What makes things similar?

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- Objectively everything is equally similar to everything else (Goodman, Watanabe)

- Similarity is a subjective, psychological notion

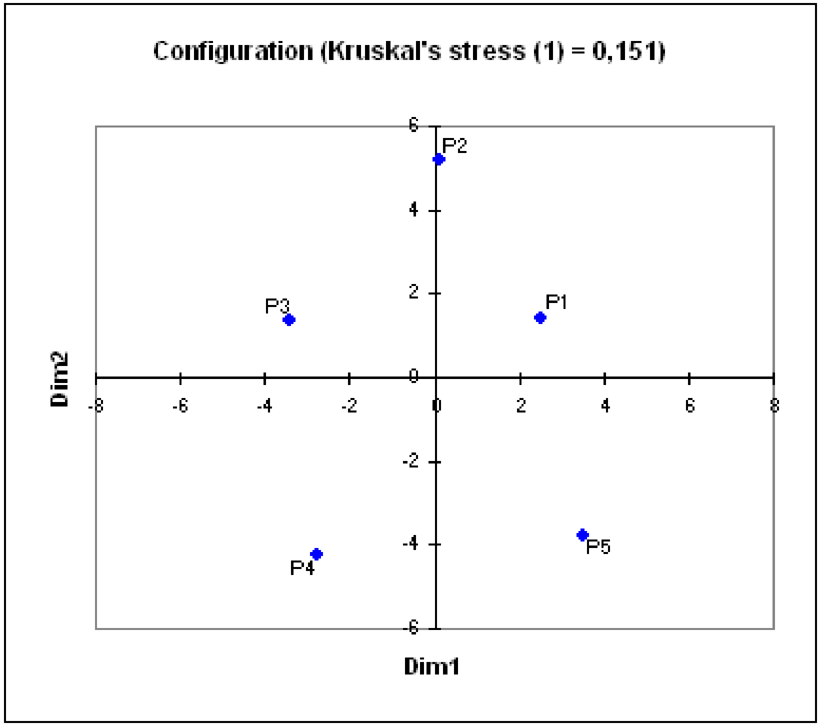
- Similarity is a relationship between mental representations of objects not objects per se

- Much of debate on similarity centers on mental representation

Shepard's spatial model

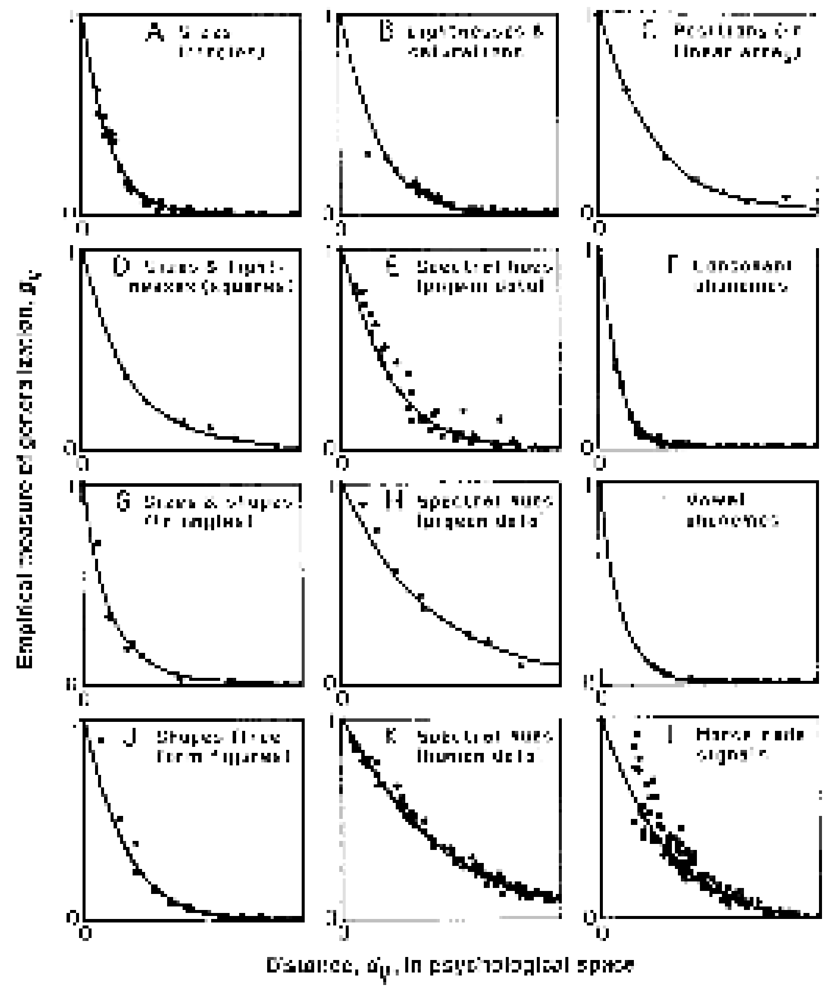
Matrix (Euclidean distance):

	P1	P2	P3	P4	P5
P1	0	5.477	6.633	6.403	7.141
P2	5.477	0	5.477	9.849	8.426
P3	6.633	5.477	0	6.557	7.416
P4	6.403	9.849	6.557	0	7.348
P5	7.141	8.426	7.416	7.348	0



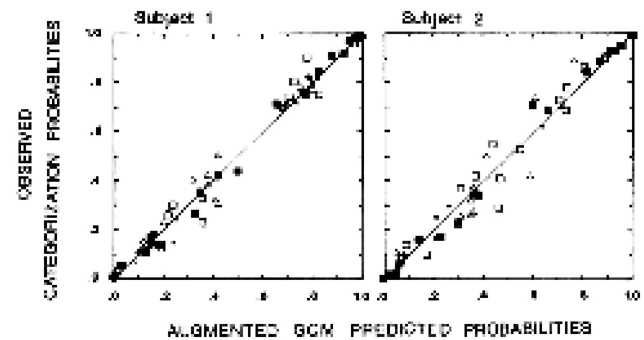
Shepard's (1987) Universal Law of Generalization

- Sizes, shapes,
- Phonemes
- Morse code signals



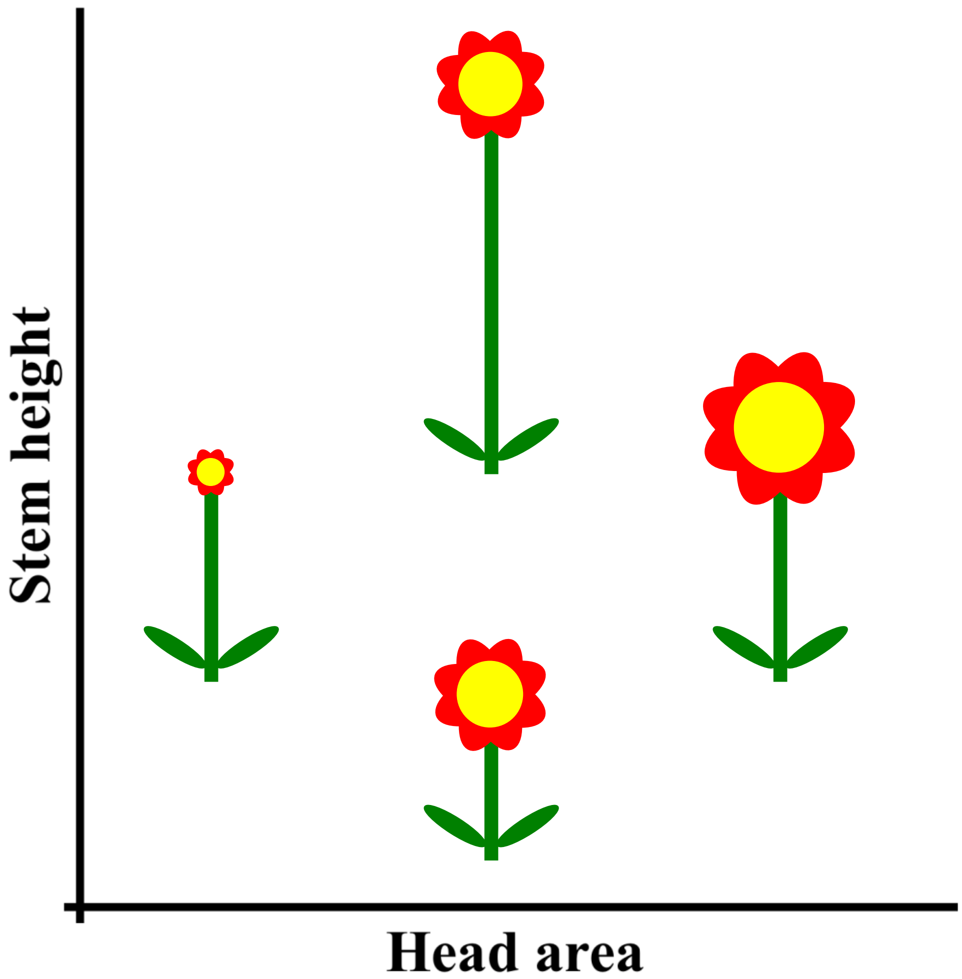
Nosofsky's (1986)
Generalized Context Model

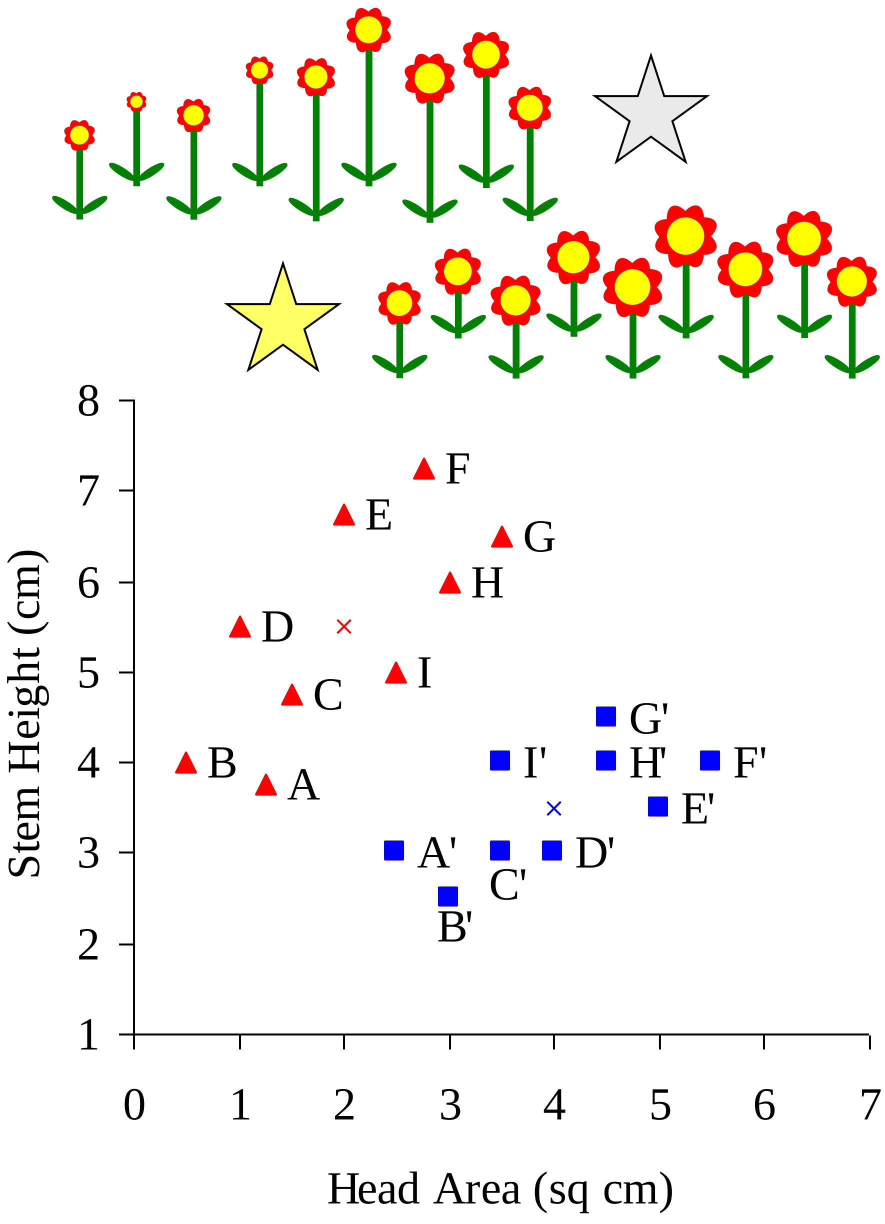
$$P(A) = \frac{sim(A)}{sim(A) + sim(B)}$$



- Extremely successful exemplar model of categorization
- Similarity = exponential function of psych. distance
- Can systematically relate performance across tasks

An Example: Effects of Category Diversity (Hahn et al., 2005)




8
7
6
5
4
3
2
1
0
1
2
3
4
5
6
7
Stem Height (cm)
Head Area (sq cm)
A
B
C
D
E
F
G
H
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A'
B'
C'
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E'
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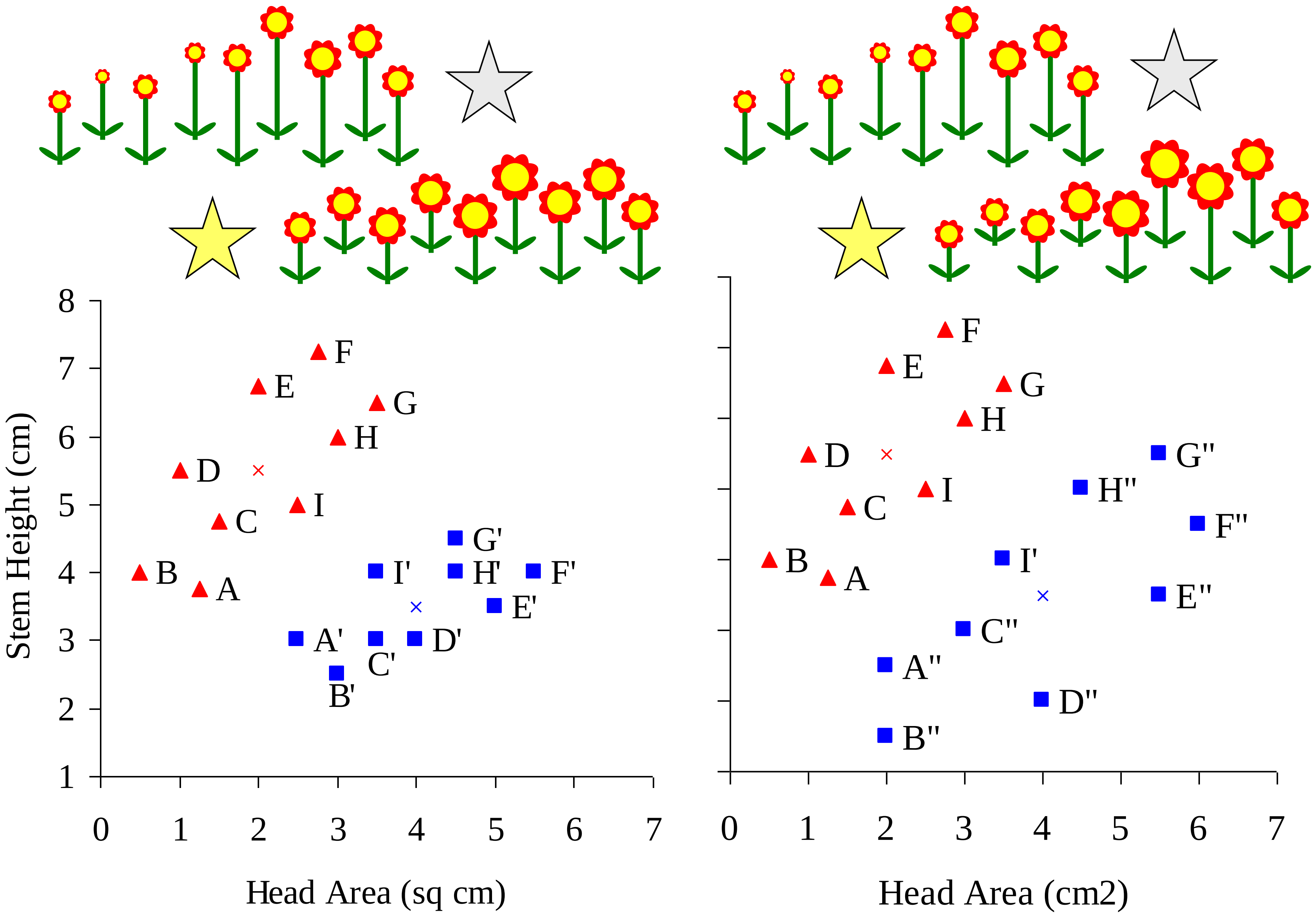
H'
I'



