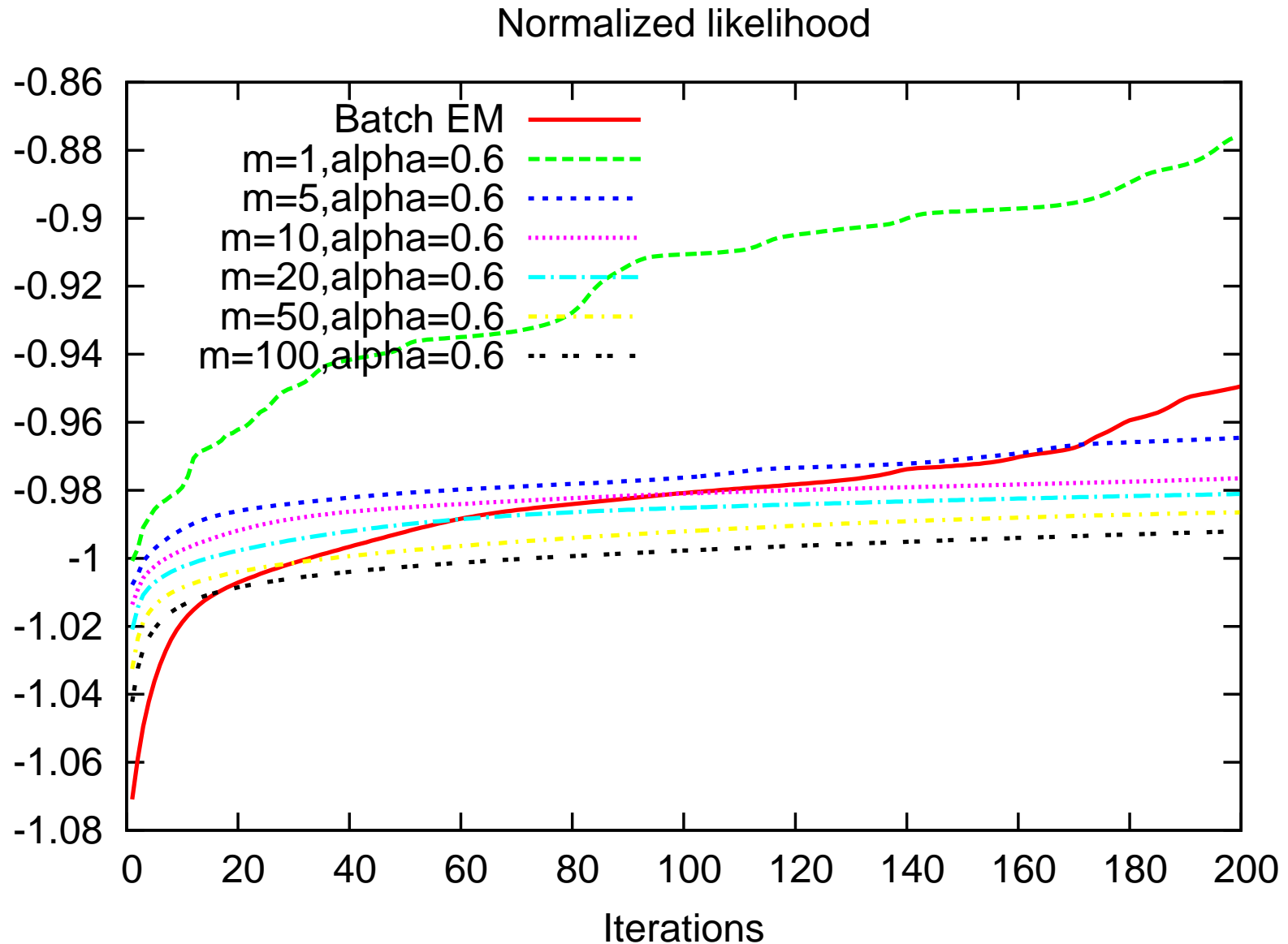
## 6.1 ON-LINE LEARNING OF SYNTACTIC MODELS

Palindrome language (15 random initializations,  $\alpha = 0.5$ )



