INSTABILITIES OF RELATIVISTIC STARS

John Friedman

Center for Gravitation and Cosmology
University of Wisconsin-Milwaukee

I. A GENERAL CRITERION

II. LOCAL INSTABILITIES:
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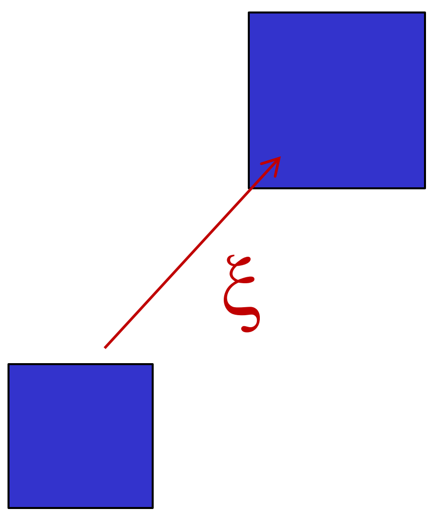
III. AXISYMMETRIC INSTABILITY AND
TURNING POINTS

IV. NONAXISYMMETRIC INSTABILITIES

To obtain an action for the Euler equation, one introduces a Lagrangian displacement ξ, joining initial and perturbed fluid elements.



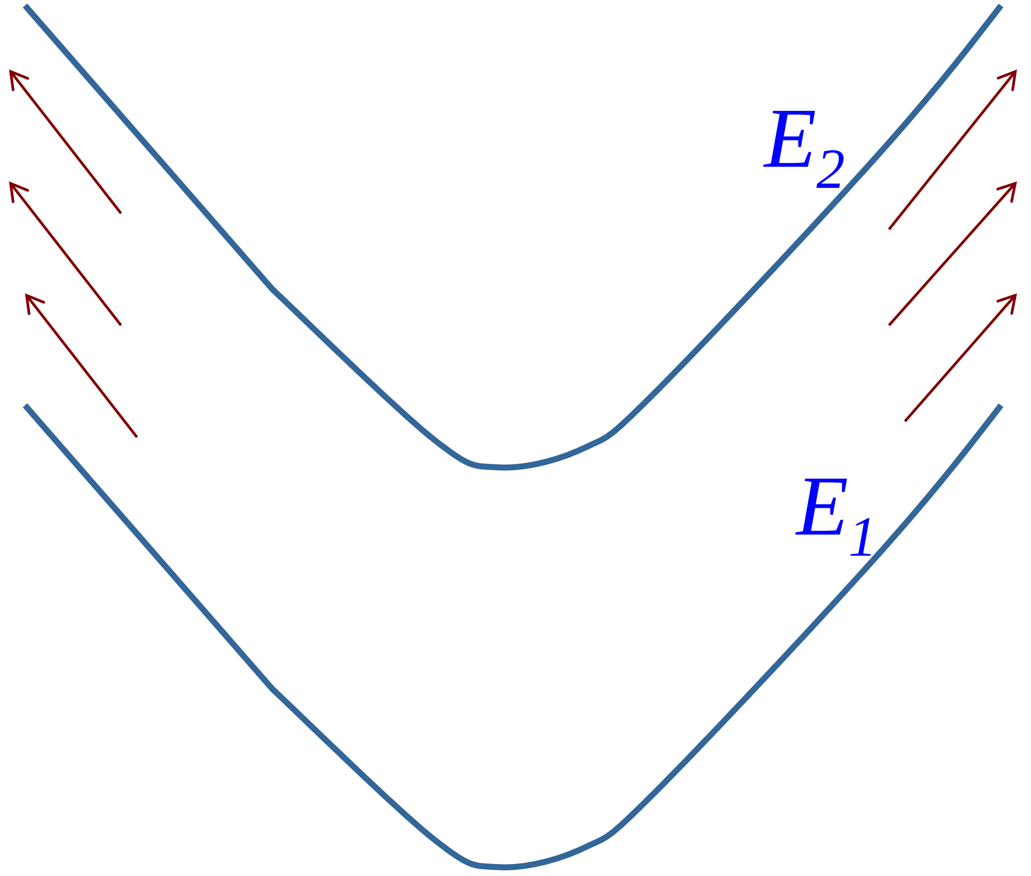
Actions for the perturbation equations by Chandra and students, culminating in work by Lynden-Bell and Ostriker.

Gauge-freedom associated in ξ associated by Noether's theorem with conservation of circulation.

The peturbed fluid and metric has a conserved current j^α associated with the time-translation symmetry of the equilibrium star. The corresponding conserved energy is

$$E = \int_S d\sigma_\alpha j^\alpha = \int_S d^3x\, \alpha \left(\Pi^\alpha \pounds_{\mathbf{t}} \xi_\alpha + \pi^{\alpha\beta} \pounds_{\mathbf{t}} h_{\alpha\beta} - t^\alpha \mathcal{L}^{(2)} \right),$$

where Π^α and $\pi^{\alpha\beta}$ are the canonical momenta associated with ξ^α and $h_{\alpha\beta}$.



$$\nabla_\alpha j^\alpha = 0 \Rightarrow$$

$$E_2 - E_1 = -\text{energy radiated to } \mathscr{I}^+ < 0$$

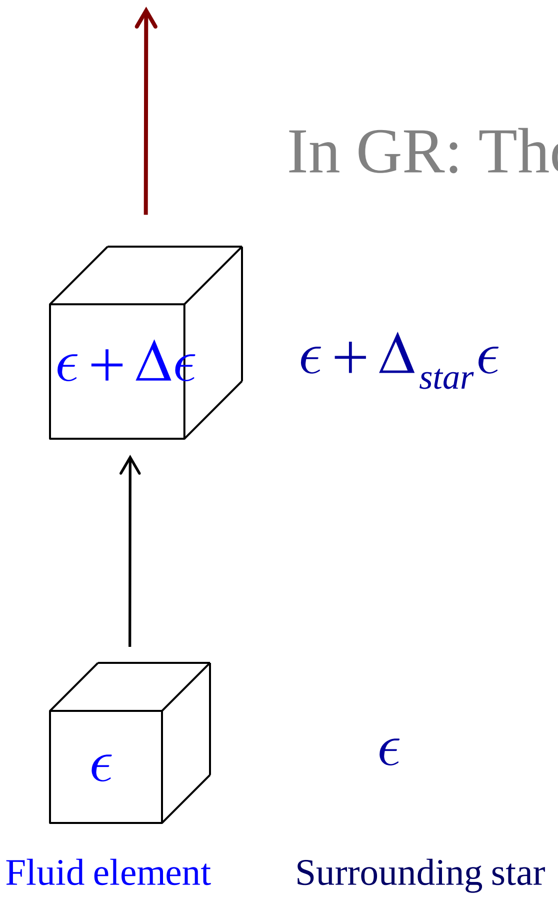
It follows that

If $E < 0$ for some data on S preserving circulation and baryon number, the configuration is unstable or marginally stable: There exist perturbations on a family of asymptotically null hypersurfaces that do not die away in time.

If $E > 0$ for all such data on S, $|E|$ is bounded in time and only finite energy can be radiated.

Local stability

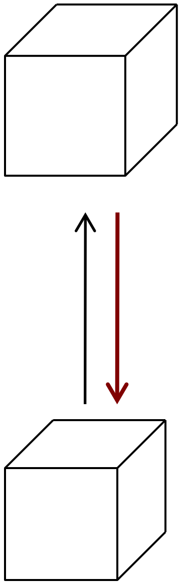
In GR: Thorne, Kovetz, Bardeen, Schutz, Seguin



- When a fluid element is displaced upward, if its density decreases more rapidly than the density of the surrounding fluid, then the element will be buoyed upward and the star will be unstable.

Unstable if

$$|\Delta\epsilon| > |\Delta_{star}\epsilon|$$



- If the fluid element expands less than its surroundings it will fall back, and the star will be stable against convection.

$$\Delta\epsilon = \left(\frac{\partial\epsilon}{\partial p}\right)_s \Delta p \quad \text{adiabatic}$$

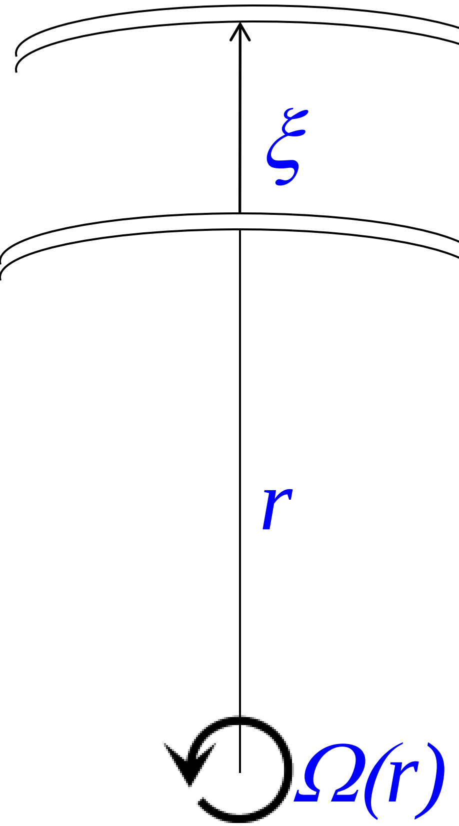
$$\Delta_{star}\epsilon = \left(\frac{d\epsilon}{dp}\right)_{star}$$

Stable if

$$\left(\frac{\partial\epsilon}{\partial p}\right)_s < \left(\frac{d\epsilon}{dp}\right)_{star}$$

Within minutes after their birth, neutron stars cool to a temperature below the Fermi energy per nucleon, below 10^{12} K. Their neutrons are then degenerate, with a nearly isentropic equation of state: Convectively stable, but with convective modes having nearly zero frequency.

INSTABILITY FROM DIFFERENTIAL ROTATION



Differential rotation is stable if a ring of fluid that is displaced outward, conserving angular momentum and mass, will fall back.

The ring of fluid displaced from r to $r+\xi$ will continue to move outward if its centripetal acceleration is larger that the restoring force

Marginal stability:

If the displaced ring has the same value of v^2/r as the surrounding fluid, then, like the surrounding fluid, it will be in equilibrium.

Unstable if

$$\Delta \frac{v^2}{r} > \Delta_{star} \frac{v^2}{r}$$

$\Delta j = 0$ and $j = vr$ imply

$$\Delta_{star}\frac{v^2}{r} - \Delta\frac{v^2}{r} = \xi\frac{1}{r^2}\frac{dj^2}{dr} \quad \Rightarrow$$

Stable if j increases outward

Exactly the same criterion for GR

(Bardeen, Seguin, Abramowicz, Prasanna)

Caveat: for large enough frame dragging – e.g., near a rotating black hole, the criterion is reversed.

This is a simplest example of the *turning-point* criterion governing axisymmetric stability:
An instability point along a sequence of circular orbits of a particle of fixed baryon mass is a point at which j is an extremum.

Instability to collapse

The pioneering paper

THE DYNAMICAL INSTABILITY OF GASEOUS MASSES APPROACHING THE SCHWARZSCHILD LIMIT IN GENERAL RELATIVITY

S. CHANDRASEKHAR
University of Chicago
Received May 11, 1964

ABSTRACT

In this paper the theory of the infinitesimal, baryon-number conserving, adiabatic, radial oscillations of a gas sphere is developed in the framework of general relativity. A variational base for determining the characteristic frequencies of oscillation is established. It provides a convenient method for obtaining sufficient conditions for the occurrence of dynamical instability. The principal result of the analysis is the demonstration that the Newtonian lower limit $\frac{4}{3}$, for the ratio of the specific heats γ, for insuring dynamical stability is increased by effects arising from general relativity; indeed, is increased to an extent that, so long as γ is finite, dynamical instability will intervene before a mass contracts to the limiting radius ($\geq 2.25\, GM/c^2$) compatible with hydrostatic equilibrium. Moreover, if γ should exceed $\frac{4}{3}$ only by a small amount, then dynamical instability will occur if the mass should contract to the radius

$$R_c = \frac{K}{\gamma - \frac{4}{3}} \frac{2GM}{c^2} \qquad (\gamma \to \tfrac{4}{3}),$$

where K is a constant depending, principally, on the density distribution in the configuration. The value of the constant K is explicitly evaluated for the homogeneous sphere of constant energy density and the polytropes of indices $n = 1$, 2, and 3.

I. INTRODUCTION

It is well known that the gravitational field external to a spherical distribution of matter is described by Schwarzschild's metric

$$-ds^2 = -\left(1 - \frac{2GM}{rc^2}\right)(dx^0)^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2) + \frac{dr^2}{1 - 2GM/rc^2}, \qquad (1)$$

where M is the inertial mass of the source of the gravitation. The metric (1) obtains, whether or not the matter which gives rise to the field is endowed with radial motions, so long as the spherical symmetry is preserved.

From the form of the metric (1), it is apparent that the spacelike and the timelike

In the Newtonian approximation, the canonical energy has the form (for $\partial_t \xi = 0$)

$$E_c = \int_0^R dr \left\{ \frac{4}{r} p' r^2 \xi^2 + \frac{1}{r^2} \Gamma p \left[(r^2 \xi)' \right]^2 \right\}$$

Choosing as initial data $\xi = r$ gives

$$E_c = 9 \int_0^R dr\, r^2 p \left(\Gamma - \frac{4}{3} \right)$$

implying instability for $\Gamma < 4/3$.

By (in effect) deriving the relativistic version

$$E_c = \int_0^R dr\, e^{\lambda+\nu} \left\{ \left[\frac{4}{r} p' - \frac{p'^2}{\epsilon + p} + 8\pi p(\epsilon + p) \right] r^2 \xi^2 + \frac{e^{3\lambda-\nu}}{r^2} \Gamma p \left[(e^{-\nu} r^2 \xi)' \right]^2 \right\}$$

Chandra showed that the stronger gravity of the full theory gives a more stringent condition:

$$\Gamma < \frac{4}{3} + K \frac{M}{R}.$$

Because a gas of photons has $\Gamma = 4/3$, and massive stars are radiation-dominated, the instability can be important for stars with $M/R >> 1$.