Stem Height (cm)
8
7
6
5
4
3
2
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4
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6
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Head Area (sq cm)
F
E
G
H
D
I
C
B
A
G'
I'
H'
F'
E'
A'
D'
C'
B'
0
1
2
3
4
5
6
7
Head Area (cm2)
F
E
G
H
D
I
C
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I'
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C"
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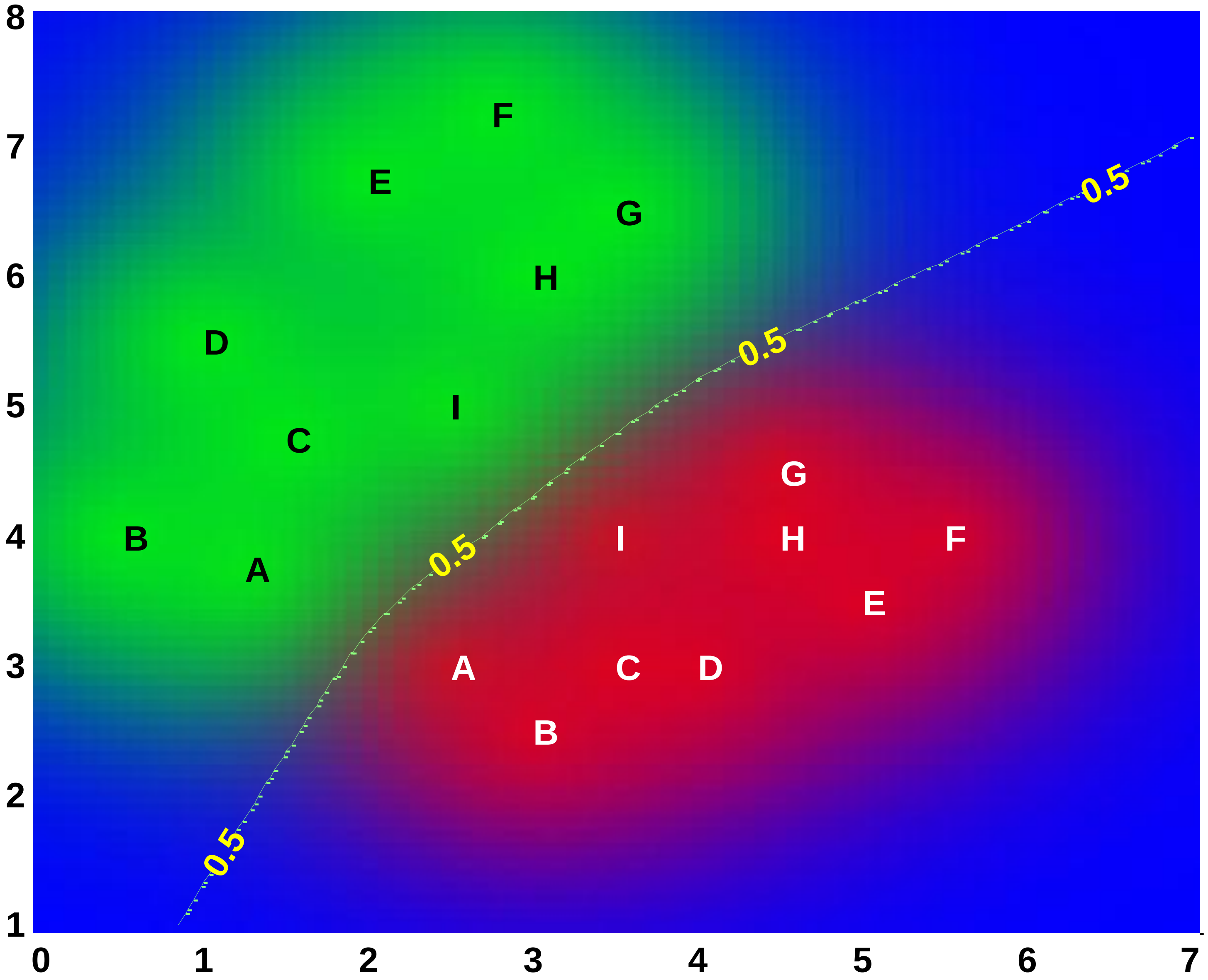
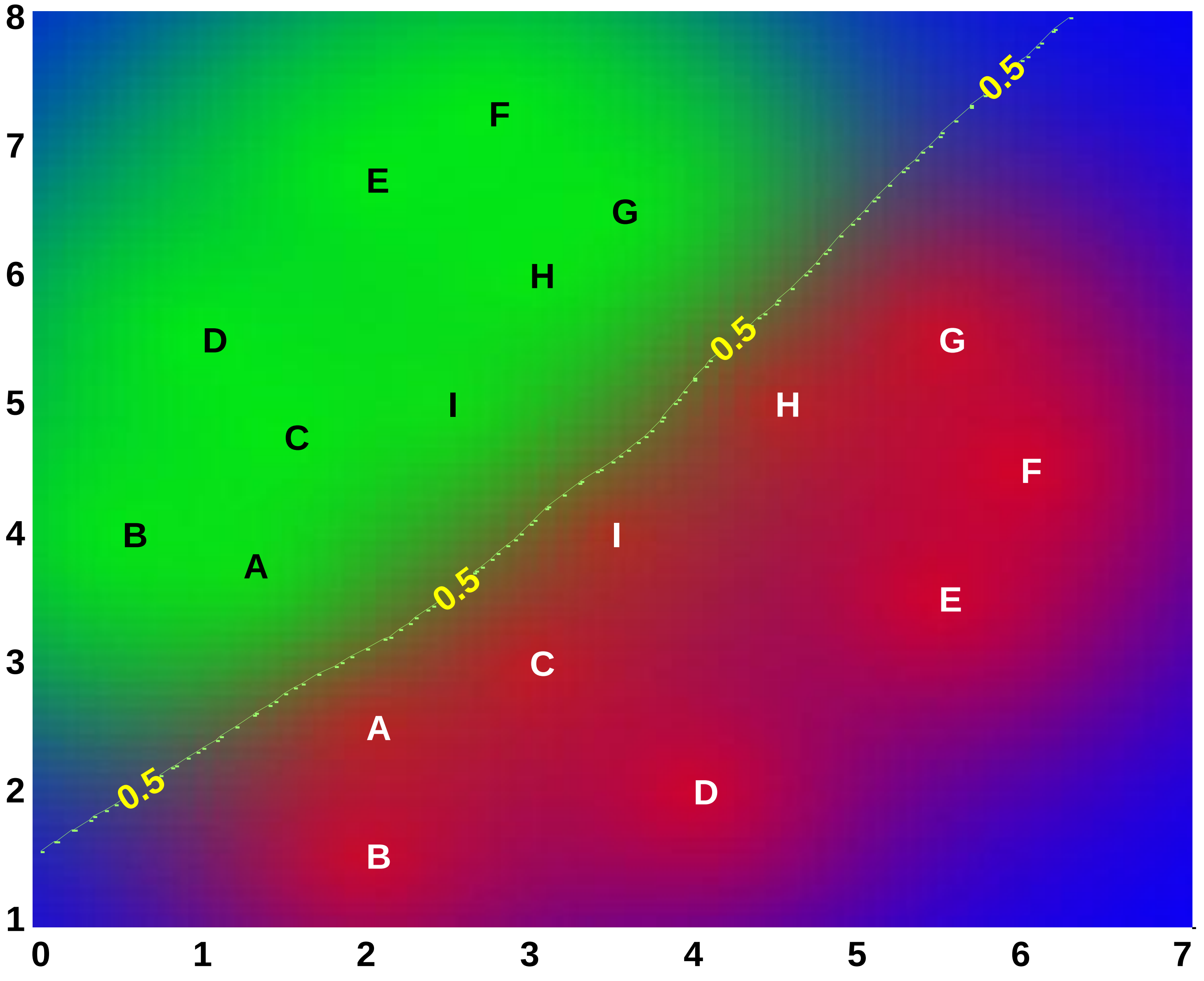
Predicted Probabilities: Clustered

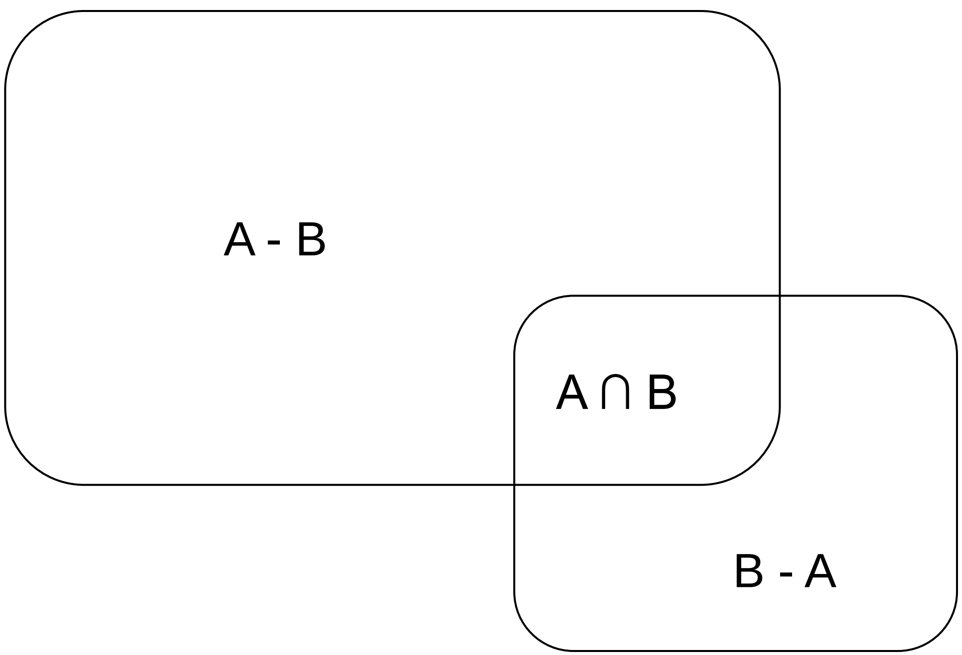


Predicted Probabilities: Diverse



An Alternative View of Similarity: Tversky's (1977) Contrast Model

$$SIM(A,B) = \theta f(A \cap B) - \alpha f(A - B) - \beta f(B - A)$$



Tversky's Critique of Spatial Models

- Fundamental assumptions of spatial similarity models:

  – Minimality
  – Symmetry
  – Triangle Inequality

…true of human similarity judgments??

Assumptions Spatial Models I: Minimality

$$d(x,x) = d(y,y) = 0$$

- Everything is most similar (proximate) to itself
- All things are equally similar to themselves

Spatial Assumptions II: Symmetry

$$d(x,y) = d(y,x)$$

…*X* is as similar to *Y* as *Y* is to *X*

Distinguish:

Non-directional comparisons:

"How similar **<u>are</u>** *X* and *Y*?"

Directional comparisons:

"How similar is *X* **<u>to</u>** *Y*?"

Violations of Symmetry?

in directional comparisons distinctive features of one object weighted more heavily than those of the other (i.e., $\alpha > \beta$).

This will give rise to asymmetries, if the objects differ in salience or complexity, $f(A) \neq f(B)$.

$$SIM(A,B) = \theta f(A \cap B) - \alpha f(A - B) - \beta f(B - A)$$

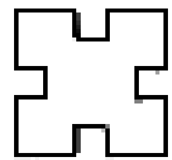
- Nb. Such differential salience also means differences in self-similarity (minimality!)

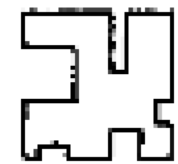
Examples

- Violations?
  - Rosch's (1975) "A is essentially B" study:
    - a line of 85° is almost vertical. A vertical line is not almost 85°.
    - a penguin is more similar to a robin than a robin to a penguin.
  - Similarity of good and bad forms (Tversky, 1977)

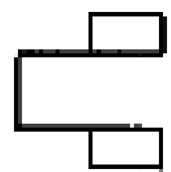
Asymmetries Tversky (1977)

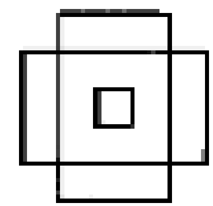
"Goodness of Form"





"Complexity"

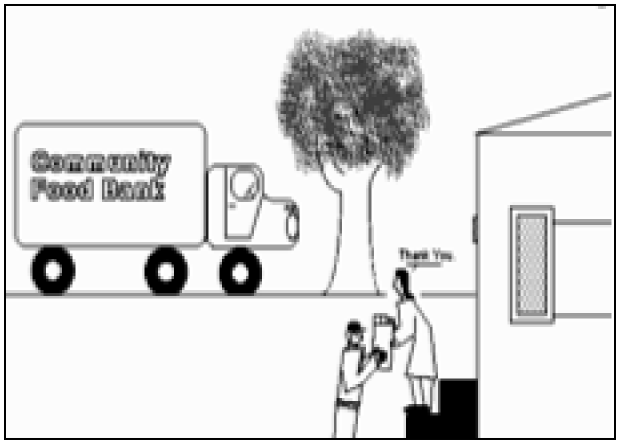


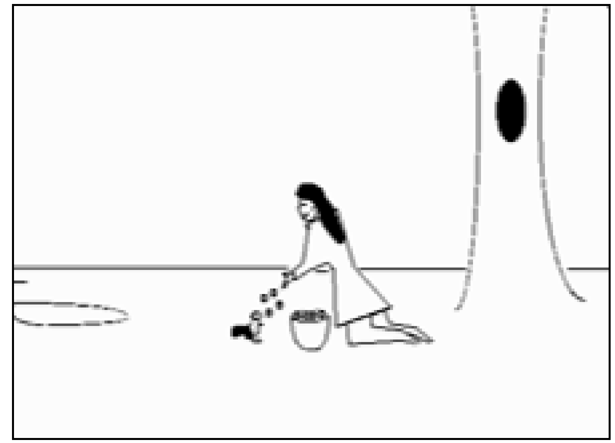


…give rise to asymmetries as predicted

$$SIM(A,B) = \theta f(A \cap B) - \alpha f(A - B) - \beta f(B - A)$$

Limitations of Spatial and Contrast Models





- Comparing naturalistic scenes requires features *and* relations

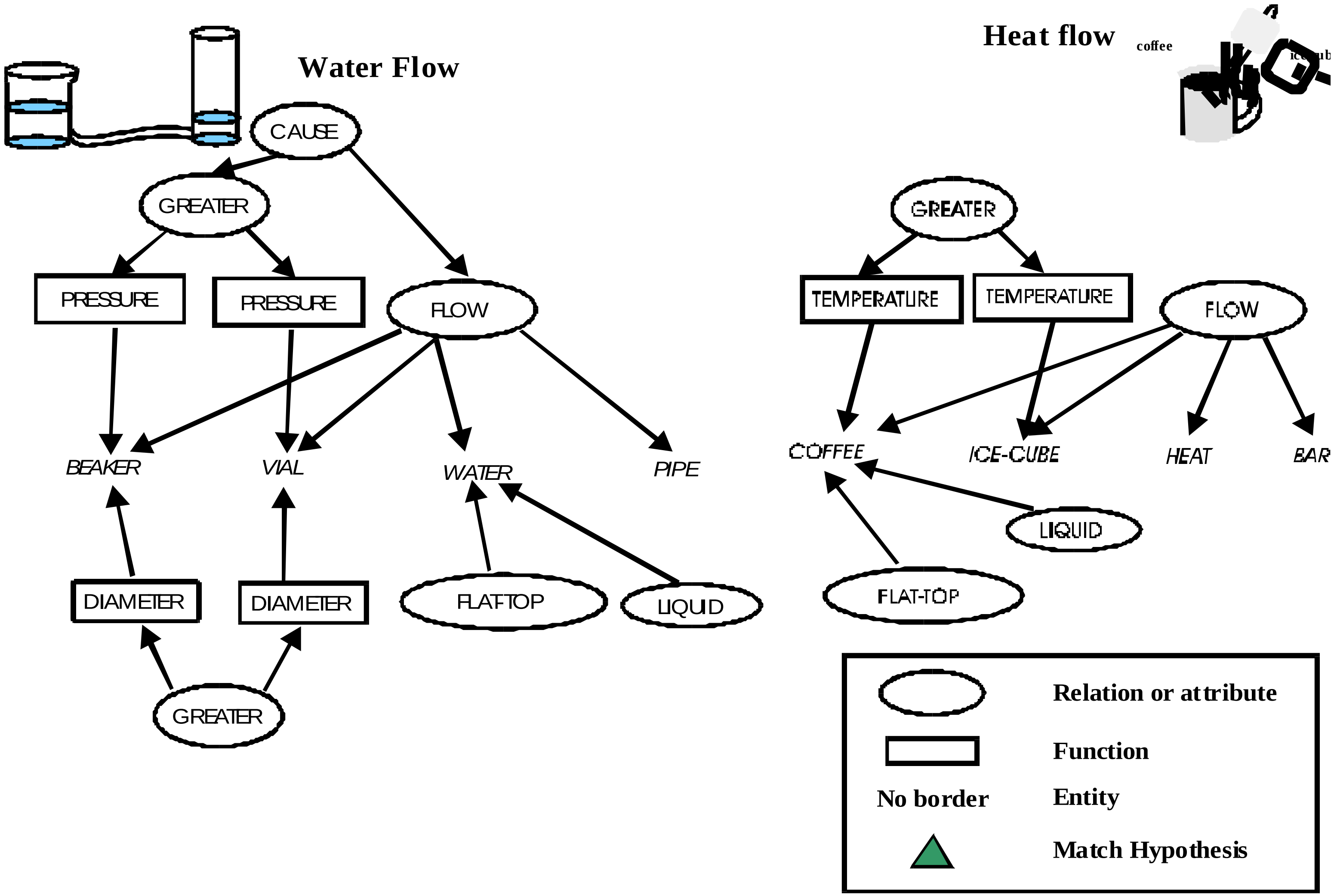
- naturalistic stimuli require **structured** representations (see Biederman, 1995; Fodor, 1999)

