PRECISIATION OF MEANING— A KEY TO SEMANTIC COMPUTING

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Full version. Available from http://www.cs.berkeley.edu/~zadeh/presentations.html

NOTE

- ***To facilitate understanding of the basic concepts which underlie precisiation of meaning, a clarification dialogue is included in the Appendix.***

INTRODUCTIO N

PRECISIATION OF MEANING—PREAMBLE
KEY POINTS

- What is information?
- Information is a restriction (constraints) on the values which a variable can take.
- Information is carried by propositions.
- Examples

p: Vera is middle-aged

p: Carol lives in a small city near San Francisco

p: It is not very likely that Robert is rich

CONTINUED

- ***What is the information which is carried by p?***
- ***To answer this question it is necessary to understand the meaning of p.***
- ***To compute with the information carried by p it is necessary to precisiate the meaning of p.***
- ***Precisiation of meaning of p= construction of a computational model of p.***

SIMPLE EXAMPLESOF PROBLEM-SOLVING WITH INFORMATION DESCRIBED IN A NATURAL LANGUAGE

- *Probably John is tall. What is the probability that John is short?*
- *Most Swedes are tall. What is the average height of Swedes?*
- *Usually Robert leaves office at about 5pm. Usually it takes Robert about an hour to get home from work. At what time does Robert get home?*

PRECISIATION OF MEANING—A KEY TO EVERYDAY REASONING AND DECISION-MAKING

- ***The coming decade is likely to be a decade of automation of everyday reasoning and decision-making. In the world of automated reasoning and decision-making, computation with information described in a natural language is certain to play a prominent role.***

CONTINUED

- ***Precisiation of meaning is a prerequisite to computation with information described in a natural language. In turn, understanding of meaning is a prerequisite to precisiation of meaning.***

MEANING VS. PRECISIATION OF MEANING—EXAMPLES

- *Robert: Keep under refrigeration.*

Lotfi: I understand what you mean, but could you precisiate your meaning of "Keep under refrigeration?"

- *Robert: Vera is middle-aged*

Lotfi: I understand what you mean, but could you precisiate your meaning of "middle-aged?"

IMPRECISION OF NATURAL LANGUAGES

- ***Natural languages are intrinsically imprecise. Basically, a natural language is a system for describing perceptions. Perceptions are imprecise, reflecting the bounded ability of human sensory organs and ultimately the brain, to resolve detail and store information. Imprecision of perceptions is passed on to natural languages.***

NATURAL LANGUAGE AND PERCEPTIONS

- *p: perception*
- *NL(p): description of p; semantic entity*
- *p^+: perceptions evoked by NL(p)*
- *p^+: meaning of p; denotation of p*

IMPRECISION OF NATURAL LANGUAGES

- ***There are many different forms of imprecision in natural languages. A principal source of imprecision is unsharpness of class boundaries.***

Everyday examples:

Words(phrases, predicates)

- ***tall***
- ***near***
- ***not very tall***
- ***mountain***
- ***hand***
- ***high fever***
- ***several large balls***
- ***recession***

CONTINUED

Propositions

- Most Swedes are tall
- Icy roads are slippery
- Speed limit is 65 mph
- Check out time is 1pm

Commands

- Keep under refrigeration
- Handle with care

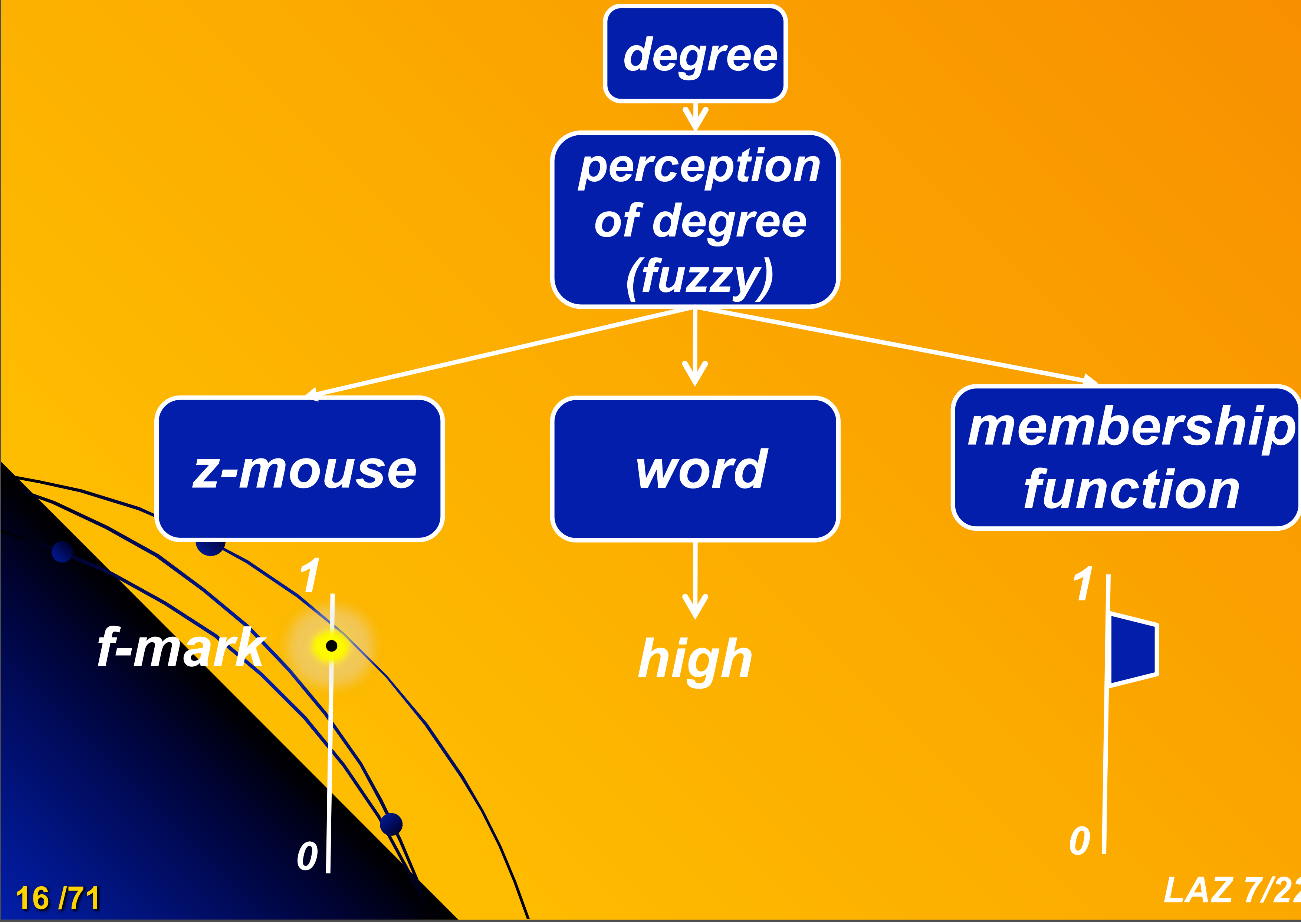
UNSHARPNESS OF CLASS BOUNDARIES=FUZZINESS

- *Words and phrases are labels of classes with unsharp boundaries.*
- *Fuzziness of words is a concomitant of fuzziness of perceptions.*
- *Fuzziness of natural languages is rooted in unsharpness of class boundaries.*
- *Fuzzy set= precisiated (graduated) class with unsharp boundaries.*

CONTINUED

- ***Graduation (precisiation)= association of a class which has unsharp boundaries with a scale of degrees—more concretely, with a membership function. Degrees are allowed to be fuzzy (fuzzy sets of type 2).***

