

First, my fear; then my curtsy; last my speech.

My fear, is your displeasure,

my curtsy, my duty,

and my speech, to beg your pardon.

Henry IV



Srinivasa Ramanujan  
1887-1920

16 January 1913

Dear Sir,

I beg to introduce myself to you as a clerk in the Accounts Department of the Port Trust Office at Madras on a salary of only 20 pounds per annum. I am now 23 years of age. I have no University education but have undergone the ordinary school course.

After leaving school I have been employing the spare time at my disposal to work at mathematics.....I would request you to go through the enclosed papers.

Being poor, if you are convinced that there is anything of value I would like to have my theorems published.....

requesting to be excused for the trouble I give you,  
I remain, Dear Sir, Yours truly,

S. Ramanujan

(4)

$$\int_0^{\infty} \frac{dx}{(1+x^2)(1+r^2x^2)(1+r^4x^2)(1+r^6x^2)\cdots} = \frac{\pi}{2(1+r+r^3+r^6+r^{10}+\cdots)}$$

where 1, 3, 6, 10, ... are sums of natural numbers.

$$(5) \quad \int_0^{\infty} \frac{\sin 2nx}{x(\cosh \pi x + \cos \pi x)} dx = \frac{\pi}{4} - 2 \left( \frac{e^{-n} \cos n}{\cosh \frac{\pi}{2}} - \frac{e^{-3n} \cos 3n}{3 \cosh \frac{3\pi}{2}} + \frac{e^{-5n} \cos 5n}{5 \cosh \frac{5\pi}{2}} - \cdots \right).$$

$$(6) \quad \int_0^{\infty} \tan^{-1} \frac{2nz}{n^2 + x^2 - z^2} \frac{dz}{e^{2\pi z} - 1}$$

can be exactly found if  $2n$  is any integer and  $x$  any quantity.

(5)

V. Theorems on summation of series; e.g.

$$(1) \quad \frac{1}{1^3} \cdot \frac{1}{2^1} + \frac{1}{2^3} \cdot \frac{1}{2^2} + \frac{1}{3^3} \cdot \frac{1}{2^3} + \frac{1}{4^3} \cdot \frac{1}{2^4} + \cdots = \frac{1}{6} (\log 2)^3 - \frac{\pi^2}{12} \log 2 + \left( \frac{1}{1^3} + \frac{1}{3^3} + \frac{1}{5^3} + \cdots \right).$$

$$(2) \quad 1 + 9 \cdot \left(\frac{1}{4}\right)^4 + 17 \cdot \left(\frac{1 \cdot 5}{4 \cdot 8}\right)^4 + 25 \cdot \left(\frac{1 \cdot 5 \cdot 9}{4 \cdot 8 \cdot 12}\right)^4 + \cdots = \frac{2\sqrt{2}}{\sqrt{\pi} \{\Gamma(\frac{3}{4})\}^2}.$$

$$(3) \quad 1 - 5 \cdot \left(\frac{1}{2}\right)^3 + 9 \cdot \left(\frac{1 \cdot 3}{2 \cdot 4}\right)^3 - \cdots = \frac{2}{\pi}.$$

$$(4) \quad \frac{1^{13}}{e^{2\pi} - 1} + \frac{2^{13}}{e^{4\pi} - 1} + \frac{3^{13}}{e^{6\pi} - 1} + \cdots = \frac{1}{24}.$$

$$(5) \quad \frac{\coth \pi}{1^7} + \frac{\coth 2\pi}{2^7} + \frac{\coth 3\pi}{3^7} + \cdots = \frac{19\pi^7}{56700}.$$

$$(6) \quad \frac{1}{1^5 \cosh \frac{\pi}{2}} - \frac{1}{3^5 \cosh \frac{3\pi}{2}} + \frac{1}{5^5 \cosh \frac{5\pi}{2}} - \cdots = \frac{\pi^5}{768}.$$

$$(7) \quad \frac{1}{(1^2 + 2^2)(\sinh 3\pi - \sinh \pi)} + \frac{1}{(2^2 + 3^2)(\sinh 5\pi - \sinh \pi)}$$

$$+ \frac{1}{(3^2 + 4^2)(\sinh 7\pi - \sinh \pi)} + \cdots = \frac{1}{2 \sinh \pi} \left( \frac{1}{\pi} + \coth \pi - \frac{\pi}{2} \tanh^2 \frac{\pi}{2} \right).$$

“They defeated me completely.  
I had never seen anything in the least like them before.  
A single look at them is enough to show that they could  
only be written down by a mathematician of the highest class.