Structural Alignment

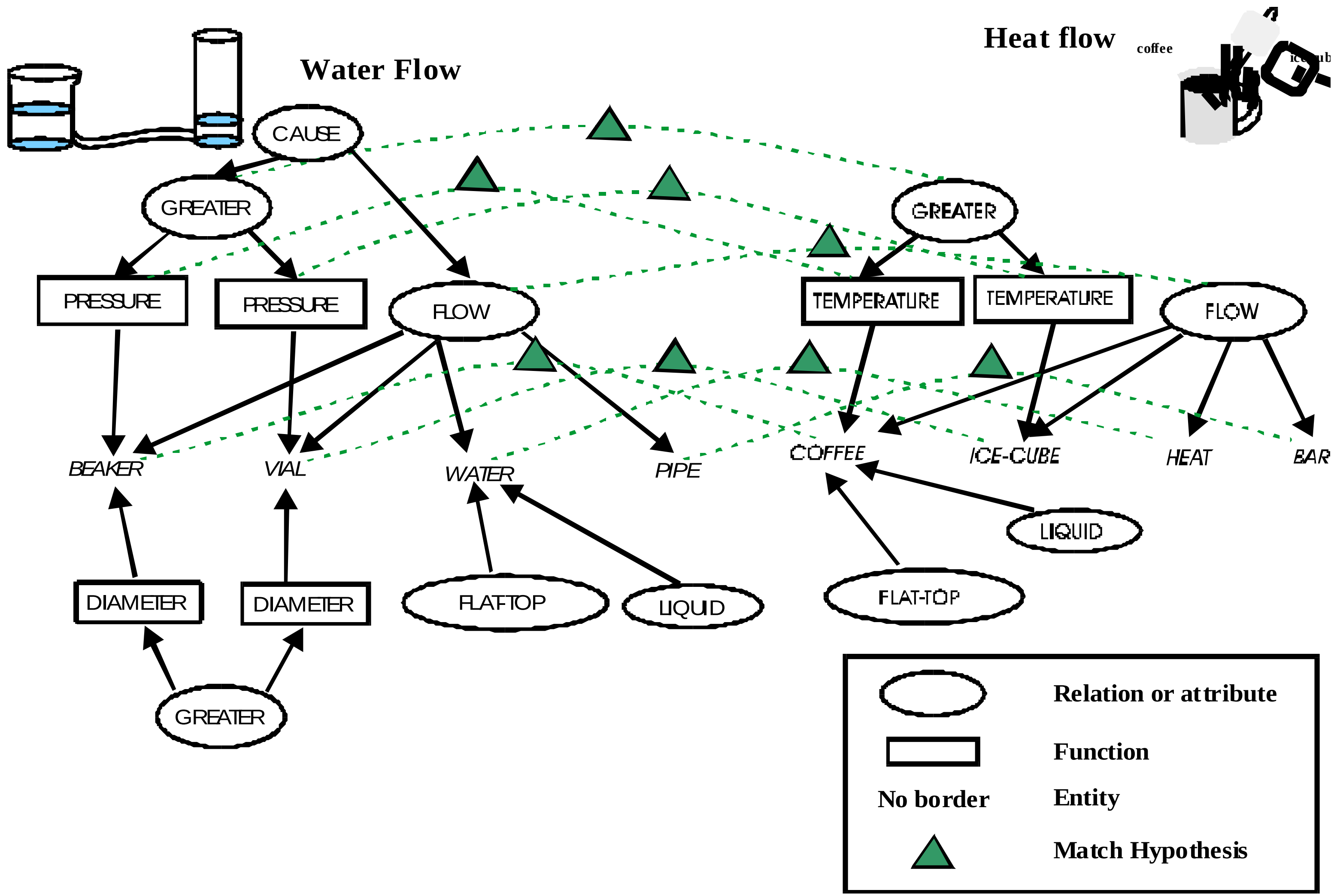
- Markman & Gentner (1990):
  - Similarity comparisons made in a manner akin to analogical mapping (Markman & Gentner, 1993a, 1993b).

  - Similarity involves mapping between corresponding features and relations.

Water flow is like heat flow….

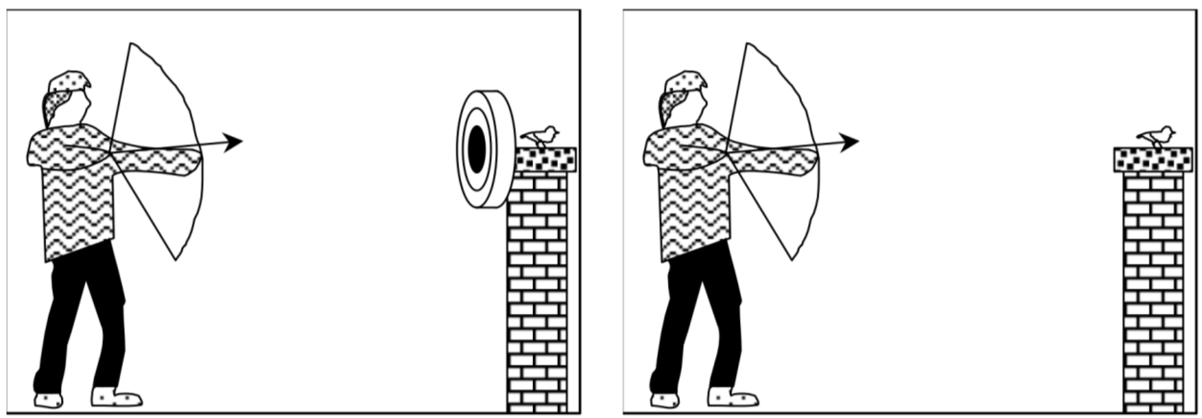


Mapping between descriptions



Experimental evidence for structural alignment

- Matches in place (MIPS) vs. Matches out of place (MOPS)



- Similarity increased more by MIPS than MOPS

Gentner & Markman, 1997; Goldstone, 1994

Transformational Approach to Similarity

similarity between objects depends on ease with which we can transform mental representation of one object into that of the other

"Representational Distortion" (RD)

(Hahn & Chater, 1997; Chater & Hahn, 1997; Chater & Vitányi, 2003; Hahn et al., 2003; Hahn et al. 2009).

Similarity as Transformations

- Kolmogorov complexity theory (Li & Vitanyi, 1993).
- In computational terms, the complexity of a transformation is the shortest program that can execute it.
- simpler transformations can be specified with shorter codes

Similarity as Transformations

‘1 2 3 4’ and ‘2 3 4 5’ (*add 1 to each number*).

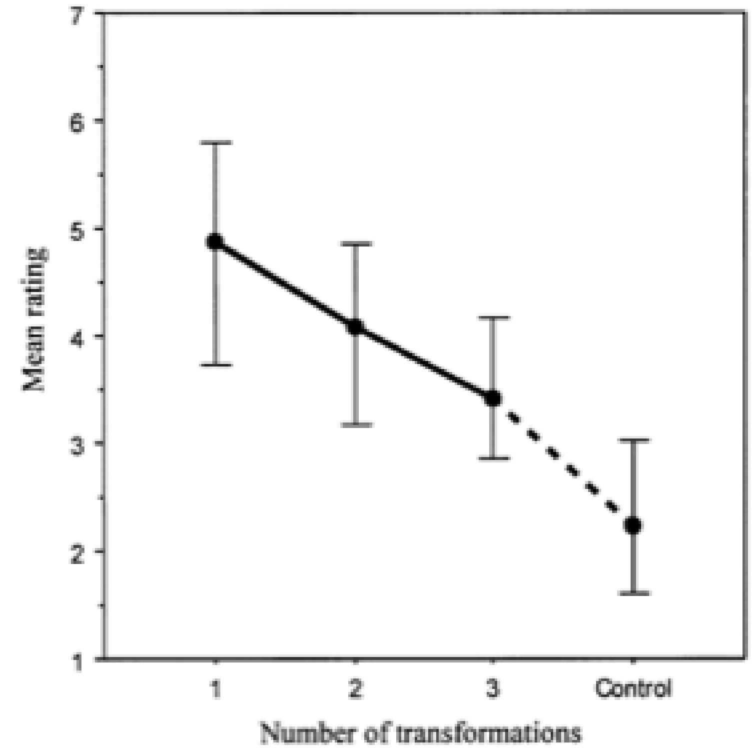
versus

‘1 2 3 4’ and ‘3 5 7 9’ (*multiply by two and add one*)

Hahn, Chater & Richardson (2003)

- Reversal ○●●●●●○○●○○○
- Mirror ○●●○●○○○●●●○ ¦ ○●●●○○○●○●●○
- Phase shift ●○○●●○●○●○●○
- Deletion ●●○●●●○●●○●○

No. of transformations	Type	Stimuli: Item one	Stimuli: Item two
1	Reversal	●○○○○○●●○●●● ●●●○●○●○●○●● ●●●●○○●●●●○○ ○○●●○●○●○○○○	○●●●●●○○●○○○ ○○○●○●○●○●○○ ○○○○●●○○○○●● ●●○○●○●○●●●●
1	Mirror	○●●●○○○●○●●○ ○○○●○○○●●○●● ●●○○●●●○○○○○ ●○○●○○●○○○●○	○●●○●○○○●●●○ ●●○●●○○○●○○○ ○○○○○●●●○○●● ○●○○○●○○●○○●
1	Phasic	●●○○●●○●○●○● ●●●●○○○○●●●● ●○○○○○○○●●●● ●○○●●○○●●○○●	●○○●●○●○●○●○ ○●●●●○○○○●●● ●●○○○○○○○●●● ●●○○●●○○●●○○
1	Deletion	●●○●●●○●●○●○ ●●●○○●○●○●●● ○●○●○●○●○●○● ○○○○●●●●○○○○	○●●●○●●○●○ ●●○○●○●○●●● ○●○●○●○●○● ○○○○●●●○○○○
2	Reversal & Mirror	●○○○●○●○●○○● ●○○○●●○○●○●● ●●●●○●●●○●○● ●●●○○●●●○●○●	○●●○●○●○●●●○ ○○●○●●○○●●●○ ○●○●○○○●○○○○ ○●○●○○○●●○○○
2	Deletion & Mirror	●●○○○●●○○○○● ○●●○●●○●○○●● ●●○●○○●○●○○○ ○●●●●○○●●●●●	●○○○○●●○○○ ●●○○●○●●○●● ○○●○●○○●○●● ●●●○○●●●●○
2	Reversal & Phasic	●●○●●○●●○●●○ ○○○●○○○●○○○● ○○○○○●○○○○○○ ●○○●●●●●●●●●	○●○○●○○●○○●○ ●○●●●○●●●○●● ●●●●●●●●○●●● ○○○○○○○●●○○○
2	Insertion & Phasic	●●●●○○●●●●● ○●●○○●○●○●● ●●○●●●●○●● ●●●○●●●○●●●	○○●●●●●○○●●● ●○●○●●○○●○●○ ●●●○●●●●○●●● ●●○●●●○●●●○●
3	Deletion, Reversal & Phasic	○○○●●○○○●●○○ ●○●○●●○○○●○● ●○○○○●○○○○○● ●●●○●●●○●●●○	○●●●○○●●●○○ ●○●○●○○●●●○ ●●○●●●●●○●● ○●○○○●○○○●○
3	Deletion, Reversal & Mirror	●○○●○●○○●○○○ ●○○●●●○●●○○● ○○○○●●●●●●○● ○●○●○○○○●●●●	●●●○●●○●○●● ○●●○○●○○○●● ○●○○○○○○●● ○○○○●●●●○●○
3	Insertion, Reversal & Phasic	●○○○●○●○○○● ●○○●○○●○○● ●●○○●●●●○○● ○○○●●○○○●●○	○●○●●●○●○●●● ●○●●○●●○●●○● ○○○○●●○○○○●● ○○●●●○○●●●○○
3	Insertion, Reversal & Mirror	○●●●○○●●●●● ○○○●●○○○○● ●○●●○●○○●● ●○●○○●○●○○○	○○○○○●●○○○○● ○●●●●○○●●●○○ ○○●●○●○○●○○● ●●●○●○●●○●○○
Control	-----	●●●●●●●●●●●● ○○●○●○○○●○●○ ○●●●●●○○○○○○ ●●○●○●○●○●●● ●●●●●○○○●○●○ ○○○●○○○●○○○● ○●●●●○●●●●○○ ●●○○●○○●●○●●	○●●○○○●○●●○● ○●○○○○○●○○●● ○●●○○●○○●○○● ○●●●●●○○○●●○ ○●○○●●○○○●○○ ●○●○○○○○●●●○ ●○○●○●●○○●○● ○●●●●●●●○●○○



Significant correlation between similarity & transformational distance Spearman's rho = -0.69 (P<0.005)

Featural model fared worse: Spearman's rho = -0.28 (P<0.05)

Exp. 2

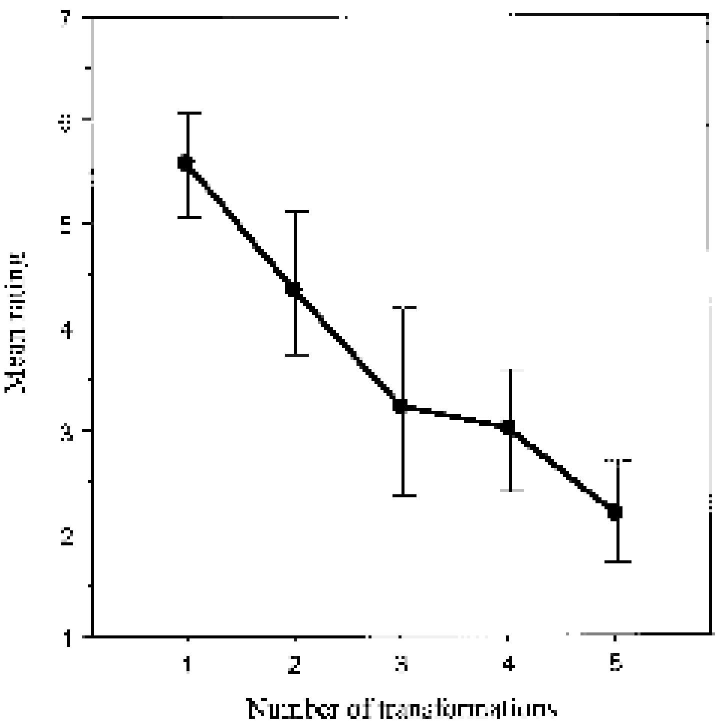
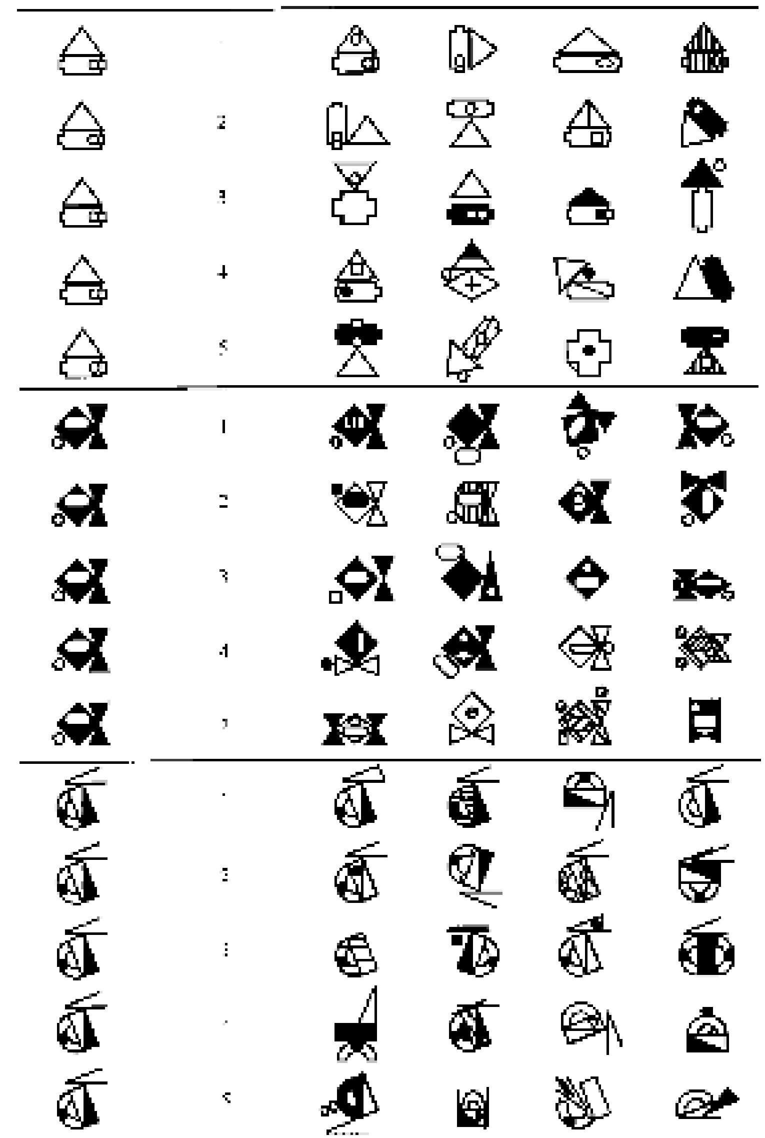
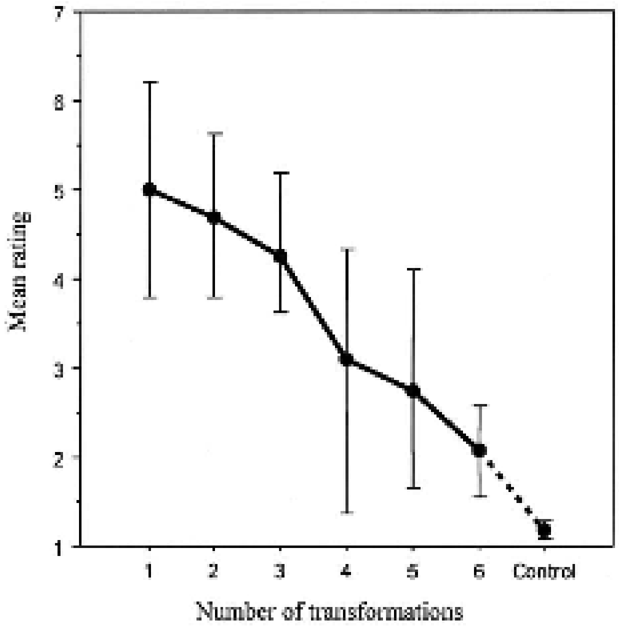