KEY POINT—REPRESENTATION OF FUZZY DEGREES



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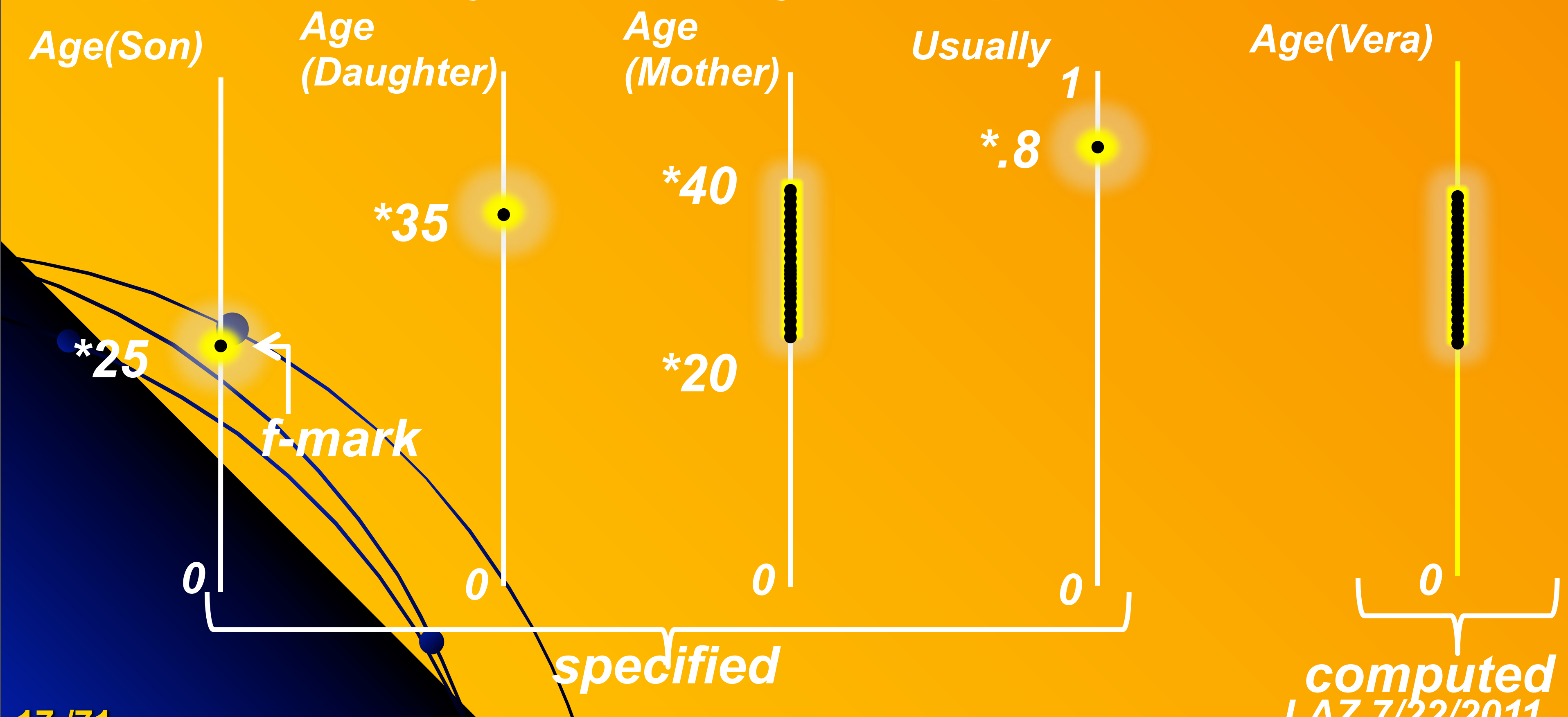
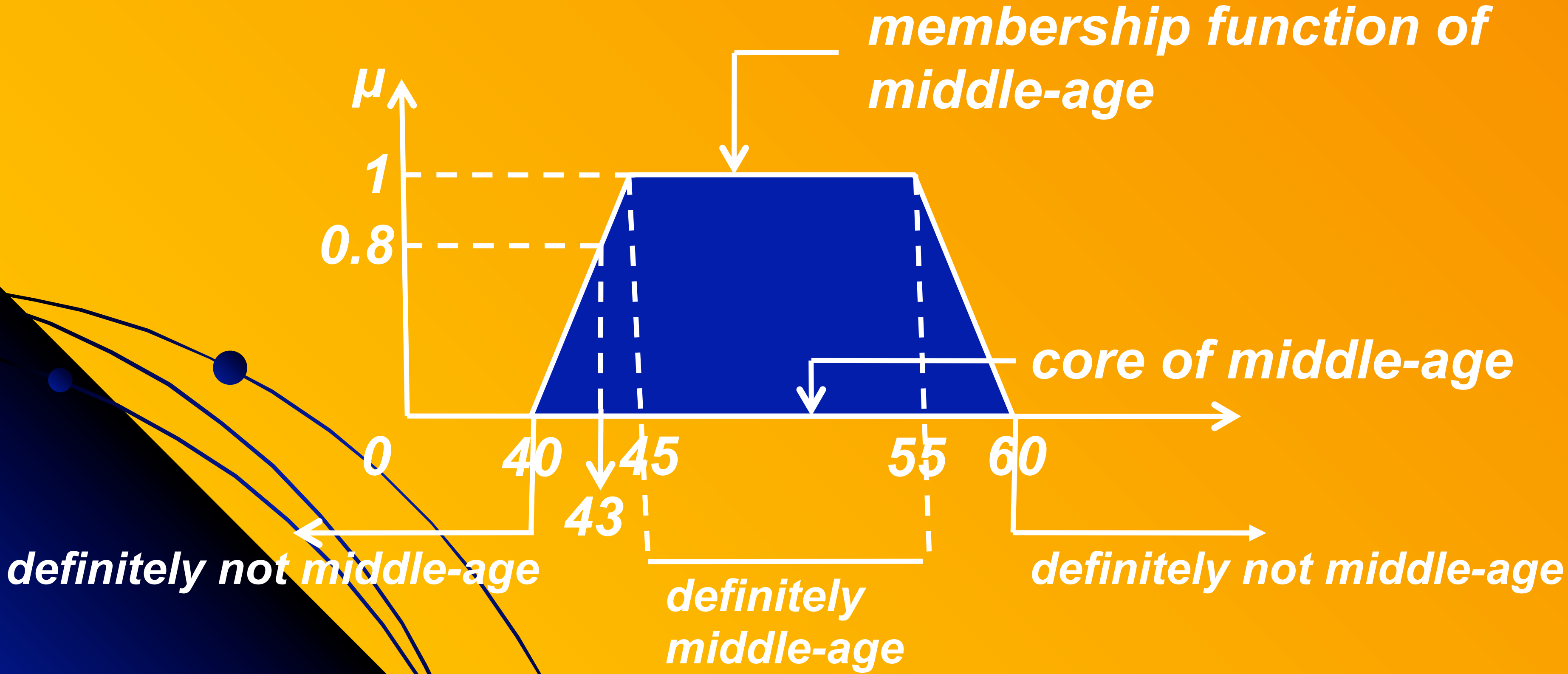
Z-MOUSE—VISUAL FUZZY DATA ENTRY AND RETRIEVAL

- *A Z-mouse is an electronic implementation of a spray pen. The cursor is a round fuzzy mark called an f-mark. The color of the mark is a matter of choice. A dot identifies the centroid of the mark. The cross-section of an f-mark is a trapezoidal fuzzy set with adjustable parameters.*