They must be true because, if they were not true,  
no one would have had the imagination to invent them.”

G.H. Hardy



“In his insight into algebraical formulae, transformation of infinite series, and so forth, that was most amazing. On this side most certainly I have never met his equal, and I can compare him only with Euler or Jacobi”.

G.H. Hardy, 1921

“I do not think now that this extremely strong language is extravagant. It is possible that great days of formulae are finished, and that Ramanujan ought to have been born 100 years ago; but he was by far the greatest formalist of his time”.

G.H. Hardy, 1936

“Srinivasa Ramanujan was a mathematician so great that his name transcends jealousies, the one superlatively great mathematician whom India has produced in the last thousand years”.

Eric Neville (1940)  
Trinity College

# Ramanujan Institute of Mathematics

- Founded in 1950 by **Sir Alagappa Chettiar**, with enthusiastic support from Chandrasekhar.
- **T. Vijayaraghavan**, an illustrious student of Hardy was the first Director.
- Strong support from **Chandra, Andre Weil, Norbert Wiener...**
- Succeeded by **CT Rajagopal** in 1955, on the recommendation of Andre Weil and Chandrasekhar.
- In 1957, Chandra writes to **Jawaharlal Nehru** to save the institute. Nehru ensures the survival of the institute.
- In 1967, the institute was merged with the Mathematics Department of the University of Madras.





The temple at Kumbakonam where Ramanujan was born

“....And my own view is that, at the bottom and to a first approximation, R. was (intellectually) as sound an infidel as Bertrand Russell or Littlewood....

One thing I am sure. R. was not in the least the ‘inspired idiot’ that some people seem to have thought him. On the contrary, he was (except for a period when his mental equilibrium was definitely upset by illness) a very shrewd and sensible person: very individual, of course, and with a reasonable allowance of the minor eccentricities of genius, but fundamentally normal and sane.”

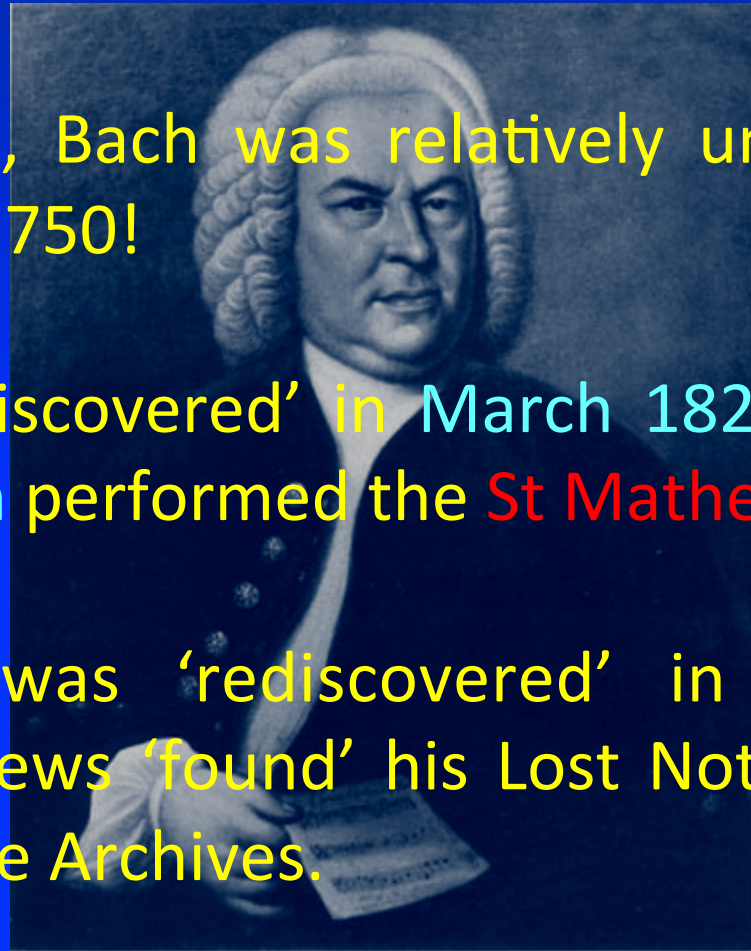
G. H. Hardy  
Letter to Chandra (1936)

Bruce Berndt – the author of five volumes on “Ramanujan’s Notebooks” has compared Ramanujan with **Johann Sebastian Bach**.