The criterion for *dynamical* instability to collapse is $E_c < 0$, with

$$\Gamma \equiv \left.\frac{\partial \log p}{\partial \log \rho}\right|_{\text{fixed composition}} .$$

In an equilibrium neutron star,

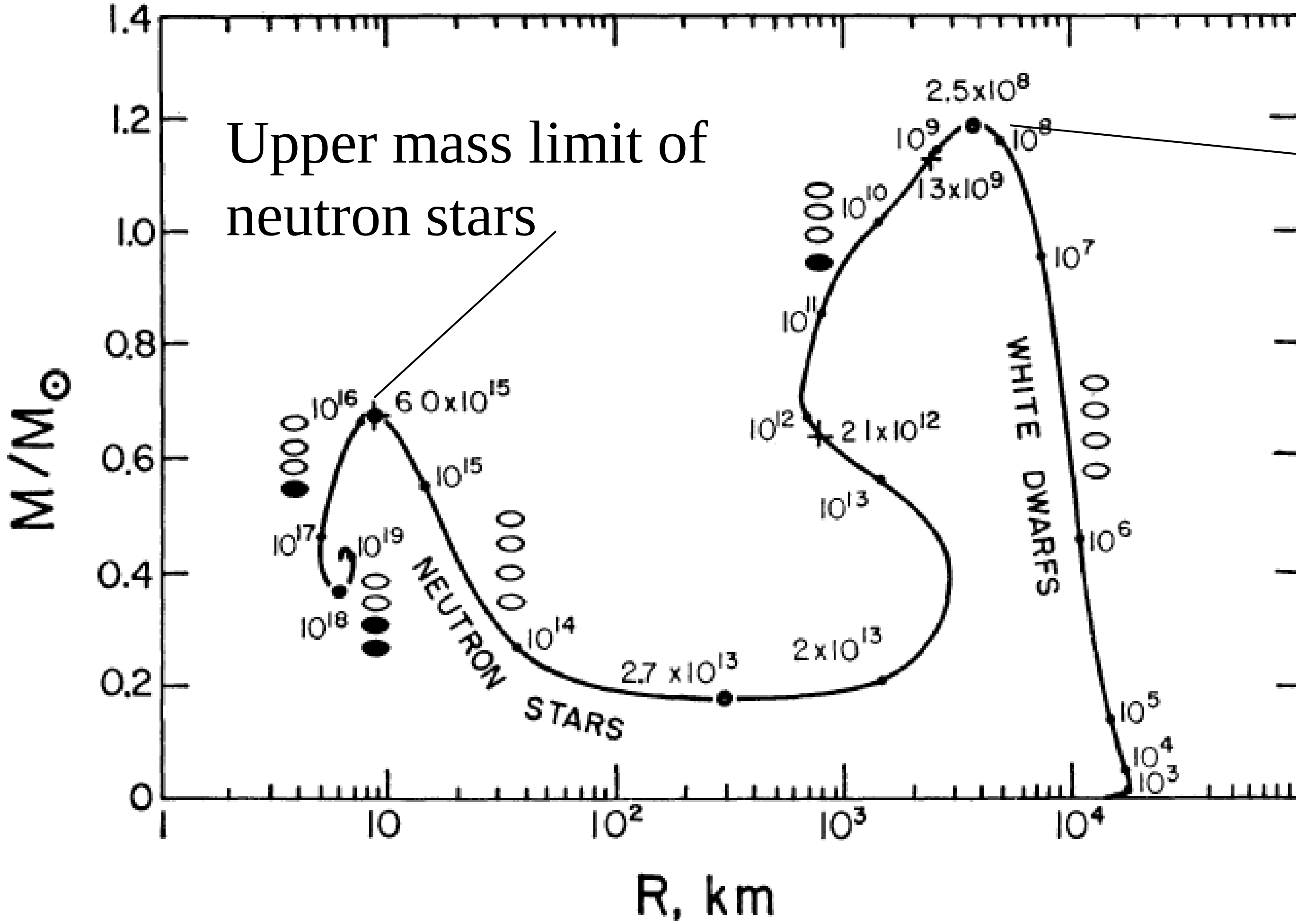
$$\Gamma \neq \left.\frac{\partial \log p}{\partial \log \rho}\right|_{\text{star}} ,$$

primarily because of a gradual change of composition (proton/neutron ratio) with radius.

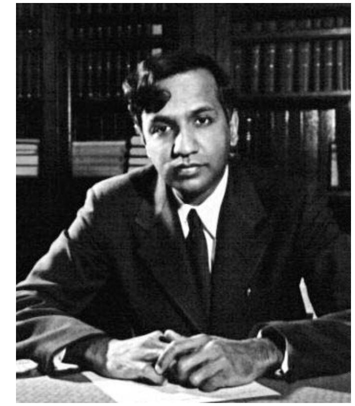
The dynamical timescale is too rapid to allow the composition of a perturbed fluid element to reach chemical equilibrium as its density is changed.

But a neutron star will be *secularly* unstable – unstable on a longer timescale –
if there are lower energy equilibrium configurations with the same baryon number that can be reached by perturbations that change the entropy of a fluid element.

For perturbations of this kind, governed by the equilibrium $p(\rho)$, instability of a uniformly rotating star to collapse sets in at a *turning point*:
The Chandrasekhar limit for white dwarfs and the corresponding upper mass limit for neutron stars.



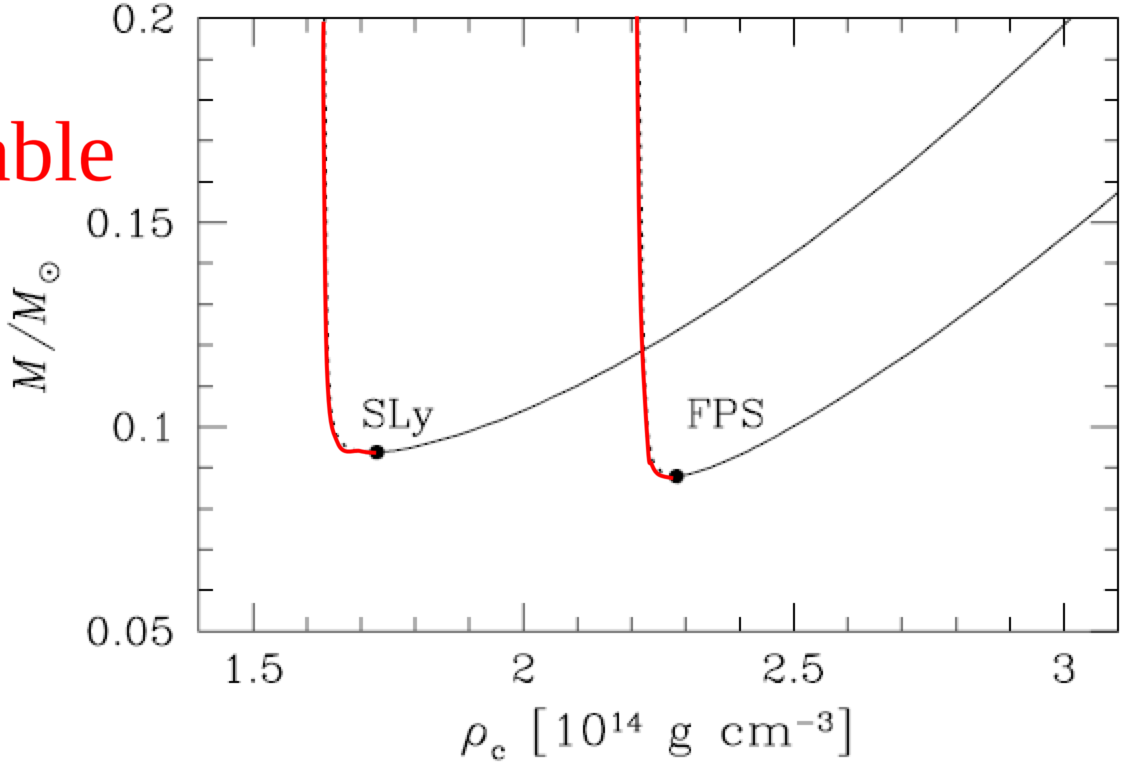
Chandrasekhar limit



(Thorne-Meltzer ’66)

Sequences of neutron stars near minimum mass for two recent EOS candidates (Haensel, Zdunik, Douchin '02)

unstable side



stable side

No other instabilities of spherical stars:

Stable against convection and

stable against collapse implies

$$E_c > 0.$$

(Lebovitz – Newtonian, Ipser-Detweiler GR)

TURNING POINT INSTABILITY

Along a sequence of uniformly rotating stars with constant angular momentum, instability sets in at the maximum-mass configuration.

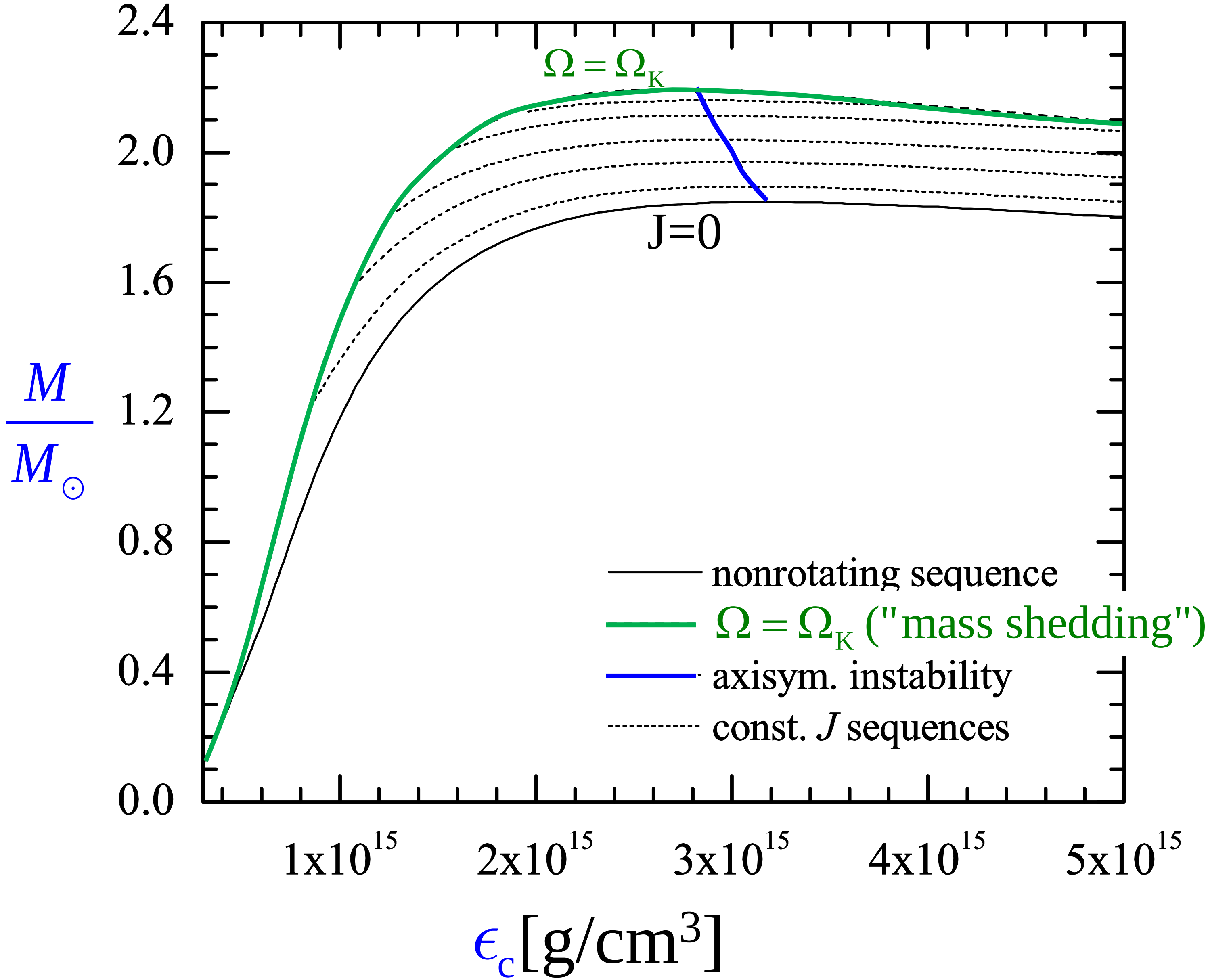
Or:

Along a sequence of uniformly rotating stars with constant baryon number, instability sets at a maximum angular-momentum configuration.

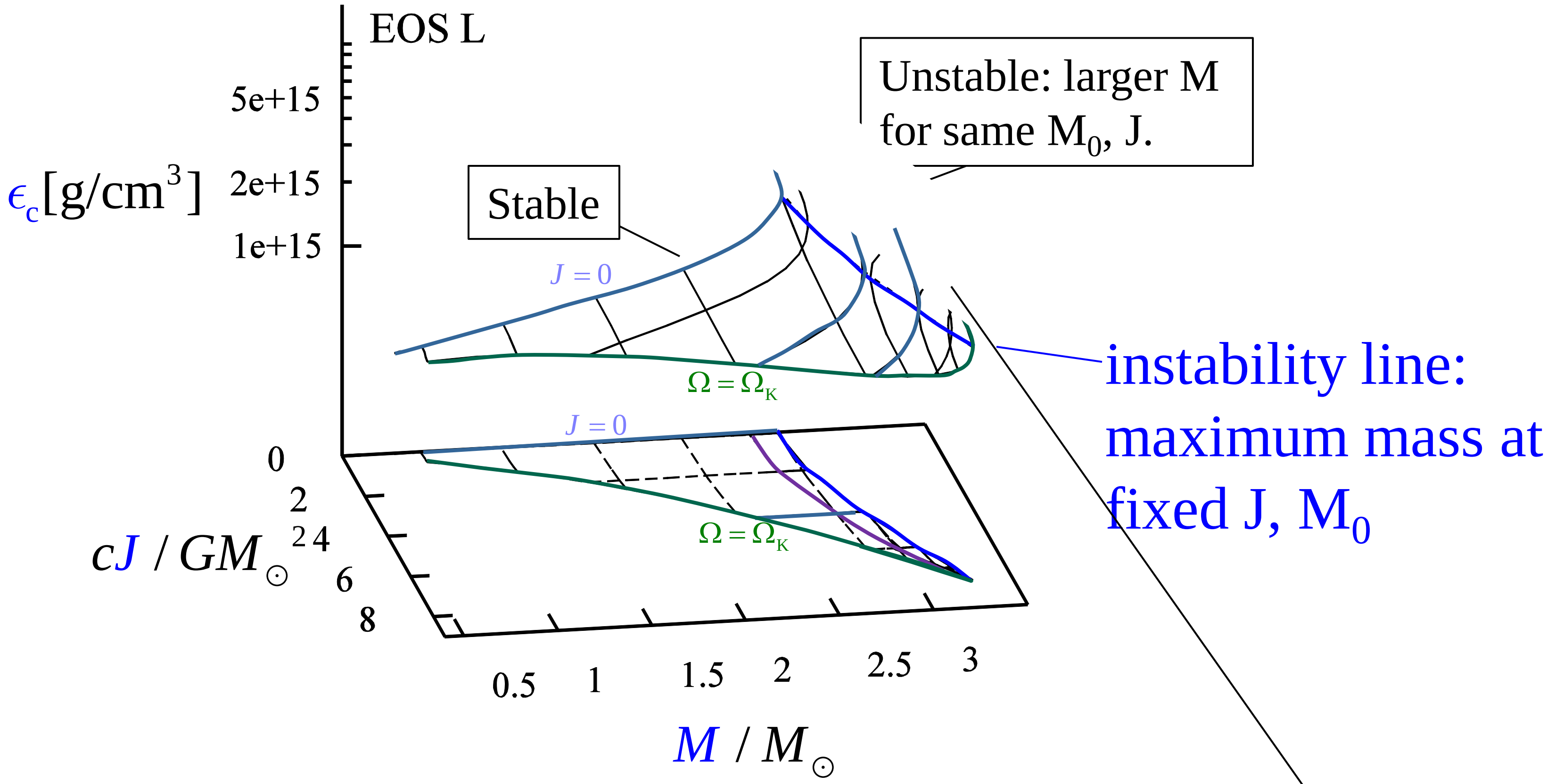
(JF, Ipser, Sorkin, using Sorkin's turning-point theorem. The J-M_0 symmetry was pointed out by FIS, but was first used by Cook, Shapiro,Teukolsky).

