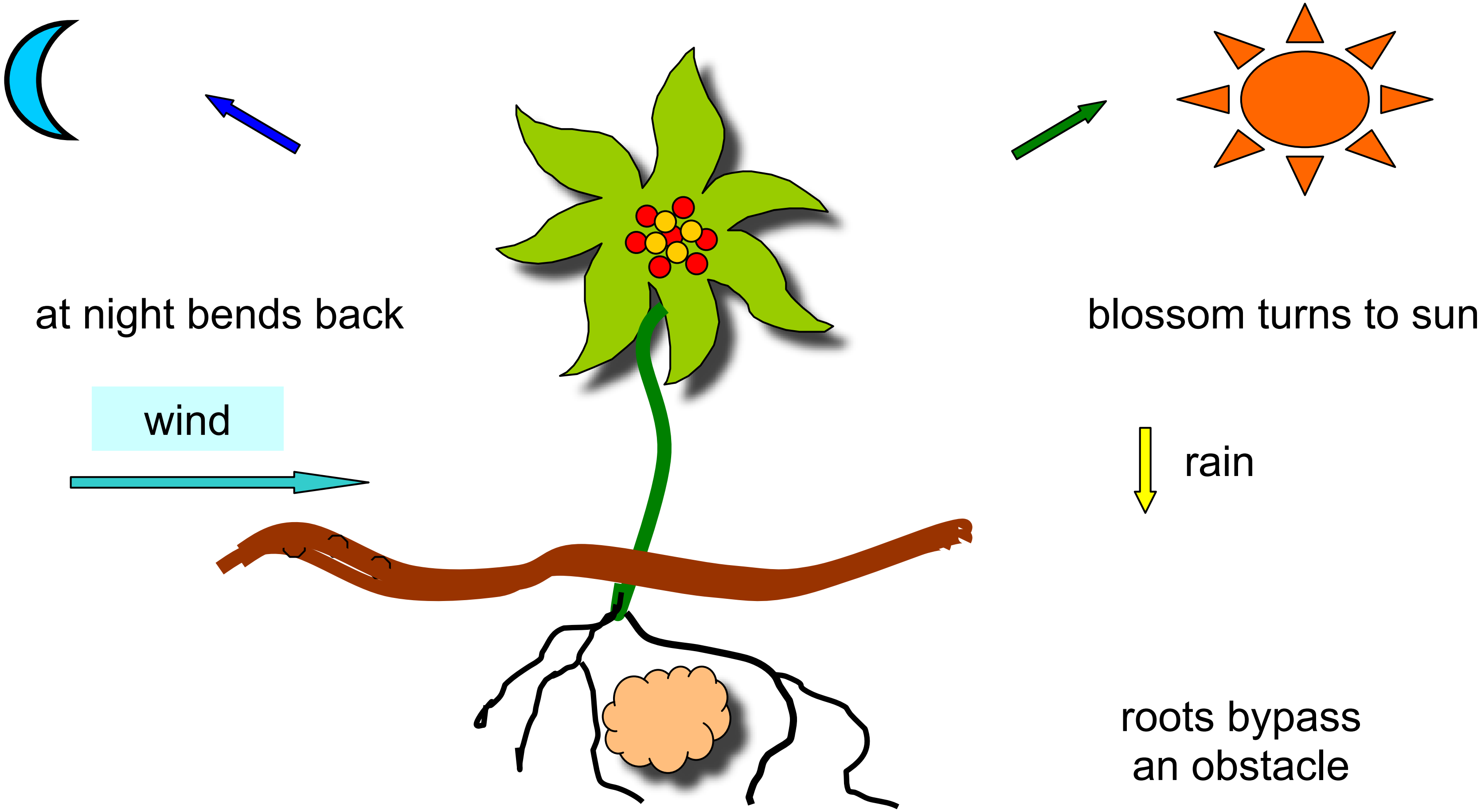
Some Basic Principles of Adaptive Computation