Astonishingly, Bach was relatively unknown after his death in 1750!

JSB was ‘rediscovered’ in March 1829 when Felix Mendelssohn performed the **St Mathew Passion**.

Ramanujan was ‘rediscovered’ in 1976 when George Andrews ‘found’ his Lost Notebook in the Trinity College Archives.



# In 1976, George Andrews discovered the Lost Notebook of Ramanujan

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357.37  $e^\lambda = 10^6 \times 10^6$

$$\lambda = 13.82 + 10^6 \log x$$

$$= 10^6 \log x (1 + \epsilon)$$

$$\log x = 13.82 + \log \log x + \epsilon' \quad (0 < \epsilon' < \epsilon)$$

$$\lambda = 10^6 \left\{ 13.82 + \log \log x + \epsilon' \right\} (1 + \epsilon)$$

$$\log \log x = \log 13.82 + \log \left( 1 + \frac{\log \log x + \epsilon'}{13.82} \right)$$

Now

$$\log x < 20 < e^3$$

$$\log \log x < 3$$

$$\frac{\log \log x + \epsilon'}{13.82} < \frac{1}{4}$$

$$\log \log x = 2.63 + \eta \quad \eta < \frac{1}{4}$$

$$\lambda = 10^6 \left[ 13.82 + 2.63 + \eta' \right] (1 + \epsilon) \quad (\eta' < \frac{1}{4})$$

$$= 10^6 \left[ 16.45 + \eta'' \right] \quad \eta'' < \frac{1}{4}$$

$$u = \frac{f(-x)}{\sqrt{x}f(-x^2)}; v = \frac{f(-x)}{\sqrt{x}f(-x^2)}$$

$n=2$   $uv + \frac{f}{uv} = \left(\frac{u}{v}\right)^2 + \left(\frac{v}{u}\right)^2$   
 $n=3$   $(uv)^3 + \left(\frac{f}{uv}\right)^3 = -\left\{\left(\frac{u}{v}\right)^6 - \left(\frac{v}{u}\right)^6\right\} - 9\left\{\left(\frac{u}{v}\right)^3 + \left(\frac{v}{u}\right)^3\right\}$   
 $n=4$   $(uv)^4 + \left(\frac{f}{uv}\right)^4 = \left(\frac{u}{v}\right)^8 + \left(\frac{v}{u}\right)^8 - 8\left\{\left(\frac{u}{v}\right)^4 + \left(\frac{v}{u}\right)^4\right\} + 4\left\{\frac{u}{v} + \frac{v}{u}\right\}$   
 $n=5$   $(uv)^5 + \left(\frac{f}{uv}\right)^5 = 5\left\{uv + \frac{f}{uv}\right\} + 15 = \left(\frac{u}{v}\right)^5 + \left(\frac{v}{u}\right)^5$   
 $n=7$   $(uv)^7 + \left(\frac{f}{uv}\right)^7 = -\left\{\left(\frac{u}{v}\right)^{14} - \left(\frac{v}{u}\right)^{14}\right\} - 7\left\{\left(\frac{u}{v}\right)^7 + \left(\frac{v}{u}\right)^7\right\}$   
 $n=7$   $\left\{\left(\frac{u}{v}\right)^7 - \left(\frac{v}{u}\right)^7\right\} + 14\left\{\frac{u}{v} + \frac{v}{u}\right\}$

$$u^2 = \frac{f(-x^2)}{\sqrt{x}f(-x^2)}; v^2 = \frac{f(-x^2)}{\sqrt{x}f(-x^2)}$$

$n=2$   $uv + \frac{f}{uv} = \left(\frac{u}{v}\right)^2 + \left(\frac{v}{u}\right)^2 - 2\left\{\frac{u}{v} + \frac{v}{u}\right\}$   
 $n=3$   $(uv)^3 + \left(\frac{f}{uv}\right)^3 = 3\left\{\frac{u}{v} + \frac{v}{u}\right\}(uv + \frac{f}{uv})$   
 $= \left(\frac{u}{v}\right)^3 + \left(\frac{v}{u}\right)^3 - 6\left\{\frac{u}{v} + \frac{v}{u}\right\} - 9$

