What is non-deductive logic

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Purpose

Our purpose is to propose how to integrate symbolic logic with network (neural and argumenation) reasoning

Let us consider the human agent in his daily activity.

We ask: what 'logic' does he have in his head?

Current relevant buzz words circulating in the community are, among others: time, action, knowledge, belief, revision, deduction, learning, context, neural nets, probabilistic nets, argumentation nets, consistency, etc.

We want to understand what kind of integrated logic engine the human uses in his daily activity.

Short Story

Mother goes into her teenage daughter's bedroom. Her instant impression is that it is a big mess. There is stuff scattered everywhere.

Mother's impression is that it is not characteristic of the girl to be like this.

What has happened?

***Conjecture*: The girl has boyfriend problems.**

***Further Analysis*: Mother noticed a collapsed shelf. Did the girl smash it? Upon further observation, mother notices that the pattern of chaos shows that a shelf has collapsed because of excessive weight and scattered everything around, giving the impression of a big mess. But, actually, it is not a mess, it does make some (gravitational) sense.**

There are several modes of reasoning

1. **Neural nets type of reasoning.**
   She recognises the mess instantly, like we recognise a face.

2. **Nonmonotonic deduction.**
   Mother reasons from context and her knowledge of her daughter is that the girl is not disorganised like this. She asks 'what happened?'.

3. **Abduction/conjecture.**
   She offers a reasonable explanation that the girl has boyfriend problems. This is common to that age.

4. **She then applies a database AI deduction**
   and recognises that the mess is due to gravity. This deduction is no longer a neural net impression. It is a careful calculation.

5. **It could have been a neural net impression.**
   For example, a man who sees many shelf collapsing mess cases may recognise the pattern like it were a face.

How do we model and integrate what is going on?

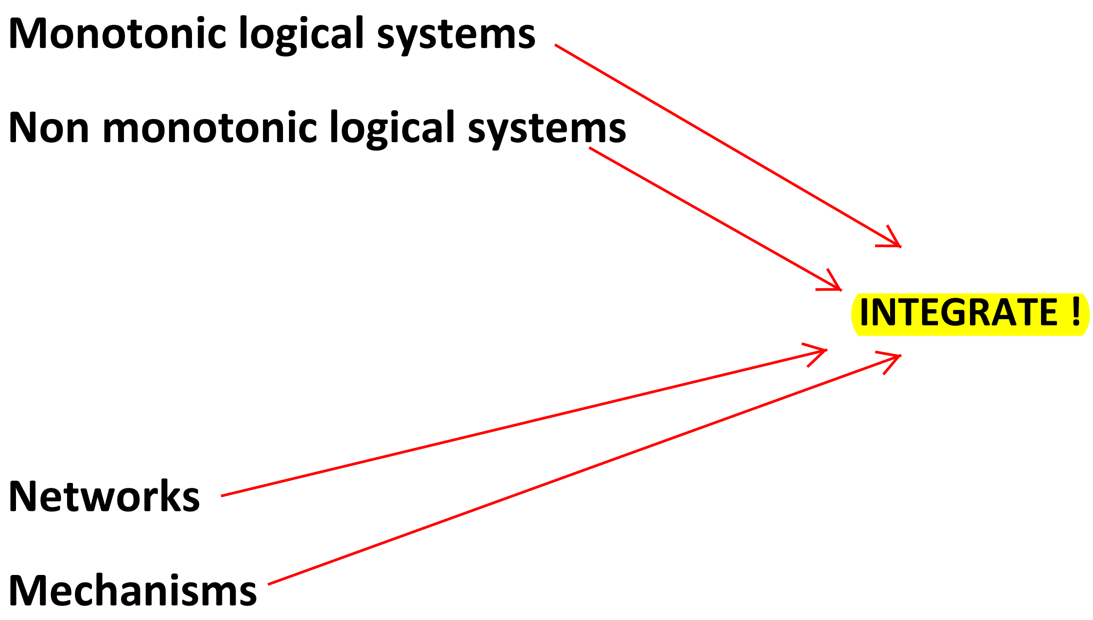
How can we view network logic and discrete symbolic logic from a common point of view?