Rudolf F. Albrecht
University of Innsbruck, Austria
Rudolf.Albrecht@uibk.ac.at

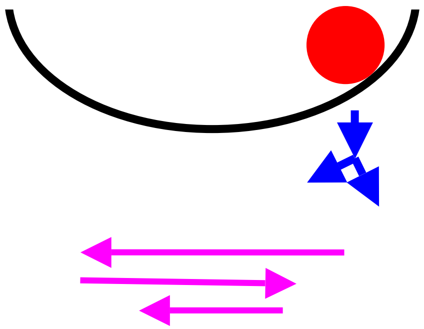
ICANNGA '11

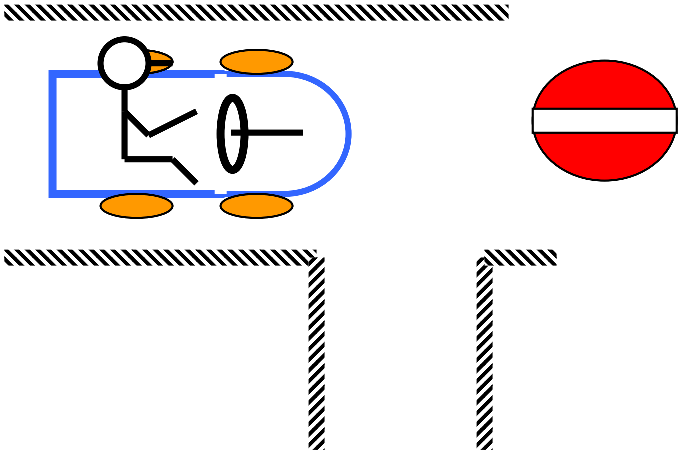
ADAPTATION is a UNIVERSAL PHENOMENON

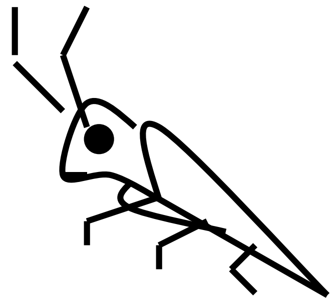
Illustrations



a ball in a bowl | steering a car | animals







oscillates to a stable position

autonomous control

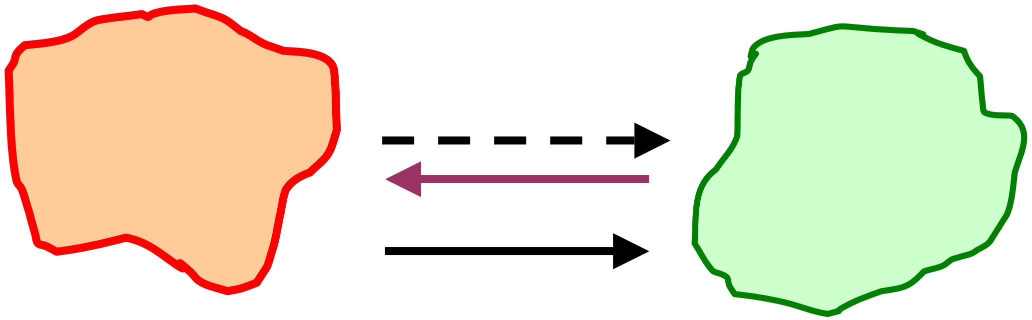
active sensing, grasp, touch

Visualization

INTERACTION PROCESSES

OBJECT STATES

OBJECT



ENVIRONMENT: FEATURES, ACTIONS

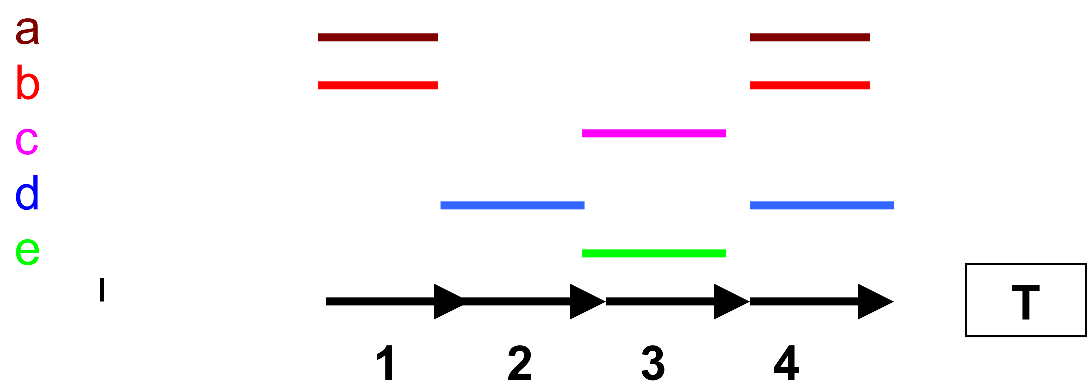
ENVIRON-MENT

The **OBJECT** has time dependent STATES and can observe and influence the **ENVIRONMENT.**
The **ENVIRONMENT** can time dependent influence the **OBJECT.**

TIME and PROCESSES

As TIME any (partial) ordered set **(T, <)** can be defined, a single / multiple (=parallel) PROCESS on a set **M** is a time indexed family: $(M_t \mid M_{[t]} \subseteq \mathbf{M})_{t \in \mathbf{U} \subseteq \mathbf{T}}$.

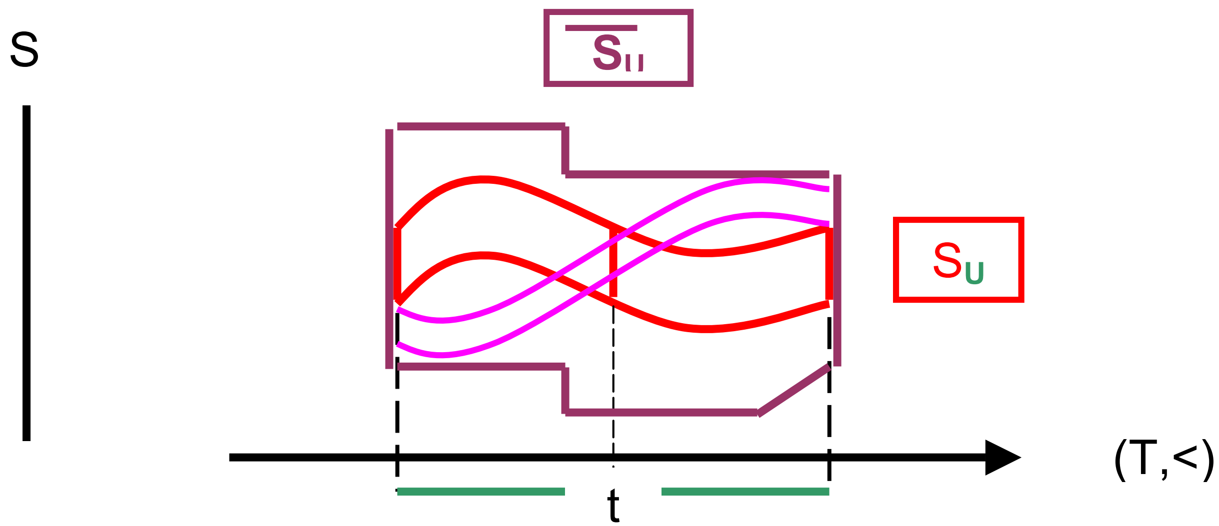
Illustration: **M** = {a,b,c,d,e}, **T** = integers, **U** = {1,2,3,4}



same color: is a single part - process, at $t = 3$ is $M_{[3]} = \{e,c\}$
A *special case* is a constant process: $M_{[t]}$ = const. If $(\mathbf{M},<)$ is an additive group: periodic and time shifted processes can be defined.

For an object OBJ let be **S** a set of possible states of OBJ, let $(\mathbf{T}, \mathbf{<})$ be a time set, let $\varnothing \neq \mathbf{U} \subseteq \mathbf{T}$, let $\mathbf{S_U} =_{def} (S_t)_{t \in \mathbf{U}}$ be a process on pow **S** (pow means power set), i.e. $S_{[t]} \subseteq \mathbf{S}$.

Illustration:



For existing constraints, requirements, etc. assume $A = \{(S_t(i))_{t\in U} \mid i\in I\}$ is a set of admissible processes $(S_t(i))_{t\in U}$. By definition, these processes are all equivalent with respect to requirements. A forms itself a process $(\overline{S}_{[t]})_{t\in U}$. A family of "concatenations $\mathbf{K}_{[t]}$" can exist with $S^*_{[t]} =_{def} \mathbf{K}_{[t]}(S_t(i), S_t(j))$, $i,j\in I$. S^*_U can be $\in A$.