Table 2
Stimuli used in Experiment 2

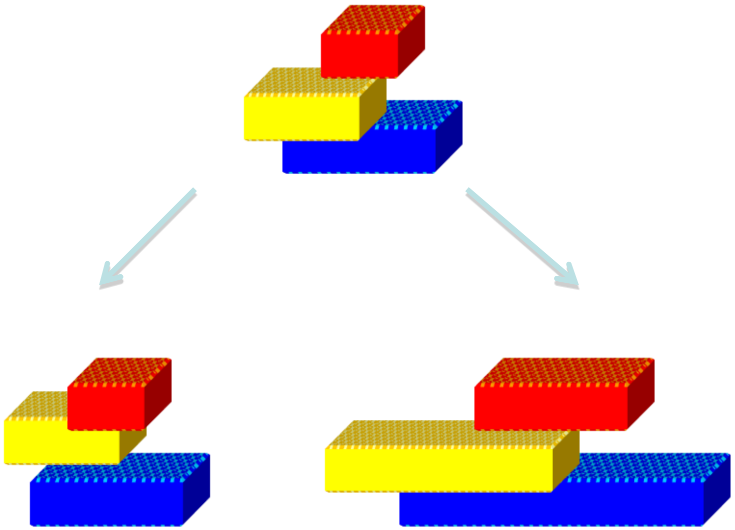
Original stimulus	No. of transformations	Stimuli



Exp. 3



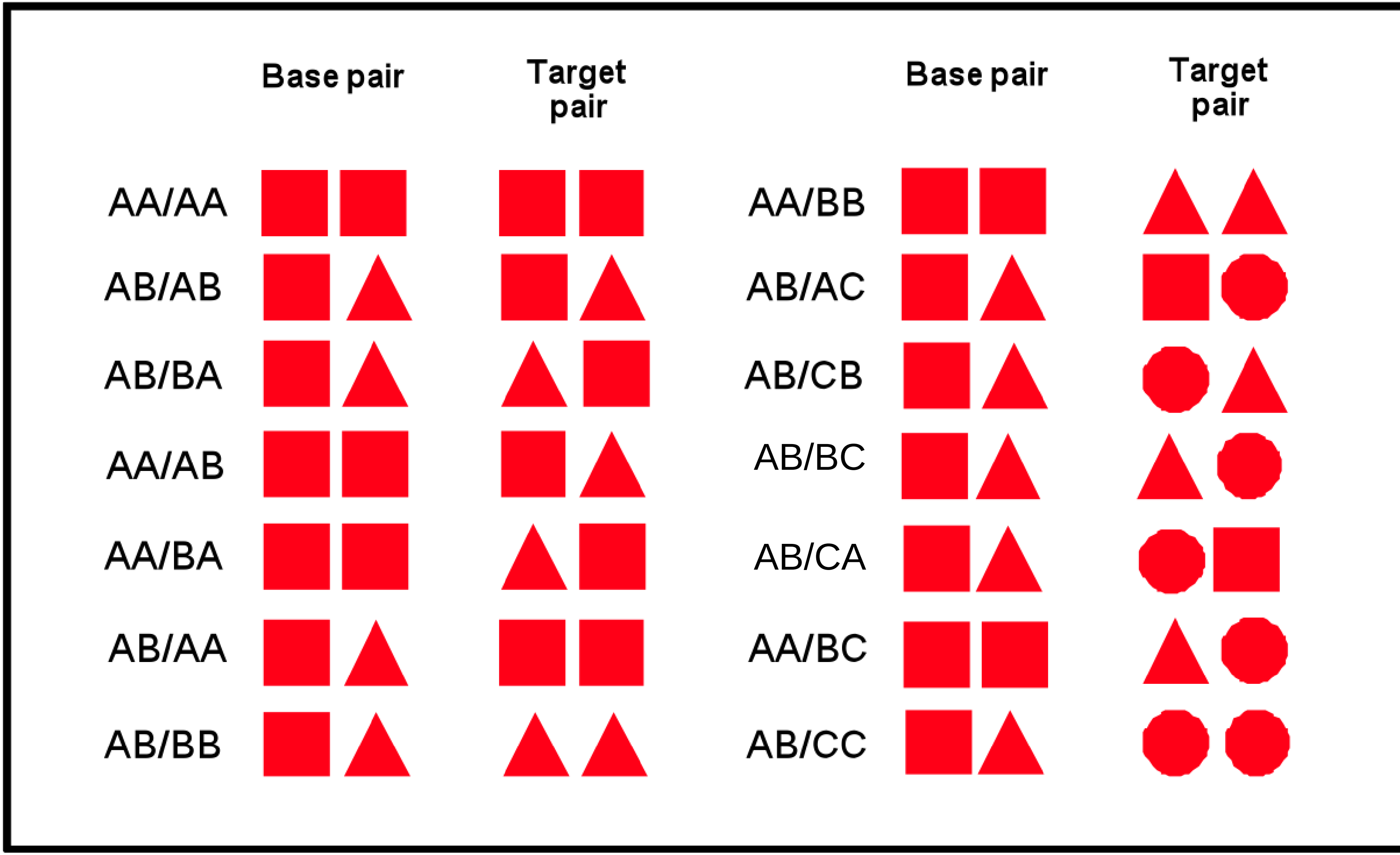
Mean rating
7
6
5
4
3
2
1
1
2
3
4
5
6
Control
Number of transformations



Larkey & Markman (2005): A Test of Structural Models of Similarity

- Similarity judgments for pairs of geometric objects as might be found in analogy tasks

- Compared SA models and RD


Base pair
Target pair
Base pair
Target pair
AA/AA
AB/AB
AB/BA
AA/AB
AA/BA
AB/AA
AB/BB
AA/BB
AB/AC
AB/CB
AB/BC
AB/CA
AA/BC
AB/CC

A psychologically plausible set of transformations for this domain

1) **Create feature** –create a novel feature.

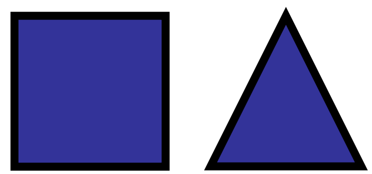
2) **Apply feature** – takes available feature applies it to either one or both objects in target pair.

3) **Swap** – swaps features between objects or swaps object in entirety.

An example

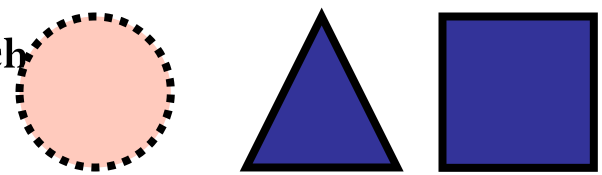
Initial comparison

Triangle has changed location so that is one operation: swap

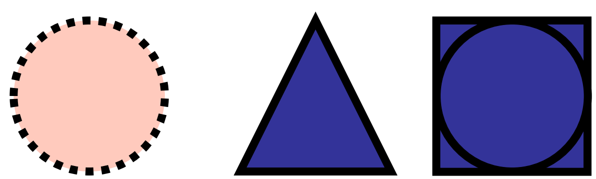


So…in total we have **three** transformations for this comparison.

Create new feature which is unique to the target pair.



Apply the new feature



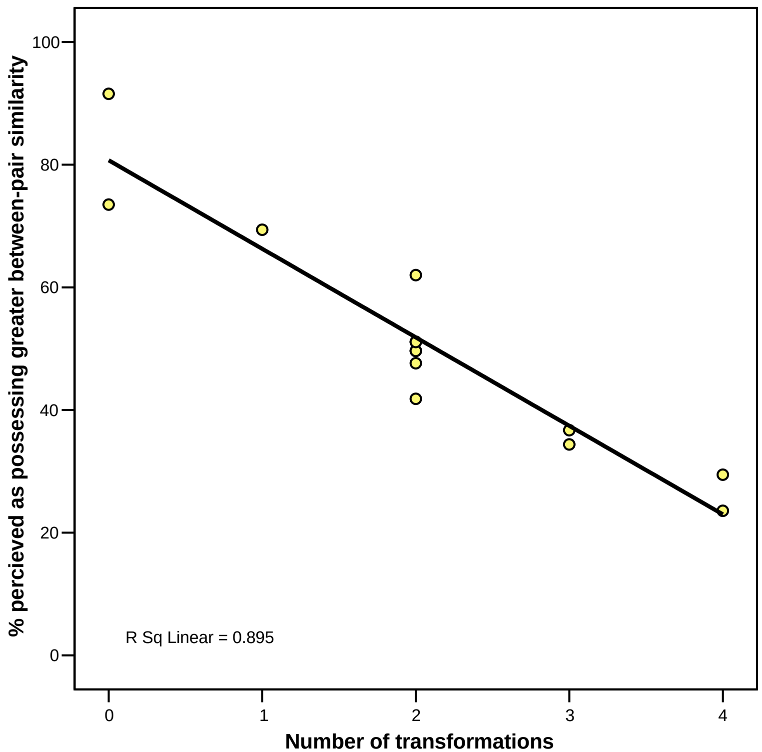
Hodgetts, Hahn & Chater, 2009

- Experiment 1 – we just changed one dimension (shape) and kept colour invariant as an initial test of our coding language.
  - Used a two alternative forced choice paradigm for all possible 1D comparisons.

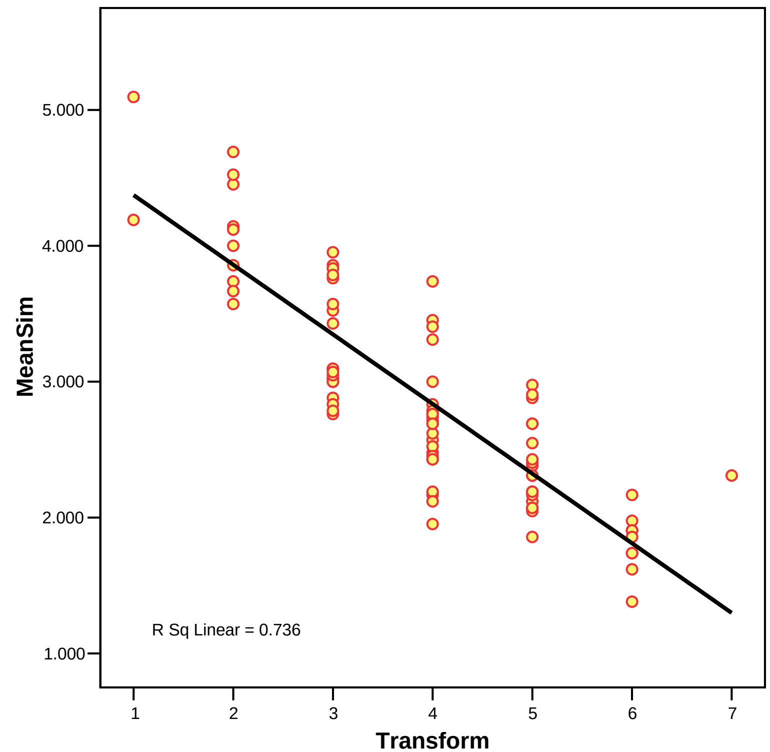
- Experiment 2 – scaled up to more complex stimuli which varied across two dimensions.
  - whole set arduous for participants so a rating scale (1-6) and a random subset of all possible shape and colour combinations (81 comparisons).

Results

One dimension (e.g., shape)



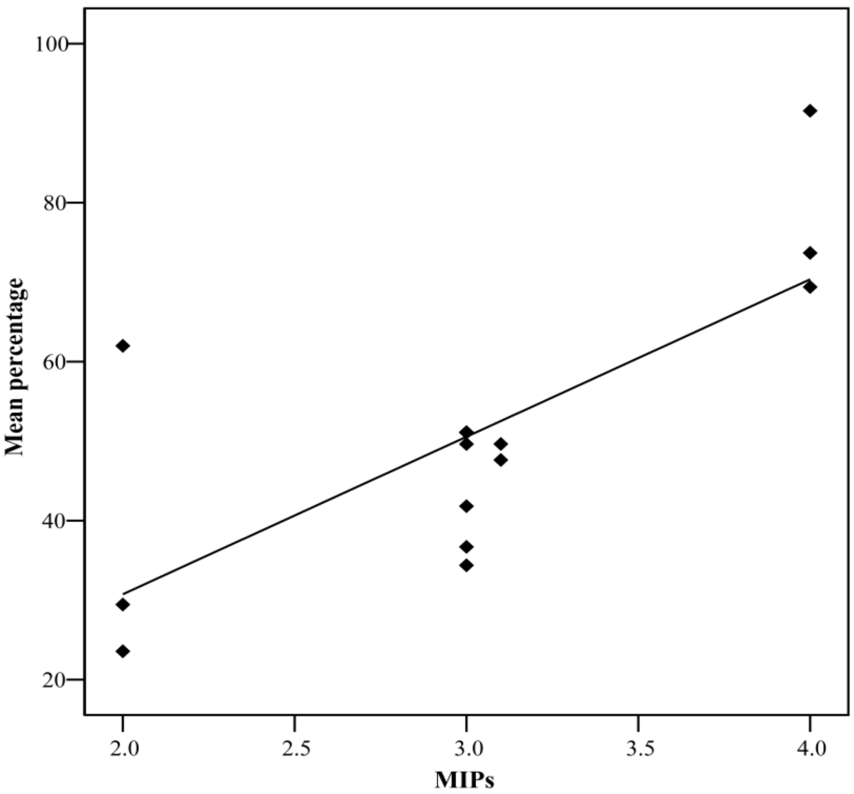
Two dimensions (e.g. colour/shape)



- Strong correlation between transformations and perceived similarity.
- ($r = -0.95$, $p < 0.01$)
- $R^2 = 0.895$