EXAMPLE—GRADUATION OF MIDDLE-AGE

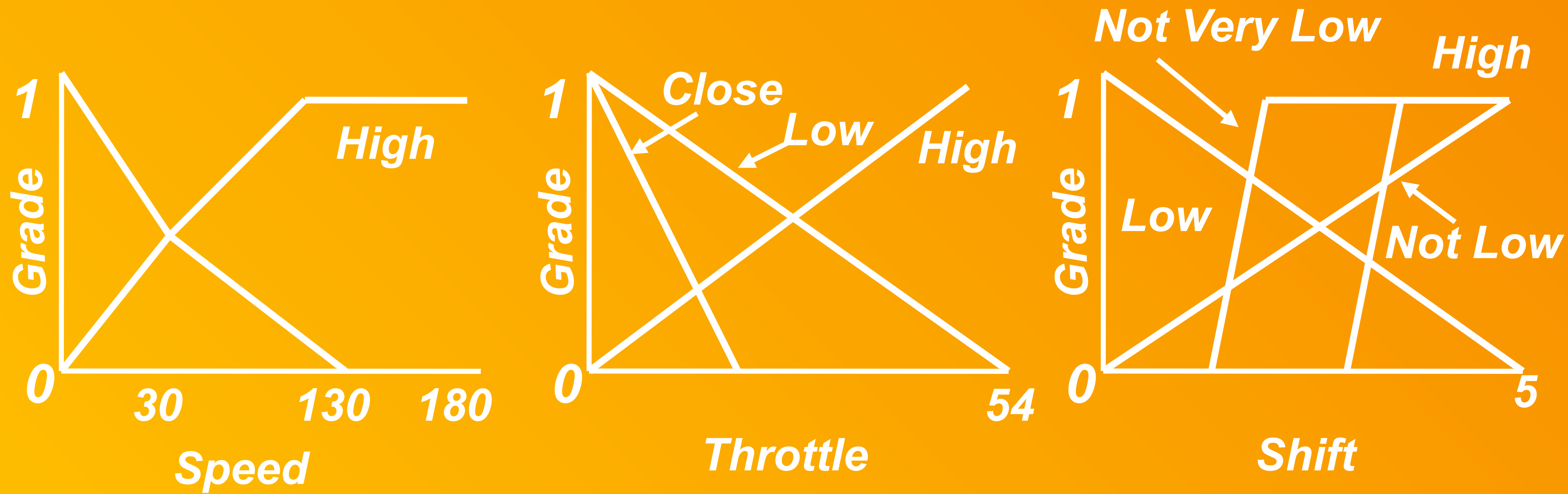
- *Imprecision of meaning = fuzziness of meaning*
- *Computational model of middle-age (trapezoidal fuzzy set)*



IMPORTANT POINT

- *Assume that Vera is 43 years old.*
- *The statement "Vera's grade of membership in middle-age is 0.8," may be interpreted as "the truth-value of the proposition "Vera is middle-age" given that she is 43, is 0.8. An equivalent interpretation is: Given that Vera is middle-age, the possibility that she is 43 is 0.8.*

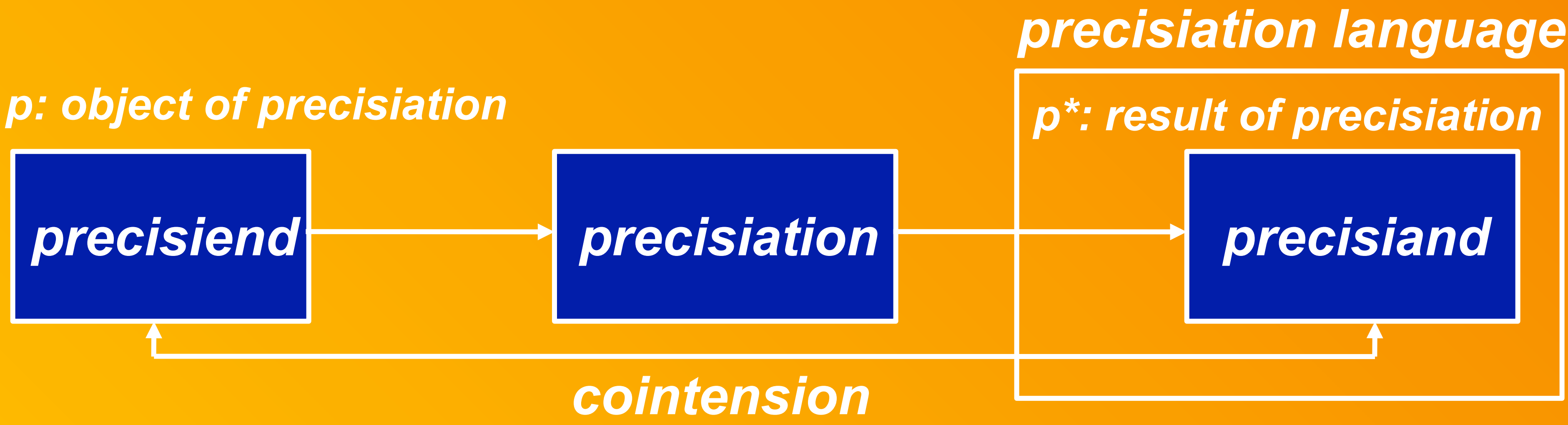
Fuzzy Sets



Control Rules:

1. *If (speed is low) and (shift is high) then (-3)*
2. *If (speed is high) and (shift is low) then (+3)*
3. *If (throt is low) and (speed is high) then (+3)*
4. *If (throt is low) and (speed is low) then (+1)*

BASIC STRUCTURE OF PRECISIATION



- precisiand = model of meaning
- extension= name-based meaning
- intension = attribute-based meaning
- cointension = qualitative measure of proximity of meanings
  = qualitative measure of proximity of the model (precisiand) and the object of modeling (precisiend)

GRADUATION OF PERCEPTIONS

- ***Humans have a remarkable capability to graduate perceptions without any measurements or any computations. More specifically, assume that I am given an object, a, and a class, A, and am asked to put a mark on a scale from 0 to 1 indicating my perception of the degree to which a fits A. Generally, I would have no difficulty in doing this.***

CONTINUED

- *This is what I do when I am asked to rate a restaurant on the scale from 0 to 10.*

MORE ON Z-MOUSE

- *If I am not sure what the degree is, and I am allowed to use a Z-mouse, I will put a fuzzy f-mark on the scale.*
- *A fuzzy f-mark reflects imprecision of my perception.*
- *In most cases, a crisp mark should be interpreted as the centroid of a fuzzy mark.*

CONTINUED

- For example, if I am asked to estimate the probability that Obama will be able to solve the financial crisis, and I put a crisp mark at .7, the crisp mark should be interpreted as the centroid of my fuzzy perception of the probability that Obama will be able to solve the financial crisis. What this points to is that more often than not fuzzy real-world probabilities are treated as if they were precise.