Or: $dM_0 \wedge dJ = 0$ contours of constant M_0 and J are parallel

(J. Read)



2.4
2.0
1.6
1.2
0.8
0.4
0.0
$\frac{M}{M_\odot}$
$\Omega = \Omega_K$
J=0
nonrotating sequence
$\Omega = \Omega_K$ ("mass shedding")
axisym. instability
const. J sequences
1x10^15
2x10^15
3x10^15
4x10^15
5x10^15
ϵ_c[g/cm^3]



The 2-dimensional surface of rotating stars doubles over, curving up and left larger central density smaller *M*

NONAXISYMMETRIC INSTABILITY OF ROTATING STARS

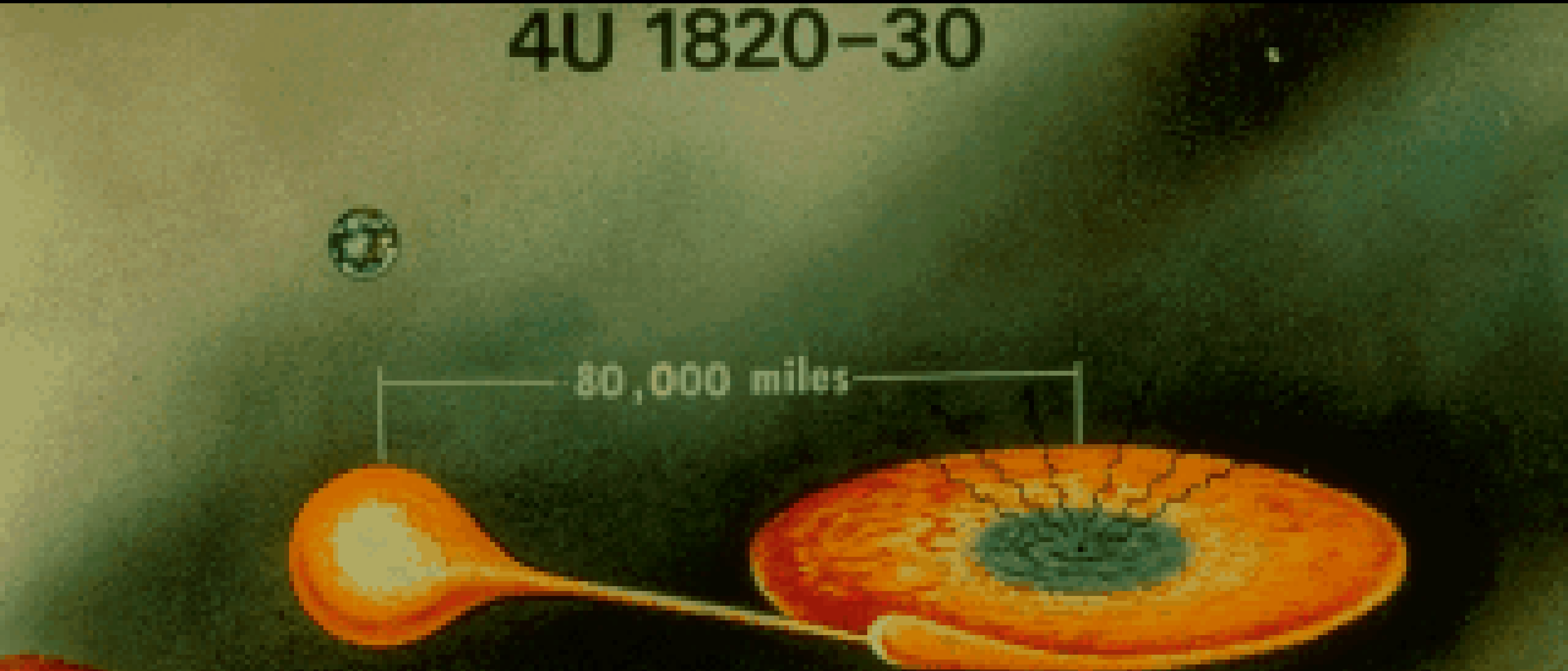
SOLUTIONS OF TWO PROBLEMS IN THE THEORY OF GRAVITATIONAL RADIATION*

S. Chandrasekhar
University of Chicago, Chicago, Illinois 60637
(Received 30 January 1970)

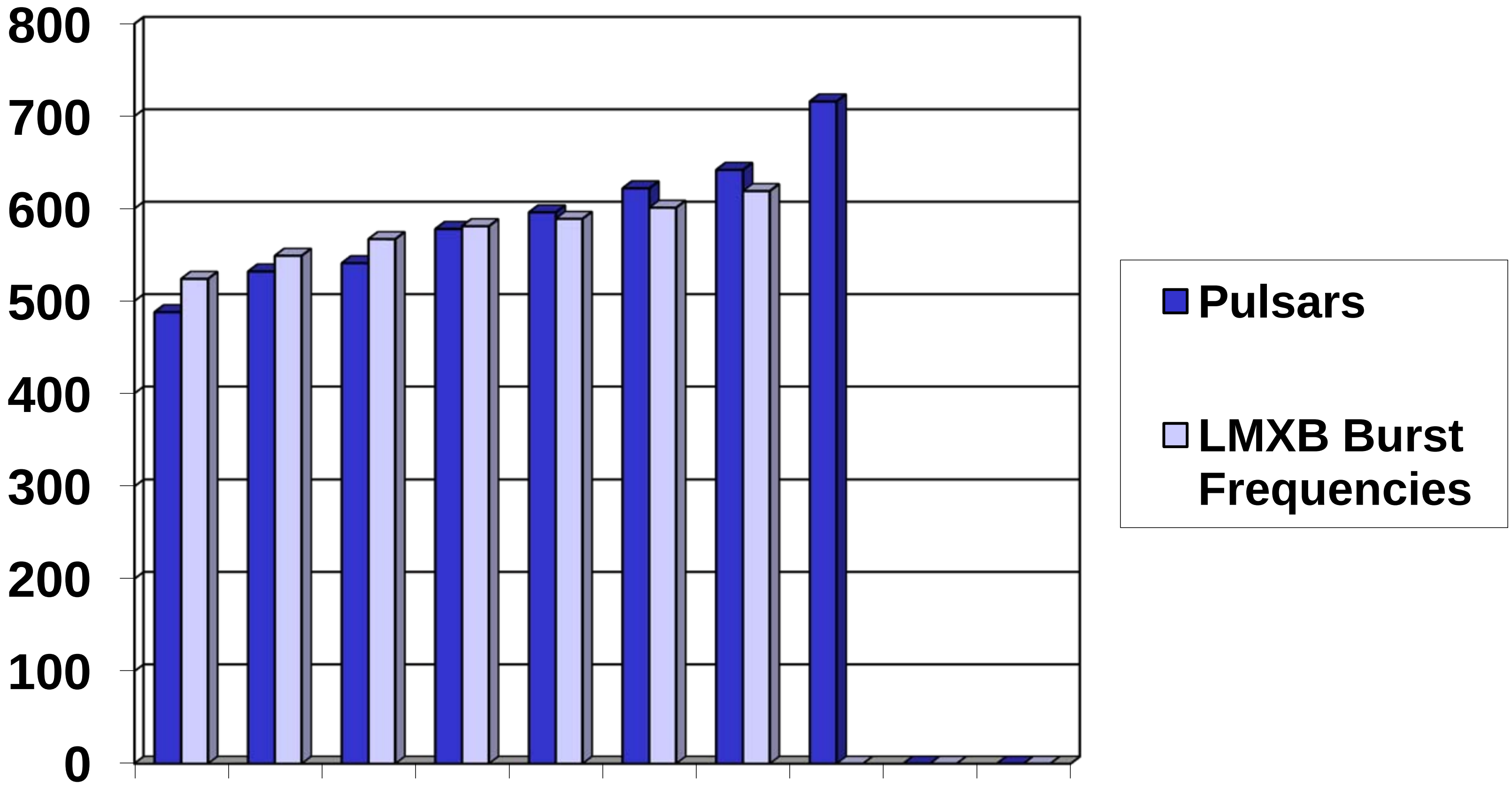
The evolution of an elongated rotating configuration by gravitational radiation and the possibility of a secular instability being induced by it are considered in the context of the classical homogeneous figures of Maclaurin and Jacobi.

From Eq. (20) it follows that while the mode $\sigma_0^{(2)}$ is damped by gravitational radiation prior to the point of bifurcation at $\Omega^2 = 2B_{11}$, it is amplified in the interval $4B_{11} > \Omega^2 > 2B_{11}$. Thus radiation reaction, like viscosity, makes the Maclaurin spheroid unstable beyond the point of bifurcation; but the mode that is made unstable by radiation reaction is not the same one that is made unstable by viscosity.

Old neutron stars in binary systems can be observed via x-rays emitted by matter that spirals that onto the neutron star. The accreting matter spins up the neutron star.



Observed frequencies of old neutron stars spun up by accretion have been observed only up to 716 Hz: Is the frequency limited below 800 Hz?



There is a sharp cutoff in the [accreting millisecond x-ray pulsar] population for spins above 730 Hz. RXTE has no significant selection biases against detecting oscillations as fast as 2000 Hz, making the absence of fast rotators extremely statistically significant

D. Chakrabarty 2008

Even for a $1.4 M_{\odot}$ star, 800 Hz is well below the maximum spin of the star – the Kepler limit Ω_K at which the star's equator rotates at the speed of an orbiting satellite

(for all but the stiffest EOS candidates)

Magnetically limited spins?

Inside the magnetosphere, matter corotates with the star. Only matter that accretes from outside of the magnetosphere can spin up the star.

Equilibrium spin at the period P of a Keplerian orbit at the magnetosphere:

With μ the magnetic dipole moment of the star,

$$P = \left(\frac{10^9 M_\odot / \mathrm{yr}}{\dot{M}} \right)^{3/7} \left(\frac{\mu}{10^{27}\,\mathrm{G\,cm}^3} \right)^{6/7}$$

Gosh&Lamb

Magnetically limited spins?

But a sharp cutoff in frequency of accreting millisecond x-ray pulsars is not an obvious prediction of magnetically limited spins for a wide variety of accretion rates and for a range of magnetic field strengths presumed to extend below 10^8 G.

The cutoff and a fairly narrow range of frequencies has made gravitational-wave limited spin a competitive possibility for accreting neutron stars.

NONAXISYMMETRIC INSTABILITY

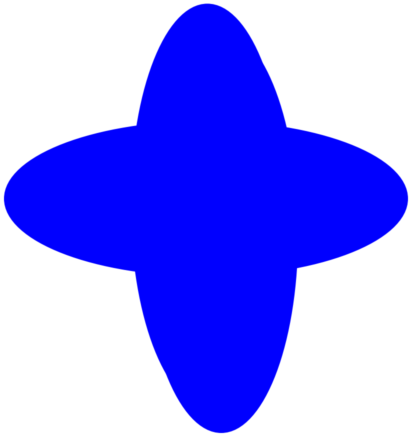
DYNAMICAL INSTABILITY – PRESENT WITHOUT DISSIPATION, DYNAMICAL TIMESCALE

GRAVITATIONAL-WAVE DRIVEN INSTABILITY

GRAVITATIONAL-WAVE DRIVEN INSTABILITY

Chandrasekhar, Schutz, JF

Outgoing nonaxisymmetric modes radiate
angular momentum to ∞

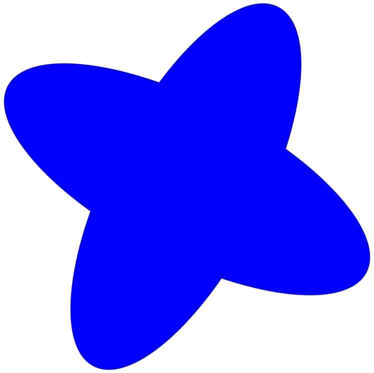


If the pattern rotates
forward relative to ∞, it radiates positive J to ∞

GRAVITATIONAL-WAVE DRIVEN INSTABILITY

Chandrasekhar, Schutz, JF

Outgoing nonaxisymmetric modes radiate angular momentum to ∞



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GRAVITATIONAL-WAVE DRIVEN INSTABILITY

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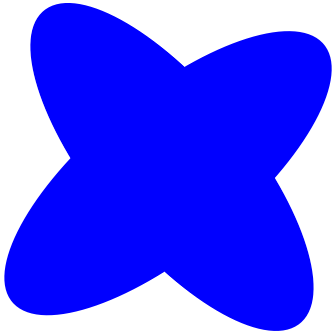

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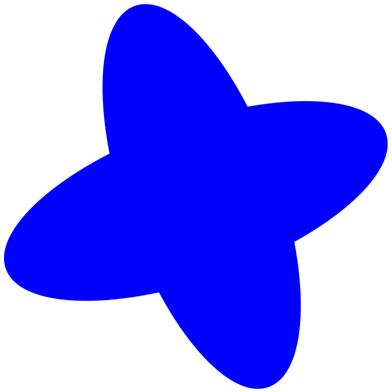

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GRAVITATIONAL-WAVE DRIVEN INSTABILITY