- Strong correlation:
- ($r = -0.86$, $p < 0.01$)
- $R^2 = 0.74$

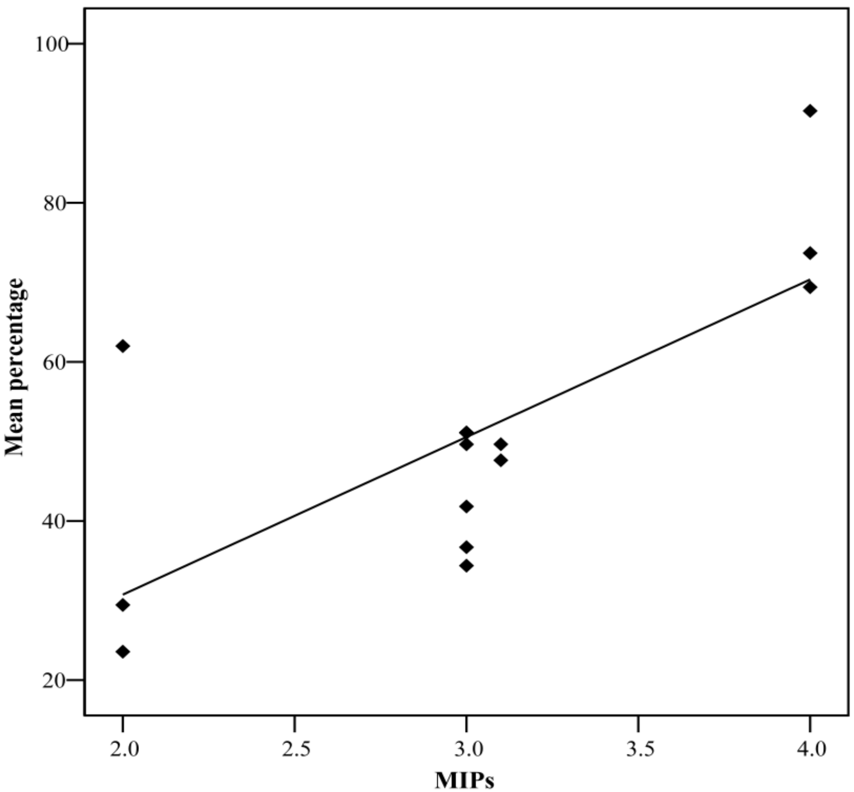
Structural Alignment Models: Exp.1

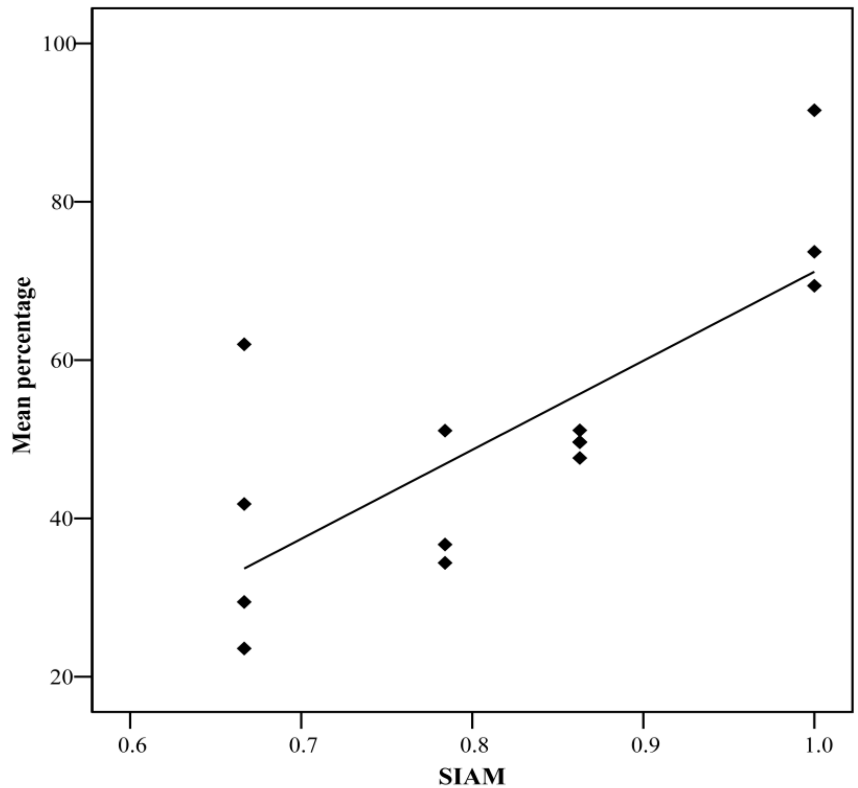


SME $R^2 = .54$

(RD $R^2 = .9$)

Structural Alignment Models: Exp.1



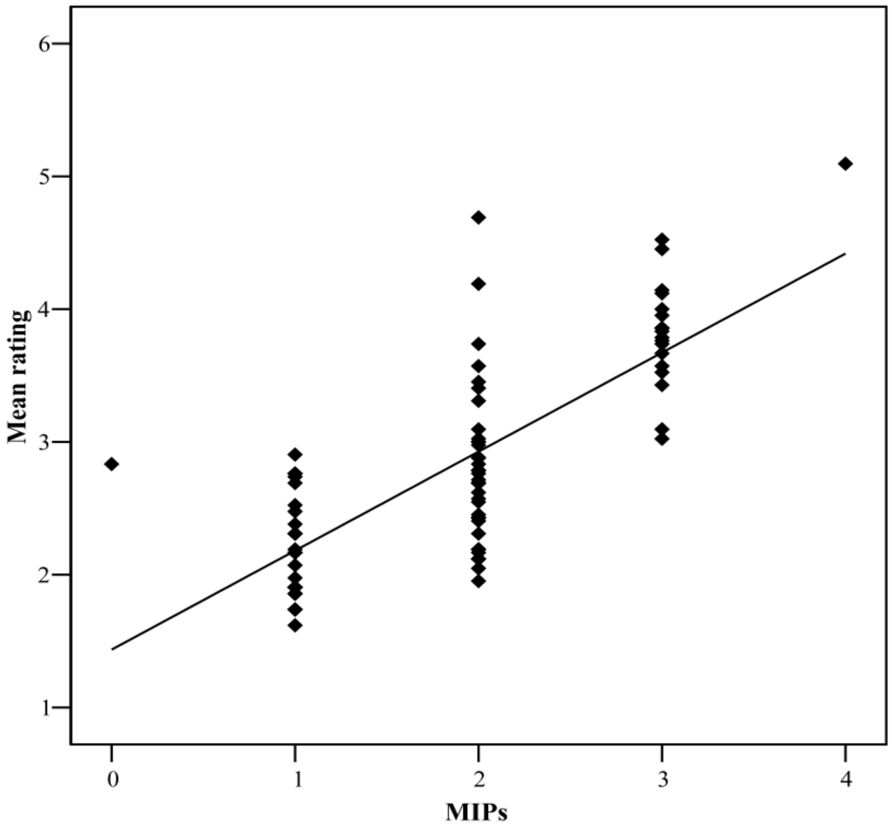


SME R^2 =.54

SIAM R^2 =.58

(RD R^2 = .9)

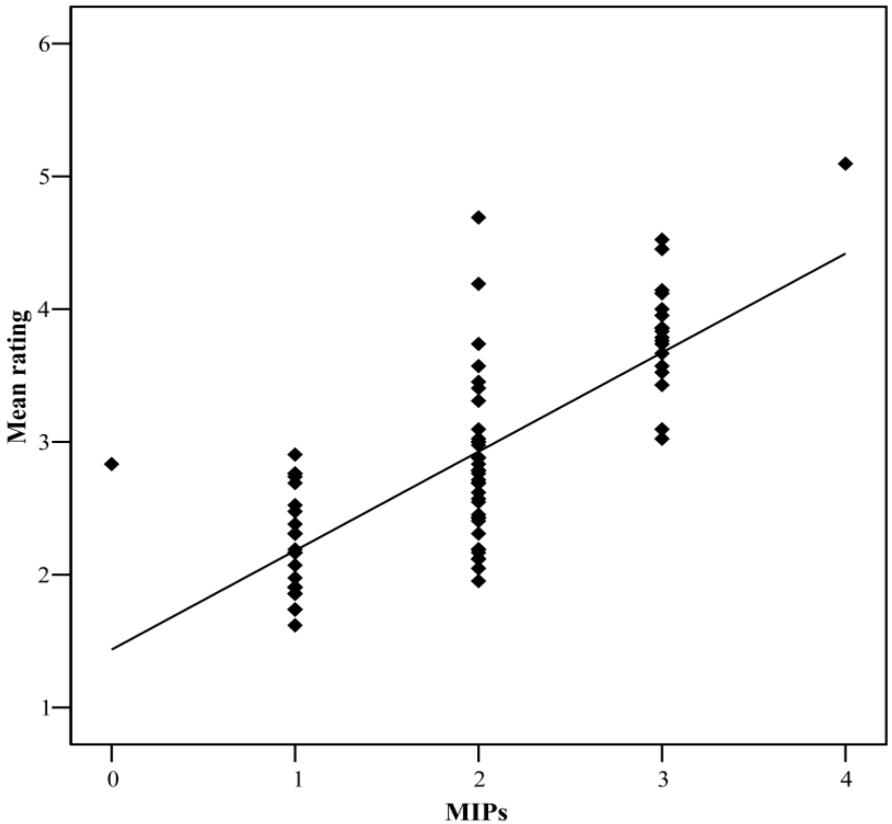
Structural Alignment Exp. 2

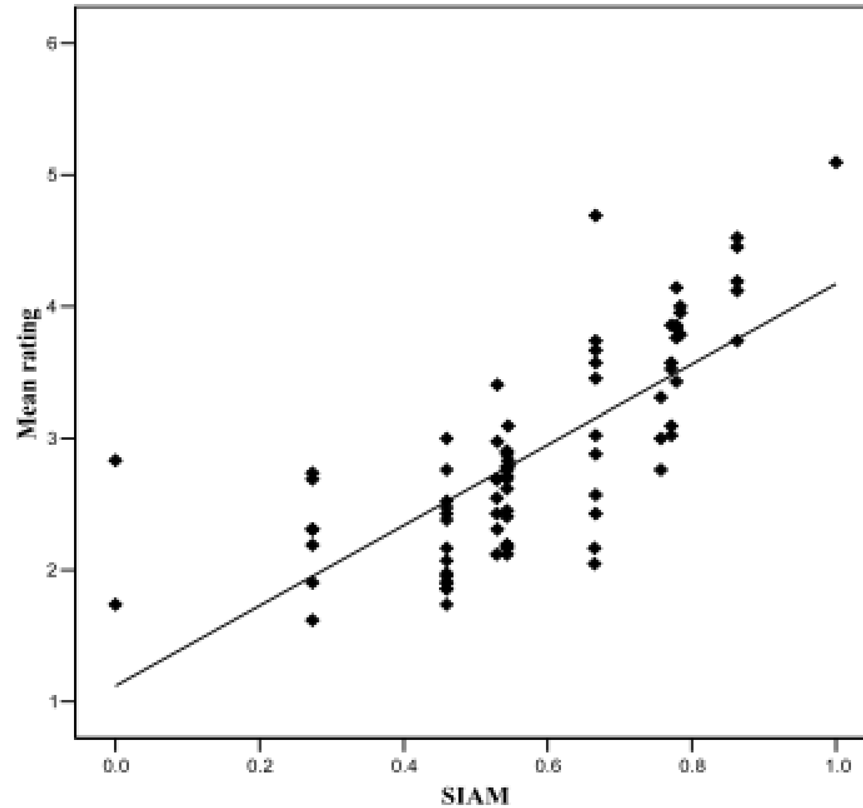


SME $R^2 = .55$

($RD\ R^2 = .74$)

Structural Alignment Exp. 2





SME $R^2 = .55$

SIAM $R^2 = .54$

(RD $R^2 = .74$)

Summary: Hodgetts, Hahn & Chater (2009)

Findings of Larkey & Markman (2005) are reversed on a representative sample of the domain once simple psychological transformations are considered:

RD considerably outperforms Structural Alignment models

…What about (more) real world categories?

Transformations in object recognition and categorization

- Object constancy across rotation, size scaling and translation
- Effects of transformation on recognition (for review see Graf, 2006)

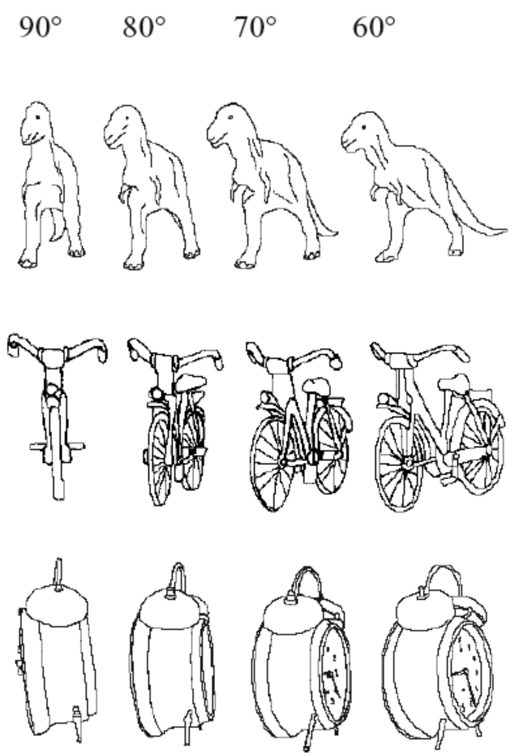
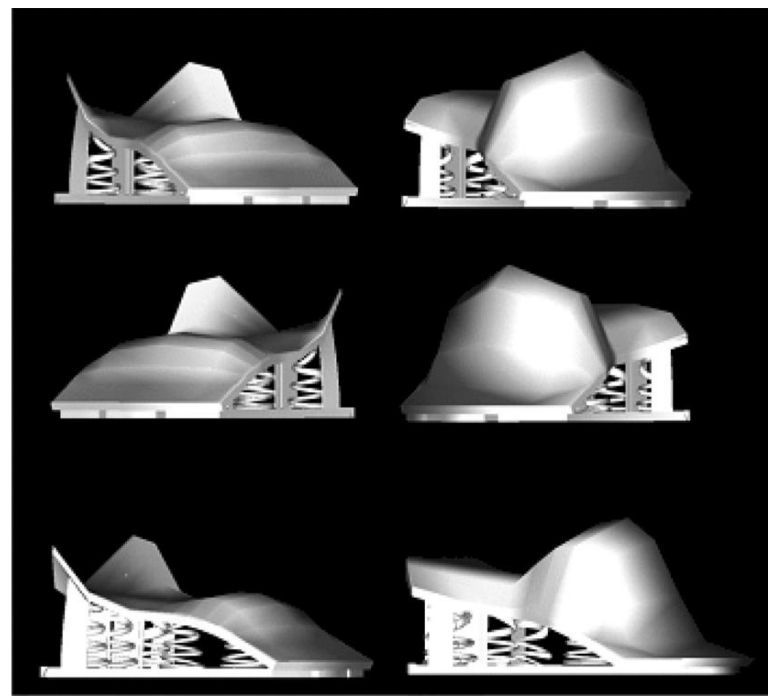


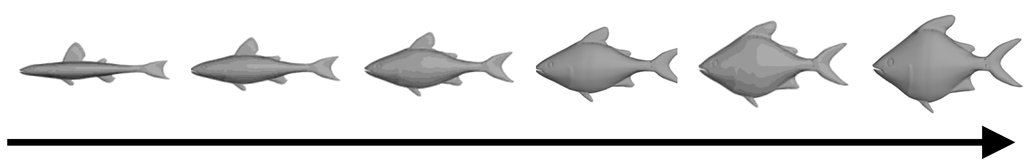
Figure 3. Examples of the four different views of the dinosaur, bicycle, and clock presented in Experiments 2 and 3.



Influence of transformation *direction*?

- Many real world transformations have a ‘natural’ direction, such that transformation in one direction is simpler than in the other

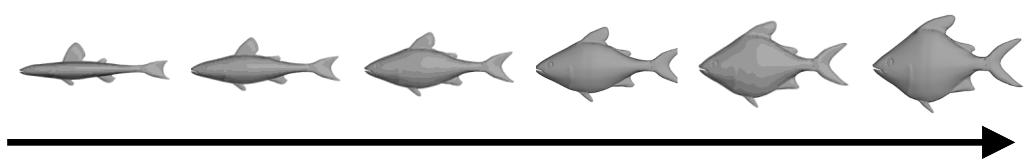
- Can such a sense of inherent direction be artificially induced?



Influence of transformation *direction*?

- Directional similarity ratings can be asymmetrical: sim(A,B) ≠ sim(B,A) (Rosch, 1975; Tversky, 1977).