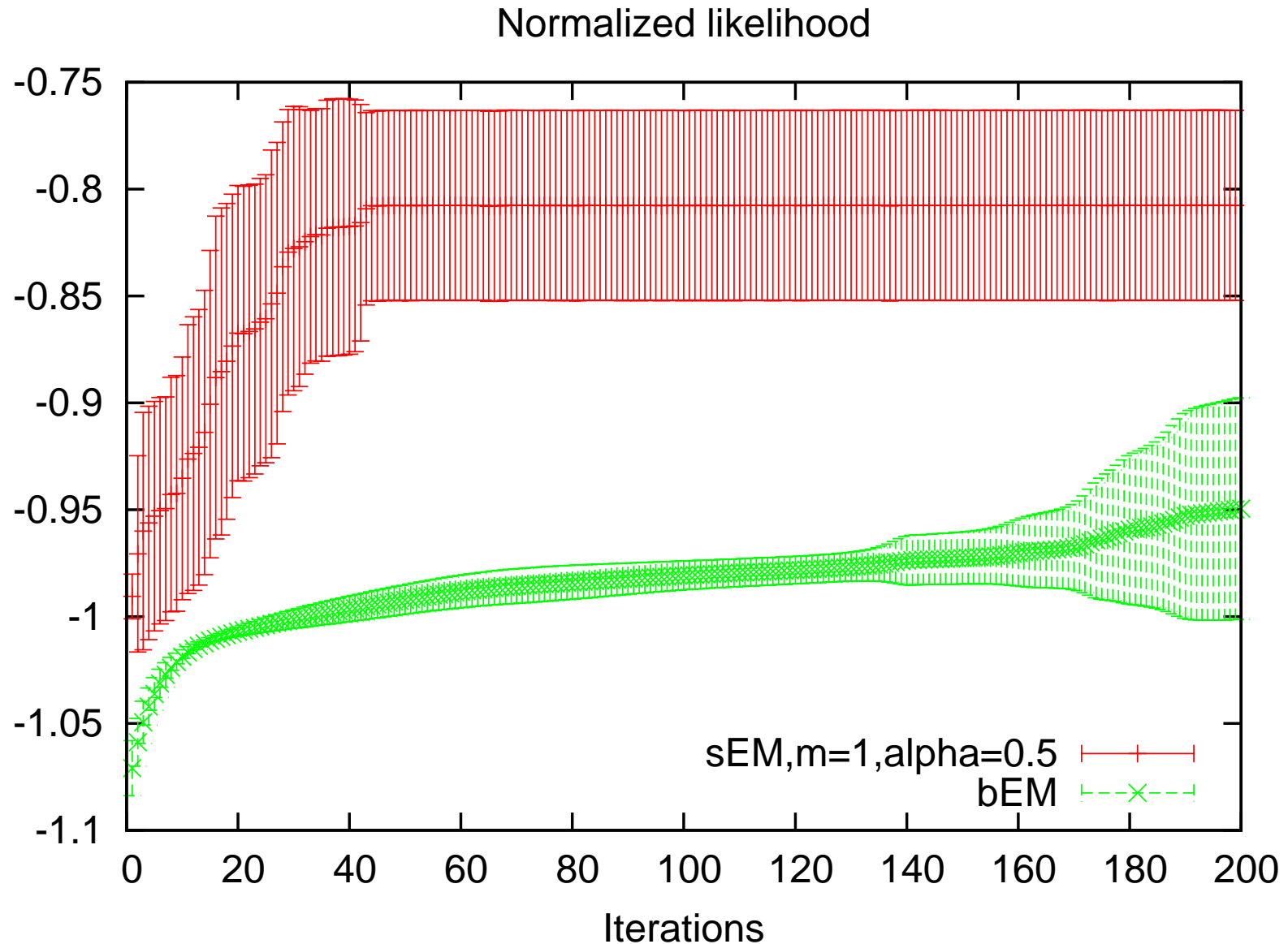
## 6.1 ON-LINE LEARNING OF SYNTACTIC MODELS

Palindrome language (15 random initializations,  $\alpha = 0.6$ )