$$u = 1 + \frac{x}{1-x} + \frac{x^2}{(1-x)(1-x^2)} + \dots$$

$$v = 1 + \frac{x^2}{1-x} + \frac{x^4}{(1-x)(1-x^2)} + \dots$$

$$uv = 1 + 11x + 6x^2 + x^4 + x^6 + \dots$$

$$u = \frac{f(-x)}{x^2 f(-x^2)}; v = \frac{f(-x^2)}{x^2 f(-x^2)}$$

$n=2$   $(uv)^2 + \left(\frac{2}{uv}\right)^2 = \left(\frac{u}{v}\right)^4 - \left(\frac{v}{u}\right)^4$   
 $n=3$   $(uv)^3 + \left(\frac{3}{uv}\right)^3 = \left(\frac{u}{v}\right)^6 - \left(\frac{v}{u}\right)^6$   
 $n=4$   $(uv)^4 + \left(\frac{4}{uv}\right)^4 = \left(\frac{u}{v}\right)^8 + \left(\frac{v}{u}\right)^8 - 5\left\{\frac{u}{v} + \frac{v}{u}\right\}$   
 $n=5$   $(uv)^5 + \left(\frac{5}{uv}\right)^5 = 5\left\{uv + \frac{f}{uv}\right\} + 15 = \left(\frac{u}{v}\right)^5 + \left(\frac{v}{u}\right)^5$   
 $n=7$   $(uv)^7 + \left(\frac{7}{uv}\right)^7 = 5 = \left(\frac{u}{v}\right)^7 - \left(\frac{v}{u}\right)^7 - 5\left\{\left(\frac{u}{v}\right)^4 + \left(\frac{v}{u}\right)^4\right\}$

$$u = \frac{f(-x^2)}{x^2 f(-x^2)}; v = \frac{f(-x)}{x^2 f(-x^2)}$$

$n=2$   $uv - \frac{5}{uv} = \left(\frac{u}{v}\right)^2 - \left(\frac{3u}{v}\right)^2$   
 $n=3$   $(uv)^3 - \left(\frac{3}{uv}\right)^3 = \left(\frac{u}{v}\right)^6 - \left(\frac{3u}{v}\right)^6 + \left(\frac{u}{v}\right)^2 - \left(\frac{3u}{v}\right)^2$

$$\frac{1}{2} \left\{ \frac{v^{1/2}}{(1-v+v^2)} + \frac{v^{3/2}}{(1-v+v^2)(1-v^2)} + \frac{v^{5/2}}{(1-v+v^2)(1-v^2)(1-v^4)} + \dots \right\}$$

$$+ \frac{1}{3} \left\{ \frac{v^2}{(1+v)^2} + \frac{v^4}{(1+v)^2(1+v^2)^2} + \frac{v^6}{(1+v)^2(1+v^2)^2(1+v^4)^2} + \dots \right\}$$

$$= v^{1/2} \frac{(1-v^2-v^4)^2}{(1-v^2)(1-v^4)(1-v^8)} + v^2(1+v)(1+av) + v^4(1+v)(1+av^2) + \dots$$

$$= (1+v)(1+v^2) \dots (1+av)(1+av^2) \dots$$

$$\times \left\{ \frac{1}{1+av} + \frac{v^2(1-v)}{(1+av)(1+av^2)(1+av^4)} + \frac{v^4(1-v)(1-v^2)}{(1+av)(1+av^2)(1+av^4)(1+av^8)} + \dots \right\}$$

$$2 \left\{ v + v^2(1+v)(1+av) + v^3(1+v)(1+av^2)(1+av)(1+av^2) + \dots \right\}$$

$$+ \frac{1}{1+a} \left\{ 1 + \frac{v}{(1+v)(1+av)} + \frac{v^2}{(1+v)(1+v^2)(1+av)(1+av^2)} + \dots \right\}$$

$$= \frac{(1+av)(1+av^2)(1+av^4) \dots}{(1-v^2)(1-v^4)(1-v^8) \dots} \left\{ \frac{1}{1+a} + \left( \frac{v}{1+av} + \frac{v^2}{1+av^2} + \frac{v^4}{1+av^4} + \dots \right) + \left( \frac{v^2}{a+v} + \frac{v^4}{a+v^2} + \frac{v^6}{a+v^4} + \dots \right) \right\}$$