Z-MOUSE—AN EXAMPLE OF APPLICATION

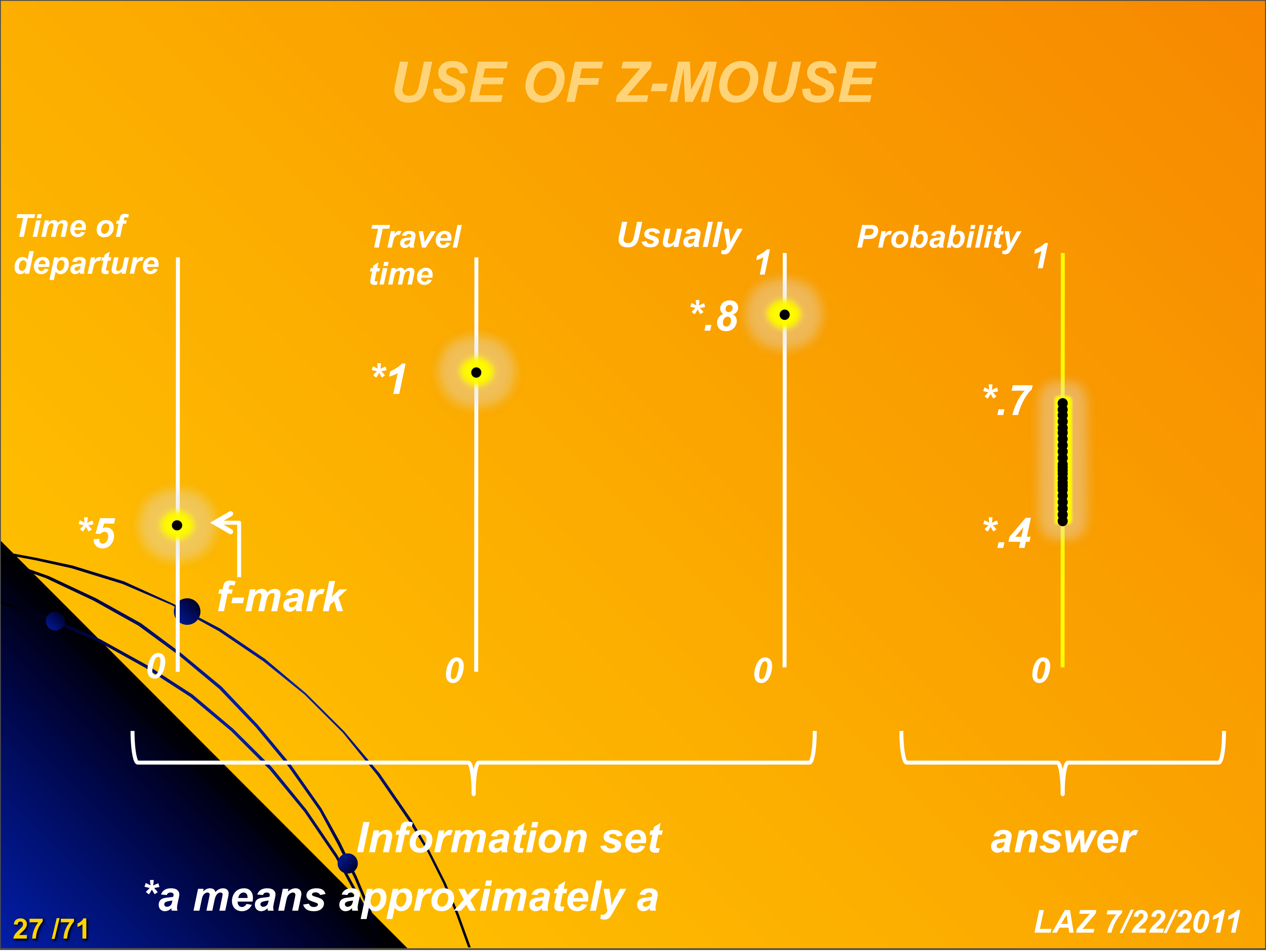
- Consider the following problem.

- Question: What is the probability that Robert is home at 6:15pm?

- Information set—information from which the answer is to be inferred:

  p_1: Usually Robert leaves office at about 5pm.

  p_2: Usually it takes Robert about an hour to get home from work.


USE OF Z-MOUSE
Time of departure
Travel time
Usually
Probability
*5
*1
1
*.8
1
*.7
*.4
f-mark
0
0
0
0
Information set
answer
*a means approximately a
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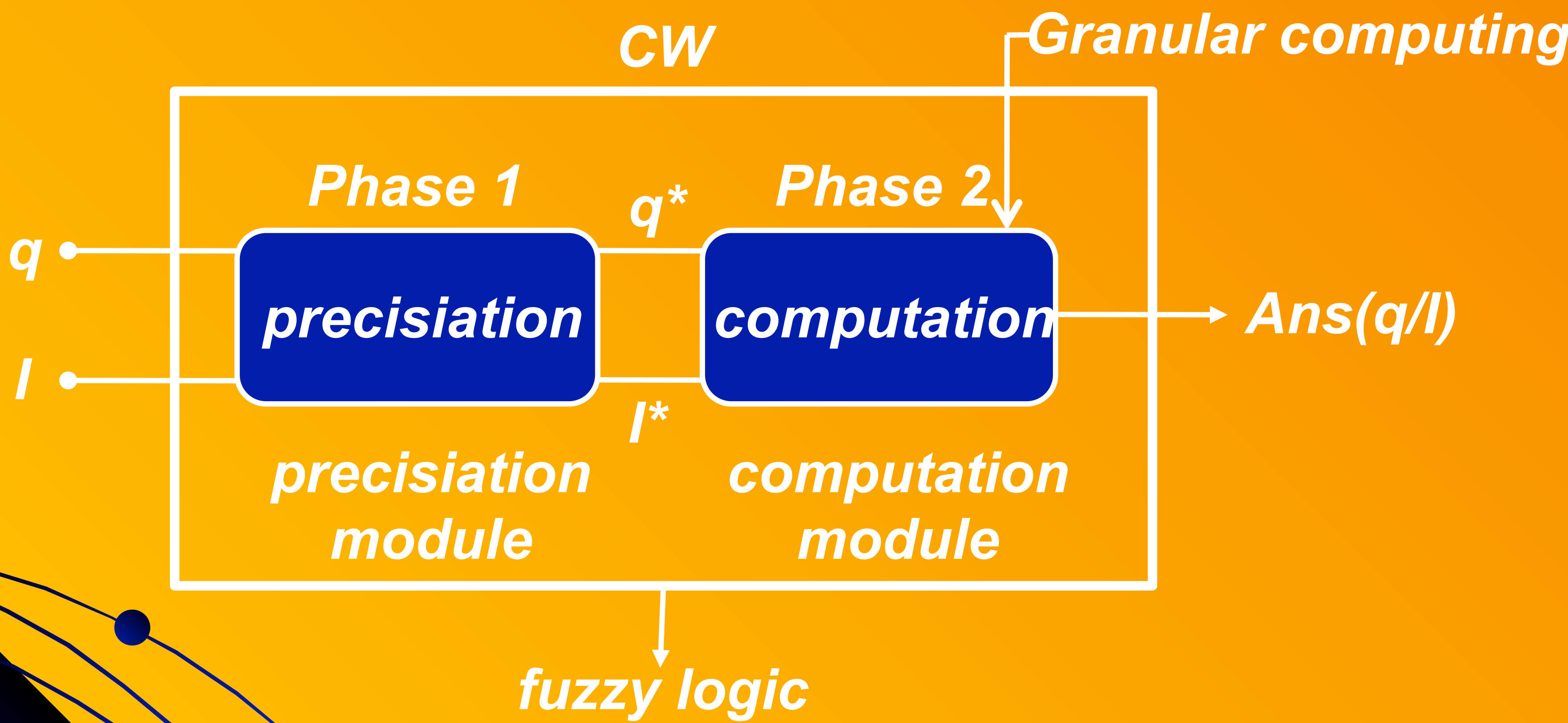
NOTE

- A Z-mouse serves primarily as a means of visual fuzzy data entry and retrieval. Computation of an answer to a question is carried out through the use of the machinery of Computing with Words (CW or CWW).
- Precisiation of meaning is a prerequisite to computation with information described in natural language.

PHASES OF CW
CW= [PRECISIATION → COMPUTATION]



- Precisiation and computation employ the machinery of fuzzy logic.

GRADUATION OF PROPOSITIONS?

- ***What is meant by graduation of propositions? If I were asked to graduate the proposition, p: Most Swedes are tall, what would I do? What is the connection between graduation of p and precisiation of p?***

CONTINUED

- ***In general, a proposition, p, may be associated with a variety of attributes. A basic attribute is the truth-value of p, t(p). In this perspective, graduation of p may be related to graduation of truth-value of p. As will be seen later, graduation of truth-value of p is a byproduct of precisiation of p.***

CONTINUED

- ***The truth-value of p cannot be assessed in isolation. If I were asked what is the truth-value of p: Most Swedes are tall, I would have to know how most and tall are defined, and be given the distribution of heights of Swedes. Let us call the needed knowledge the Information Base, IB(p).***

CONTINUED

- ***The question is: How can the truth-value be computed given the information base, IB(p)? What is needed for this purpose is restriction-based semantics, RS. Restriction-based semantics is rooted in test-score semantics (Zadeh 1981, 1986.)***

CONTINUED

- ***Restriction-based semantics is a generalization of truth-conditional and possible-world semantics. In the following, precisiation of propositions through the use of restriction-based semantics is discussed in greater detail.***

PRECISIATION OF PROPOSITIONS

THE BASICS OF RESTRICTION-BASED SEMANTICS

- *The point of departure in restriction-based semantics, RS, is an unconventional definition of the concept of a proposition.*

DEFINITION OF A PROPOSITION

- *A proposition, p, is a carrier of information.*
- *Information = a restriction on the values of a variable*
- *A proposition, p, is a restriction (generalized constraint) on the values of a variable, X, which is implicit in p. In symbols,*

$$X \text{ isr } R$$

CONTINUED

where R is a relation which restricts the values of X and, r, is an indexical variable which defines the way in which R restricts X.