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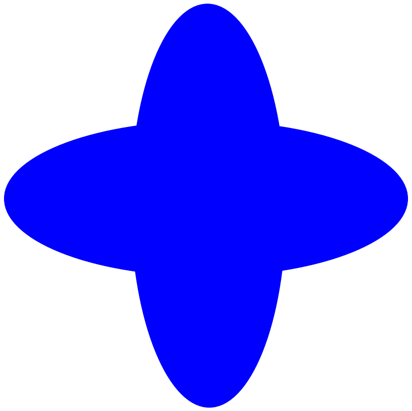

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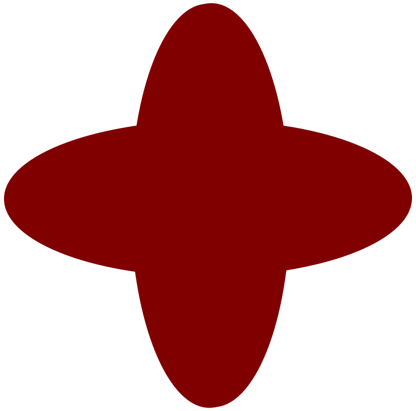

If the pattern rotates
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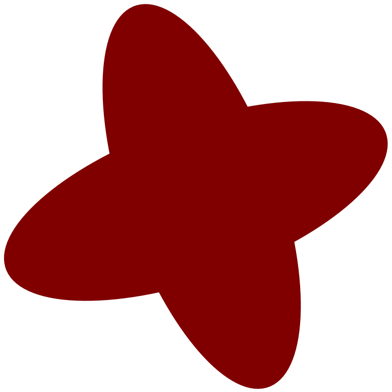

If the pattern rotates
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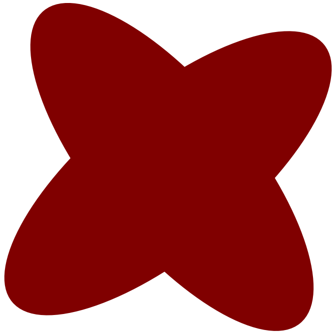

If the pattern rotates
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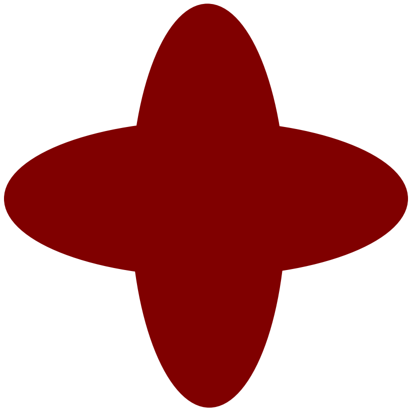

If the pattern rotates
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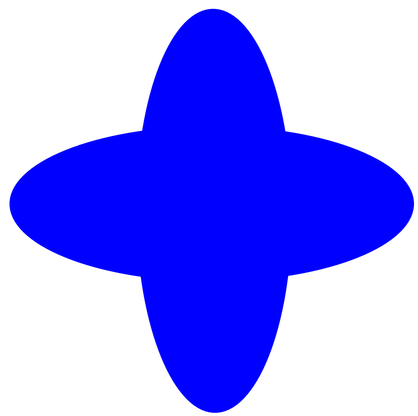


That is:

A forward mode, with $J > 0$, radiates positive J to ∞

A backward mode, with $J < 0$, radiates negative J to ∞

Radiation damps all modes of a spherical star

But, a rotating star drags a mode in the direction of the star's rotation:

A mode with behavior $e^{i(m\phi-\omega t)}$ that moves *backward* relative to the star is dragged *forward* relative to infinity, when

$$m\,\Omega > \omega\,.$$

The mode still has $J < 0,$ because

$$J_{\rm star} + J_{\rm mode} < J_{\rm star}\,.$$

But

this backward mode, with $J < 0$, radiates positive J.

Thus J becomes increasingly negative, and

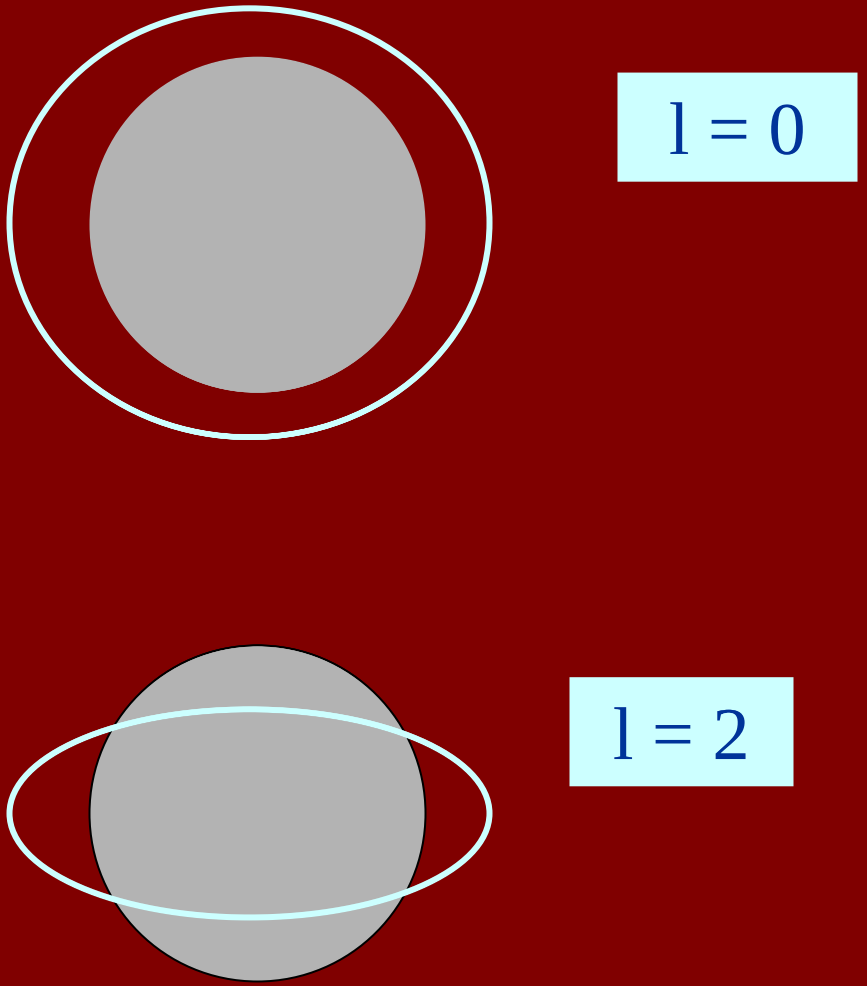
THE AMPLITUDE OF THE MODE GROWS

PERTURBATIONS WITH ODINARY (POLAR) PARITY

modes with pressure and gravity providing the restoring force

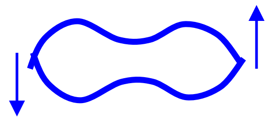
$$\delta p,\, \delta\epsilon, \delta u^r \propto Y_{lm}$$
$$(\delta u^\theta, \delta u^\phi) \propto (\nabla^\theta Y_{lm}, \nabla^\phi Y_{lm})$$

Parity is that of Y_{lm}


l = 0
l = 2

INSTABILITY OF POLAR MODES

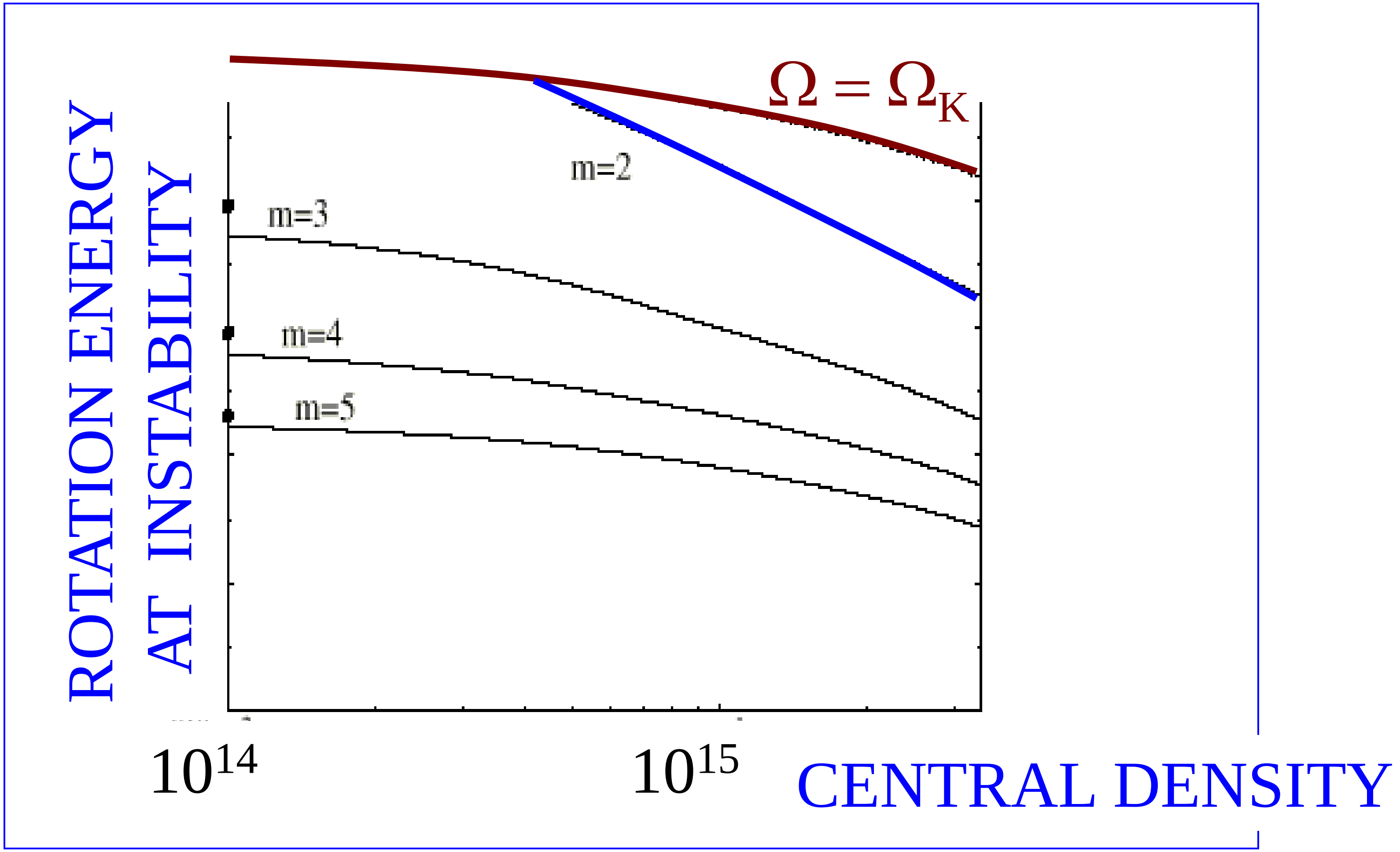
THE BAR MODE (l=m=2)



HAS FREQUENCY σ OF ORDER THE MAXIMUM

ANGULAR VELOCITY Ω_K OF A STAR.

IT IS DRAGGED BACKWARD ONLY
WHEN A STAR ROTATES NEAR ITS MAXIMUM
ANGULAR VELOCITY, Ω_K



Polar modes unstable only for Ω near Ω_K

$\Omega > 1000$ Hz (unless neutron matter *very* stiff)

but observed cutoff in spins < 750 Hz

But it's worse than that:

Old accreting stars are too cold for polar modes to be unstable at any Ω

Instability of polar modes does not explain the cutoff in neutron-star spins.

PERTURBATIONS WITH AXIAL PARITY

Parity is *opposite* to that of Y_{lm}

Axial perturbations of a spherical star do not change density or pressure, because scalars have the parity of Y_{lm}

$$\delta p,\ \delta\epsilon, \delta u^r = 0 \ \Rightarrow$$

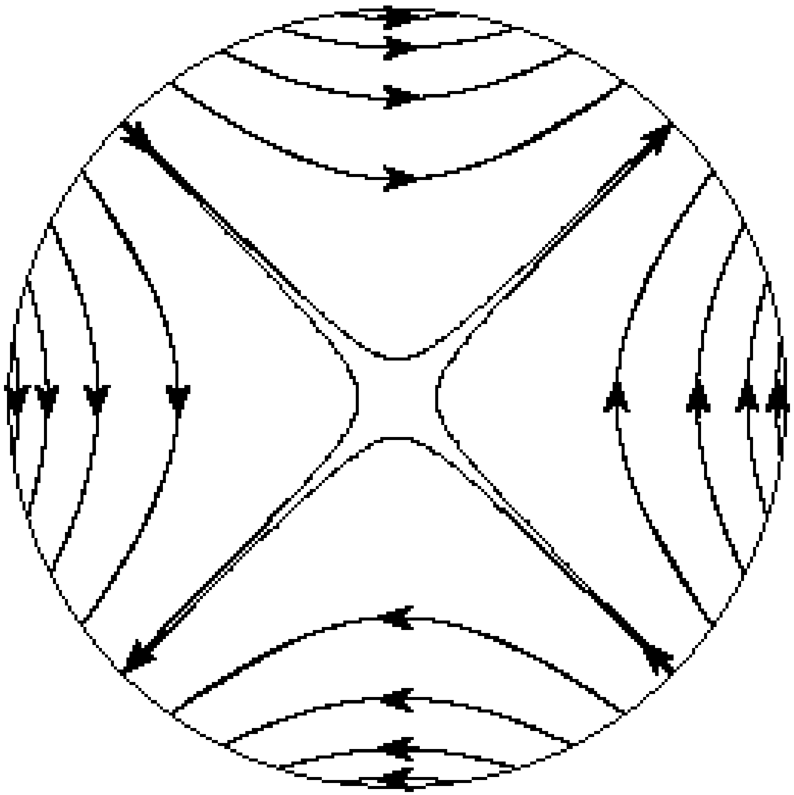
No restoring force in Euler equation:

For spherical stars, axial parity perturbations are time independent currents

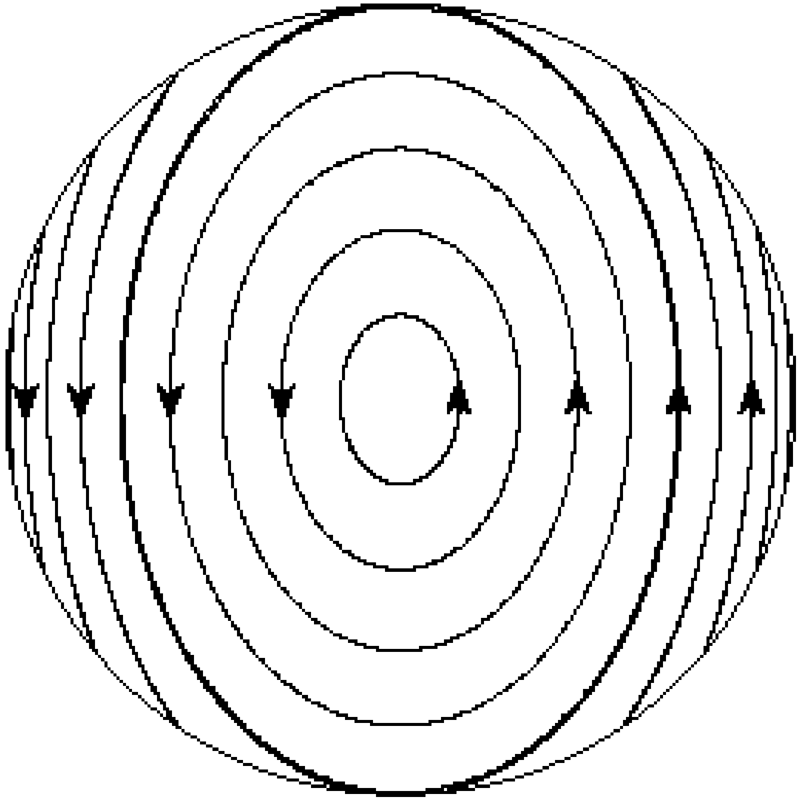
$$\delta \boldsymbol{u} \propto \boldsymbol{r} \times \nabla Y_{lm}$$

THE UNSTABLE l = m = 2 r-MODE

View from pole



View from equator



Because their frequency is already zero for a nonrotating star, any slowly rotating star has backward-moving r-modes for each l that are dragged forward by the rotation.