## 6.1 ON-LINE LEARNING OF SYNTACTIC MODELS

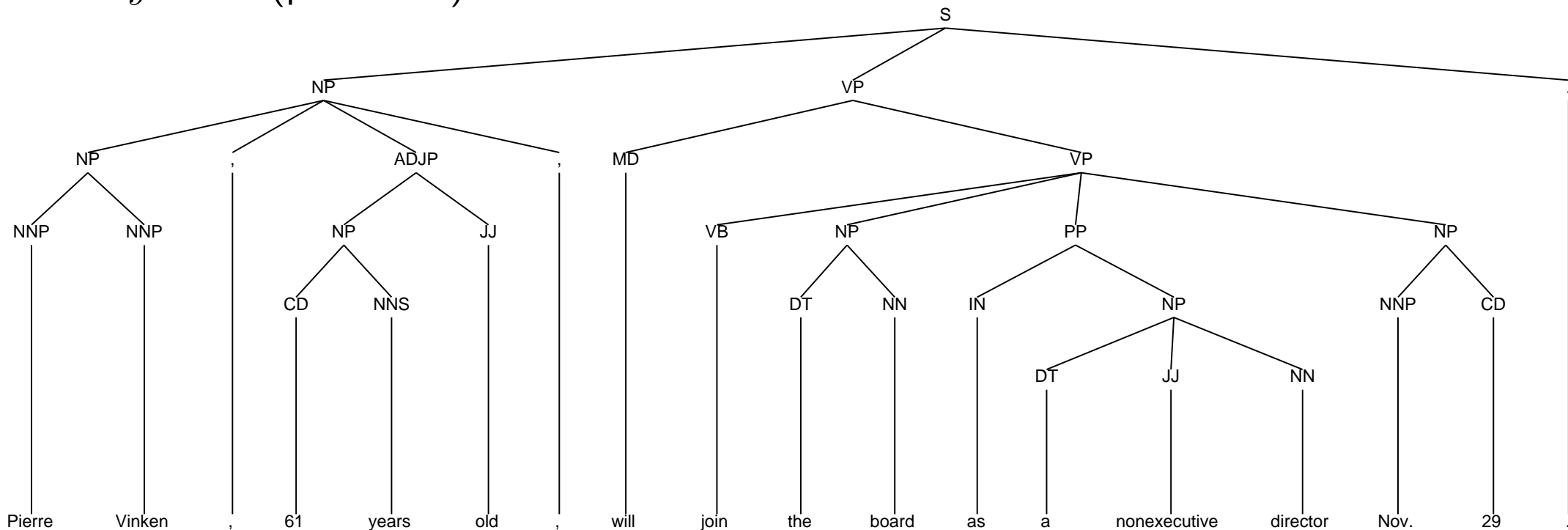
Palindrome language (15 random initializations,  $\alpha = 0.5$ , confidence interval)



## 6.2 ACTIVE LEARNING OF SYNTACTIC MODELS

### Problem definition:

- Supervised learning:  $(x, y)$ 
  - $x$ : input data (sentence)
  - $y$ : label (parse tree)



- Problem: to annotate data is slow and expensive
- **Active learning: to annotate just the necessary data**

## 6.2 ACTIVE LEARNING OF SYNTACTIC MODELS

---

Pool-based active learning [Settles 08, Settles 10]:

```
Given: Labeled set  $\mathcal{L}$ , unlabeled pool  $\mathcal{U}$ ,  
query strategy  $\phi(\cdot)$ , query batch size  $B$   
repeat  
  // learn a model using the current  $\mathcal{L}$   
   $\theta = \text{train}(\mathcal{L})$   
  for  $b = 1$  to  $B$  do  
    // query the most informative instance  
     $\mathbf{x}_b^* = \arg \max_{\mathbf{x} \in \mathcal{U}} \phi(\mathbf{x})$   
    // move the labeled query from  $\mathcal{U}$  to  $\mathcal{L}$   
     $\mathcal{L} = \mathcal{L} \cup \langle \mathbf{x}_b^*, \text{label}(\mathbf{x}_b^*) \rangle$   
     $\mathcal{U} = \mathcal{U} - \mathbf{x}_b^*$   
  end  
until some stopping criterion
```

➤ Similar scheme for parsing in [Hwa 04]

### Query strategies:

➤ **Uncertainty sampling:** to query the instance that is most uncertainty how to label

➤ Sequence entropy:

$$\phi^{SE}(\mathbf{x}) = - \sum_{\hat{\mathbf{y}}} P(\hat{\mathbf{y}}|\mathbf{x}; \theta) \log P(\hat{\mathbf{y}}|\mathbf{x}; \theta)$$