KEY POINT

- ***In restriction-based semantics, the meaning of a proposition, p, is defined by answers to three questions. First, what is the restricted variable, X? Second, what is the restricting relation, R? Third, how does R restrict X? In natural languages restrictions are predominantly possibilistic, expressed as X is R.***

EXAMPLES

p: Vera is middle-aged.

p ⟶ Age(Vera) is middle-age

↑ X — *↑ R (fuzzy set)*

p: Most of Robert's friends are rich.

p ⟶ Proportion (rich.friends.Robert/friends. Robert) is Most

↑ X — *↑ R (fuzzy set)*

CANONICAL FORM of p: CF(p)

- When p is represented as a restriction, the expression X isr R is referred to as the canonical form of p, CF(p). Thus,

  CF(p): X isr R

- The concept of a canonical form of p has a position of centrality in precisiation of meaning of p.
- The canonical form of p may be interpreted as a generalized assignment statement.

CONCLUSION

- *In conclusion, the concept of a restriction is the centerpiece of restriction-based semantics. The importance of the concept of a restriction derives from the fact that it makes it possible to standardize precisiation of meaning by expressing a precisiated form of p as a restriction.*

THE CONCEPT OF EXPLANATORY DATABASE (ED)

- *In restriction-based semantics, the restricted variable, X, and the restricting relation, R, are described in a natural language. The concept of explanatory database, ED, serves to precisiate the meaning of X and R.*
- *Generally, ED is represented as a collection of relations, with the names of relations drawn, but not exclusively, from the constituents of p. (Zadeh 1984)*

CONTINUED

- *For example, for the proposition, p: Most Swedes are tall, ED may be represented as:*

ED=POPULATION.SWEDES[Name; Height]+TALL[Height;μ]+ MOST[Proportion;μ],

where + plays the role of comma.

CONTINUED

- *In relation to possible-world semantics, ED may be viewed as the description of a collection of possible-worlds, with the understanding that an instantiated ED is the description of a possible-world.*

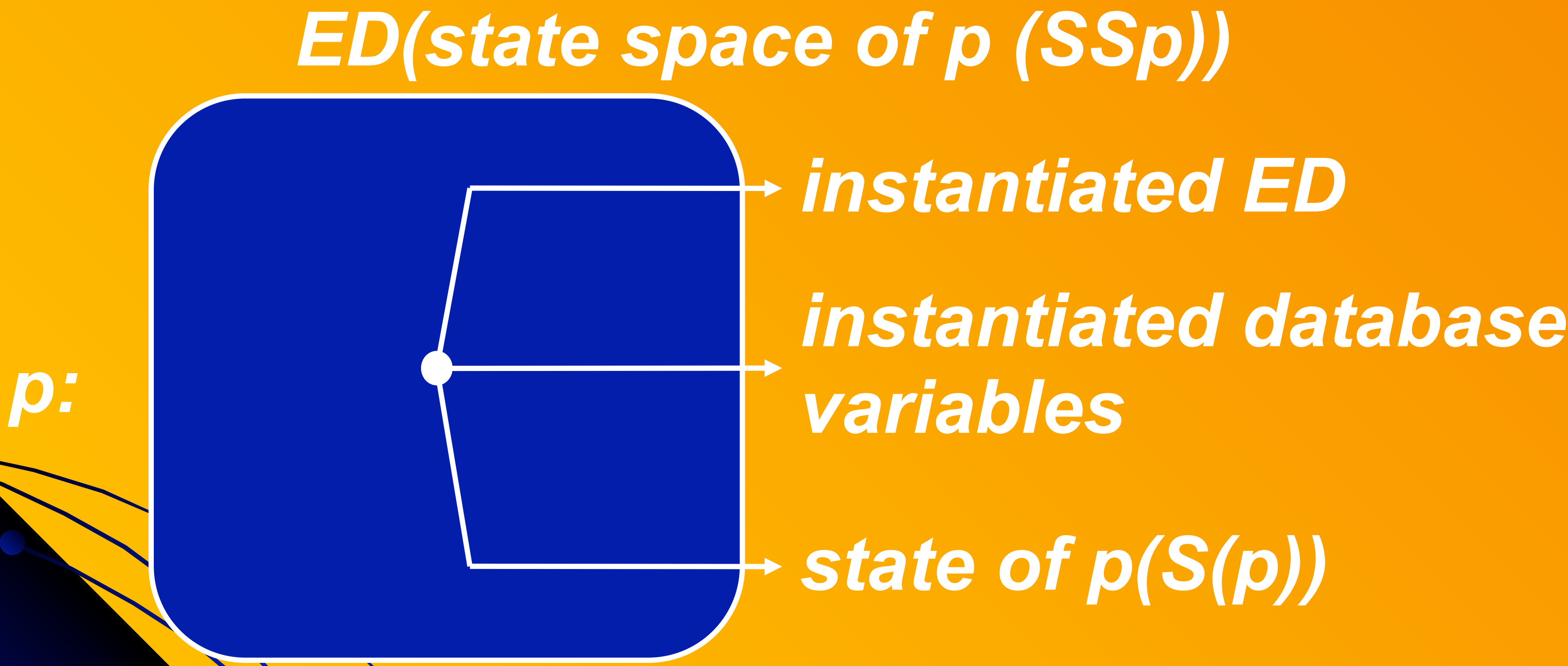
CONTINUED

- ***More generally, an instantiated ED may be viewed as the description of a scenario.***
- ***In the spirit of Carnap, (<u>Meaning and Necessity</u>. Chicago: University of Chicago Press, 1952) an instantiated ED may be viewed as a state of p, with ED playing the role of the state space of p, SS(p).***

CONTINUED

- *It is important to note that relations in ED are uninstantiated, that is, the values of database variables—entries in relations—are not specified. A database variable may be a scalar variable, and n-ary variable, a function or a relation. As an illustration, the database variables in p: Most Swedes are tall, are μ_{tall}, μ_{most} and $h_1, \ldots, h_n$, where h_i is the height of $Name_i$, $i = 1, \ldots, n$. Instantiated database variables constitute a state of p.*

SUMMARY


ED(state space of p (SSp))
p:
instantiated ED
instantiated database variables
state of p(S(p))

THE CONCEPT OF A PRECISIATED CANONICAL FORM, CF*(p)

- *After X and R have been identified and the explanatory database, ED, has been constructed, X and R may be defined as functions of ED. As was noted earlier, definition of X and R may be viewed as precisiation of X and R. Precisiated X and R are denoted as X* and R*, respectively.*

CONTINUED

- *A canonical form, CF*(p), with precisiated values of X and R, X* and R*, will be referred to as a precisiated canonical form.*
- *In the following, construction of the precisiated canonical form of p is discussed in greater detail.*

FROM p TO CF*(p):
X* isr R*

- The concepts discussed so far provide a basis for a relatively straightforward procedure for constructing the precisiated canonical form of a given proposition, p. The precisiated canonical form may be viewed as a computational model of p. Effectively, the precisiated canonical form may be interpreted as a representation of precisiated meaning of p.

- ***A summary of the procedure for computing the precisiated canonical form of p is presented in the following.***

PROCEDURE

- **Step 1. Clarification**

  The first step is clarification, if needed, of the meaning of p. This step requires world knowledge.

Examples:

- **Overeating causes obesity** $\xrightarrow{\text{clarification}}$

  Most of those who overeat are obese.

- **Obesity is caused by overeating** $\xrightarrow{\text{clarification}}$

  Most of those who are obese, overeat.