$$\frac{1}{1+a} \left\{ 1 + \frac{v}{(a+v)(1+v)} + \frac{v^2}{(a+v)(1+v)} + \dots \right\}$$

$$+ (1+a) \left\{ v + v^2(1+av)(1+av^2) + v^3(1+av)(1+av^2)(1+av^4) + \dots \right\}$$

$$= \frac{1}{(1-v)(1-v^2)(1-v^4) \dots} \frac{(1+av)(1+av^2)(1+av^4) \dots}{(1+\frac{v}{a})(1+\frac{v^2}{a})(1+\frac{v^4}{a}) \dots}$$

$$\times \left\{ \frac{1}{1+a} + \frac{av}{1+av} + \frac{a^2v^2}{1+av^2} + \frac{a^3v^4}{1+av^4} + \dots \right. \\ \left. + \frac{v/a}{a+v} + \frac{v^2/a}{a+v^2} + \frac{v^4/a}{a+v^4} + \dots \right\}$$

$$(1+\frac{a}{2}) \left\{ \frac{1}{1+v} + \frac{v(1-av)(1-\frac{v}{a})}{(1+v)(1+v^2)(1+v^4)} + \frac{v^2(1-av)(1-av^2)(1-\frac{v}{a})(1-\frac{v^2}{a})}{(1+v)(1+v^2)(1+av^4)(1+av^8)} + \dots \right\}$$

$$= (1+\frac{a}{2}) - (a+\frac{1}{2})v^2 + (a^2+\frac{1}{2}a)v^4 - \dots$$

$$\frac{1}{1+a} \left\{ 1 + \frac{v}{(1+av)(1+\frac{v}{a})} + \frac{v^2}{(1+av)(1+av^2)(1+\frac{v}{a})(1+\frac{v^2}{a})} + \dots \right\}$$

$$\times (1-v)(1-v^2)(1-v^4) \dots$$

$$= \frac{1}{1+a} - \frac{v^2}{1+av} + \frac{v^4}{1+av^2} - \frac{v^6}{1+av^4} + \dots$$

$$- \frac{v^2}{a+v} + \frac{v^4}{a+v^2} - \frac{v^6}{a+v^4} + \dots$$

Two pages from Ramanujan's Lost Notebook



$$\int_0^{\infty} e^{-3\pi x^2} \frac{\sinh \pi x}{\sinh 3\pi x} dx = \frac{1}{e^{\frac{2}{3}\pi} \sqrt{3}} \sum_{n=0}^{\infty} \frac{e^{-2n(n+1)\pi}}{(1+e^{-\pi})^2 (1+e^{-3\pi})^2 \dots (1+e^{-(2n+1)\pi})^2}$$

‘The discovery of the Lost Notebook is as sensational a discovery for the mathematicians as a complete draft of a tenth symphony of Beethoven would have been to the musicians’.