➤ Approach:  $N$ -best Sequence entropy:

$$\phi^{NSE}(\mathbf{x}) = - \sum_{\hat{\mathbf{y}} \in \mathcal{N}} P(\hat{\mathbf{y}}|\mathbf{x}; \theta) \log P(\hat{\mathbf{y}}|\mathbf{x}; \theta)$$

➤ **Information density:** to query the instance that is the most “informative” in average

$$\phi^{ID}(\mathbf{x}) = \phi^{NSE}(\mathbf{x}) \times \left( \frac{1}{U} \sum_{u=1}^U \text{sim}(\mathbf{x}, \mathbf{x}^{(u)}) \right)^{\beta}$$

### Query strategies for parsing [Hwa 04]:

➤ Problem space:

- Based on novelty and frequencies of word pair co-occurrences
- Based on sentence length:  $f_{\text{len}}$

➤ Performance of the hypothesis:

- Error-driven function:

$$f_{\text{err}}(\mathbf{w}, G) = 1 - P(\hat{d}_{\mathbf{w}} | \mathbf{w}, G)$$

- Normalized tree entropy (similar to  $\phi^{SE}(\mathbf{x})$ ):  $f_{\text{unc}}$



## 6.2 ACTIVE LEARNING OF SYNTACTIC MODELS

---

Experiments on WSJ UPenn Treebank reported in [Hwa 04]:

- Collins' model 2 parser
- Learning algorithm: statistics directly over the treebank
- Data:
  - Training: sections 02-21
  - Test: section 23
- Initial model trained on 500 sentences
- Batch size: 100
- Parsing performance:  $F$  score

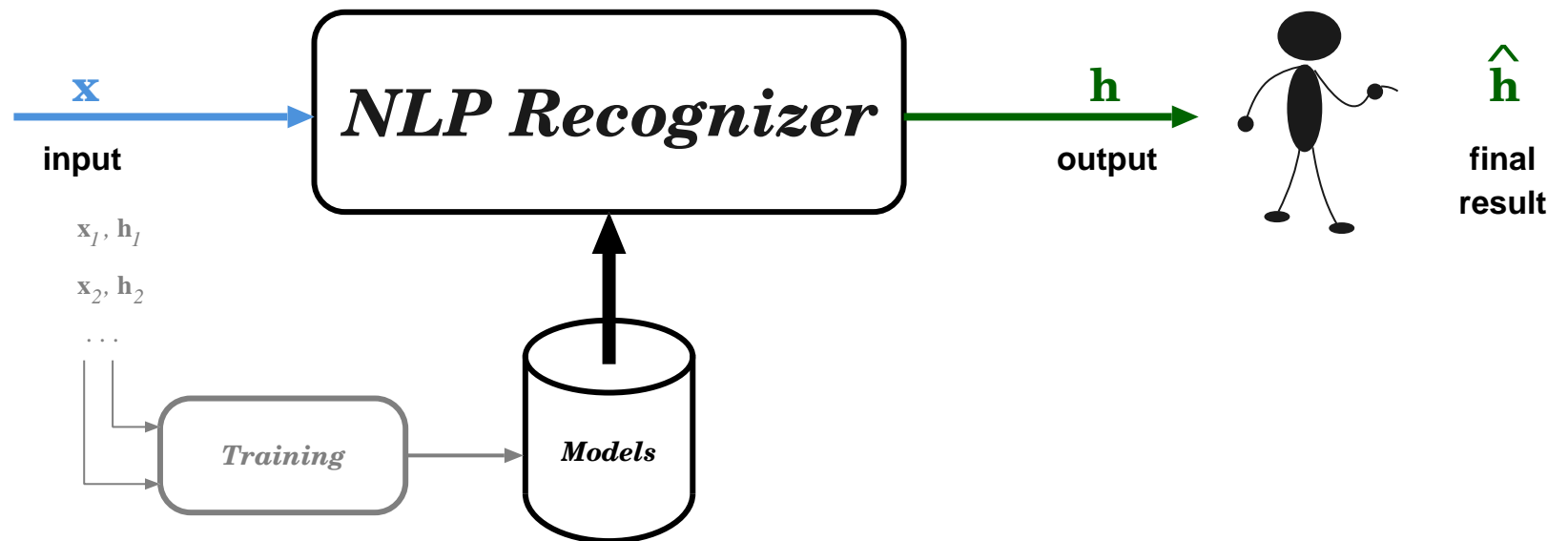
Number of labelled samples at the test performance level of 88%:

	$f_{\text{ran}}$	$f_{\text{len}}$	$f_{\text{err}}$	$f_{\text{unc}}$
# sentences	30,500	-	20,500 (33%)	17,500 (43%)
# constituents	695,000	625,000 (10%)	577,000 (17%)	505,000 (27%)

## 6.3 IPP: A FRAMEWORK FOR ACTIVE LEARNING

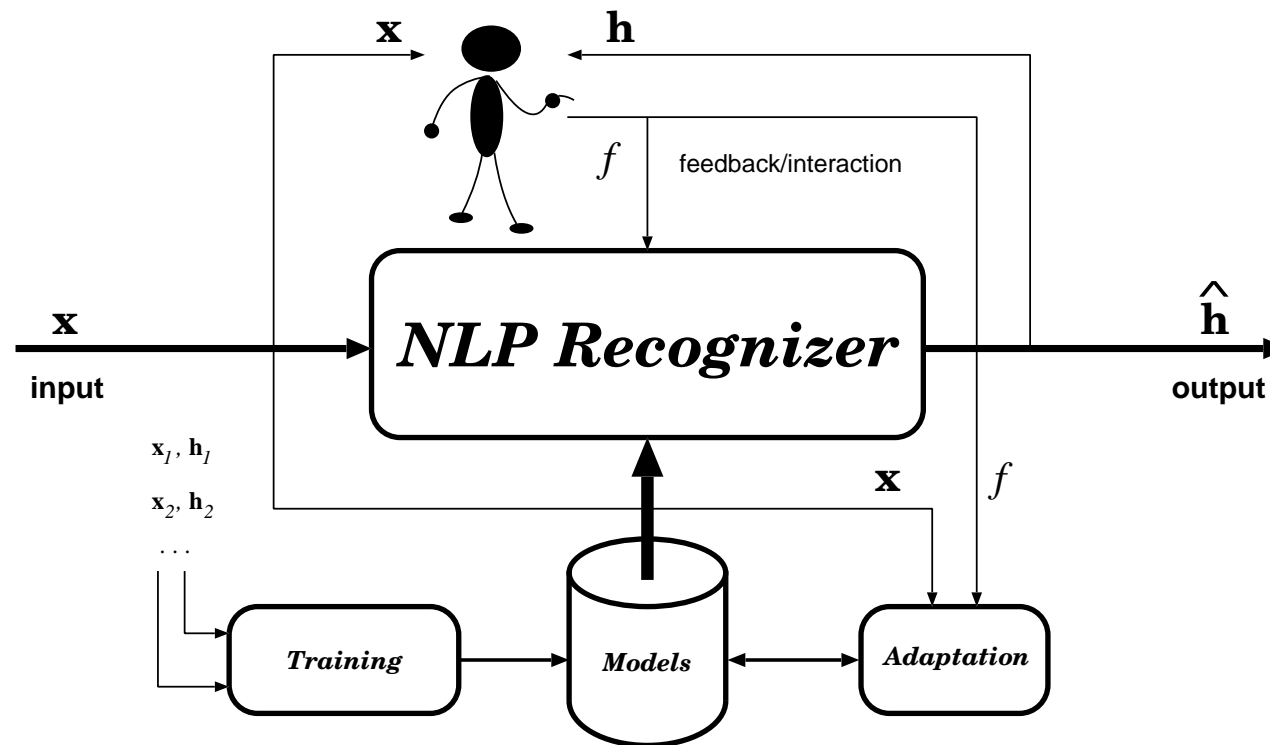
Problem definition [Sánchez 09, Sánchez 10]: Annotation parse tree is expensive and requires skilled expert humans

- Classical two-step approach:
  - 1 Apply an automatic system
  - 2 Manually validate/correct the output



## 6.3 IPP: A FRAMEWORK FOR ACTIVE LEARNING

- Interactive Predictive approach:
  - Formally integrate the user into the recognition process
  - The system reacts to user feedback



- New opportunities:
  - Feedback information can be used to create efficient interactive systems
  - Each interaction step yields *ground-truth data*, which allows building *active learning systems*

## 6.3 IPP: A FRAMEWORK FOR ACTIVE LEARNING

---

### Classical parsing

$$\hat{t} = \arg \max_{t \in \mathcal{T}} p_G(t|x)$$

$x \rightarrow$  input sentence

$G \rightarrow$  mode (e.g. PCFG)