CONTINUED

- *Young men like young women* —clarification→ *Most young men like mostly young women.*
- *Swedes are much taller than Italians* —clarification→ *Most Swedes are much taller than most Italians.*
- *Step 2. Identification (explicitation) of X and R.*

Identify the constrained variable, X, and the corresponding constraining relation, R.

CONTINUED

- ***Step 3. Construction of ED.***

What information is needed—but not necessarily minimally—to precisiate (define) X and R? An answer to this question identifies the explanatory database, ED. Equivalently, ED may be viewed as an answer to the question: What information is needed—but not necessarily minimally—to compute the truth-value of p?

CONTINUED

- ***Step 4. Precisiation of X and R.***

  How can the information in ED be used to precisiate the values of X and R? This step leads to precisiated values of X and R, X* and R*, and thus results in the precisiated canonical form, CF*(p).

- ***Precisiated X* and R* may be expressed as functions of ED and, more specifically, as functions of database variables, v_1, …, v_n.***

A KEY POINT

- *It is important to observe that in the case of possibilistic constraints, CF*(p) induces a possibilistic constraint on database variables, $v_1, \ldots, v_n$, in ED. This constraint may be interpreted as the possibility distribution of database variables in ED or, equivalently, as a possibility distribution on the state space, SS(p), of p—a possibility distribution which is induced by p. The possibility distribution induced by p may be viewed as the intension of p.*

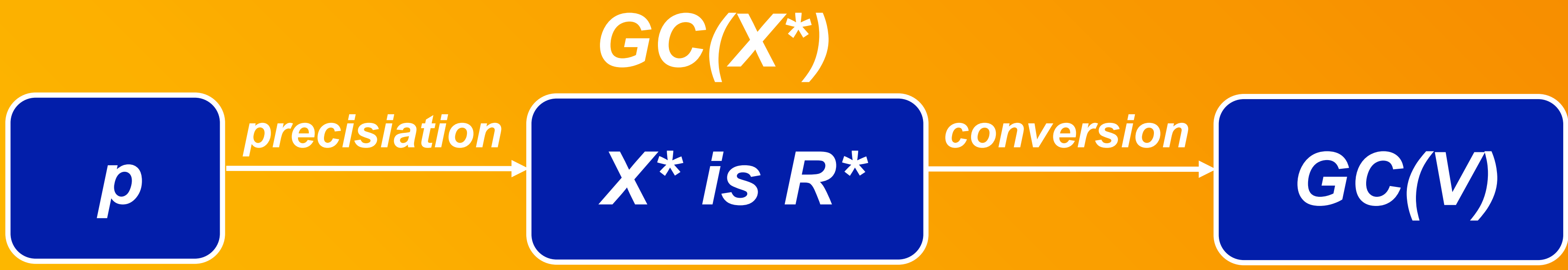
CONTINUED

- ***Step 5. (Optional) Computation of truth-value of p. The truth-value of p depends on ED. The truth-value of p, t(p, ED), may be computed by assessing the degree to which the generalized constraint, X* isr R*, is satisfied. It is important to observe that the possibility of an instantiated ED given p is equal to the truth value of p given instantiated ED (Zadeh 1981).***
- ***End of procedure.***

NOTE

- ***It is important to note that humans have no difficulty in learning how to use the procedure. The principal reason is: Humans have world knowledge. It is hard to build world knowledge into machines.***

SUMMARY



- The generalized constraint on X*, GC(X*), induces (converts into) a generalized constraint, GC(V), on the database variables, $V=(v_1, \ldots, v_n)$. For possibilistic constraints, GC(V) may be expressed as:

$$f(V) \text{ is } A$$

where f is a function of database variables and A is a fuzzy relation (set) in the space of database variables.

EXAMPLE

- *Note. In the following example r=blank, that is, the generalized constraints are possibilistic.*

1. p: Most Swedes are tall

Step 1. Clarification. Clarification not needed

Step 2. Identification (explicitation) of X and R.

X is identified as the proportion of tall Swedes among Swedes.

CONTINUED

Correspondingly, R is identified as Most.

Digression.

In fuzzy logic, proportion is defined as a relative ΣCount. (Zadeh 1983) More specifically, if A and B are fuzzy sets in U, U={u_1, …, u_n}, the ΣCount (cardinality) of A is defined as:

$$\Sigma Count(A) = \Sigma_i \mu_A(u_i)$$

CONTINUED

The relative ΣCount of B in A is defined as:

$$\Sigma Count(B/A) = \frac{\Sigma Count(A \cap B)}{\Sigma Count(A)}$$

$$= \frac{\Sigma_i(\mu_A(u_i) \wedge \mu_B(u_i))}{\Sigma_i \mu_A(u_i)}$$

where $\cap$ *=intersection and* $\wedge$ *=min*

CONTINUED

In application to the example under consideration, assume that the height of ith Swede, $Name_i$, is h_i and that the grade of membership of h_i in tall is $\mu_{tall}(h_i)$, i=1, …, n. X may be expressed as:

$$X = \frac{1}{n}\left(\Sigma_i \mu_{tall}(h_i)\right)$$

Step 3. Construction of ED.
The needed information is contained in the explanatory database, ED, where

CONTINUED

ED= POPULATION.SWEDES[Name; Height]+
TALL[Height; μ]+
MOST[Proportion; μ]

Step 4. Precisiation of X and R.
In relation to ED, precisiated X and R may be expressed as:

$$X^* = \frac{1}{n}\left(\sum_i \mu_{tall}(h_i)\right)$$

R* = MOST[Proportion; μ]

CONTINUED

- The precisiated canonical form is expressed as:

CF*p=X* is R*

where

$$X^* = \frac{1}{n}\left(\sum_i \mu_{tall}(h_i)\right)$$

R* = MOST[Proportion; µ]

CONTINUED

Step 5. The truth-value of p, t(p, ED), is the degree to which the constraint in Step 4 is satisfied. More concretely,

$$t(p,ED) = \mu_{most}\left(\frac{1}{n}\Sigma_i \mu_{tall}(h_i)\right)$$

Note. The right-hand side of this equation may be viewed as a constraint on database variables $h_1, \ldots, h_n$, μ_{tall} and μ_{most}.

SUMMATION

- ***Natural languages are pervasively imprecise, especially in the realm of meaning. The primary source of imprecision is unsharpness of class boundaries. In this sense, words, phrases, propositions and commands in natural languages are preponderantly imprecise.***

CONTINUED

- ***Precisiation of meaning is a prerequisite to achievement of higher levels of mechanization of natural language understanding. Precisiation of meaning plays a particularly important role in communication between humans and machines. Furthermore, precisiation of meaning is a prerequisite to problem-solving with information which is described in a natural language.***

CONTINUED

- *Despite its intrinsic importance, precisiation of meaning has drawn little, if any, attention within linguistics and computational linguistics. There is a reason. In large measure, theories of natural languages are based on bivalent logic.*

APPENDIX

INFORMAL EXPOSITION OF GCS—CLARIFICATION DIALOGUE

- ***The basic ideas which underlie precisiation of meaning and, more particularly, generalized-constraint-based semantics, are actually quite simple. To bring this out, it is expedient to supplement a formal exposition of GCS with an informal narrative in the form of a dialogue between Robert and Lotfi. In large measure, the narrative is self-contained.***

DIALOGUE

Robert: Lotfi, generalized-constraint-based semantics looks complicated to me. Can you explain in simple terms the basic ideas which underlie GCS?

CONTINUED

Lotfi: I will be pleased to do so. Let us start with an example, p: Most Swedes are tall. p is a proposition. As a proposition, p is a carrier of information. Without loss of generality, we can assume that p is a carrier of information about a variable, X, which is implicit in p. If I asked you what is this variable, what would you say?

CONTINUED

Robert: As I see it, p tells me something about the proportion of tall Swedes among Swedes.

Lotfi: Right. What does p tell you about the value of the variable?

Robert: To me, the value is not sharply defined. I would say it is fuzzy.

Lotfi: So what is it?

Robert: It is the word "most."