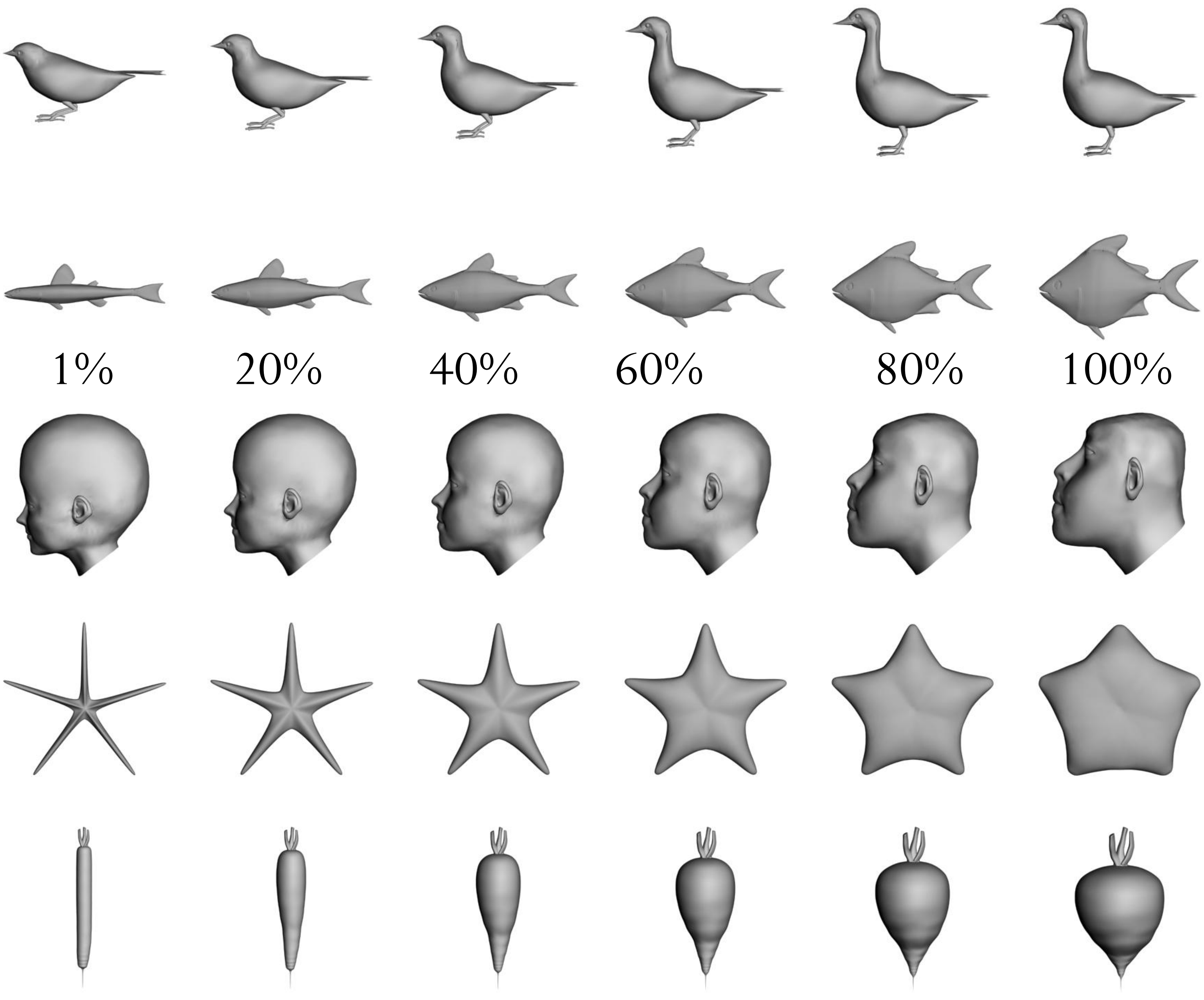
- Morph **animations** are **directional**.



➔ **Can** the **direction** of a morph animation influence subsequent similarity ratings?

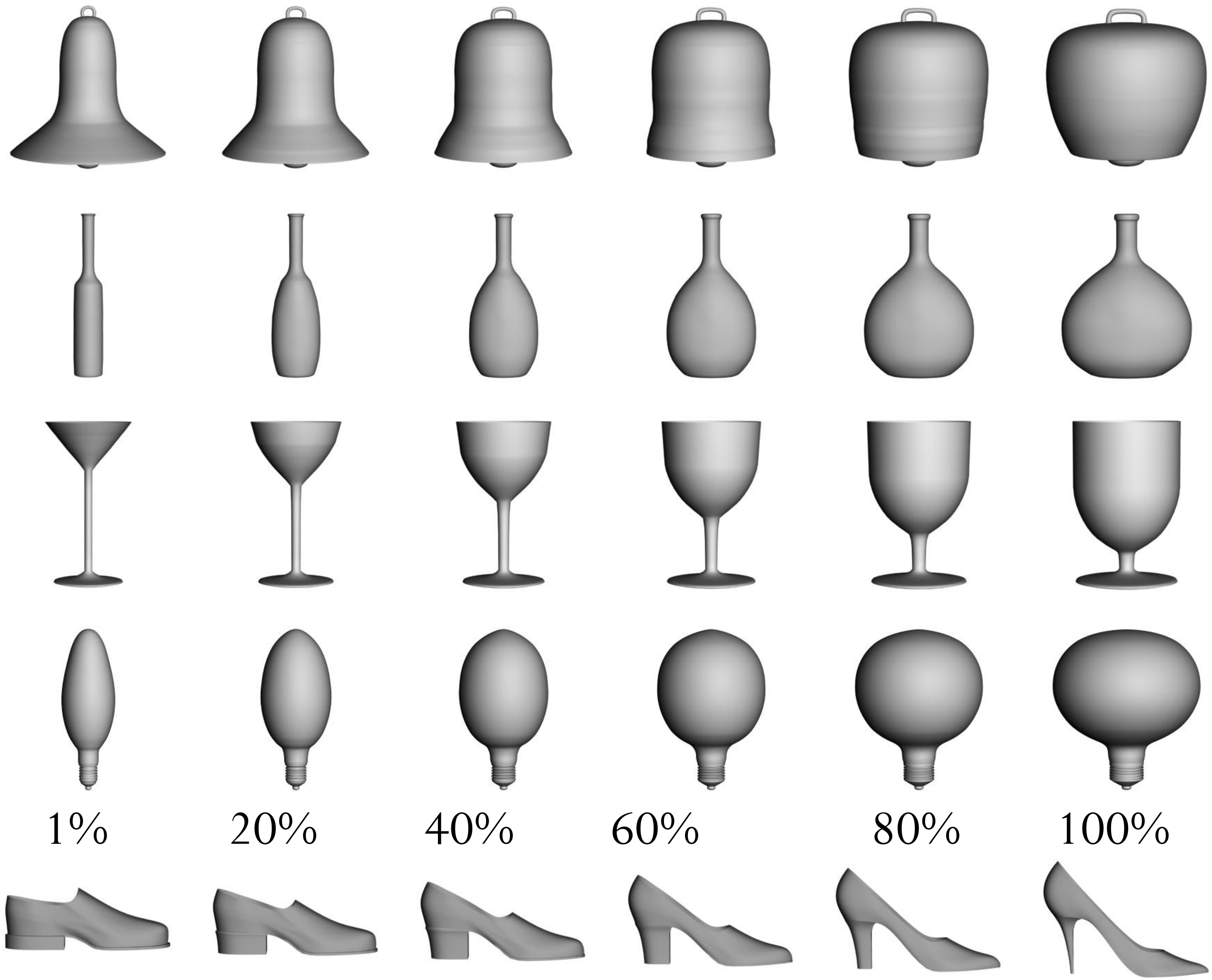
Materials used Hahn, Close & Graf, 2009

Biological categories

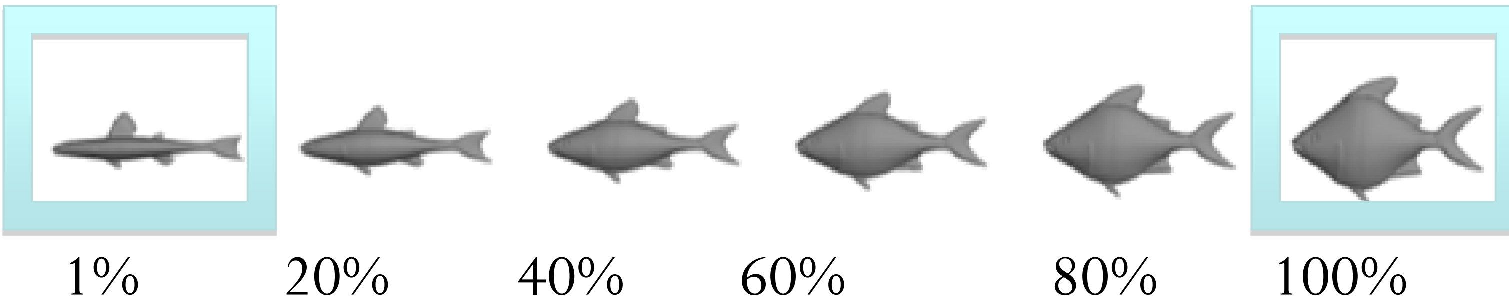


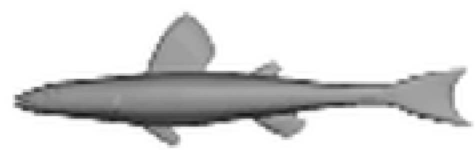
Materials used in Hahn, Close & Graf, 2009

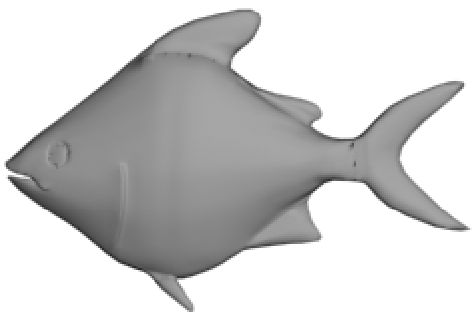
Man-made categories

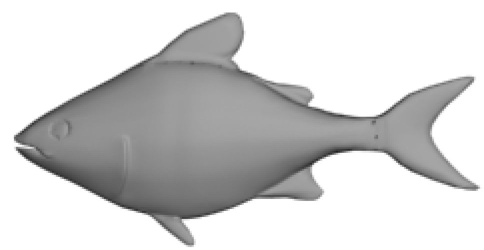


Materials

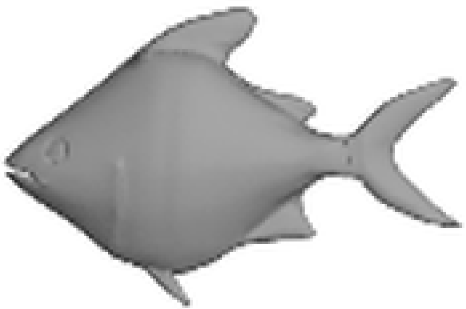


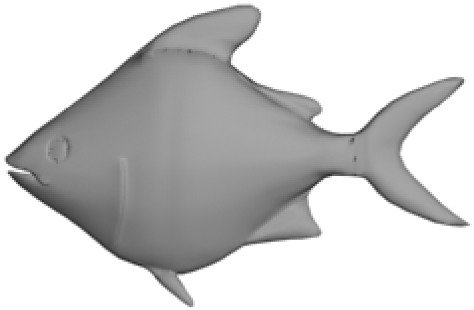


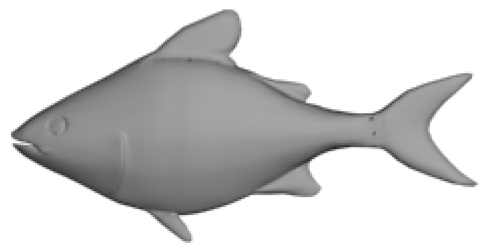




How similar is the centre image to the image in the corner of the screen?



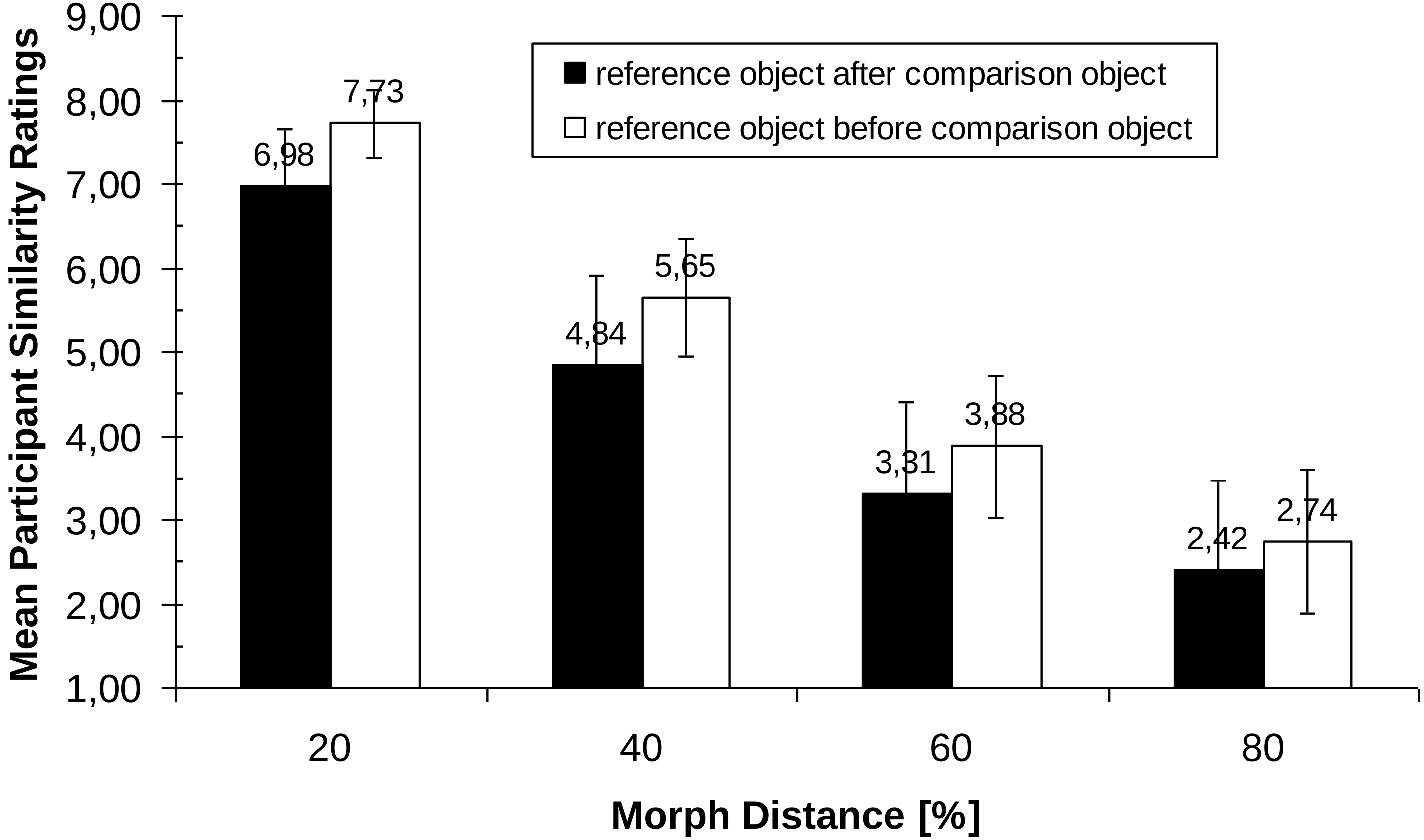




How similar is the centre image to the image in the corner of the screen?

Hahn, Close & Graf, 2009

n=40



- Sign. effect of morph distance
- Similarity ratings were higher when reference object came before the comparison object in the morph sequence

Result cannot be explained by any other model of similarity:

- Nothing about objects under comparison changes

Back to the beginning…

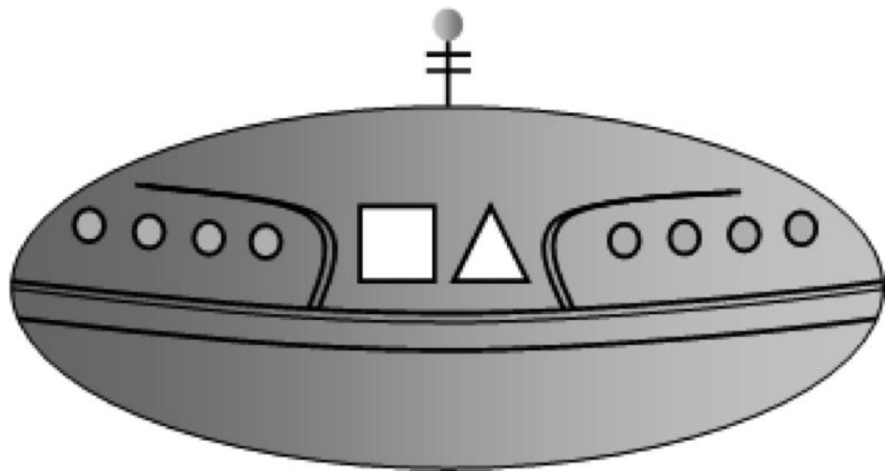
What about categorization?

- Spontaneous categorization
- Participants given initial category “seed”,
- They then decide which other items also ‘go with’ this item

"A group of UFOs infiltrated Earth's atmosphere at approximately 2am. There is a danger that they will wreak havoc on mankind unless they are swiftly identified and stopped. The biggest problem is that only some of these ships are hostile. Your task is to help the armed forces by identifying which ships you believe are hostile based on the evidence. The only distinguishing marks are two symbols on the side-panel of each craft. Based on these features alone, which other ships would you consider a possible danger?