$\mathcal{T} \rightarrow$  set of all possible trees for  $x$  with  $G$

$\hat{t} \rightarrow$  obtained parse tree

### Interactive predictive parsing

$$\hat{t} = \arg \max_{t \in \mathcal{T}: t_p \in t} p_G(t|x, t_p)$$

The tree prefix  $t_p$  is:

- the corrected constituent, plus
- all its ancestors, plus
- all the constituents to its left

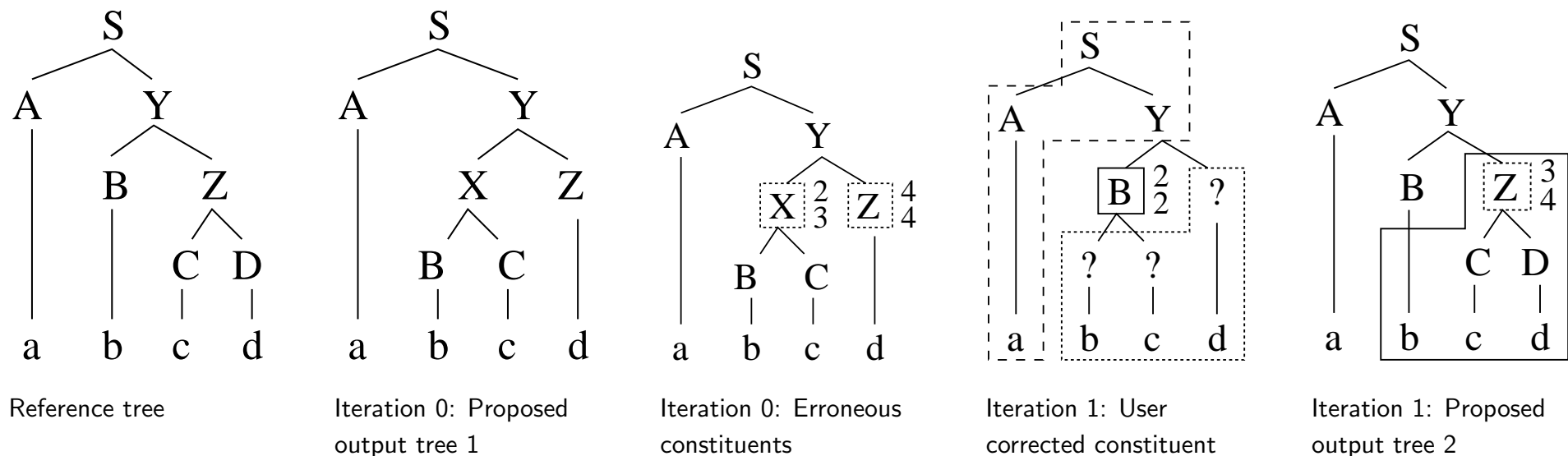
$$t_p(c'_{ij}) = \{c^B_{mn} : m \leq i, n \geq j, \text{depth}(c^B_{mn}) \leq \text{depth}(c'_{ij})\} \cup \{c^D_{pq} : p \geq 1, q < i\}$$

## 6.3 IPP: A FRAMEWORK FOR ACTIVE LEARNING

### IPP parsing

1. The system propose a parse tree  $\hat{t}$
2. The user finds an incorrect constituent  $c$  and corrects it, implicitly validating the prefix tree  $t_p(c)$
3. The system propose a parse tree  $\hat{t}'$  taking into account the prefix tree  $t_p(c)$
4. Go to step 2
- n. The user keeps iterating until an error free parse tree is achieved

### Example:



### Experiments [Sánchez 09]:

- Experiments were performed using the WSJ Treebank and a modified CYK parser
- Vanilla CNF PCFG obtained from sections 02-21. Test set: section 23
- The system simulates user interaction:
  1. Explore the proposed tree and find the first wrong constituent
  2. Replace it with the correct gold constituent
  3. Perform the predictive step (obtain new tree)
  - n. Repeat until the gold tree is achieved

## 6.3 IPP: A FRAMEWORK FOR ACTIVE LEARNING

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### Evaluation and results:

- Tree Constituent Error Rate (TCER): Normalized edit distance between the proposed parse tree and the gold tree  
→ *User effort when manually postediting the erroneous tree*
- Tree Constituent Action Rate (TCAC): Ration of user constituent corrections performed to obtain the reference tree using the IPP system  
→ *User effort when using the IPP system*

PCFG	Baseline		IPP	RelRed
	$F_1$	TCER	TCAC	
h=0,v=1	0.67	0.40	0.22	45%
h=0,v=2	0.68	0.39	0.21	46%
h=0,v=3	0.70	0.38	0.22	42%

## 6.3 IPP: A FRAMEWORK FOR ACTIVE LEARNING

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IPP-ANN tool: <http://cat.iti.upv.es/ipp/>

### Parser server

- Custom Viterbi implementation
- Using PCFG in CNF
- Allows requesting subtrees with
  - a root span
  - a complete root constituent

### Parser client

- Light Web-client using Flash plugin
- Decodes user feedback
- Requests subtrees to the parse server based on user corrections

### Communication

- Client-server communication via sockets
- Using a library specifically tailored for interactive predictive applications



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# APPENDICES

A growth transformation can be defined as:

$$\bar{p}(A \rightarrow \alpha) = \frac{p(A \rightarrow \alpha) \left( \frac{\partial \text{Pr}_{G_s}(\Omega, \Delta_\Omega)}{\partial p(A \rightarrow \alpha)} \right)_p}{\sum_{i=1}^{n_A} p(A \rightarrow \alpha_i) \left( \frac{\partial \text{Pr}_{G_s}(\Omega, \Delta_\Omega)}{\partial p(A \rightarrow \alpha_i)} \right)_p}$$

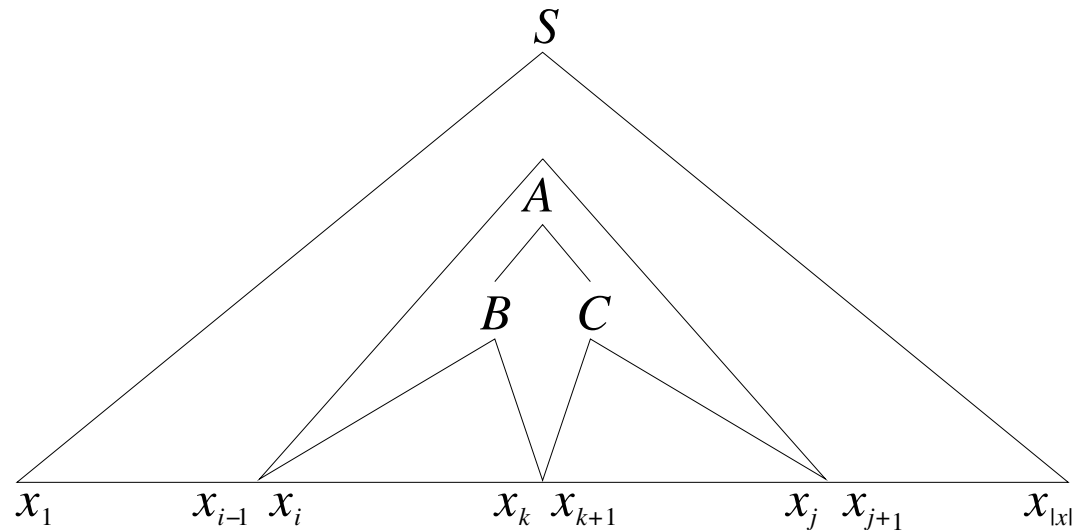
Numerator:

$$\begin{aligned} p(A \rightarrow \alpha) \left( \frac{\partial \text{Pr}_{G_s}(\Omega, \Delta_\Omega)}{\partial p(A \rightarrow \alpha)} \right)_p &= \text{Pr}_{G_s}(\Omega, \Delta_\Omega) \sum_{x \in \Omega} \frac{p(A \rightarrow \alpha)}{\text{Pr}_{G_s}(x, \Delta_x)} \left( \frac{\partial \text{Pr}_{G_s}(x, \Delta_x)}{\partial p(A \rightarrow \alpha)} \right)_p \\ &= \text{Pr}_{G_s}(\Omega, \Delta_\Omega) \sum_{x \in \Omega} \frac{p(A \rightarrow \alpha)}{\text{Pr}_{G_s}(x, \Delta_x)} \sum_{\forall d_x \in \Delta_x} \left( \frac{\partial \text{Pr}_{G_s}(x, d_x)}{\partial p(A \rightarrow \alpha)} \right)_p \\ &= \text{Pr}_{G_s}(\Omega, \Delta_\Omega) \sum_{x \in \Omega} \frac{1}{\text{Pr}_{G_s}(x, \Delta_x)} \sum_{\forall d_x \in \Delta_x} \text{N}(A \rightarrow \alpha, d_x) \text{Pr}_{G_s}(x, d_x) \end{aligned}$$