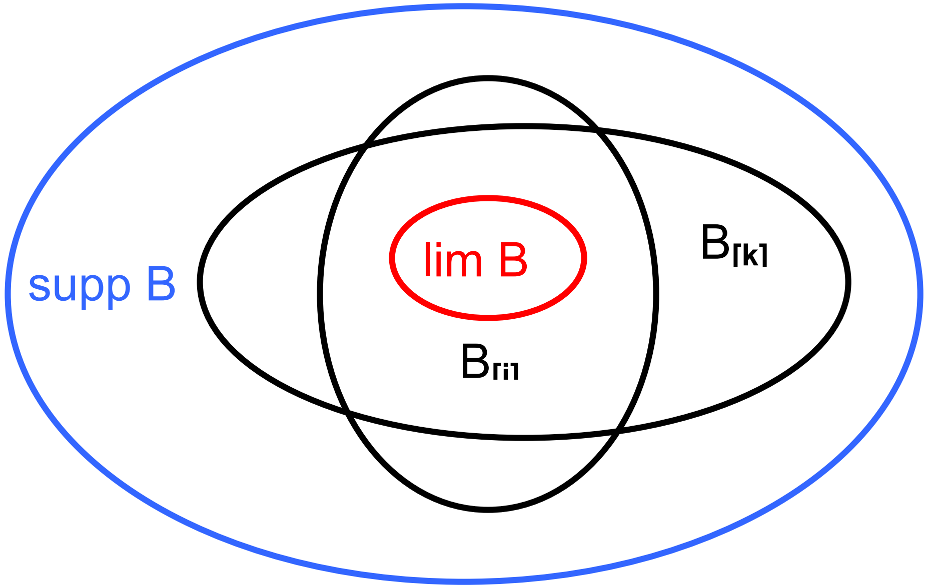
Examples: $\mathbf{K}_{[t]} = \cup, \cap$, if arithmetic exists: a mean value.

$\mathbf{K}$ maps parallel processes onto one single process.

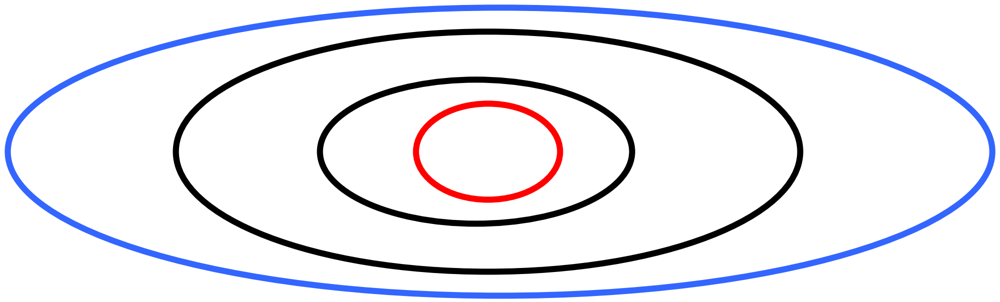
TOPOLOGICAL CONCEPTS

Given a non-empty set M and a family $\mathbf{B} = (B_{[i]} \mid i\in I)$ of subsets $\emptyset \neq B_{[i]} \subseteq M$ with the property : for any pair $(i,j) \in I\times I$ exists $k\in I$ with $B_{[i]}\subseteq B_{[k]}$ and $B_{[j]}\subseteq B_{[k]}$. **B** is a "filter base on M", For our purpose we assume $\lim \mathbf{B} =_{def} \cap\ \mathbf{B} \in \mathbf{B}$, and $\text{supp}\ \mathbf{B} =_{def} \cup\ \mathbf{B} \in \mathbf{B}$, and say, **B** is a "neighborhood system to lim **B**" with support supp **B.**

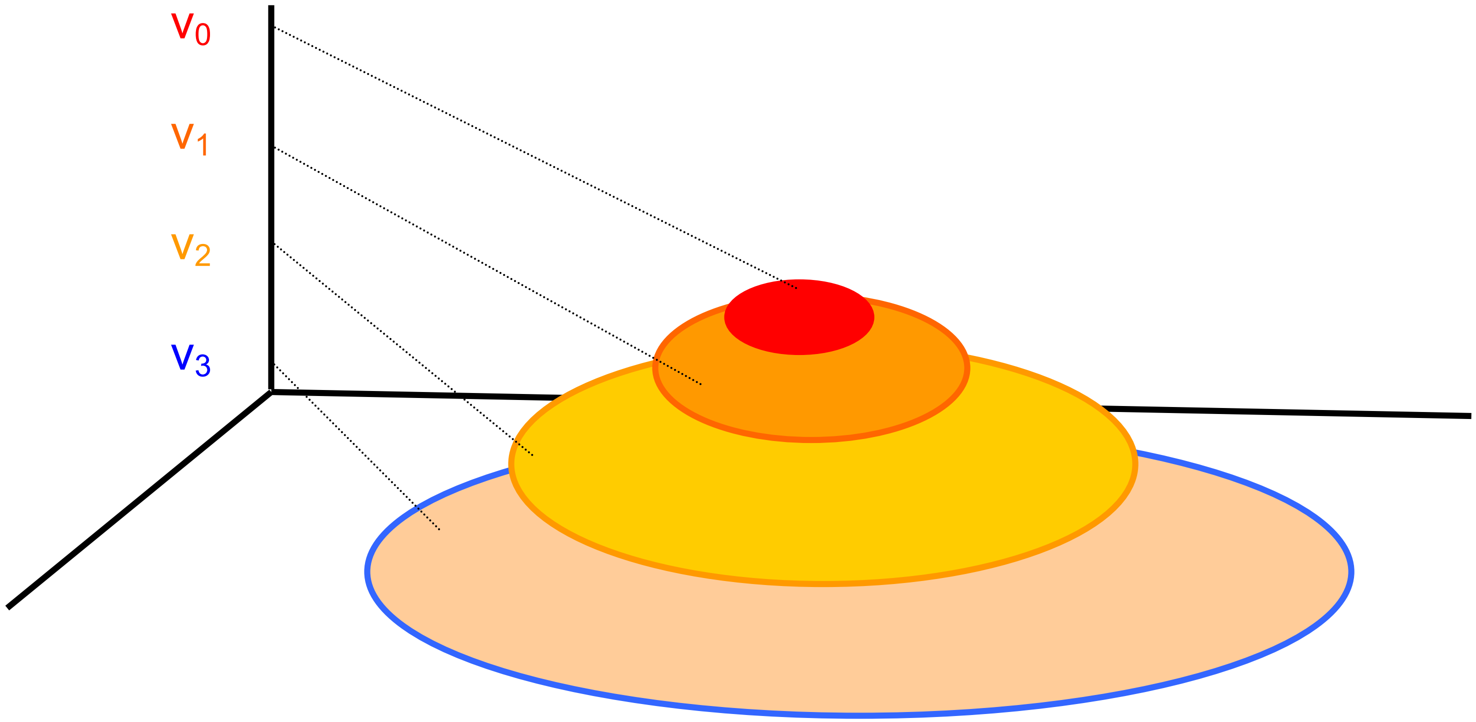
<u>*Visualization*</u>: neighborhood system to lim B with supp **B** ∈ **B**