Rely on your intuition and select as naturally as possible. Good luck…"

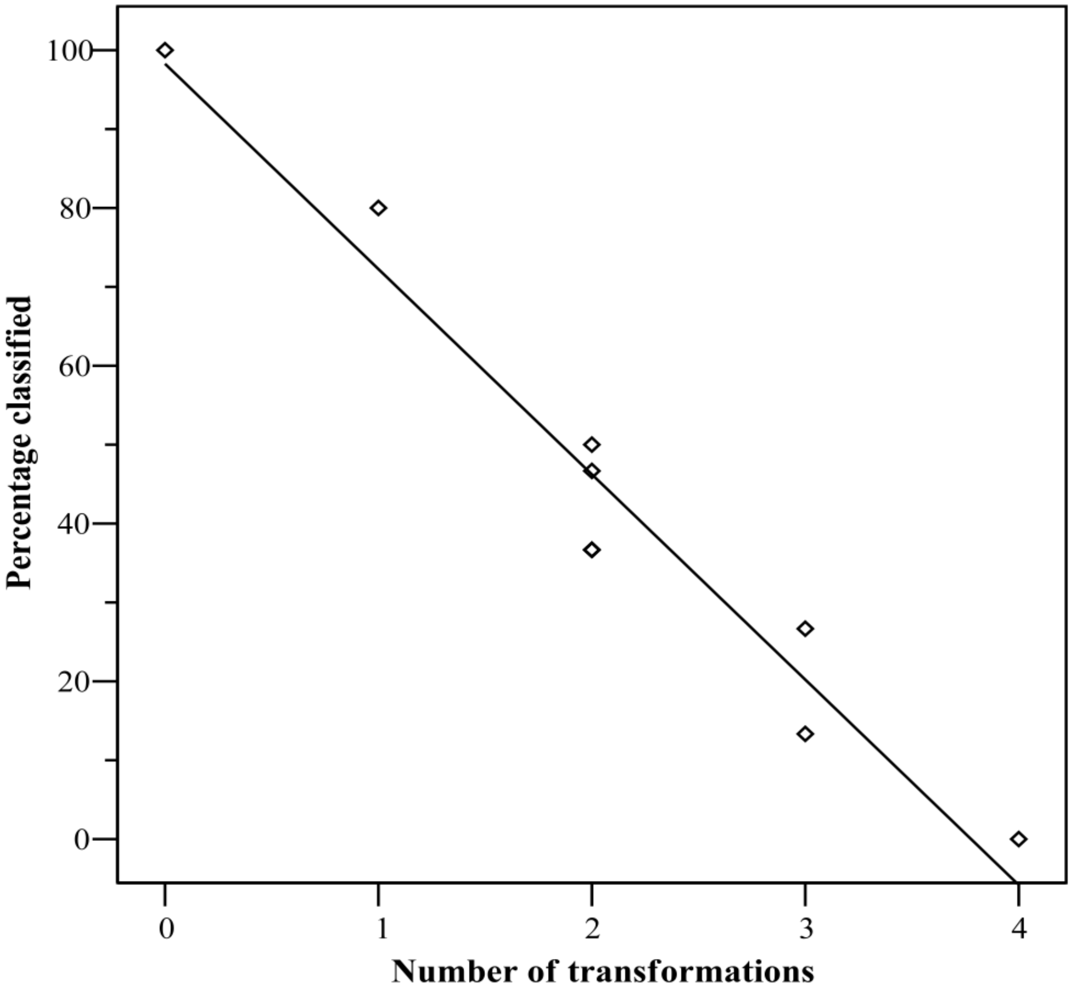
L&M (2005) shapes again…



Results

n=30

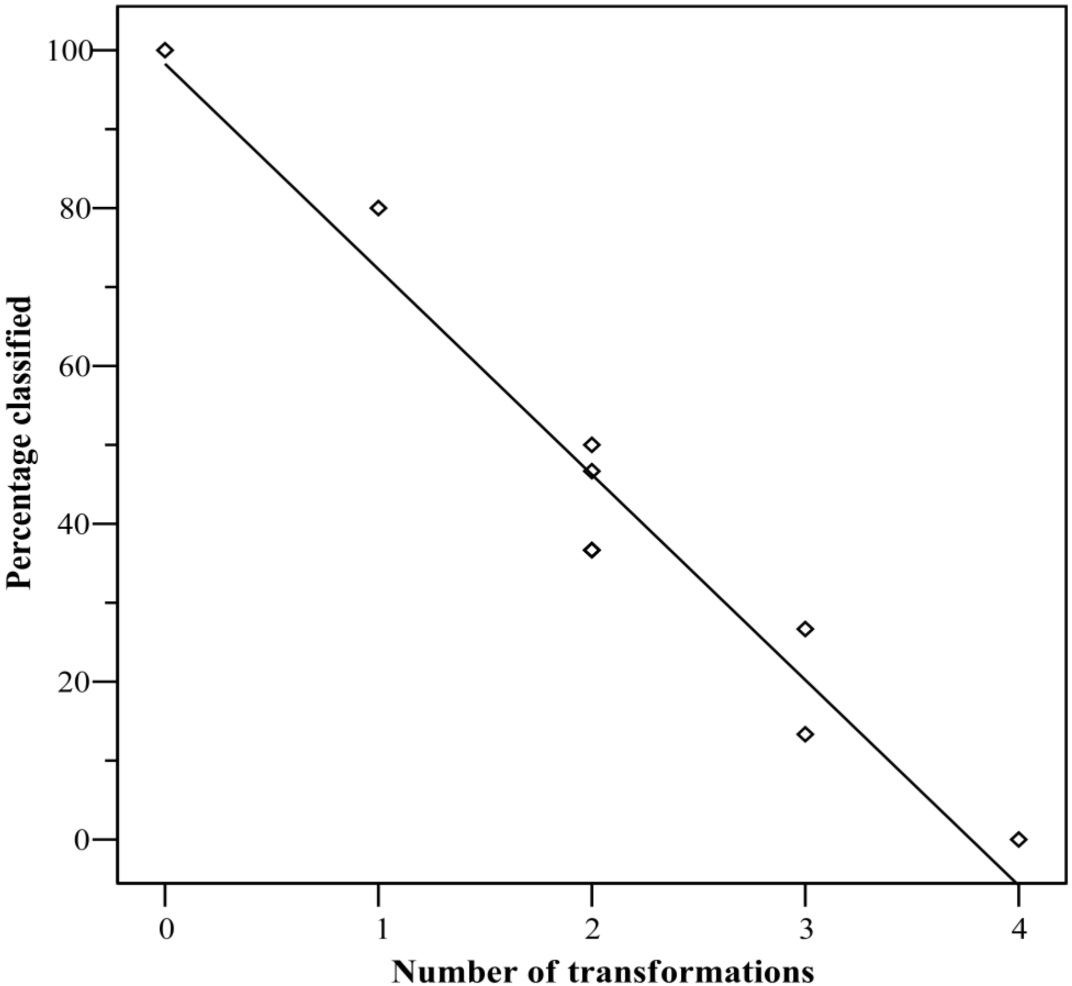
R^2=.95



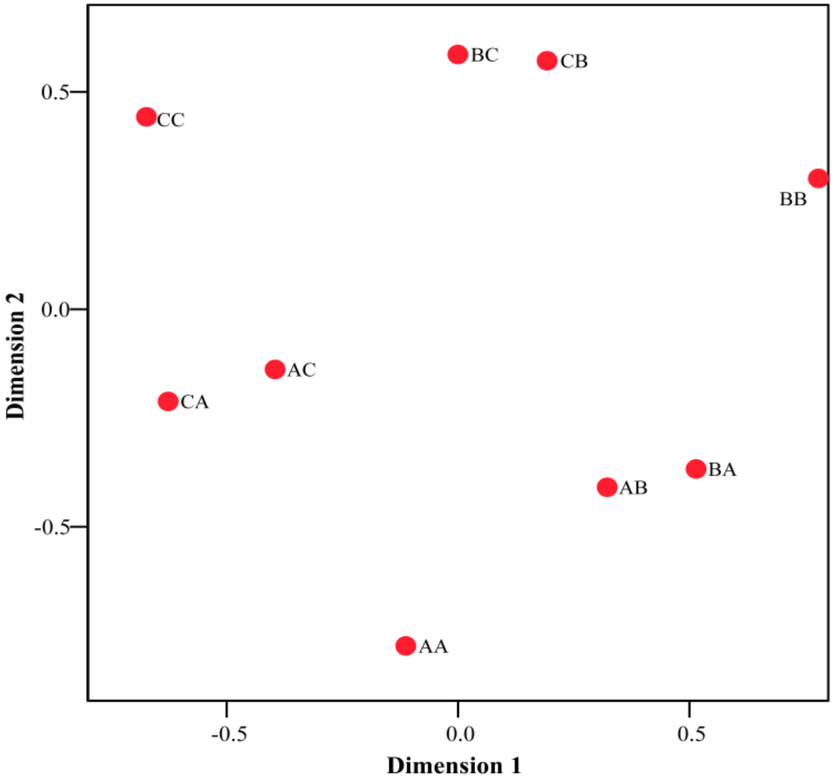
Results

n=30

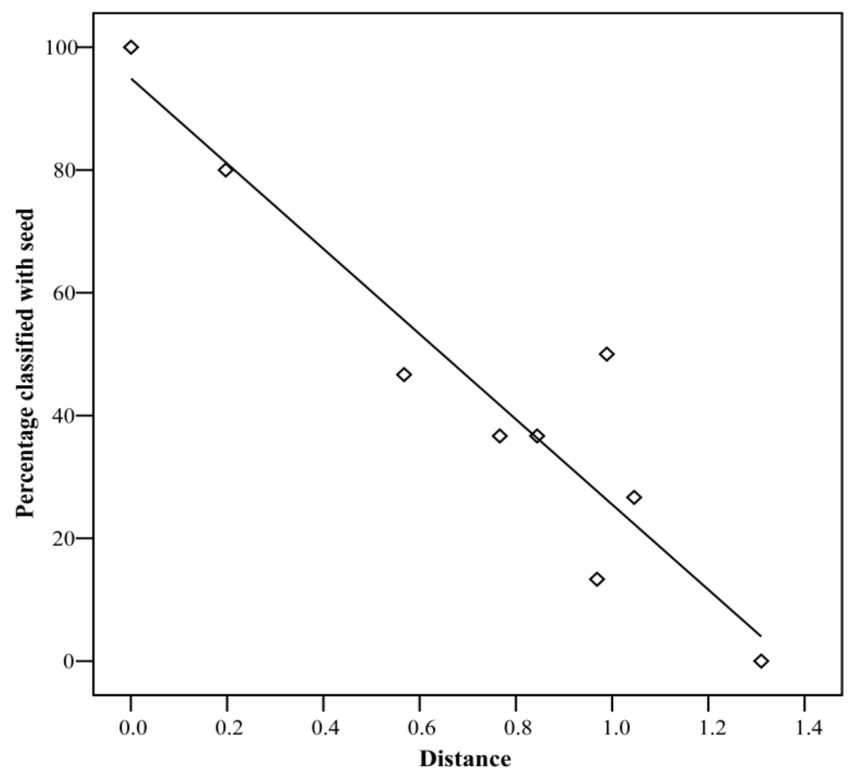
R^2=.95



MDS (n=10)



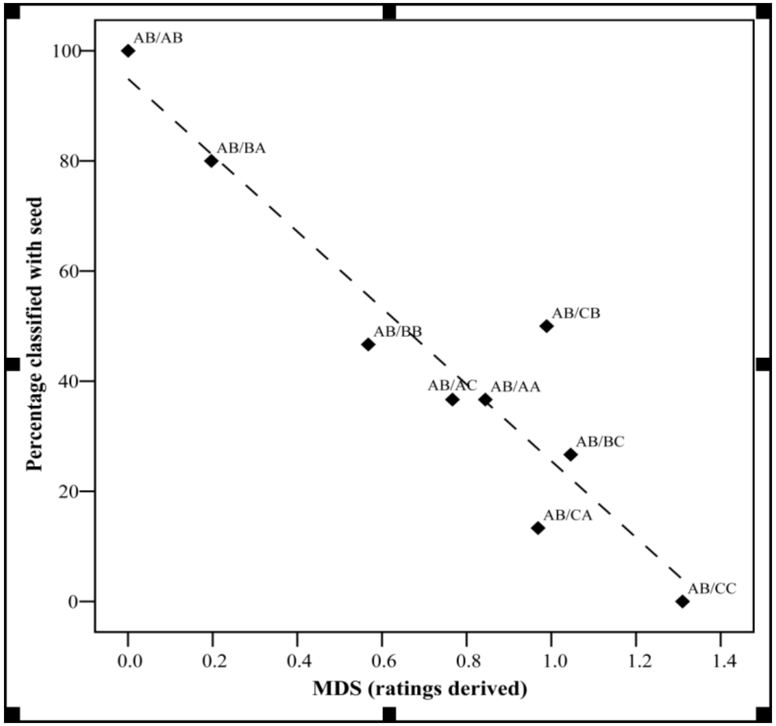
BC
CB
CC
BB
AC
CA
BA
AB
AA
0.5
0.0
-0.5
Dimension 2
-0.5
0.0
0.5
Dimension 1

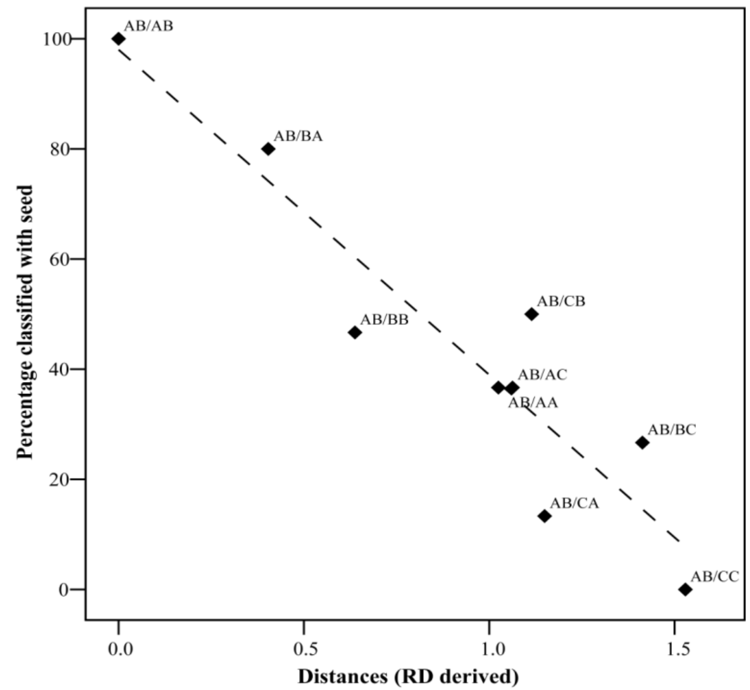


100
80
60
40
20
0
Percentage classified with seed
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Distance