What are the principles involved?

A unifying view for discrete symbolic logic and networks systems

neural, argumentation, Bayesian, fuzzy, biological

Transportation networks, flow networks, inheritance nets, mathematical graphs, etc.


Monotonic logical systems
Non monotonic logical systems
INTEGRATE !
Networks
Mechanisms

1983 A logical system is a consequence relation $\vdash$ together with an algorithmic proof system.

Gödel		+
classical	✓	+
Intuitionistic	✓	
	Gentzen	Truth Tables

Example of goal directed algorithmic proof for intuitionistic →.

1. $(c \to a) \to c$	$\vdash ?\ a$
2. $c \to a$	
3. From 2 delete 2.	$?\ c$
4. From 1. delete 1	$?\ c \to a$
5. c	$?\ a$

The database has only 5. c in it.

6. use bounded restart	$?c$

success.

$(a \to b) \to a$? a

? $a \to b$

a ? b

use restart ? a

Bounded restart will not help

Book 2000

1989 LDS.

structured data Goedel Logic.

t: Accident $\rightarrow$ pay

s: Accident

---

r: pay.

must have $t < s$

or $t_i < s+1$

$$\frac{\begin{array}{l} t : A \to B \\ s : A \\ \varphi(t, s) \end{array}}{\underset{\sim}{f}(t, s) : B.}$$

$$\frac{\begin{array}{ll} 1. & A \to (A \to B) \\ 2. & A \end{array}}{}$$

3. $(1, 2) : A \to B$

4. $(1, 2, 2) : B$

not in linear logic.

$$\Delta = \left(t_i : A_i \right) \vdash s : B$$

manipulate Δ to get to $s:B$

Mechanisms:

Abduction, analogy

Meta-level Vs Object level

Reactivity

Equations

⋮

LDS book OUP 1996

Fibred Semantics book 1998

non-Monotonic Jumping to Conclusions

$$\Delta \mathrel{|}\!\!\sim A$$

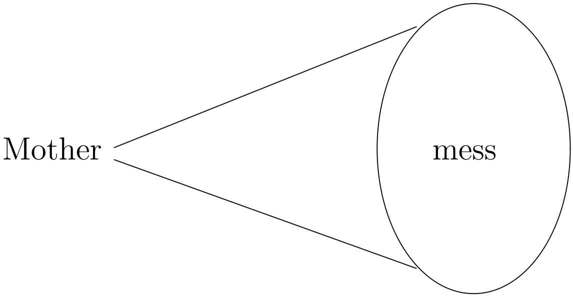
Jump

English A but not B

logic $A \mathrel{|}\!\!\sim B$.

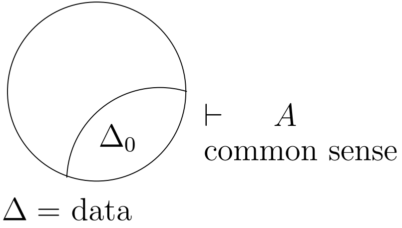
Jump

Neural nets



Neural nets can learn.
Show several examples of mess and train the net.