Olga Taussky-Todd

$$\begin{aligned}
 & \frac{1}{3} \left\{ \frac{v^{1/2}}{(1-v+v^2)} + \frac{v^{3/2}}{(1-v+v^2)(1-v^2+v^4)} + \frac{v^{5/2}}{(1-v+v^2)(1-v^2+v^4)(1-v^4+v^8)} + \dots \right\} \\
 & + \frac{1}{3} \left\{ \frac{v^{1/2}}{(1+v)^2} + \frac{v^{3/2}}{(1+v)^2(1+v^2)^2} + \frac{v^{5/2}}{(1+v)^2(1+v^2)^2(1+v^4)^2} + \dots \right\} \\
 & = v^{1/2} \frac{(1-v^3-v^9+\dots)^2}{(1-v^2)(1-v^4)(1-v^8)} \\
 & \quad \cdot \left[ 1 + v(1+v)(1+v^2) + v^2(1+v)(1+v^2)^2 + \dots \right] \\
 & = (1+v)(1+v^2)\dots(1+v^7)(1+v^{14}) \dots \\
 & \quad \times \left\{ \frac{1}{1+av} + \frac{v^2(1-v)}{(1+av)(1+av^2)(1+av^4)} + \frac{v^4(1-v)(1-v^2)}{(1+av)\dots(1+av^8)} + \dots \right\} \\
 & \quad + \frac{1}{1+a} \left\{ 1 + \frac{v}{(1+v)(1+v^2)} + \frac{v^2}{(1+v)(1+v^2)(1+v^4)} + \dots \right\} \\
 & = \frac{(1+av)(1+av^2)(1+av^4)\dots}{(1-v)(1-v^2)(1-v^4)\dots} \cdot \left\{ \frac{1}{1+a} + \left( \frac{v}{1+av} + \frac{v^2}{1+av^2} + \frac{v^4}{1+av^4} + \dots \right) \right\} \\
 & \quad + \frac{1}{1+a} \left\{ 1 + \frac{v}{(1+v)(1+v^2)} + \frac{v^2}{(1+v)(1+v^2)(1+v^4)} + \dots \right\} \\
 & \quad + (1+l) \left\{ v + v^2(1+av)(1+lv) + v^3(1+av)(1+av^2)(1+lv)(1+lv^2) + \dots \right\} \\
 & = \frac{1}{(1-v)(1-v^2)(1-v^4)\dots} \cdot \frac{(1+av)(1+av^2)(1+av^4)\dots}{(1+\frac{v}{2})(1+\frac{v^2}{2})(1+\frac{v^4}{2})\dots} \\
 & \quad \times \left\{ \frac{1}{1+a} + \frac{lv}{1+av} + \frac{l^2v^2}{1+av^2} + \frac{l^3v^3}{1+av^3} + \dots \right. \\
 & \quad \left. + \frac{v/l}{a+v} + \frac{v^2/l^2}{a+v^2} + \frac{v^3/l^3}{a+v^3} + \dots \right\} \\
 & (1+\frac{l}{2}) \left\{ \frac{1}{1+v} + \frac{v(1-av)(1-\frac{v}{2})}{(1+v)(1+v^2)(1+v^4)} + \dots \right. \\
 & \quad \left. + \frac{v^2(1-av)(1-av^2)(1-\frac{v}{2})(1-\frac{v^2}{2})}{(1+v)(1+v^2)\dots(1+v^8)} + \dots \right\} \\
 & = (1+\frac{l}{2}) - (a+\frac{l}{2})v^2 + (a^2+\frac{l}{2}a)v^4 - \dots \\
 & \frac{1}{1+a} \left\{ 1 + \frac{v}{(1+av)(1+\frac{v}{2})} + \frac{v^2}{(1+av)(1+av^2)(1+\frac{v}{2})(1+\frac{v^2}{2})} + \dots \right\} \\
 & \quad \times (1-v)(1-v^2)(1-v^4)\dots \\
 & = \frac{1}{1+a} - \frac{v^2}{1+av} + \frac{v^4}{1+av^2} - \frac{v^6}{1+av^3} + \dots \\
 & \quad - \frac{v^2}{a+v} + \frac{v^4}{a+v^2} - \frac{v^6}{a+v^3} + \dots
 \end{aligned}$$

“Try to imagine the quality of Ramanujan’s mind, one which drove him to work unceasingly while deathly ill, and one great enough to grow deeper while his body became weaker. I stand in awe of his accomplishments; understanding is beyond me. We would admire any mathematician whose life’s work was half of what Ramanujan found in the last year of his life while he was dying.”