That leads to much faster growth times for moderate neutron-star rotation.

GRAVITATIONAL RADIATION

MASS QUADRUPOLE

$$Q = \int \rho Y_{22} r^2 dV$$

ENERGY RADIATED

$$\frac{dE}{dt} = \dddot{Q}^2$$

AXIAL GRAVITATION

$$\vec{r} \times \nabla Y_{22}$$

CURRENT QUADRUPOLE

$$J_{22} = \int \rho \, \vec{v} \cdot \vec{Y}_{22} r^2 dV$$

ENERGY RADIATED

$$\frac{dE}{dt} = \dddot{J}^{\,2}$$

R-MODE INSTABILITY

Andersson
Kojima
Owen, Lindblom, Cutler,
Schutz, Vecchio, Andersson
Andersson, Kokkotas, Stergioulas
Ipser, Lindblom
Beyer, Kokkotas
Hiscock   Lindblom
Rezzolla, Shibata, Asada,
Baumgarte, Shapiro
Rezzola,Lamb, Shapiro
Ferrari, Matarrese,Schneider

JF, Morsink
Lindblom,Owen, Morsink
Andersson, Kokkotas,Schutz
Madsen
Levin   Bildsten
JF, Lockitch
Kojima, Hosonuma
Brady, Creighton   Owen
Lindblom, Mendell, Owen
Flanagan
Spruit   Levin
Lockitch   Rezania

Prior work on axial modes: Chandrasekhar & Ferrari

MORE RECENT

Stergioulas, Font, Kokkotas
Yoshida, Lee
Yoshida, Karino, Yoshida, Eriguchi
Andersson, Lockitch, JF
Andersson, Kokkotas, Stergioulas
Ushomirsky, Cutler, Bildsten
Andersson, Jones, Kokkotas, Stergioulas
Lindblom,Owen,Ushomirsky
Wu, Matzner, Arras
Levin, Ushomirsky
Lindblom, Tohline, Vallisneri
Arras, Flanagan, Schenk, Teukolsky,Wasserman
Morsink
Ruoff, Kokkotas,

Kojima, Hosonuma
Rezania, Jahan-Miri
Rezania, Maartens
Lindblom, Mendell
Andersson
Bildsten, Ushomirsky
Brown, Ushomirsky
Rieutord
Ho, Lai
Madsen
Stergioulas, Font
JF, Lockitch
Sa
Jones
Lindblom,Owen
Andersson,Lockitch,JF

Karino, Yoshida, Eriguchi
Watts, Andersson
Arras, Flanagan, Morsink
Shenk, Teukolsky,
Brink, Bondarescu
Wagoner, Hennawi, Liu
Jones, Andersson, Stergioulas
Lockitch, Andersson
Hehl
Gressman, Lin, Suen, Stergioulas, JF
Lin, Suen
Xiaoping, Xuewen, Miao, Shuhua, Nana
Reisenegger, Bonacic
Drago, Lavagno, Pagliari
Gondek-Rosinska, Gourgoulhon, Haensel

Hosonuma
Rezzolla, Lamb, Markovic,
Shapiro

Haensel,
Prix, Comer, Andersson

Sa, Tome
Flanagan, Racine
Lackey, Nayyar, Owen
Dias, Sa
Abramowicz, Rezzolla, Yoshida
Andersson, Comer, Glampedakis, Haskell, Passamonti

Axial perturbations of a spherical star do not change density and pressure, because scalars have parity of Y_{lm}

Then no restoring force in Euler equation:
Axial parity modes have zero frequency
for nonrotating star.

THE l = m = 2 r-MODE

Newtonian: Papaloizou & Pringle, Provost et al,
Saio et al, Lee, Strohmayer

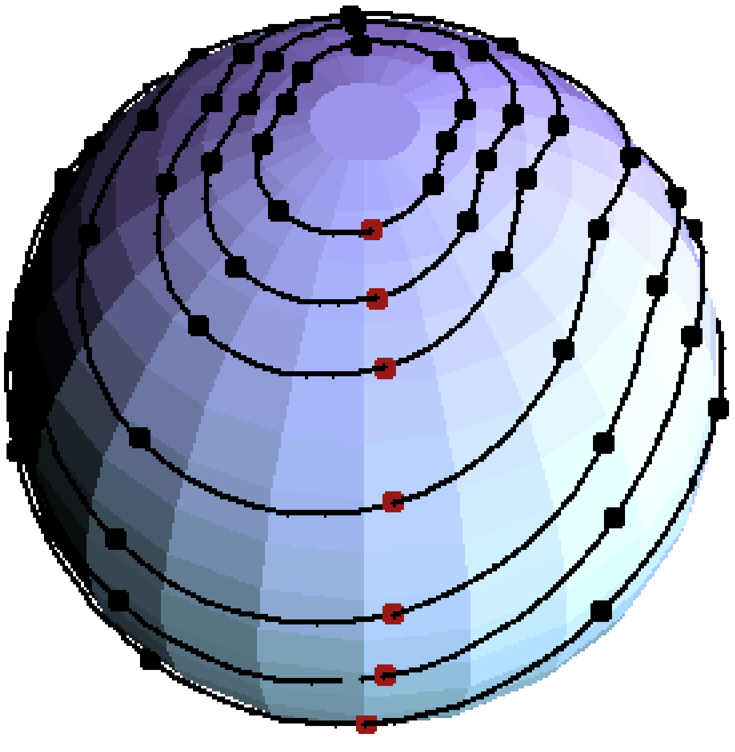
Frequency relative to a rotating observer:

$\omega_R = - 2/3\ \Omega$ COUNTERROTATING

Frequency relative to an inertial observer:

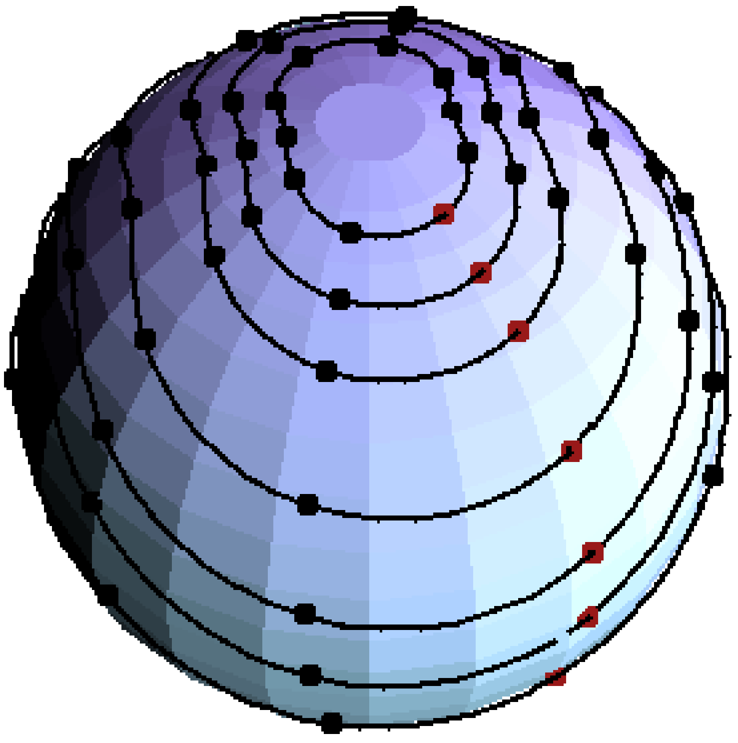
$\omega_I = 4/3\ \Omega$ COROTATING $e^{i(2\phi - \omega t)}$

corotating frame



Animations by Chad Hanna

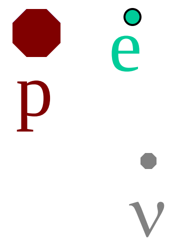
inertial frame



Animations by Chad Hanna

VISCOUS DAMPING

Above 10^{10}K, beta decay and inverse beta decay



produce neutrinos that carry off the energy of the mode:

bulk viscosity

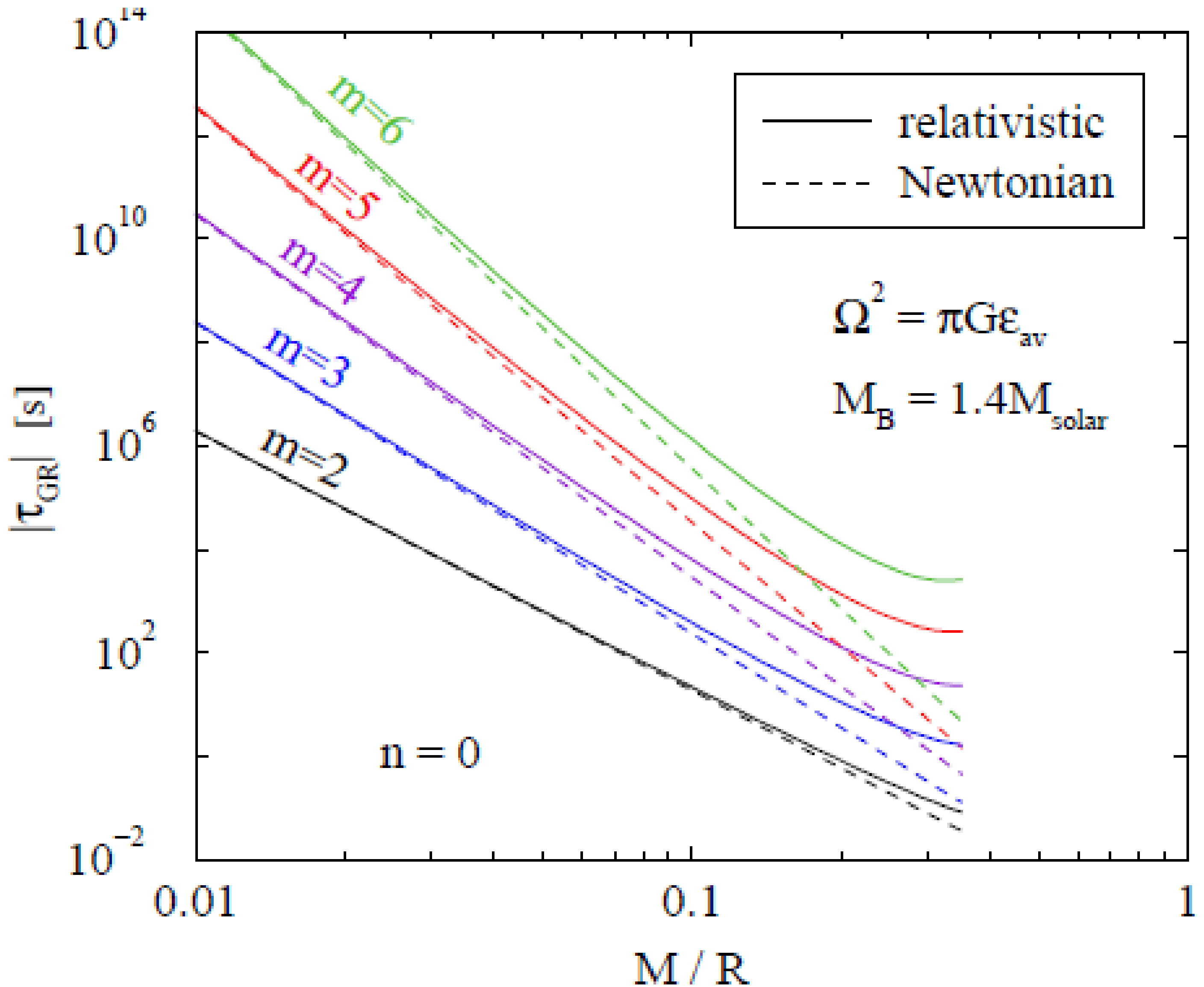
$\tau_{BULK} = CT^6$

Below 10^9K, *shear viscosity* dissipates the mode's energy in heat

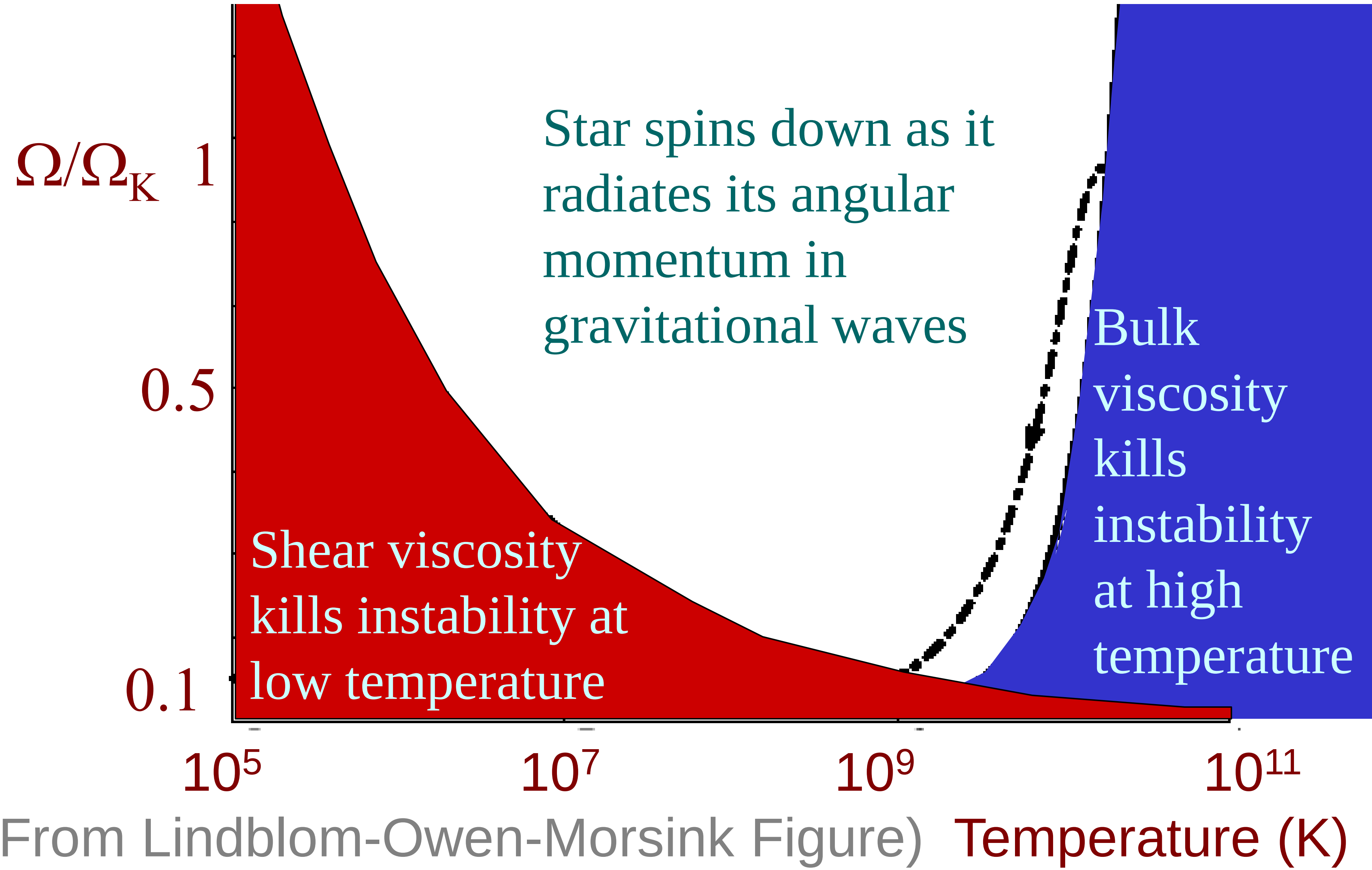
$\tau_{SHEAR} = CT^{-2}$

$$\frac{1}{\tau} = \frac{1}{\tau_{GR}} + \frac{1}{\tau_{\text{shear viscosity}}} + \frac{1}{\tau_{\text{bulk viscosity}}}$$

GRR growth times for r-modes

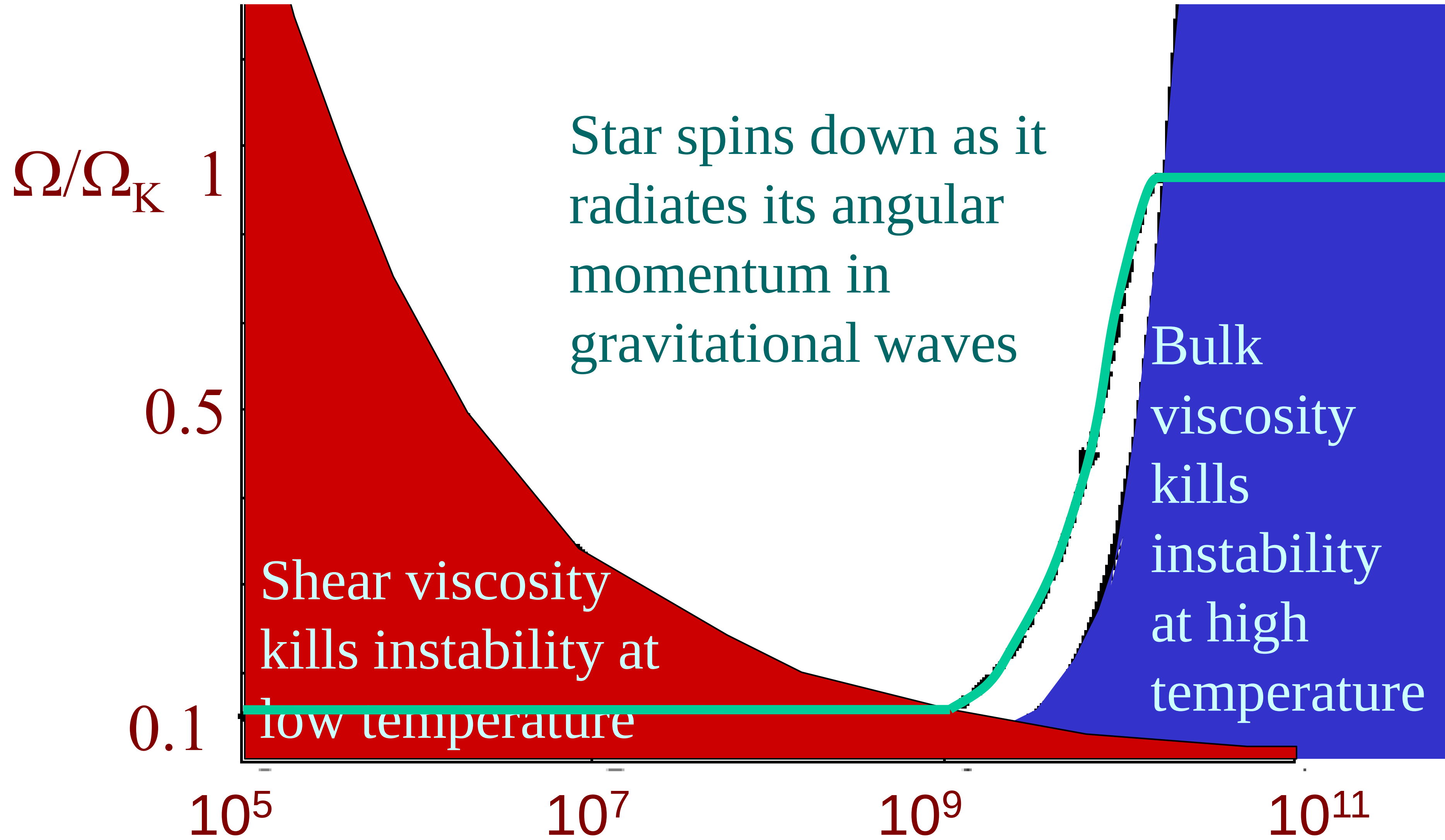


Star is unstable only when Ω is larger than critical frequency set by bulk and shear viscosity



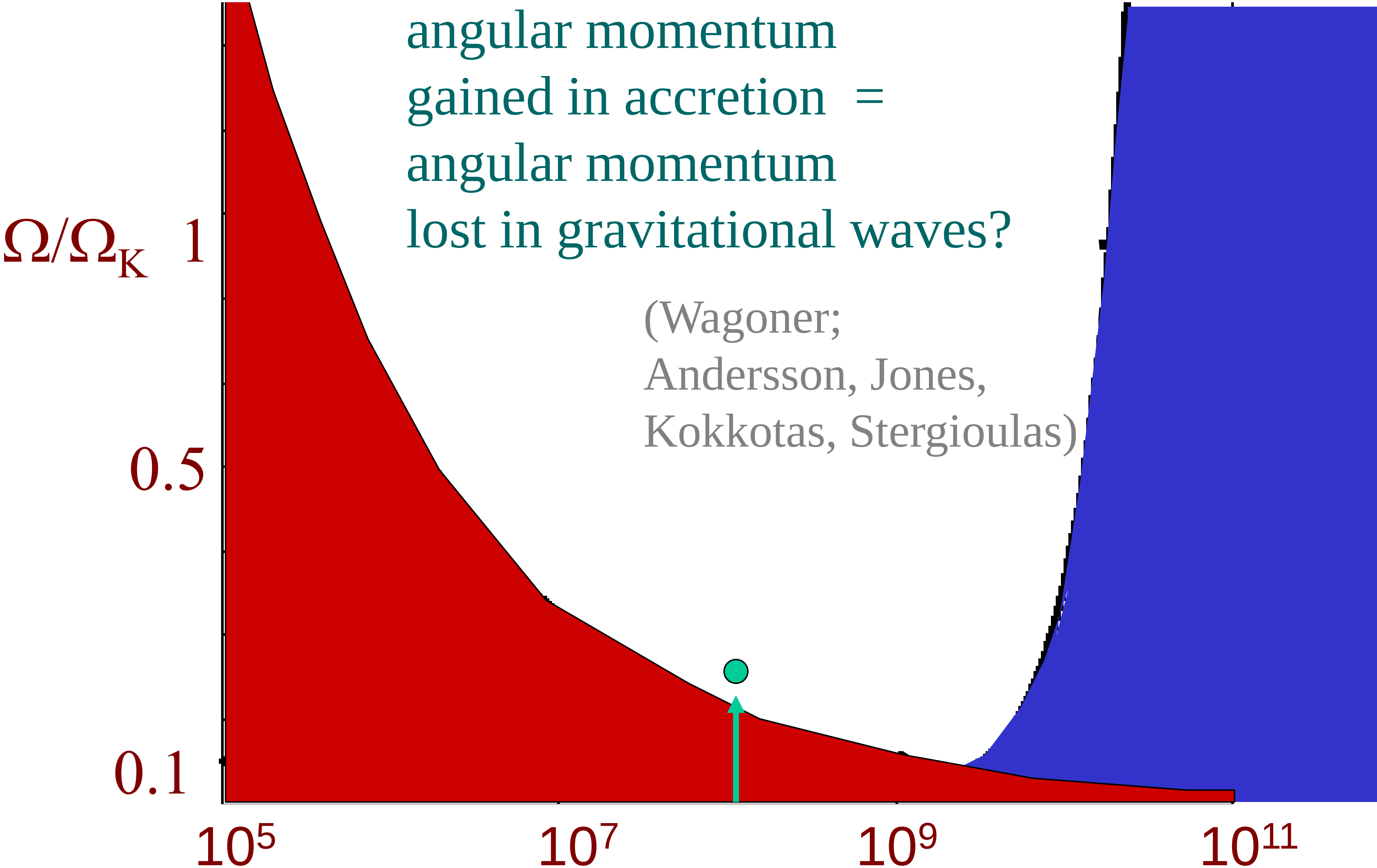
(From Lindblom-Owen-Morsink Figure)

Star is unstable only when Ω is larger than critical frequency set by bulk and shear viscosity

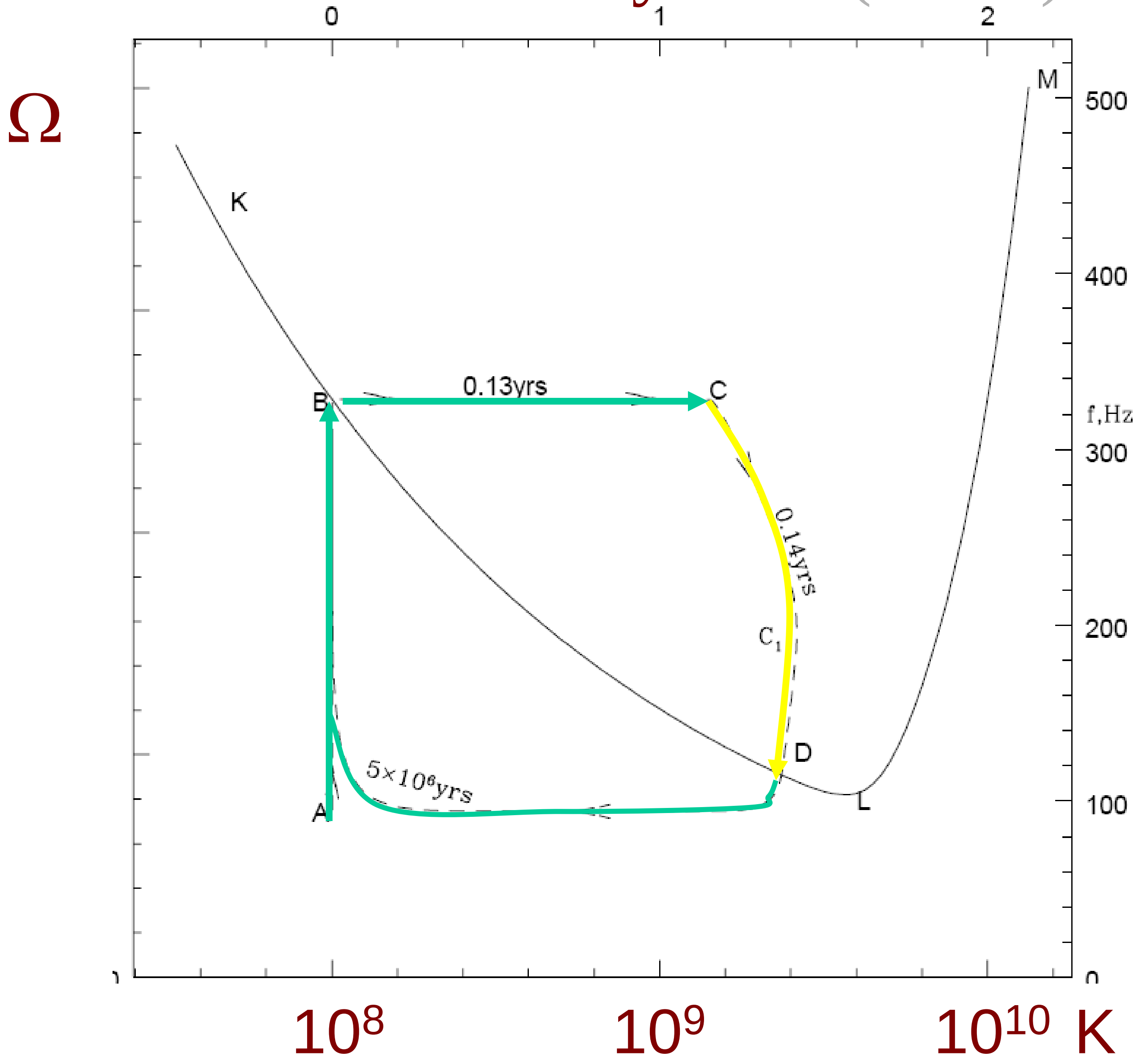


(From Lindblom-Owen-Morsink Figure)

Star spun up by accretion: Does it hover, with



Thermal runaway (Levin)



DOES THE INSTABILITY SURVIVE THE PHYSICS OF A REAL NEUTRON STAR?