Nonmonotonic Logic



Girl never late

Never forgets homework

Always fussy about how hot morning tea is

Never dresses with clothes not symmetrical

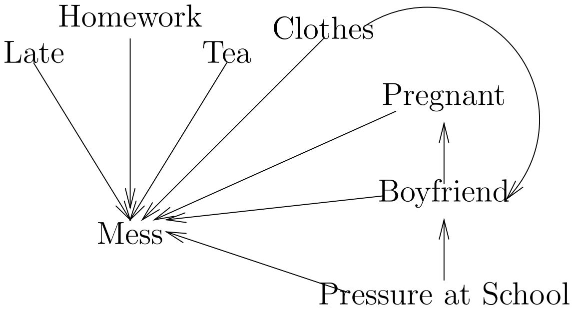
Room always tidy

Pregnant

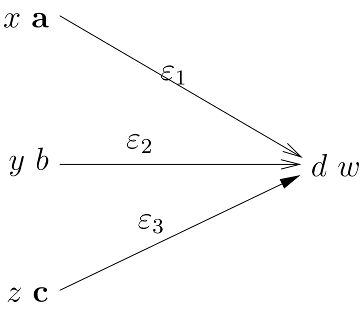
Boyfriend problems

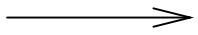
Pressure at school

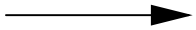
Bayesian Net



Argumentation network



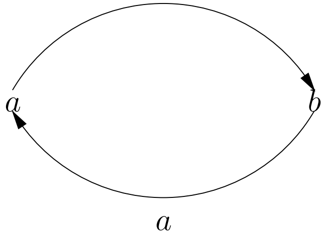
 attack

 defence

x strength $0 \leq x \leq 1$

ε transmission

Loops

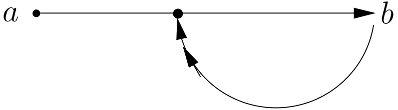


Evaluate in waves
Result depends on starting set

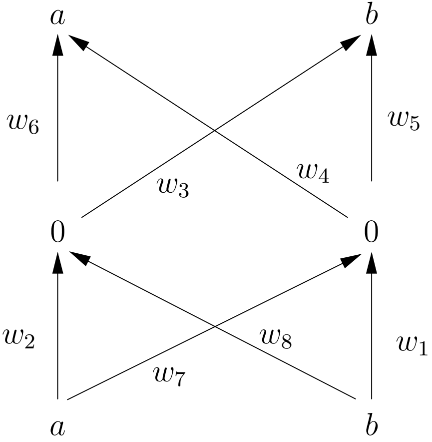
Starting set	Result
a	$\{a\}$
b	$\{b\}$
$\{a, b\}$	$\emptyset$

Compare movements in each area

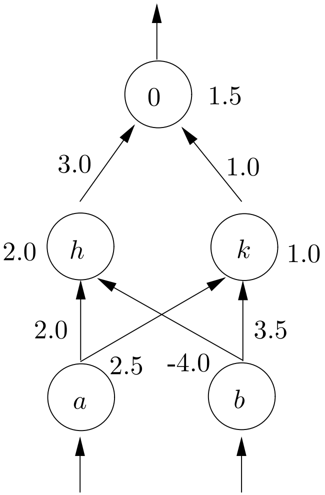
NETS	LOGIC
Value propagation	substitution
Probabilities	hypotheticals
attack defence	time
networking	strength of proof
feedback	deduction sequences
	LDS
	context
	fibring and combining
	reactivity



1. **Attack the relevance of a to attack b**
2. **Feedback learning loop**
3. **Modify probability function**
4. **Weaken effectiveness of virus**
5. **Bridge collapse. Salesman problem.**
6. **Brain**