a special case: a *monotonous* neighborhood system



The neighborhood system can be homomorphous *valuated* by values $v \in (V,<)$, V is mostly a linearly ordered set: $B \rightarrow v(B)$ The symmetric difference $D =_{der} B´\backslash B´´ \cup B´´\backslash B´$ defines a general distance of B´ from B´´ which can also be valuated.



a valuated neighborhood system

The simplest case $V = \{0,\ldots, 1= v_0\} \subseteq [0,1] \subset \mathbf{R}$ is known as "*fuzzy set*"

An example are concentric circles in the Euclidean plane around a minimum circle, v is the radius of the circles.

The set union of neighborhood systems forms again a neighborhood system; an example is a metric space in $\mathbf{R}^2$**.** If all neighborhood systems are uniformly valuated, the topology is "*uniform*". A particular case is a metric space.

A neighborhood system **B** = ($B_{[i]}$ | i∈I) on a set M with M∈**B** defines a *topology* on M: Let be

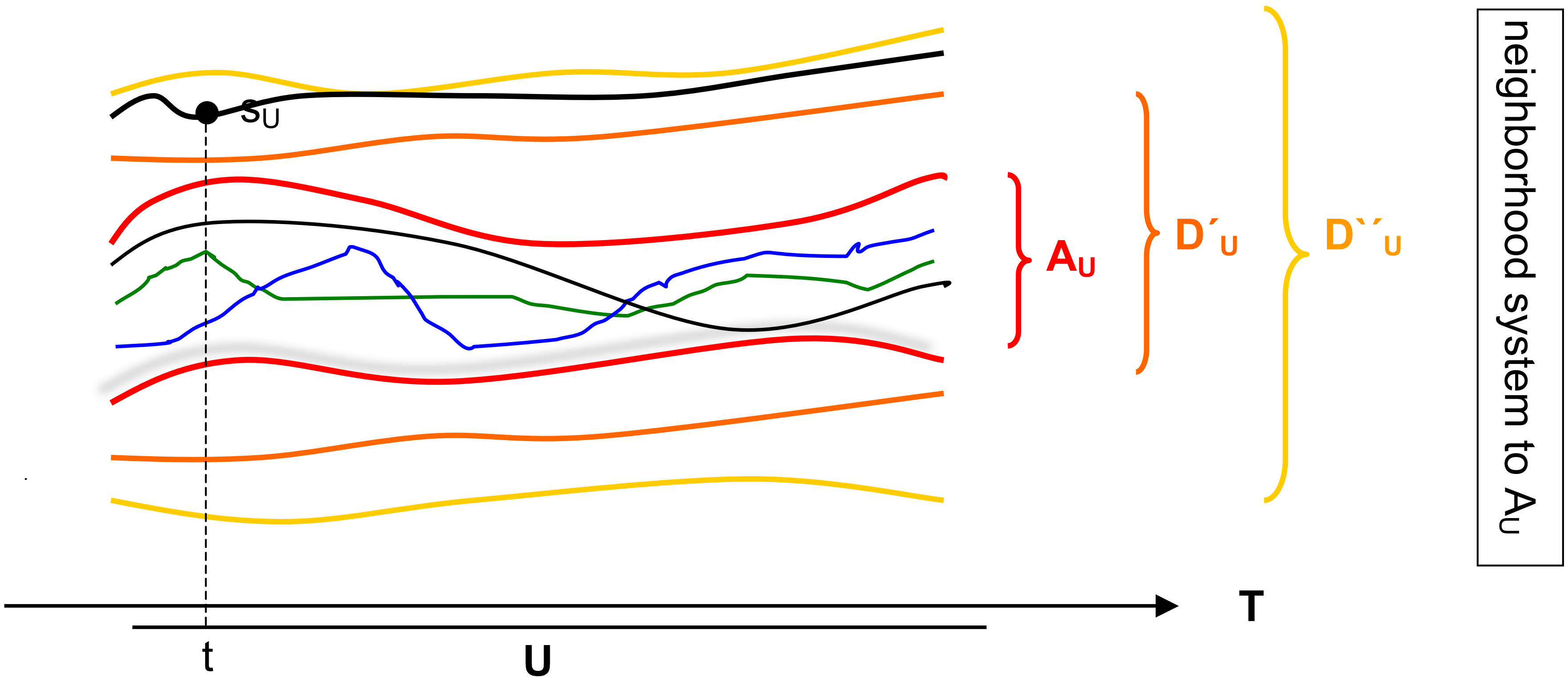
   B ∪ {∅} closed sets,

   arbitrary intersections ∩ and finite unions ∪ of closed sets are closed sets

   set complements with respect to M of closed sets are open sets

ADAPTATION of a STATE PROCESS to the ADMISSIBLE DOMAIN

A process $s_U =_{def} (s_t)_{t\in U}$, is *admissible* if it satisfies all constraints, requirements, limitations, properties etc. imposed on it bat U. In general, the admissible domain forms a process A_U, to which we consider a neighborhood system $\mathrm{N} = \{D_n \mid n\in N\}$, $\lim \mathrm{N} = A_U$.



In the illustration, s_U has distance D''_U from A_U, (*adaptation degree*), s_U to be in $A_{\mathbf{U}}$ is the "goal". A process of processes s_U passing trough a neighborhood system $\mathcal{N}$ towards $\lim\mathcal{N}$ *adapts* $\lim\mathcal{N}$. If $\mathcal{N}$ is homomorphous valuated, the adaptation process tends to a minimal/maximal value (class of *gradient* methods).