CONTINUED

Lotfi: You are right. So what we see is that p may be interpreted as the assignment of a value "most" to the variable, X: Proportion of tall Swedes among Swedes.

CONTINUED

As you can see, a basic difference between a proposition drawn from a natural language and a proposition drawn from a mathematical language is that in the latter the variables and the values assigned to them are explicit, whereas in the former the variables and the assigned values are implicit.

CONTINUED

There is an additional difference. When p is drawn from a natural language, the assigned value is not sharply defined—typically it is fuzzy, as "most" is. When p is drawn from a mathematical language, the assigned value is sharply defined.

Robert: I get the idea. So what comes next?

CONTINUED

Lotfi: There is another important point. When p is drawn from a natural language, the value assigned to X is not really a value of X—it is a constraint (restriction) on the values which X is allowed to take. This suggests an unconventional definition of a proposition, p, drawn from a natural language. Specifically, a proposition is an implicit constraint on an implicit variable.

CONTINUED

I should like to add that the constraints which I have in mind are not standard constraints—they are so-called generalized constraints.

CONTINUED

Robert: What is a generalized constraint? Why do we need generalized constraints?

Lotfi: A generalized constraint is expressed as:

$$X \text{ isr } R$$

CONTINUED

where X is the constrained variable, R is the constraining relation—typically a fuzzy set—and r is an indexical variable which defines how R constrains X. Let me explain why the concept of a generalized constraint is needed in precisiation of meaning of a proposition drawn from a natural language.

CONTINUED

Standard constraints are hard in the sense that they have no elasticity. In a natural language, meaning can be stretched. What this implies is that to represent meaning, a constraint must have elasticity. To deal with richness of meaning, elasticity is necessary but not sufficient. Consider the proposition: Usually most flights leave on time.

CONTINUED

What is the constrained variable and what is the constraining relation in this proposition? Actually, for most propositions drawn from a natural language a large repertoire of constraints is not necessary. What is sufficient are three so-called primary constraints and their combinations. The primary constraints are: possibilistic, probabilistic and veristic.

Here are simple examples of primary constraints:

- *Possibilistic constraint:*

  Robert is possibly French and possibly German

- *Probabilistic constraint:*

  With probability 0.75 Robert is German

  With probability 0.25 Robert is French

- *Veristic constraint:*

  Robert is three-quarters German and one-quarter French

CONTINUED

The role of primary constraints is analogous to the role of primary colors: red, green and blue. In most cases, constraints are possibilistic. Possibilistic constraints are much easier to manipulate than probabilistic constraints.

CONTINUED

Robert: Could you clarify what you have in mind when you talk about elasticity of meaning?

Lotfi: I admit that I did not say enough. Let me elaborate. In a natural language, meaning can be stretched. Consider a simple example, Robert is young. Assume that young is a fuzzy set and Robert is 30.

CONTINUED

Furthermore, assume that in a particular context the grade of membership of 30 in young is 0.8. To apply young to Robert, the meaning of young must be stretched. To what degree? In fuzzy logic, the degree of stretch is equated to (1 - grade of membership of 30 in young.) Thus, the degree of stretch is 0.2.

CONTINUED

Furthermore, the grade of membership of 30 in young is interpreted as the possibility that Robert is 30, given that Robert is young. What this implies is that the fuzzy set young defines the possibility distribution of the variable Age (Robert). Note that the fuzzy set young is a restriction on the values which the variable Age (Robert) can take.

CONTINUED

It is in this sense that the proposition Robert is young is a possibilistic constraint on Age (Robert).

Now, in a natural language almost all words and phrases are labels of fuzzy sets. What this means is that in a natural language the meaning of words and phrases can be stretched, as in the Robert example.

CONTINUED

It is in this sense that words and phrases in a natural language have elasticity. Another important point. What I have said so far explains why in the realm of natural languages most constraints are possibilistic. This is equivalent to saying what I said already, namely, that in a natural language most words and phrases are labels of fuzzy sets.

CONTINUED

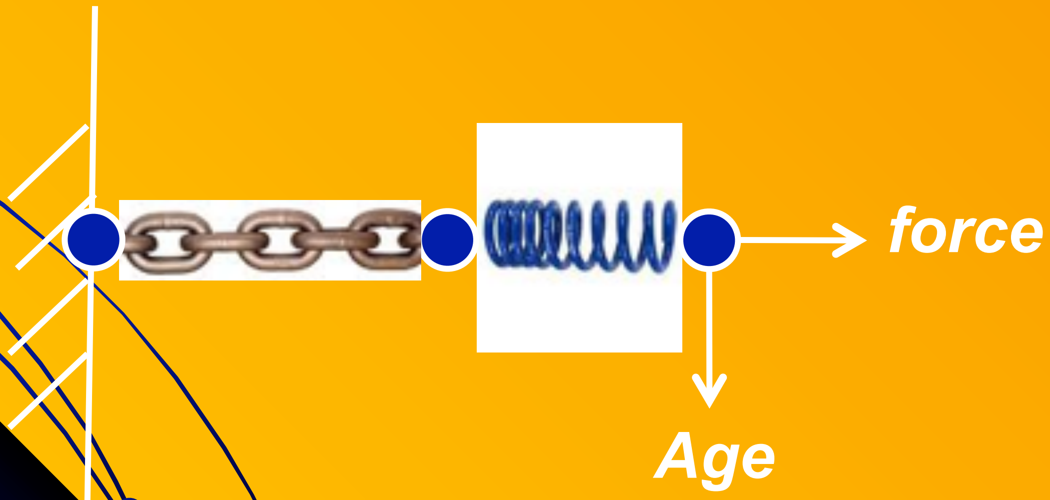
Robert: Many thanks. You clarified what was not clear to me.

CONTINUED

Lotfi: May I add that there is an analogy that may be of assistance. More specifically, the fuzzy set young may be represented as a chain linked to a spring, as shown in the next viewgraph. The left end of the chain is fixed and the position of the right end of the spring represents the value of the variable Age (Robert).

CONTINUED

The force that is applied to the right end of the spring is a measure of grade of membership. Initially, the length of the chain is 0, as is the length of the spring.



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Robert: Many thanks for the explanation. The analogy helps to understand what you mean by elasticity of meaning.

Lotfi: I should like to add that elasticity of meaning is a basic characteristic of natural languages. Elasticity of meaning is a neglected issue in the literatures of linguistics, computational linguistics and philosophy of languages. There is a reason.

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Traditional theories of natural language are based on bivalent logic. Bivalent logic, by itself or in combination with probability theory, is not the right tool for dealing with elasticity of meaning. What is needed for this purpose is fuzzy logic. In fuzzy logic everything is or is allowed to be a matter of degree.

CONTINUED

Robert: Thanks again for the clarification. Going back to where we left of suppose I figured out what is the constrained variable, X, and the constraining relation, R. Is there something else that has to be done?

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Lotfi: Yes, there is. You see, X and R are described in a natural language. What this means is that we are not through with precisiation of meaning of p. What remains to be done is precisiation (definition) of X and R.

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For this purpose, we construct a so-called explanatory database, ED, which consists of a collection of relations in terms of which X and R can be defined. The entries in relations in ED are referred to as database variables. Unless stated to the contrary, database variables are assumed to be uninstantiated.

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Robert: Can you be more specific?

Lotfi: To construct ED you ask yourself the question: What information—in the form of a collection or relations—is needed to precisiate (define) X and R? Looking at p, we see that to precisiate X we need two relations: POPULATION.SWEDES[Name; Height] and TALL[Height; μ].

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In the relation TALL[Height; μ], μ is the grade of membership of a value of Height, h, in the fuzzy set tall. So far as R is concerned, the needed relation is MOST[Proportion; μ], where μ is the grade of membership of a value of Proportion in the fuzzy set Most.

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Equivalently, it is frequently helpful to ask the question: What is the information which is needed to assess the degree to which p is true?

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At this point, we can express ED as the collection:

ED= POPULATION.SWEDES[Name; Height]+
TALL[Height; μ]+
MOST[Proportion; μ]

in which for convenience plus is used in place of comma.

