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### 5.111 Principles of Chemical Science

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### 5.111 Lecture Summary \#21

Acid-Base Equilibrium Read Chapter 10
Topics: Classification of Acid-Bases, Autoionization of Water, pH Function, Strength of Acids and Bases, Equilibrium Involving Weak Acids.

Classification of Acids and Bases

1. Arrhenius - a narrow definition of acids and bases

An acid is a substance that when dissolved in water increases the concentration of hydrogen ions. A base is a substance that increases the hydroxide concentration.

## 2. Brønsted-Lowry - a broader definition

A Brønsted-Lowry acid - a substance that can donate a hydrogen ion
A Brønsted-Lowry base - a substance that can accept a hydrogen ion

Example 1
$\underset{\text { Acid1 }}{\mathrm{CH}_{3} \mathrm{COOH}(\mathrm{aq})} \underset{\text { Base2 }}{\mathrm{H}_{2} \mathrm{O}(\mathrm{l})} \rightleftharpoons \underset{\text { Acid2 }}{\mathrm{H}_{3} \mathrm{O}^{+}(\mathrm{aq})}+\underset{\text { Base1 }}{\mathrm{CH}_{3} \mathrm{COO}^{-}(\mathrm{aq})}$
(note: hydronium ion $\mathrm{H}_{3} \mathrm{O}^{+}(\mathrm{aq})$ is used instead of $\mathrm{H}^{+}(\mathrm{aq})$ to represent the true nature of hydrogen ions in water)

Acid-bases occur as conjugate acid-base pairs. $\mathrm{CH}_{3} \mathrm{COOH}$ and $\mathrm{CH}_{3} \mathrm{COO}$ are a pair. $\mathrm{H}_{2} \mathrm{O}$ and $\mathrm{H}_{3} \mathrm{O}^{+}$are a pair. The conjugate base of an acid is the base that is formed when the acid has donated a hydrogen ion. The conjugate acid of a base is the acid that forms when base accepts a hydrogen ion.

Example 2 Which are Brønsted-Lowry acids and which are Brønsted-Lowry bases?
$\mathrm{HCO}_{3}^{-}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}(\mathrm{aq})+\mathrm{CO}_{3}^{-2}(\mathrm{aq})$
$\mathrm{HCO}_{3}{ }^{-}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \rightleftharpoons \mathrm{H}_{2} \mathrm{CO}_{3}(\mathrm{aq})+\mathrm{OH}^{-}(\mathrm{aq})$
amphoteric - molecules that can function either as acids or bases depending on the reaction conditions.
3. Lewis Acid and Base - more general definition - applies to reactions that don't involve a hydrogen ion

Lewis base - species that donates lone-pair electrons
Lewis acid - species that accepts such electrons
Example 1


Ammonia is the Lewis base. It donates lone-pair electrons to $\mathrm{BF}_{3}$, the Lewis acid and the electron acceptor.

## Autoionization of Water

$\mathrm{H}_{2} \mathrm{O}(\mathrm{l})+\mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}(\mathrm{aq})+\mathrm{OH}^{-}(\mathrm{aq})$ or $2 \mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}(\mathrm{aq})+\mathrm{OH}^{-}(\mathrm{aq})$
acid base acid base

How much $\mathrm{H}_{2} \mathrm{O}$ is in a glass of water?

$$
\begin{aligned}
\Delta \mathrm{G}^{\circ} & =\Delta \mathrm{G}_{\mathrm{f}}^{\circ}\left(\mathrm{H}_{3} \mathrm{O}^{+}, \mathrm{aq}\right)+\Delta \mathrm{G}_{\mathrm{f}}^{\circ}\left(\mathrm{OH}^{-}, \mathrm{aq}\right)-2 \Delta \mathrm{G}_{\mathrm{f}}^{\circ}\left(\mathrm{H}_{2} \mathrm{O}, \mathrm{l}\right) \\
& =(-237.13)+(-157.24)-2 \times(-237.13) \mathrm{kJ} / \mathrm{mol} \\
& =+79.89 \mathrm{~kJ} / \mathrm{mol} \\
\ln \mathrm{~K} & =-\Delta \mathrm{G}^{\circ} / \mathrm{RT}=\frac{-\left(7.989 \times 10^{4} \mathrm{~J} / \mathrm{mol}\right)}{(8.3145 \mathrm{~J} / \mathrm{Kmol})(298.0 \mathrm{~K})}=-32.24
\end{aligned}
$$