STATE- and CONTROL- PROCESSES in $\mathbf{R}^n$ and $\mathbf{R}^m$

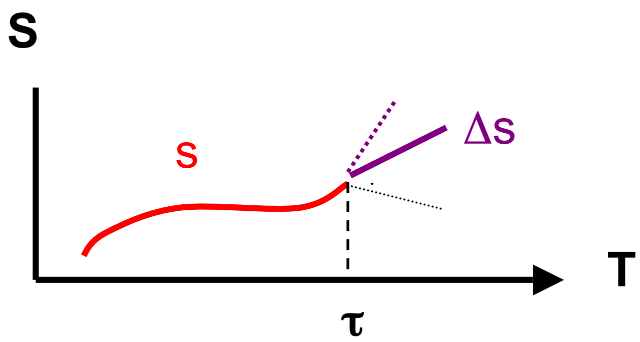
Dual incremental eqs. for same **T** , s states, c control, T(s), T(c) elapsed s-process-, c-process times, $T(s) < \tau$, $T(c) < \sigma$; $\tau, \sigma \in \mathbf{T}$, with $\Delta(s_{T(s)})_\tau$, $\Delta(c_{T(c)})_\tau$ variables on predefined domains. If assigned

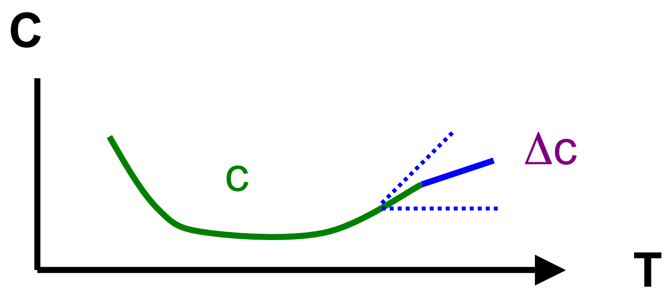
$\Delta(s_{T(s)})_\tau = F(s_{T(s)}, c_{T(c)}, \Delta(c_{T(c)})_\sigma)$, $\sigma<\tau$, $\Delta(s_{T(s)})_\tau$ has to be admissible, depends on (parts of) past s- and c- processes

$\Delta(c_{T(c)})_\tau = G(s_{T(s)}, c_{T(c)}, \Delta(s_{T(s)})_\tau)$ $\sigma>\tau$, $\Delta(c_{T(c)})_\sigma$ has to be admissible, depends on (parts of) past s- and c- processes

Illustration

$\Delta s/\Delta t$ corresponds c, branching marks possible choices

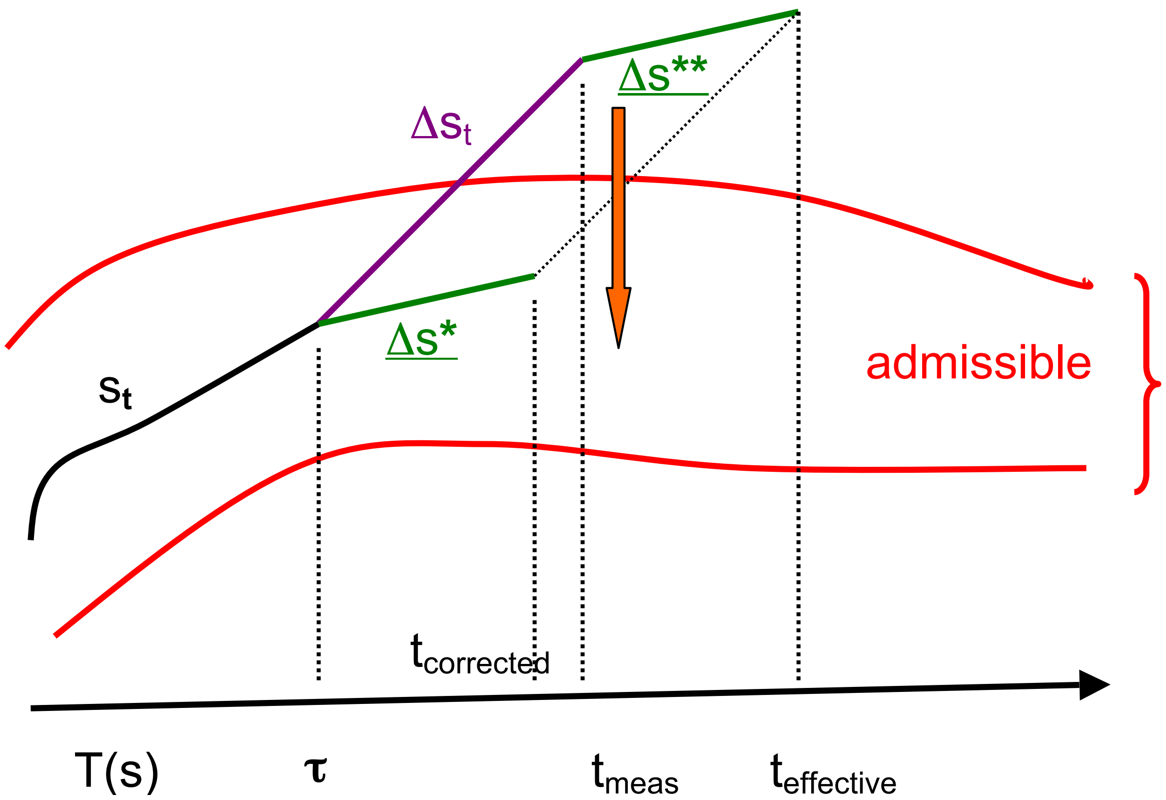




continuation of s by Δs such that the resulting process is admissible

dual for c

Feedback Based Control and Corrections



corrected and still not admissible !