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Robert: So, we have constructed ED for the proposition, p: Most Swedes are tall. More generally, given a proposition, p, how difficult is it to construct ED for p?

Lotfi: For humans it is easy. A few examples suffice to learn how to construct ED. Construction of ED is easy for humans because humans have world knowledge. At this juncture, we do not have an algorithm for constructing ED.

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Robert: Now that we have ED, what comes next?

Lotfi: We can use ED to precisiate (define) X and R. Let us start with X. In words, X is described as the proportion of tall Swedes among Swedes. Let us assume that in the relation POPULATION.SWEDES there are n names. Then the proportion of tall Swedes among Swedes would be the number of tall Swedes divided by n.

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Here we come to a problem. Tall Swedes is a fuzzy subset of Swedes. The question is: What is the number of elements in a fuzzy set? In fuzzy logic, there are different ways of answering this question. The simplest is referred to as the ΣCount. More concretely, if A is a fuzzy set with a membership function μ_A, then the ΣCount of A is defined as the sum of grades of membership in A.

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In application to the number of tall Swedes, the ΣCount of tall Swedes may be expressed as:

$$\Sigma Count(tall.Swedes) = \sum_{i=1}^{n} \mu_{tall}(h_i)$$

where h_i is the height of $Name_i$. Consequently, the proportion of tall Swedes among Swedes may be written as:

$$X = \frac{1}{n}\left(\sum_{i=1}^{n} \mu_{tall}(h_i)\right)$$

CONTINUED

This expression may be viewed as a precisiation (definition) of X in terms of ED. More specifically, X is expressed as a function of database variables h_1, …, h_n, μ_{tall} and μ_{most}.

Precisiation (definition) of R is simpler. Specifically, R=Most, where Most is a fuzzy set. At this point, we have precisiated (defined) X and R in terms of ED.

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Robert: So what have we accomplished?

Lotfi: We started with a proposition, p: Most Swedes are tall. We interpreted p as a generalized (possibilistic) constraint. We identified the constrained variable, X, as the proportion of tall Swedes among Swedes. We identified the constraining relation, R, as a fuzzy set, Most. Next, we constructed an explanatory database, ED.

CONTINUED

Finally, we precisiated (defined) X, R and q in terms of ED, that is, as function of database variables $h_1, \ldots, h_n$, μ_{tall} and μ_{most}. In this way, we precisiated the meaning of p, which was our objective. The precisiated meaning may be expressed as the constraint:

$$\frac{1}{n}\left(\sum_{i=1}^{n}\mu_{tall}(h_i)\right) \text{ is Most}$$

Robert: So, you precisiated the meaning of p. What purpose does it serve?

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Lotfi: The principal purpose is the following. Unprecisiated (raw) propositions drawn from a natural language cannot be computed with. Precisiation is a prerequisite to computation. What is important to understand is that precisiation of meaning opens the door to computation with natural language.

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Robert: Sounds great. I am impressed. However, it is not completely clear to me what you have in mind when you say "opens the door to computation with natural language." Can you clarify it?

Lotfi: With pleasure. Computation with natural language or, more or less equivalently, Computing with Words (CW or CWW), is largely unrelated to natural language processing.

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More specifically, computation with natural language is focused on computation with information described in a natural language. Typically, what is involved is solution of a problem which is stated in a natural language. Let me go back to our example, p: Most Swedes are tall. Given this information, how can you compute the average height of Swedes?

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Robert: Frankly, your question makes no sense to me. Are you serious? How can you expect me to compute the average height of Swedes from the information that most Swedes are tall?

Lotfi: That is conventional wisdom. A mathematician would say that the problem is ill-posed. It appears to be ill-posed for two reasons.

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First, because the given information: Most Swedes are tall, is fuzzy, and second, because you assume that I am expecting you to come up with a crisp answer like "the average height of Swedes is 5' 10." Actually, what I expect is a fuzzy answer—it would be unreasonable to expect a crisp answer.

Robert: Thanks for the clarification. I am beginning to see the point of your question.

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Lotfi: I should like to add a key point. The problem becomes well-posed if p is precisiated. This is the essence of Computing with Words.

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Robert: I am beginning to understand the need for precisiation, but my understanding is not complete as yet. Can you explain how the average height of Swedes can be computed from precisiated p?

Lotfi: Recall that precisiated p is a possibilistic constraint expressed as:

$$\frac{1}{n}\left(\sum_{i=1}^{n} \mu_{tall}(h_i)\right) \quad \text{is Most}$$

CONTINUED

From the definition of a possibilistic constraint it follows that the constraint on X may be rewritten as:

$$t = \mu_{most}\left(\frac{1}{n}\sum_{i=1}^{n}\mu_{tall}(h_i)\right)$$

What this expression means is that given the h_i, μ_{tall} and μ_{most}, we can compute the degree, t, to which the constraint is satisfied.

CONTINUED

It is this degree, t, that is the truth-value of p. Now, here is a key idea. The precisiated p constrains X. X is a function of database variables. It follows that indirectly p constrains database variables. This has important implications. Let me elaborate.

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What we see is that the constraint induced by p on the h_i is of the general form

$$f(h_1, \ldots, h_n) \text{ is Most}$$

What we are interested in is the induced constraint on the average height of Swedes. The average height of Swedes may be expressed as:

$$h_{ave} = \frac{1}{n}\left(\sum_{i=1}^{n} h_i\right)$$

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This expression is of the general form

$$g(h_1, \ldots, h_n) \text{ is } ?h_{ave}$$

where $?h_{ave}$ is a fuzzy set that we want to compute.

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At this stage, we can employ the Extension Principle of fuzzy logic to compute h_{ave}. (Zadeh 1975 I, II & III) In general terms, this principle tells us that from a given possibilistic constraint of the form

$$f(x_1, \ldots, x_n) \text{ is } A$$

in which A is a fuzzy set, we can derive an induced possibilistic constraint on $g(x_1, \ldots, x_n)$,

$$g(x_1, \ldots, x_n) \text{ is } ?B,$$

CONTINUED

in which B is a fuzzy set defined by the solution of the mathematical program

$$\mu_B(v)=\sup_{x_1, \ldots, x_n} \mu_A(f(x_1, \ldots, x_n))$$

subject to

$$v=g(x_1, \ldots, x_n)$$

In application to our example, what we see is that we have reduced computation of the average height of Swedes to the solution of the mathematical program

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$$\mu_B(v)=\sup_{h_1,\ldots,h_n}\mu_{most}(f(h_1,\ldots,h_n))$$

subject to

$$v=\frac{1}{n}\left(\sum_{i=1}^{n}h_i\right)$$

In effect, this is the solution to the problem which I posed to you. As you can see, reduction of the original problem to the solution of a mathematical program is not so simple.

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However, solution of the mathematical program to which the original problem is reduced, is well within the capabilities of desktop computers.

CONTINUED

Robert: I am beginning to see the basic idea. Through precisiation, you have reduced the problem of computation with information described in a natural language—a seemingly ill-posed problem—to a well-posed tractable problem in mathematical programming. I am impressed by what you have accomplished, though I must say that the reduction is nontrivial.

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Without your explanation, it would be hard to see the basic ideas. I can also see why computation with natural language is a move into a new and largely unexplored territory. Thank you for clarifying the import of your statement: precisiation of meaning opens the door to computation with natural language.

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Lotfi: I appreciate your comment. May I add that I believe—but have not verified it as yet—that in closed form the solution to the mathematical program may be expressed as:

$$h_{ave} \text{ is } \geq Most \times Tall$$

where Most × Tall is the product of fuzzy numbers Most and Tall.

Robert: This is a very interesting result, if true. It agrees with my intuition.

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Lotfi: I appreciate your comment. I would like to conclude our dialogue with a prediction. As we move further into the age of machine intelligence and automated reasoning, the complex of problems related to computation with information described in a natural language, is certain to grow in visibility and importance.

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The informal dialogue between Robert and Lotfi has come to an end.