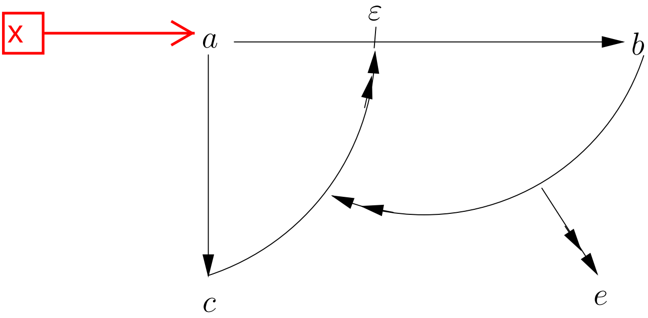


can learn weights and strength



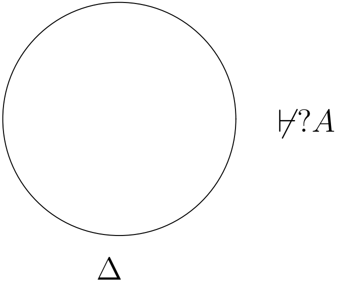
A set of weights for backpropagation

Higher complexity



Use waves of transmission

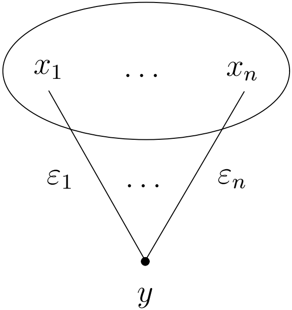
Abduction



Add hypothesis H

$$\Delta, H \vdash A$$

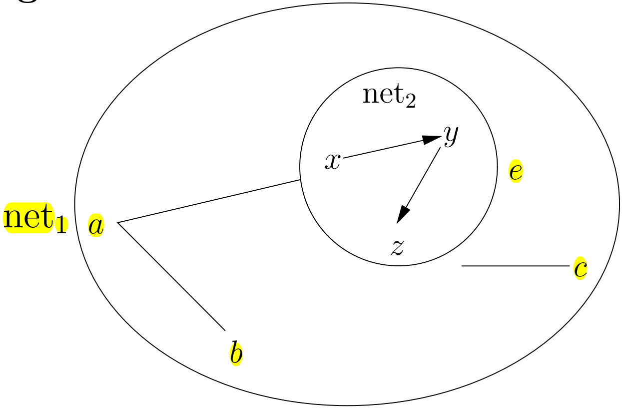
complex mechanisms



$$\textbf{new } y = f(x_1, \dots, x_n, \varepsilon_1, \dots, \varepsilon_n, y)$$

Propagation function

Fibring



net$_2$ can be:

- **another argumentation system**
- **abduction system**
- **neural net**
- **same type as net$_1$**

3. $\forall(\phi, l) \in \mathcal{S} \cup \mathcal{D}$*, there exist arguments* $\epsilon_{(\phi,l)}, \zeta_{(\phi,l)} \in \mathcal{A}_X$ *s.t.*
  - $\alpha_\phi \rightarrow \epsilon_{(\phi,l)}$, $\epsilon_{(\phi,l)} \rightarrow \zeta_{(\phi,l)}$, $\zeta_{(\phi,l)} \rightarrow \alpha_{\overline{l}}$.

4. $\forall(\phi, l) \in \mathcal{S}, (\phi', l') \in \mathcal{D}$*, if* $l = \overline{l'}$*:*
  - $\zeta_{(\phi,l)} \rightarrow \zeta_{(\phi',l')}$

5. $\forall(\phi, l) \in \mathcal{D}, (\phi', l') \in \mathcal{D}$*, if* $l = \overline{l'}$ *and* $(\phi, l) \leq (\phi', l')$*:*
  - $\zeta_{(\phi,l)} \rightarrow \zeta_{(\phi',l')}$

6. All ϵ*,* ζ *and* η*-arguments are not involved in any attack other than the ones specified above.*

We demonstrate our theory with two examples, shown in figure 4 and 5. The figures show only part of the logical argumentation framework: the δ nodes are omitted.

Figure 4 shows the famous non-flying bird example, formalized in a logical argumentation framework. The focal set consists of $p, \neg p, f, \neg f, b$ and $\neg b$ (i.e. p = 'it is a penguin', b = 'it is a bird' and f = 'it flies'); there is one fact, namely p; two strict rules: $p \rightarrow \neg f$ and $b \rightarrow f$; and one defeasible rule: $b \Rightarrow f$. There is one extension: $\{p, b, \neg f\}$.