Denominator:

$$\begin{aligned}
 & \sum_{i=1}^{n_A} p(A \rightarrow \alpha_i) \left( \frac{\partial \Pr_{G_s}(\Omega, \Delta_\Omega)}{\partial p(A \rightarrow \alpha_i)} \right)_p = \\
 &= \Pr_{G_s}(\Omega, \Delta_\Omega) \sum_{x \in \Omega} \frac{1}{\Pr_{G_s}(x, \Delta_x)} \sum_{\forall d_x \in \Delta_x} \sum_{i=1}^{n_A} N(A \rightarrow \alpha_i, d_x) \Pr_{G_s}(x, d_x) \\
 &= \Pr_{G_s}(\Omega, \Delta_\Omega) \sum_{x \in \Omega} \frac{1}{\Pr_{G_s}(x, \Delta_x)} \sum_{\forall d_x \in \Delta_x} N(A, d_x) \Pr_{G_s}(x, d_x).
 \end{aligned}$$

- Let  $A \rightarrow BC$  in a position delimited by integers  $i, j, k$ ,  $1 \leq i \leq k < j \leq |x|$



- $\Delta_{x,i,j,k,A \rightarrow BC} \subseteq D_x$ : subset of derivations of  $x$  in which the rule  $A \rightarrow BC$  appears delimited by positions  $i, j, k$
- $\Delta_{x,i,j,A}$ : subset of derivations of  $x$  in which the non-terminal  $A$  appears delimited by positions  $i, j$



$$\begin{aligned}
 \blacktriangleright \sum_{\forall d_x \in D_x} N(A \rightarrow BC, d_x) \Pr_{G_s}(x, d_x) &= \sum_{1 \leq i \leq k < j \leq |x|} \sum_{\forall d_x \in \Delta_{x,i,j,k,A \rightarrow BC}} \Pr_{G_s}(x, d_x) \\
 &= \sum_{1 \leq i \leq k < j \leq |x|} \Pr_{G_s}(S \xrightarrow{*} x_1 \dots x_{i-1} A x_{j+1} \dots x_{|x|}) \cdot \\
 &\quad p(A \rightarrow BC) \cdot \Pr_{G_s}(B \xrightarrow{*} x_i \dots x_k) \cdot \Pr_{G_s}(C \xrightarrow{*} x_{k+1} \dots x_j) \\
 &= \sum_{1 \leq i \leq k < j \leq |x|} f(A < i, j >) p(A \rightarrow BC) e(B < i, k >) e(C < k + 1, j >),
 \end{aligned}$$

$$\begin{aligned}
 \blacktriangleright \sum_{\forall d_x \in D_x} N(A, d_x) \Pr_{G_s}(x, d_x) &= \sum_{1 \leq i \leq j \leq |x|} \sum_{\forall d_x \in \Delta_{x,i,j,A}} \Pr_{G_s}(x, d_x) \\
 &= \sum_{1 \leq i \leq j \leq |x|} \Pr_{G_s}(S \xrightarrow{*} x_1 \dots x_{i-1} A x_{j+1} \dots x_{|x|}) \Pr_{G_s}(A \xrightarrow{*} x_i \dots x_j) \\
 &= \sum_{1 \leq i \leq j \leq |x|} f(A < i, j >) e(A < i, j >).
 \end{aligned}$$

## EM algorithm [Neal 98]:

E step: Compute a distribution  $\tilde{p}^{(t)}$  over the range of  $\mathbf{Z}$  such that

$$\tilde{p}^{(t)}(\mathbf{z}) = p(\mathbf{z}|\mathbf{x}; \theta^{(t-1)})$$

M step: Set  $\theta^{(t)}$  to the  $\theta$  that maximizes  $E_{\tilde{p}^{(t)}}[\log p(\mathbf{x}, \mathbf{z}; \theta)]$