$r^2 = .88$

MDS from ratings vs from RD values





Transformations meet MDS

AB

BA

AA …equidistant to

AC

CA

AB

BA

AC

CA

AB

BA

AA

AC

CA

AB

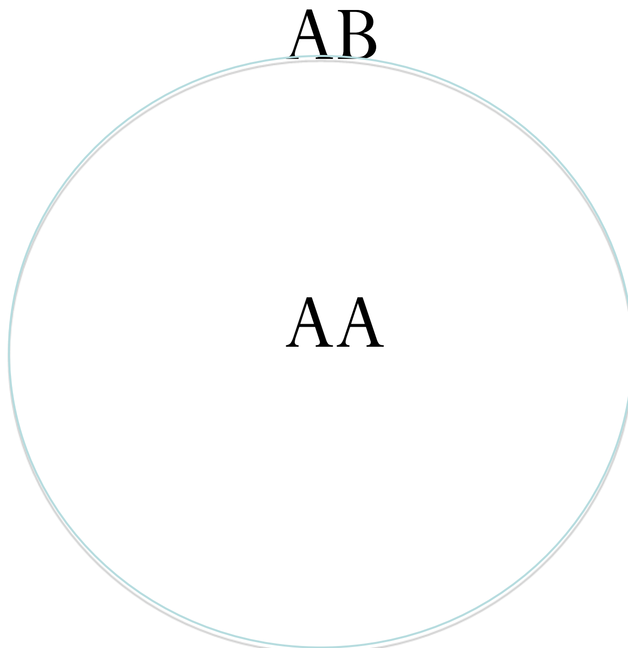
AB

BA

AA

AC

CA

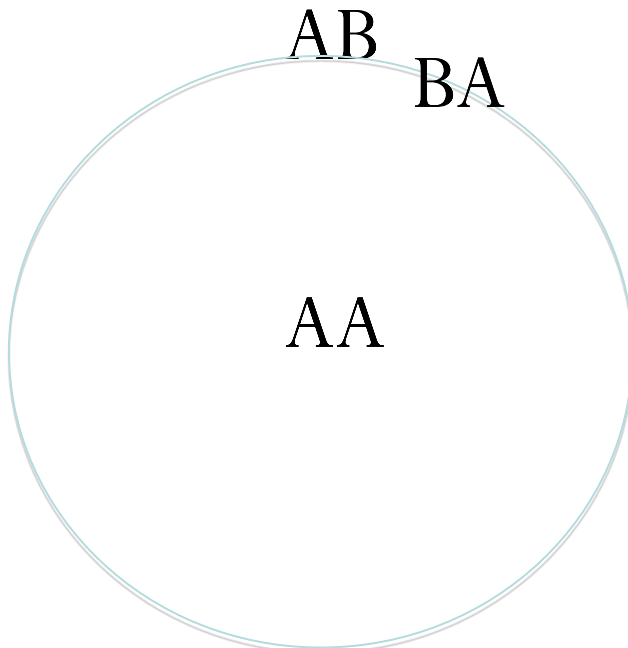


AB

BA

AC

CA

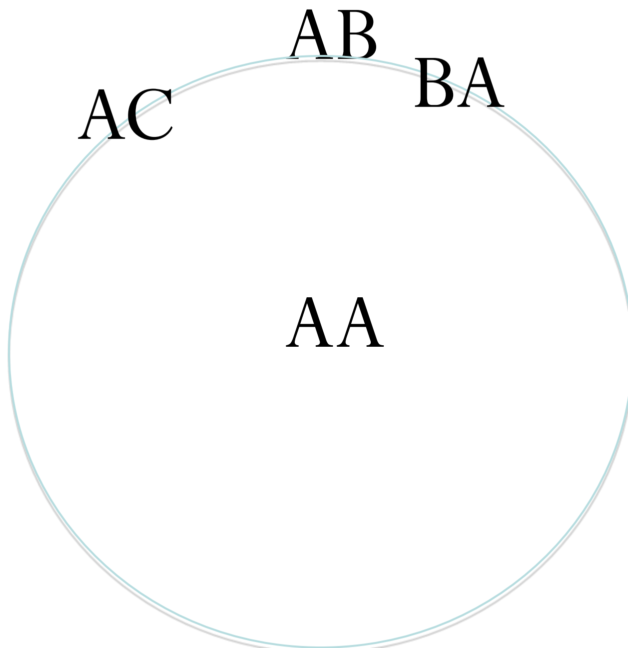


AB

BA

AC

CA



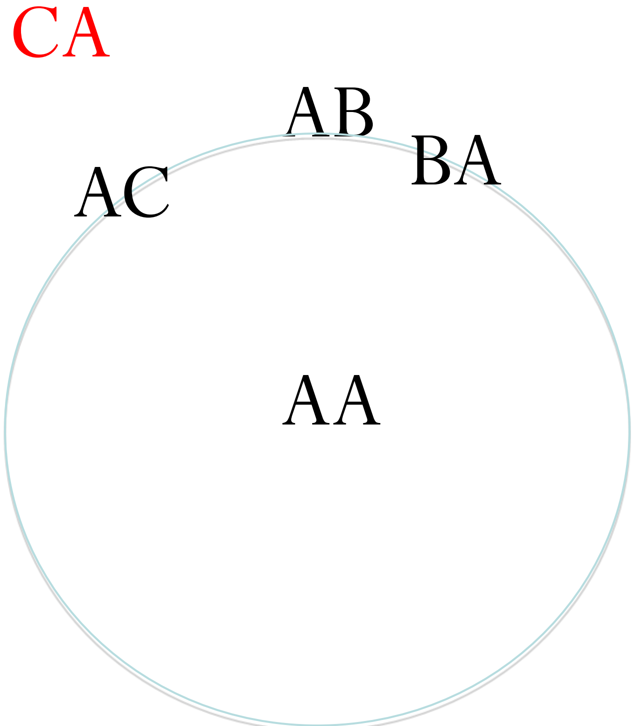
AB

BA

AC

CA

???

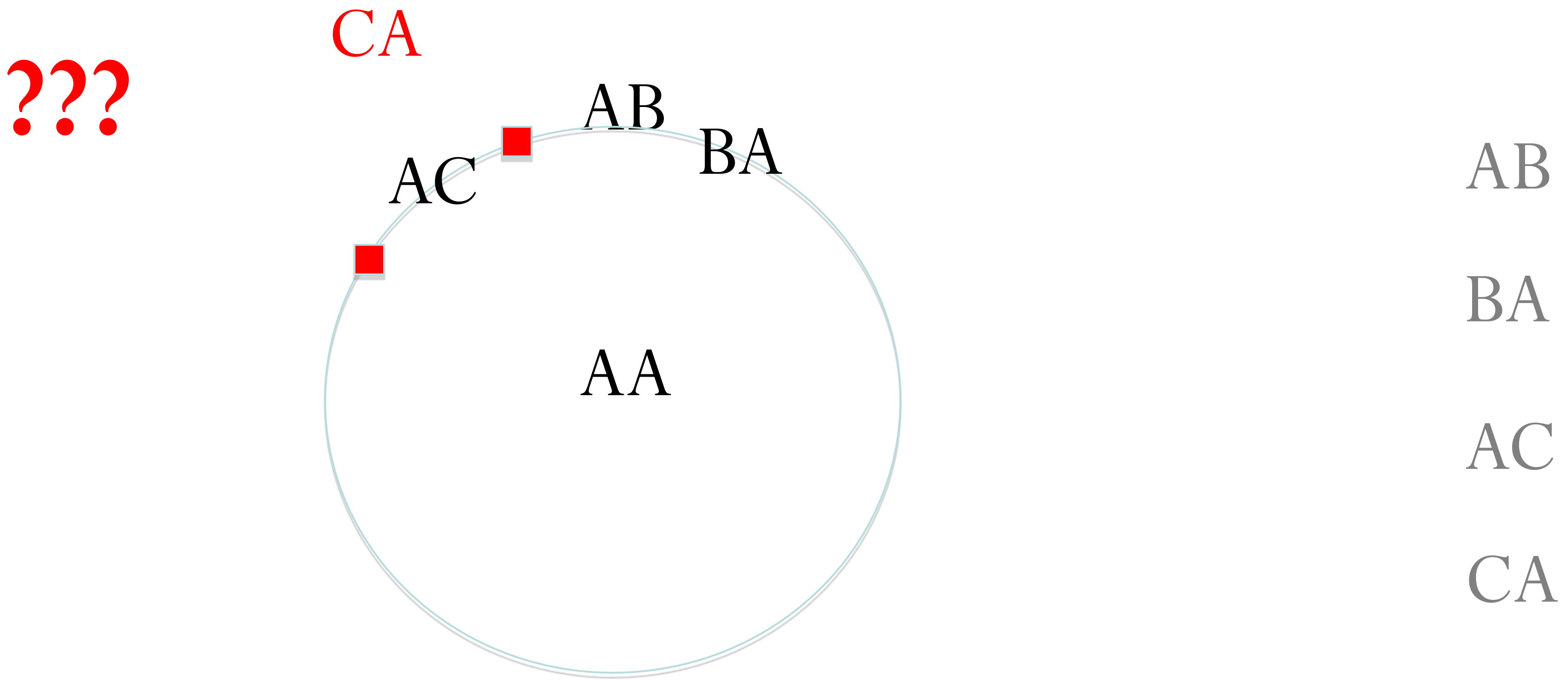


AB

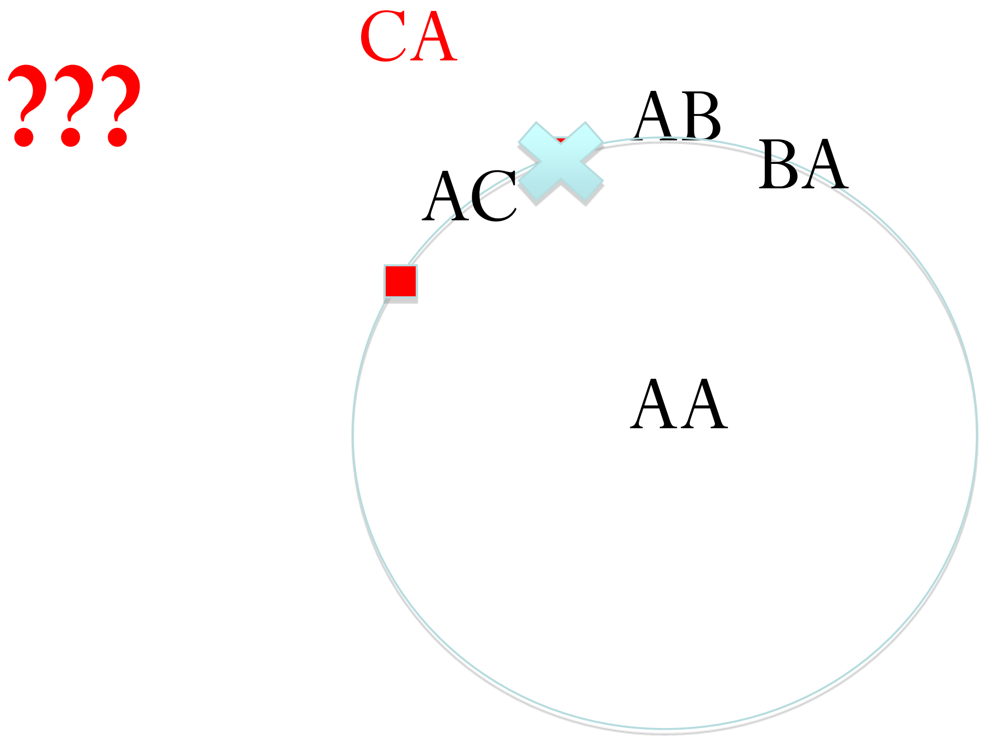
BA

AC

CA



???
CA
AB
BA
AC
AA
AB
BA
AC
CA

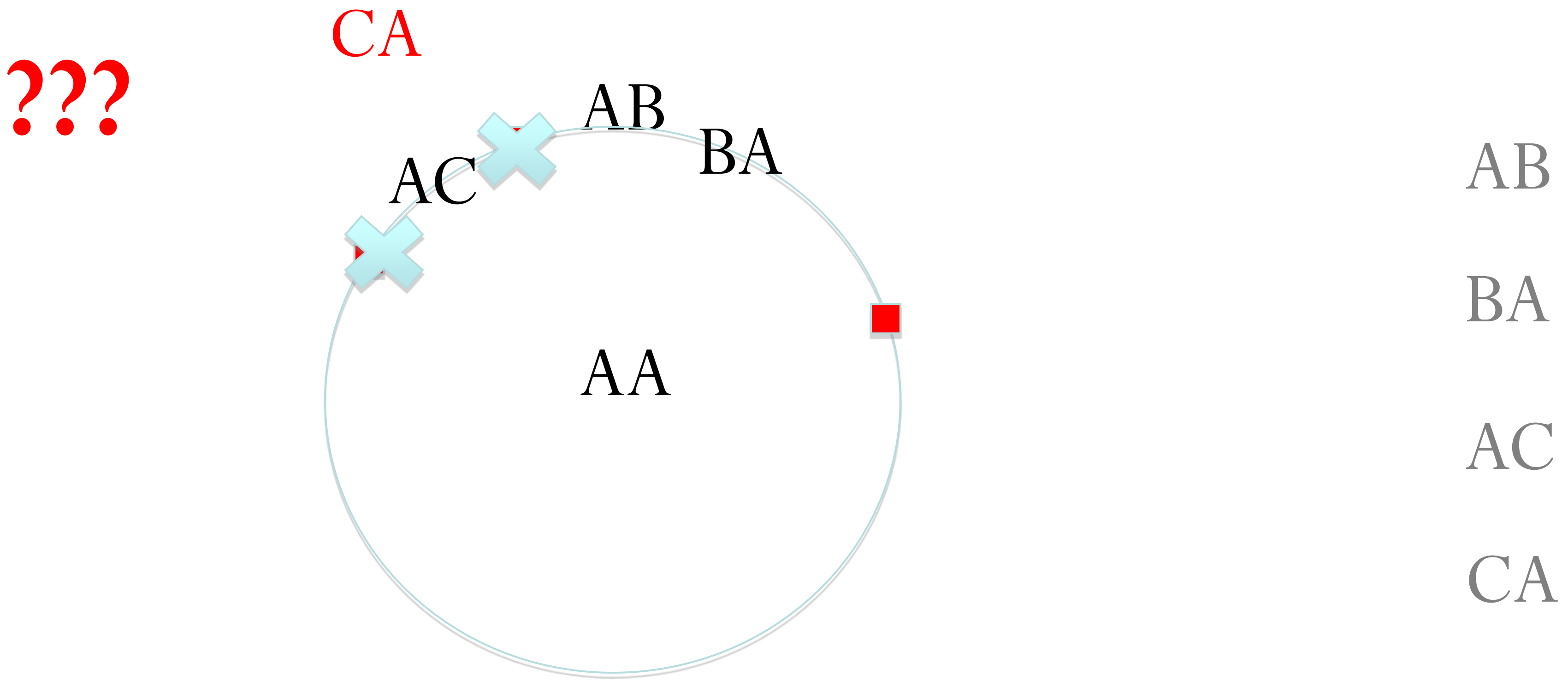


AB

BA

AC

CA



???
CA
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AA
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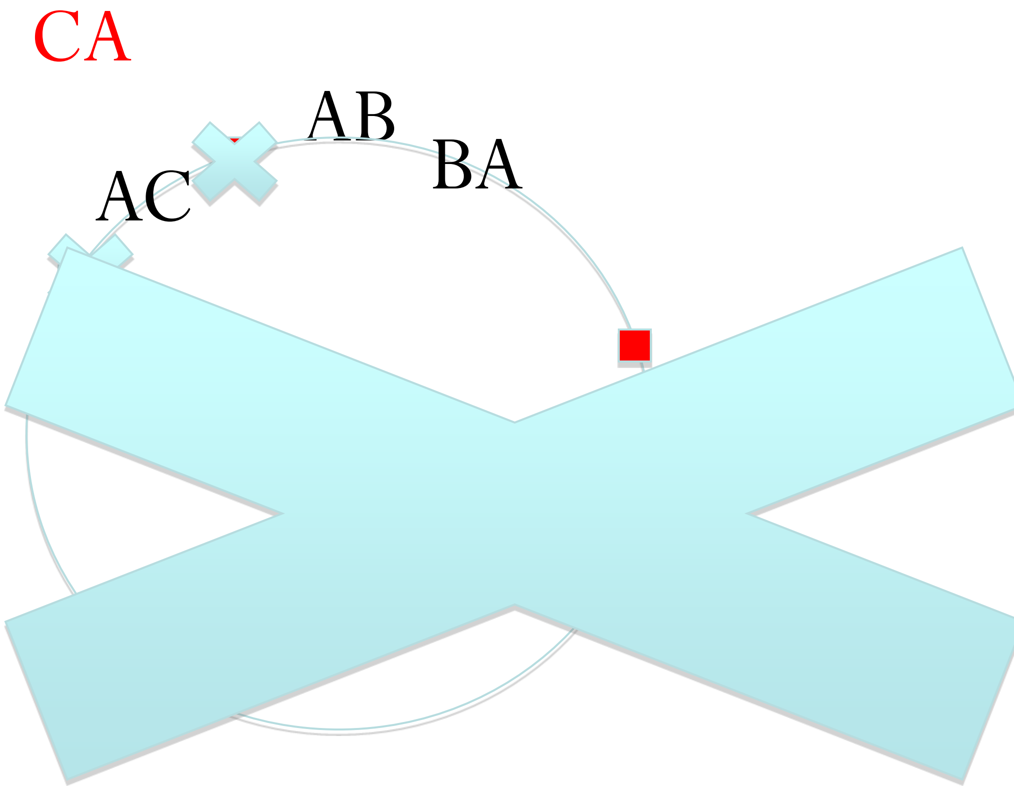
CA

???



AB

BA

AC

CA

- Transformational neighbourhood relations distinct from those of spatial models

- Tversky & Hutchinson (1986) for similar points on featural vs. spatial models

Relationship between models?

- A simple historical progression?

Spatial model

Contrast Model

Structural Alignment

Transformations

…and the winner is….

Relations between models

Transformational framework versus featural or spatial models:

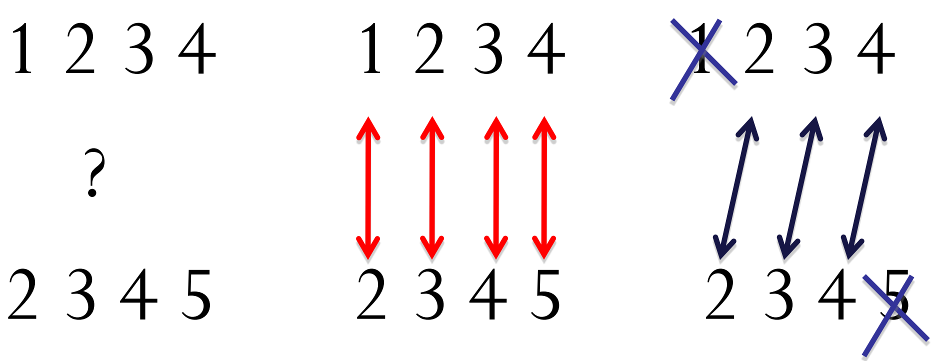
Transformational account is a ***generalization*** of featural or spatial models

Feature insertion/substitution/deletion and changes along continuous valued dimensions are *specific kinds of transformations*

Relations between models

Representational Distortion and Structural Alignment:

- Transformation and alignment (mapping) are complementary aspects/processes



Transformations & Alignment

- Though specific models compete, general notions do not!
- Transformations determine alignment and vice versa
- Minimization of transformation distance as guide to determining correspondence (perception?)

Relations between models: Insights

What we've learned from each:

1. Spatial models: tight coupling with mathematical models of categorization

=> similarity *is* an explanatory notion

2. Contrast model: asymmetric similarities, the role of context

3. Structural alignment: the importance of structured representations, MIPS/MOPS

4. Transformations: a new slant on the problem and a more general framework

Collaborators!

Nick Chater

Carl Hodgetts

James Close

Markus Graf