Richard Askey







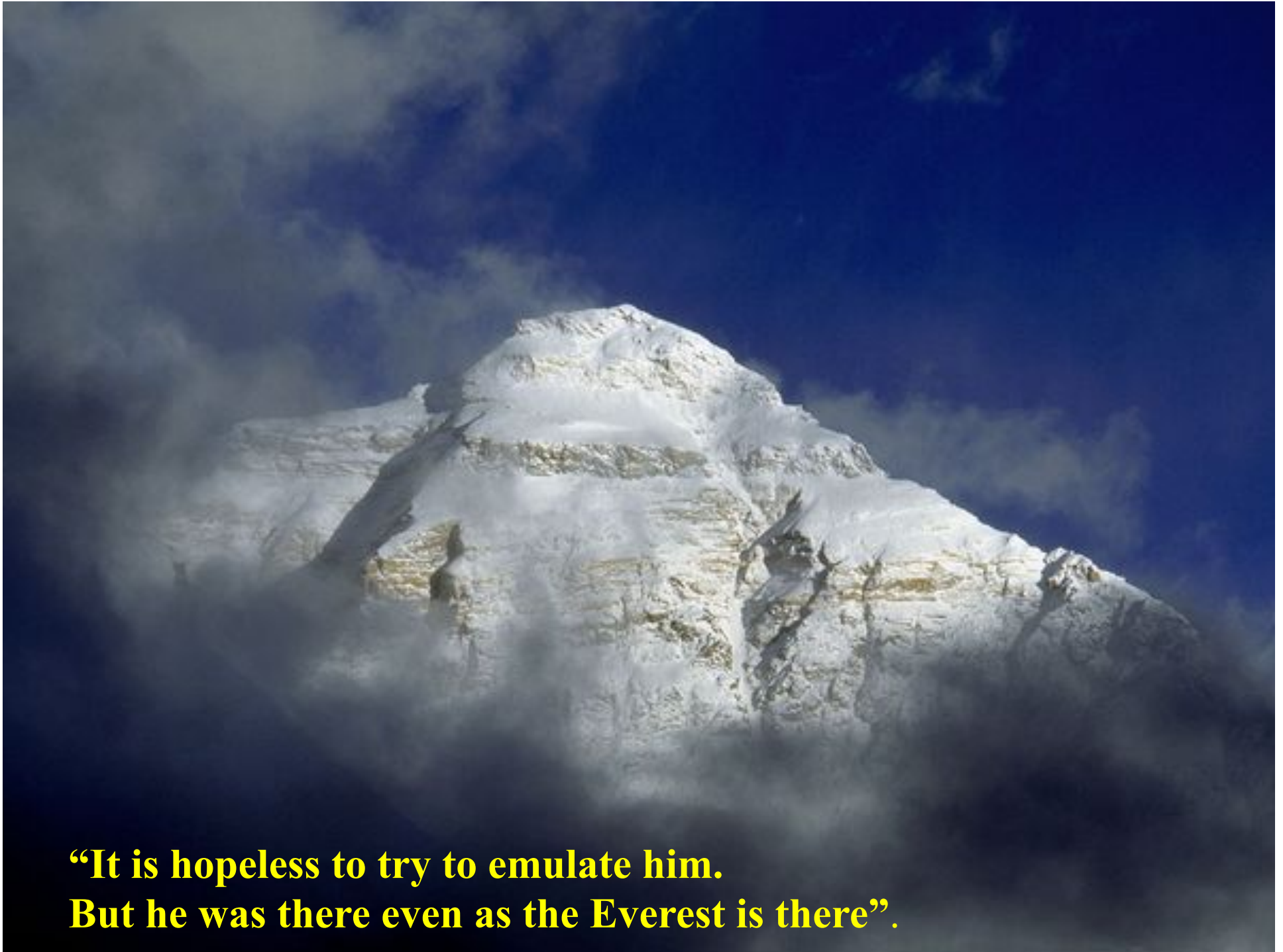


“I can still recall the gladness I felt at the assurance that one brought up under circumstances similar to my own, could have achieved what I could not grasp....”.

Chandrasekhar, 1987

“The fact that Ramanujan’s early life was spent in a scientifically sterile atmosphere, that his life in India was not without hardship, that under circumstances that appeared to most Indians as nothing short of miraculous, he had gone to Cambridge, supported by eminent mathematicians, and had returned to India with every assurance that he would be considered, in time, as one of the most original mathematicians of the century – these facts were enough – more than enough – for aspiring young Indian students to break their bonds of intellectual confinement and perhaps soar the way that Ramanujan had”.

S. Chandrasekhar , 1987



**“It is hopeless to try to emulate him.  
But he was there even as the Everest is there”.**



“The pursuit of science has often been compared to the scaling of mountains, high and not so high. But who amongst us can hope, even in imagination, to scale the Everest and reach its summit when the sky is blue and the air is still, and in the stillness of the air survey the entire Himalayan range in the dazzling white of the snow stretching to infinity? None of us can hope for a comparable vision of nature and of the universe around us. But there is nothing mean or lowly in standing in the valley below and awaiting the sun to rise over Kanchenjunga.”

S.

Chandrasekhar



Paul Granlund in his studio