Feed Back Synchronization Problem

IMPROVEMENTS

shorten time step for Δs

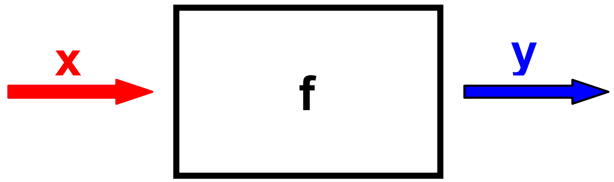
shorten time step for deviation measurements

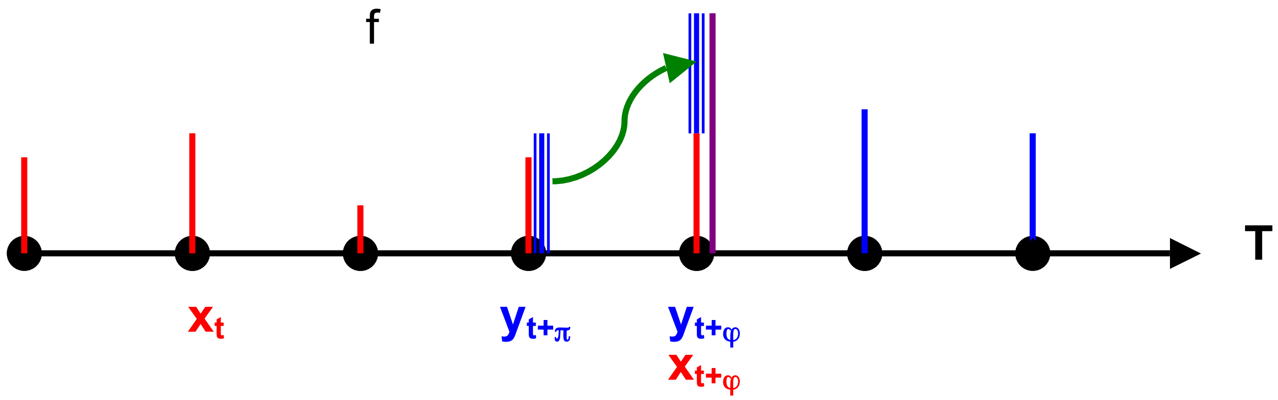
shorten operation time for correction by feed back

admission process look ahead

state/control process look ahead

Input- Output System
(multi-dimensional)





$$y_{[t+\pi]} = x^{*}_{[t+\varphi]}$$

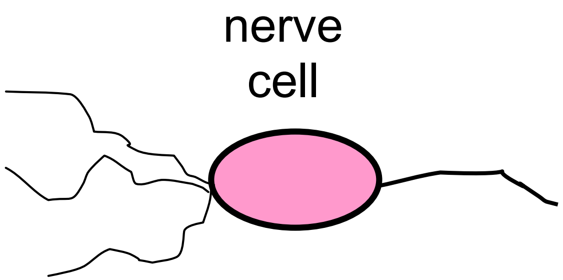
$\pi > 0$ duration of **f**
$\varphi - \pi > 0$ duration of back feeding

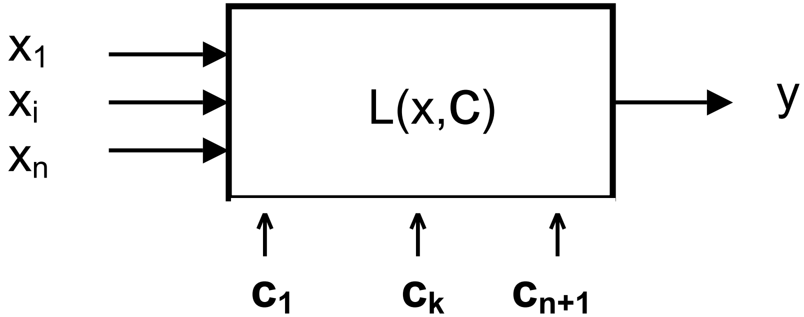
$(x_{t+\varphi}\ ,\ x^{*}_{t+\varphi})$
combined to $x^{**}_{t+\varphi}$
cases:
$x^{**}_{t+\varphi}$ =,<,> $x_{t+\varphi}$
$\tau \neq \varphi$, influences
input/ output
frequency,
enables

EXAMPLE: ARTIFICIAL NEURAL NETS

nature

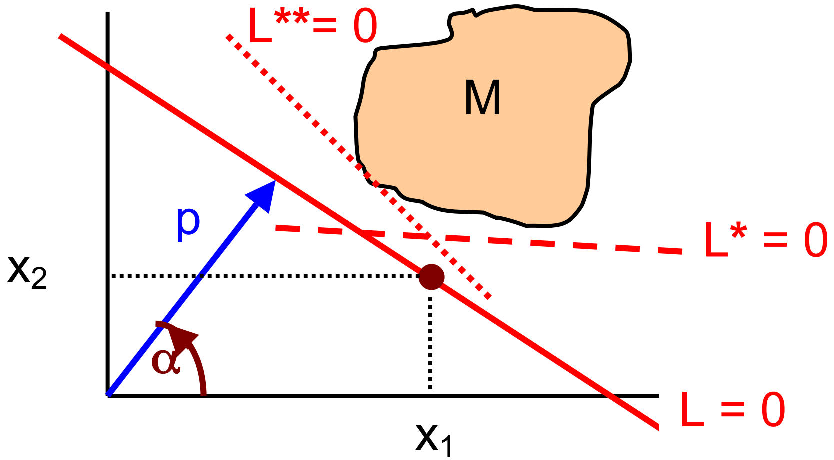
engineer





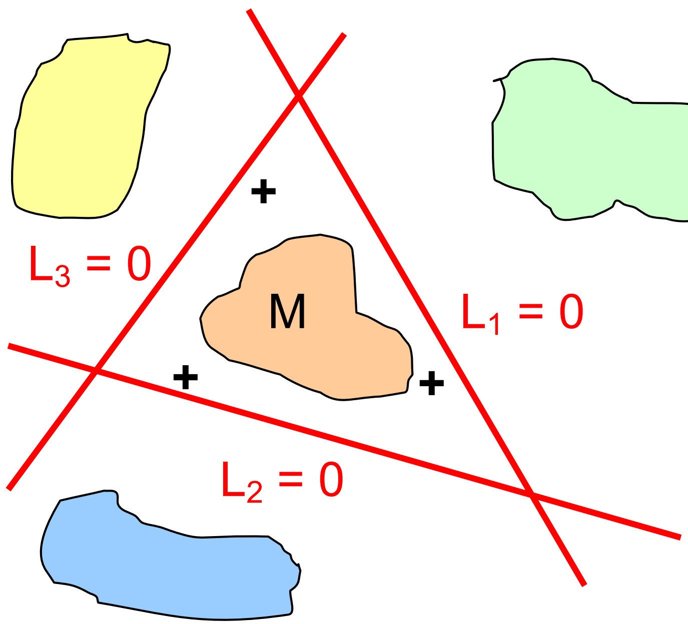
variable data $x = (x_i)_{i=1,2,\ldots n}$; variable control parameters $c = (c_k)_{k=1,2,\ldots n+1}$; given x, c is the "program for processing
$y = L(x,c) = \Sigma_{1\ldots n}\ c_i x_i + c_{n+1}$, let c be such that for all $x \in M$ $L(x,c) \geq 0$
V*isualization for* n = 2, L in Hessian form:

$$L(x, \alpha, p) = (\cos \alpha)\, x_1 + (\sin \alpha)\, x_2 - p$$



$L \geq 0$, $L^* \geq 0$, $L^{**} \geq 0$, …
are all admissible half planes forming a neighborhood system with lim containing M

more L´s for finer discrimination



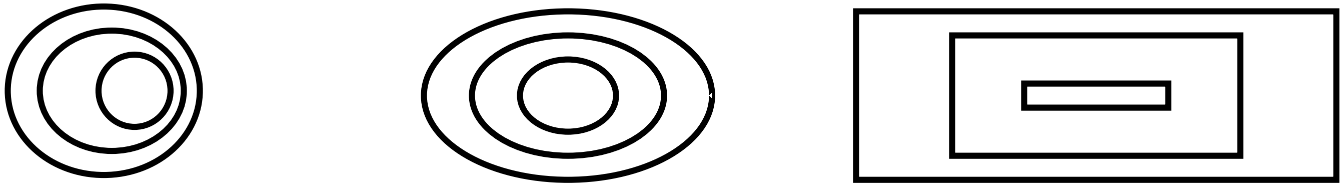
$(x_1, x_2) \Rightarrow$	$L_1 \geq 0$	y_1
$(x_1, x_2) \Rightarrow$	$L_2 \geq 0$	y_2
$(x_1, x_2) \Rightarrow$	$L_3 \geq 0$	y_3

$(x_1, x_2) \Rightarrow$	$L_1 L_2 L_3 \geq 0$	y

$K = L_1 L_2 L_3$ is a cubic fct.
M is enclosed by triangle, a convex neighborhood of M

Generalization: any neighborhood system N with $M \subseteq \lim \mathrm{N}$

Examples for N: circles, ellipses, polygons



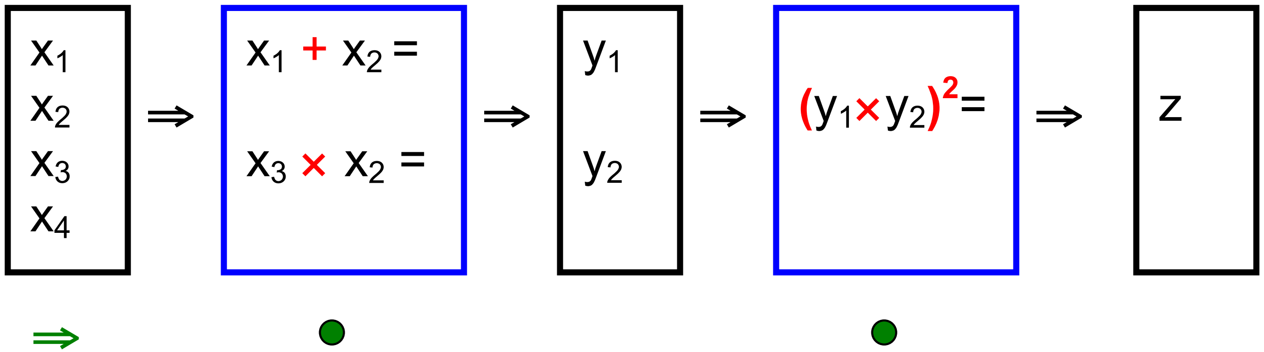
Architectural generalizations:

multiple layers $x \Rightarrow f(x) = y \Rightarrow g(y) = z \Rightarrow h(z) = w$, and so on,

any f, g, h,…; e.g. h = g (reuse of operations at later time possible)

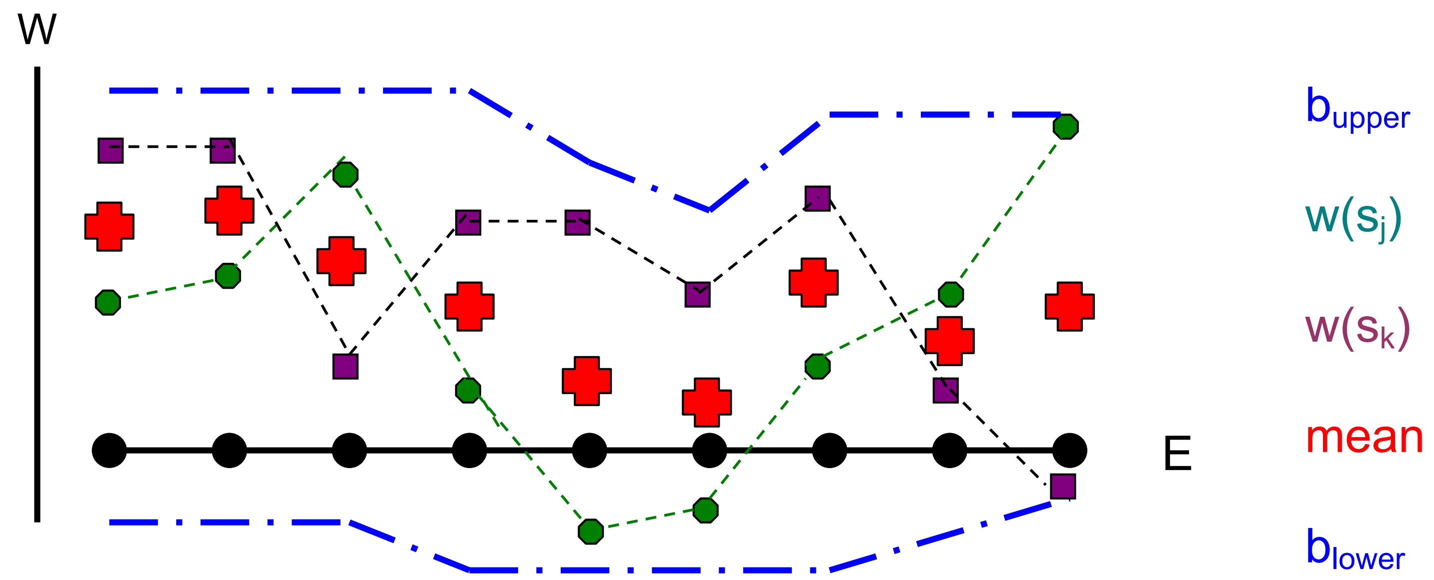
compare
with computer
architecture:
control, layers