Will nonlinear couplings limit the amplitude to $\delta v/v \ll 1$?

Will a continuous spectrum from GR or differential rotation eliminate the r-modes?
(Kojima …Ferarri et al, Andersson, Lockitch, Watts)

Will a viscous boundary layer near a solid crust
windup of magnetic-field from 2nd order differential rotation of the mode
bulk viscosity from hyperon production
kill the instability?

NONLINEAR EVOLUTION

Fully nonlinear numerical evolutions showed no evidence that nonlinear couplings limiting the amplitude to $\delta v/v < 1$:

Nonlinear fluid evolution in GR

Cowling approximation (background metric fixed)

Font, Stergioulas

Newtonian approximation, with radiation-reaction term

GRR enhanced by huge factor to see growth in 20 dynamical times.

Lindblom, Tohline, Vallisneri

BUT

Work to 2nd order in the perturbation amplitude shows

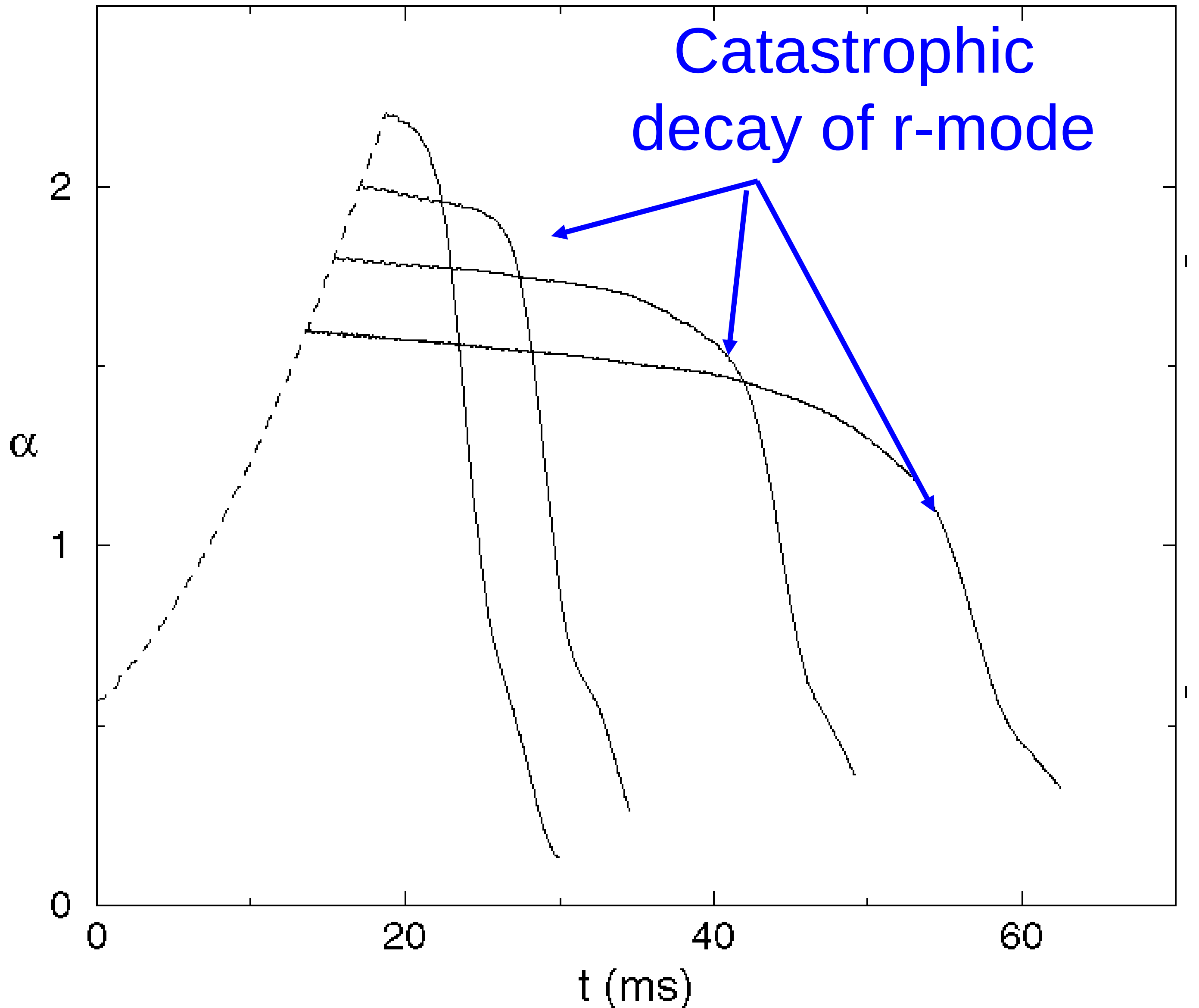
TURBULENT CASCADE

The energy of an r-mode appears in this approximation to flow into short wavelength modes, with the effective dissipation too slow to be seen in the nonlinear runs.

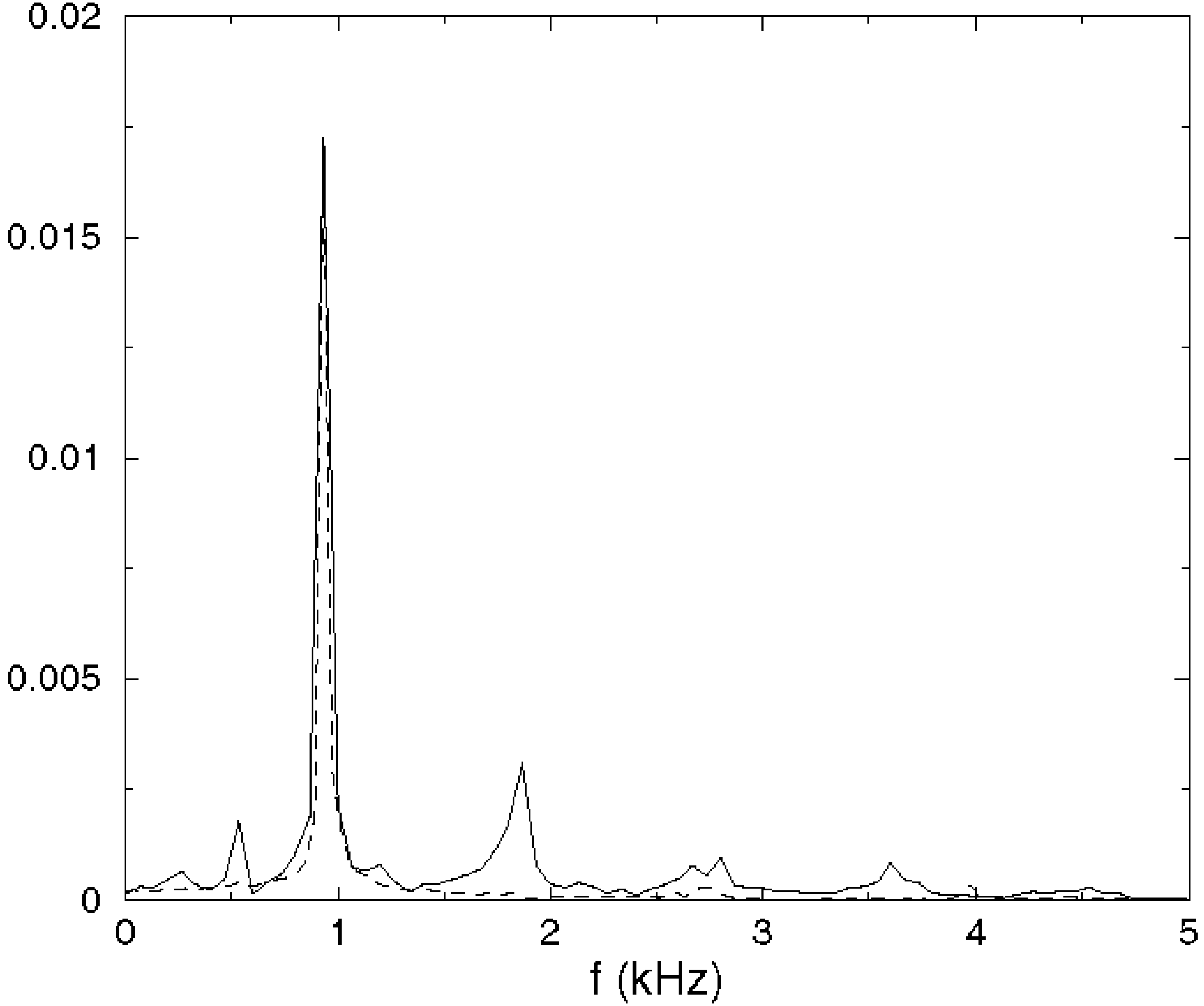
Arras, Flanagan, Morsink, Schenk, Teukolsky,Wasserman

Newtonian evolution with somewhat higher resolution, w/ and w/out enhanced radiation-driving force

(Gressman, Lin, Suen, Stergioulas, JF)

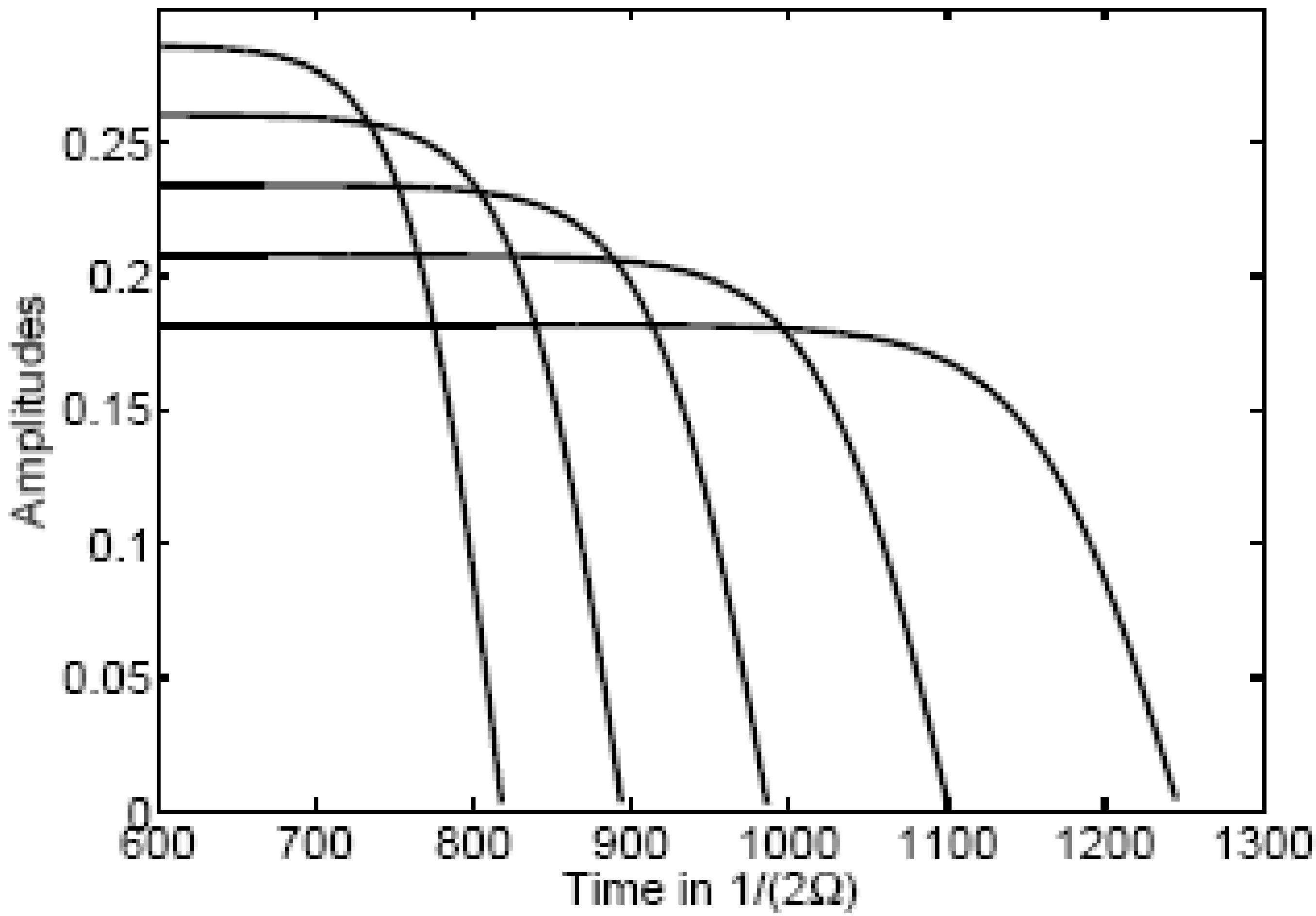


Fourier transform shows sidebands - apparent daughter modes.