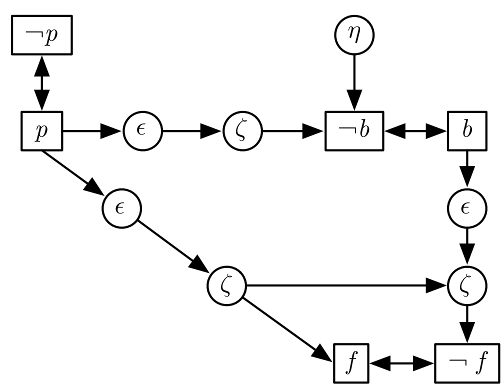
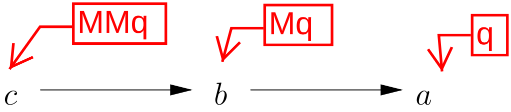


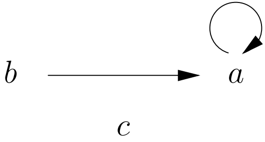
Fig. 4. The famous non-flying bird example.

Figure 5 shows the famous nixon diamond scenario, formalized in a logical argumentation framework. The focal set consists of $r, \neg r, p, \neg p, q$ and $\neg q$ (i.e. r = 'nixon is a republican', p = 'nixon is a pacifist' and q = 'nixon is a Quaker'); there are two facts, namely r and q; and two defeasible rules: $q \Rightarrow p$ (Quakers are normally pacifists) and $r \Rightarrow !p$ (republicans are normally not pacifists). There are two extensions: $\{r, q, p\}$ and $\{r, q, \neg p\}$.

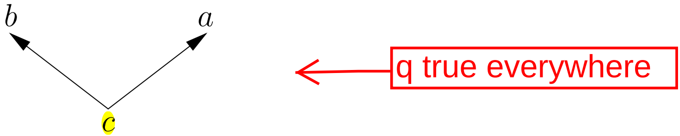
(note: also add the caminada tandem example, because these examples only use literals)



argumentation net



reject graph

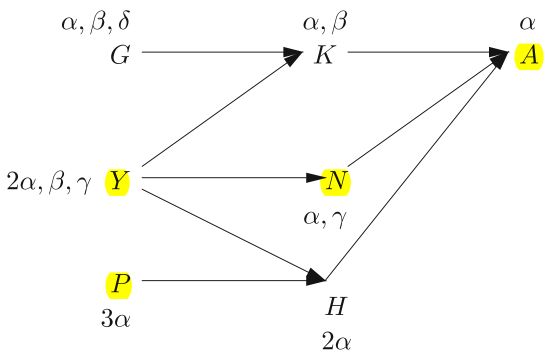


accept graph

	N	A	P	Y	H	G	K
m	0	1	1	0	1	0	0
h	1	?	0	0	0	0	0
b	1	1	0	1	1	0	1
w	0	1	0	0	0	1	1

Figure 62.

1. Graph for $x = 1$



2. Graph for $x = 0$

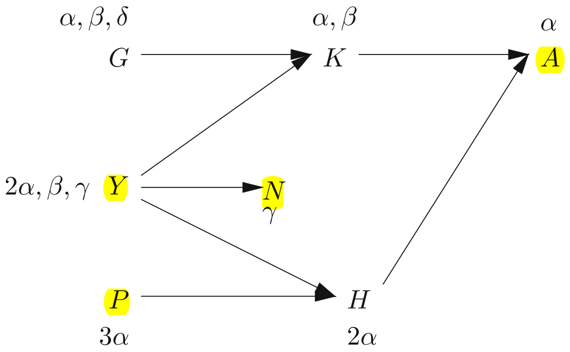


Figure 63.

We get Figure 52

	N	A	Y
b	1	1	1
h	1	$x=?$	0

Figure 52.

The graphs we get are in Figure 53

1. Graph for $x = 1$

$$\begin{array}{ccc} Y & \longrightarrow & N = A \\ 2\alpha & & \alpha \end{array}$$

2. Graph for $x = 0$

$$\begin{array}{ccc} A = Y & \longrightarrow & N \\ 2\alpha & & \alpha \end{array}$$

Figure 53.

Clearly they are of equal strength and the answer is undecided.

The proponent now combines both tables to get $x = 1$ to win(items 6–8 of the text). We get Figure 54

	N	A	P	Y
m	0	1	1	0
h	1	$x=?$	0	0
b	1	1	0	1

Figure 54.