algorithmic time



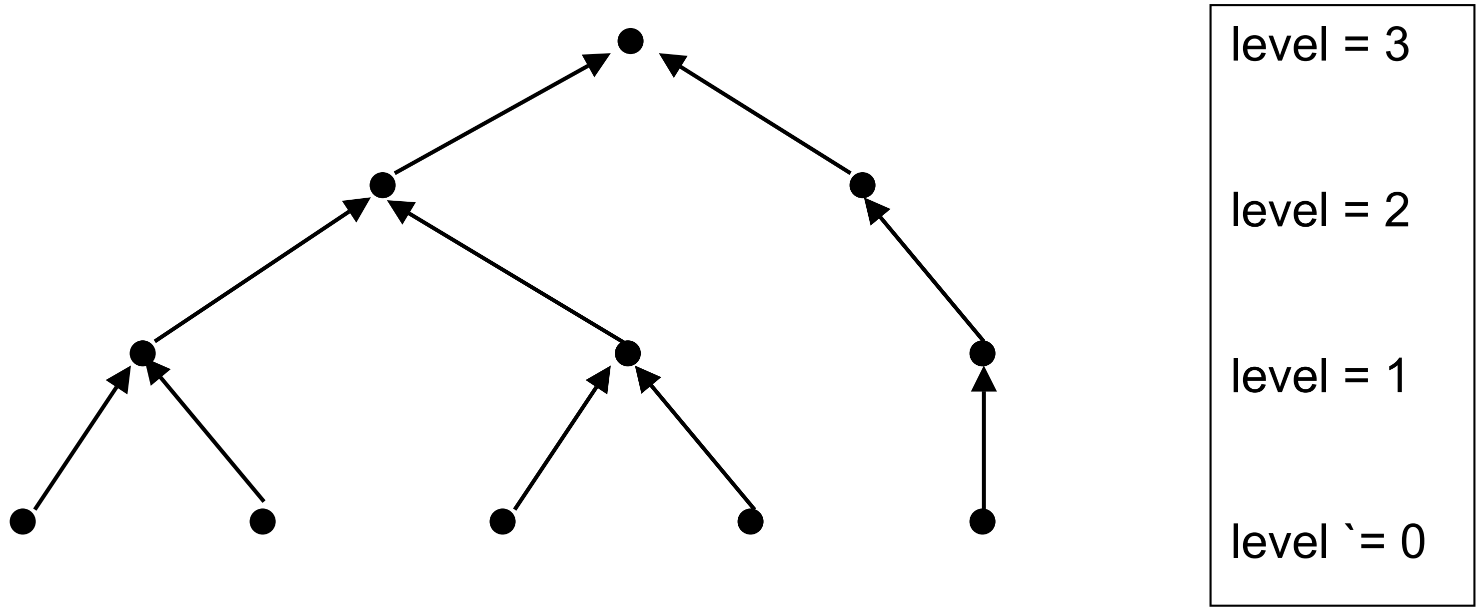
EXAMPLE: "GENETIC ALGORITHMS" (sketch of personal view)

$S = \{s_j \mid j \in J\}$, $J = \{1,2,\ldots m\}$ individuals,
$E = \{e_i \mid i \in I\}$, $I = \{1,2,\ldots n\}$ properties e_j of each individual s_i
$W \subset \mathbb{R}$, s_j has property e_i with weight $w_{[ij]} \in W$, weight function $w_j = (w_{ij})_{i \in I}$, for all j let uniform upper and lower bounds for $w_{[ij]}$ be given: $\overline{\mathbf{b}}_{[i]}$ and $\underline{b}_{[i]}$ respectively, $\underline{b}_{[i]} \leq w_{[ij]} \leq \overline{\mathbf{b}}_{[i]}$.



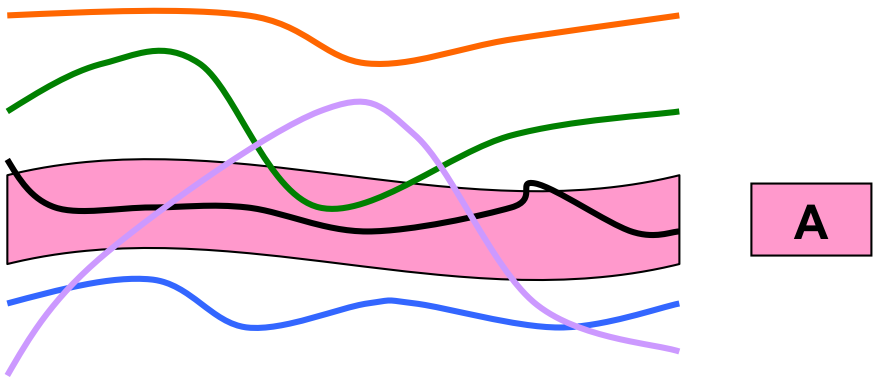
Concatenation κ of pairs (s_j, s_k), gives next level objects, for simplicity sake: 1 "child" only, generated objects on next level only.

$S^{(1)} =_{def} \{s_{jk}^{(1)} =_{def} \kappa(s_j, s_k) \mid (j,k) \in J \times J,\ j \neq k\}$, level 1,
valuation of $\kappa(s_j, s_k)$ is defined by $(w(s_{[ij]})\ \kappa_i\ (s_{[ik]}))_{i \in I}$,
κ_i fo*r example* min, max, convex mean = $a\alpha + (1 - \alpha)b$, $0 \leq \alpha \leq 1$, these κ_i are associative. For this case and for pairs with all elements distinct, levels form a hierarchy up to level L = "least integer ≥ ld n",. The valuations tend to $(\kappa_i(w_{[ij]})_{j \in J})_{i \in I}$



Adaptation Problem (breeder's problem)

For given set of individuals, properties and weight relation $(w_{ij})_{ij \in I \times J}$, and for given adaptation domain $A = \{ [a_i, b_i] \subset W \mid a_i < b_i, i \in I \}$ find within object generation 0, 1, …an object which (approximately) fits A



May have no solution, or approximations only. Is polynomial complex