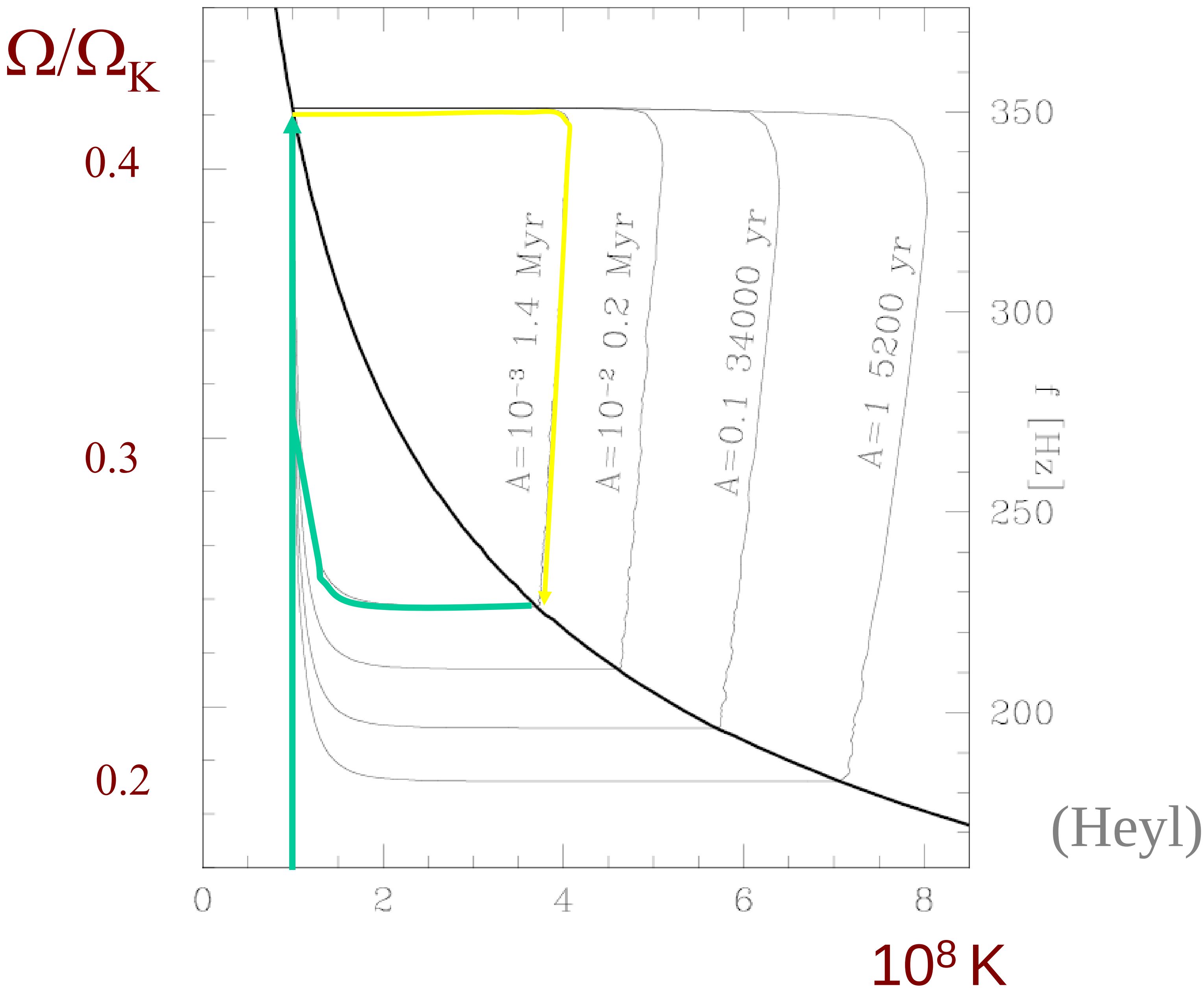
Similar picture emerges from 2nd-order coupling of modes for uniform density model (Maclaurin)

(Brink, Teukolsky, Wasserman



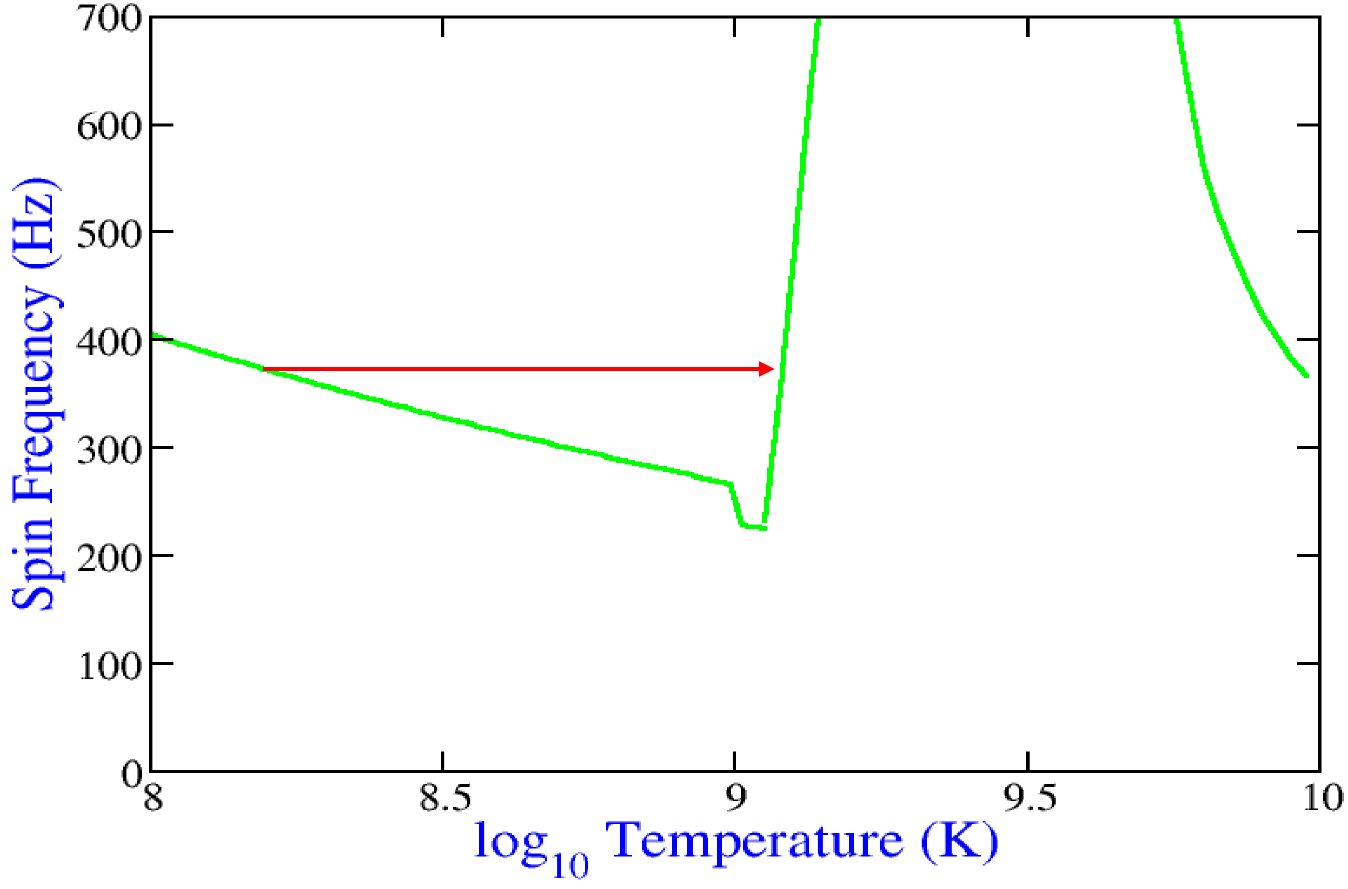
Amplitude too low to see gravitational waves from r-mode instability in newborn stars.

But a low amplitude can improve the chance of seeing gravitational waves from old stars spun up by accretion

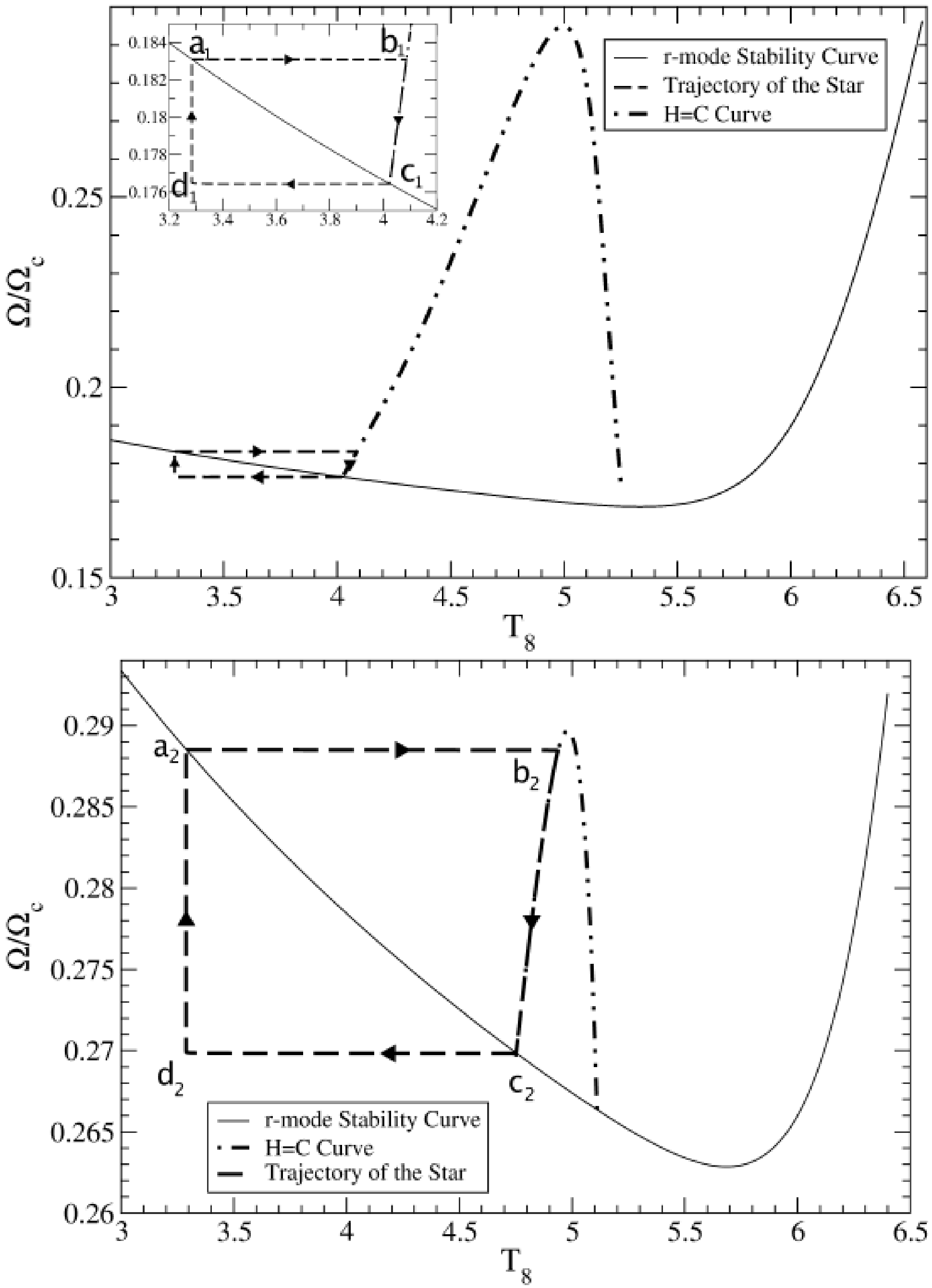


Ω/Ω_K
0.4
0.3
0.2
350
300
250
200
f [Hz]
A=10⁻³ 1.4 Myr
A=10⁻² 0.2 Myr
A=0.1 34000 yr
A=1 5200 yr
0
2
4
6
8
10⁸ K
(Heyl)

Thermal sit-there

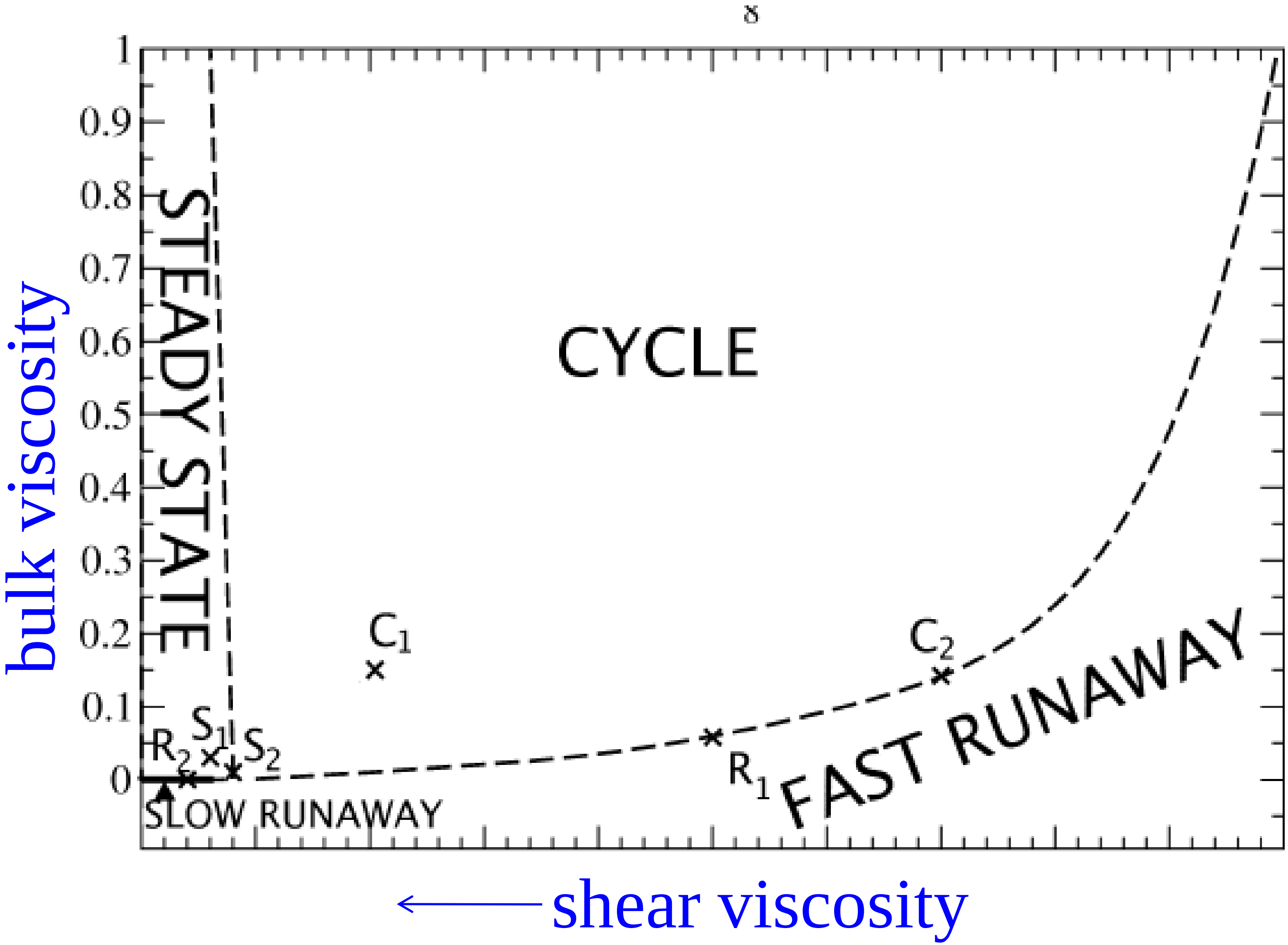


(Owen)



Bondarescu, Teukolsky, Wasserman

3-mode evolution, with viscous heating (H) and neutrino cooling (C)



higher viscosity lower viscosity

(governed by slippage at boundary layer)

Instability curve (hyperons)

(Lindblom & Owen 2002)