SARSA (λ) In RKHS

Matthew W. Robards, Peter Sunehag, Scott Sanner

THE AUSTRALIAN NATIONAL UNIVERSITY
COLLEGE OF ENGINEERING AND COMPUTER SCIENCE

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- We use kernels to automatically linearize a non-linear problem
- We introduce the first memory efficient kernel TD algorithm which allows for eligibility traces **with sparsification**
- Furthermore, this is a surprisingly easy to implement algorithm which gives a nice interpretation of the eligibility trace.

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- Assume first order Markov property. ie. $(s_{t+1}, a_{t+1}, r_{t+1})$ is independent of $(s_{t-1}, a_{t-1}, r_{t-1})$ given (s_t, a_t, r_t)

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- Where $err(s_t, a_t, R_t) = (Q(s_t, a_t) - R_t)$ and $R_t = r_t + \gamma Q(s_{t+1}, a_{t+1})$

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- For now lets assume that kernalizing our algorithm means storing all previously visited state action pairs anyway!

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- We use ideas from the projectron method of Orabona et. al to make our algorithm more efficient in memory

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  - For poly kernels, in fact, we will eventually span the RKHS and never need to add new samples
- Rather than storing all new samples, consider projecting the newest hypothesis in $\mathcal{H}_t$ onto $\mathcal{H}_{t-1}$

Projectron RKHS-SARSA(λ)

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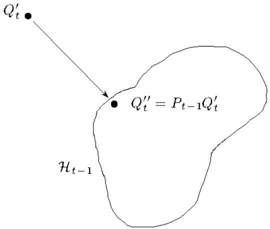

Figure: Projection of temporal hypothesis onto lower RKHS.

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- Now we can also perform the projectron method on the eligibility trace.

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- Our new update equations are given by

$$\alpha_i' = (1 - \eta\xi)\alpha_i - \eta err(s_t, a_t, R_t)\gamma\lambda\beta_i, \qquad \text{for } i = 1, \ldots, |\mathbb{S}| \tag{9}$$

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$$\beta_i' = \gamma\lambda\beta_i + \mathbf{d}_i, \qquad \text{for } i = 1, ..., |\mathbb{S}|. \tag{10}$$

- If $\delta_t < \epsilon$ where δ is the norm of the difference between the temporal hypothesis and its projection.
- Moreover d_i's are the parameters of the projection and $|\mathbb{S}|$ is the support set of stored basis functions.

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$$\alpha_{|\mathbb{S}|+1}' = \eta_t err(s_t, a_t, R_t). \tag{13}$$

and simply update β through $\beta_i' = \gamma\lambda\beta_i$ for $i = 1, \ldots, |\mathbb{S}|$ and $\beta_{|\mathbb{S}|+1} = 1$

Mountain Car

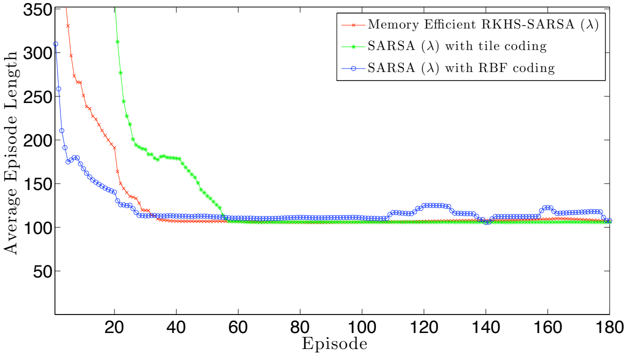

Figure: Moving average time per episode with window 10 evaluated for various algorithms at the end of each episode on mountain car.

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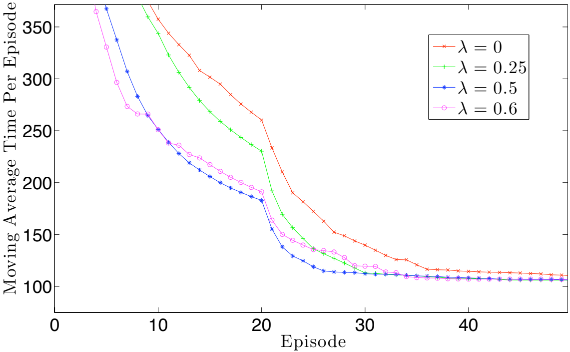
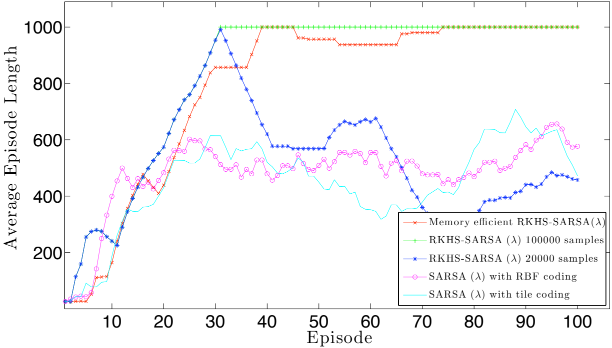


Figure: Moving average time per episode with window 10 evaluated for our algorithm with various values of λ on the mountain car problem 2.

Cart Pole



(a)

Figure: Moving average time per episode with window 10 evaluated for various algorithms at the end of each episode on the cart pole problem.

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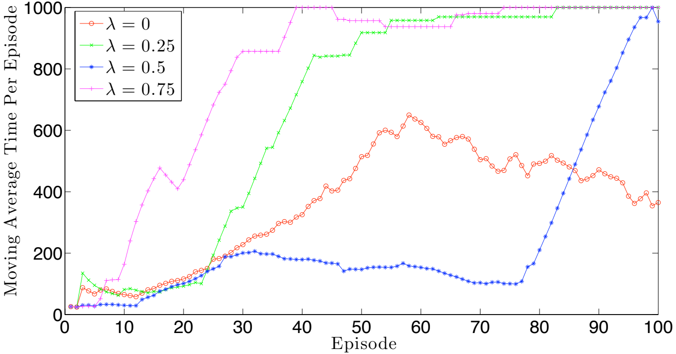

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Memory Efficiency

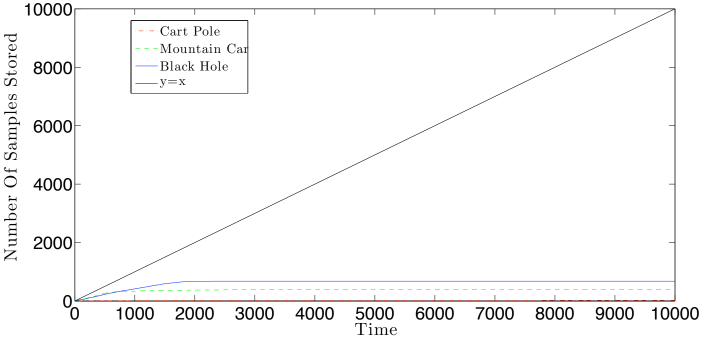

Figure: Number of samples stored by the memory efficient version of our algorithm on each problem.

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Questions

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