$\mathrm{K}=1.0 \times 10^{-14}$ at 298 K

This very small value indicates that only a small proportion of water molecules are ionized. Concentration of ions due to autoionization of water is very low, about 1 molecule in 200 million.

$$
\mathrm{K}=\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]\left[\mathrm{OH}^{-}\right] \quad \text { This } \mathrm{K} \text { is called } \mathrm{K}_{\mathrm{w}} .
$$

Because $\mathrm{K}_{\mathrm{w}}$ is an equilibrium constant, the product of $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]\left[\mathrm{OH}^{-}\right]$is always $1.0 \times 10^{-14}$ at 298 K .
Note: Because the concentration of the solvent, $\mathrm{H}_{2} \mathrm{O}$, does not change significantly in a dilute solution, it does not enter the equilibrium expression. The solvent, water, is very nearly pure, and pure liquids and pure solids are not included in equilibrium expressions.

## pH Function

$\mathrm{pH}=-\log \left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$

## pOH Function

$\mathrm{pOH}=-\log \left[\mathrm{OH}^{-}\right]$
$\mathrm{K}_{\mathrm{w}}=\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]\left[\mathrm{OH}^{-}\right]$
$\log \mathrm{K}_{\mathrm{w}}=\log \left[\mathrm{H}_{3} \mathrm{O}^{+}\right]+\log \left[\mathrm{OH}^{-}\right]$
$-\log \mathrm{K}_{\mathrm{w}}=-\log \left[\mathrm{H}_{3} \mathrm{O}^{+}\right]-\log \left[\mathrm{OH}^{-}\right]$
$\mathrm{pK}_{\mathrm{w}}=\mathrm{pH}+\mathrm{pOH}=14.00$ at $25^{\circ} \mathrm{C}$

## Strength of Acids and Bases

pH of pure water $\mathrm{pH}=-\log \left(1.0 \times 10^{-7}\right)=7.00$
pH of an acid solution is
pH of an base solution is
EPA defines waste as "corrosive" if the pH is lower than 3.0 or higher than 12.5 .

## 1. Acid in water

$$
\mathrm{CH}_{3} \mathrm{COOH}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}(\mathrm{aq})+\mathrm{CH}_{3} \mathrm{CO}_{2}^{-}(\mathrm{aq})
$$

Acid ionization constant $\mathrm{K}_{\mathrm{a}}=\underline{\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]\left[\mathrm{CH}_{3} \mathrm{CO}_{2}{ }^{-}\right]}$
$\left[\mathrm{CH}_{3} \mathrm{COOH}\right]$
$\mathrm{K}_{\mathrm{a}}$ equals $1.76 \times 10^{-5}$ at $25^{\circ} \mathrm{C}$. Small value tells us that only a small proportion of $\mathrm{CH}_{3} \mathrm{COOH}$ molecules donate their proton when dissolved in water (weak acid).
$\mathrm{HA}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}(\mathrm{aq})+\mathrm{A}^{-}(\mathrm{aq})$
ACID (HA) IN WATER
$\mathrm{BH}^{+}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}(\mathrm{aq})+\mathrm{B}(\mathrm{aq})$
ACID ( $\mathrm{BH}^{+}$) IN WATER

A strong acid has a $K_{a}>1$ which means that the acid ionizes almost completely.
A weak acid has a $\mathrm{K}_{\mathrm{a}}<1$. The reaction with water does not produce many ionized species before equilibrium is reached.
$\mathrm{pK}_{\mathrm{a}}=-\log \mathrm{K}_{\mathrm{a}}$
The lower the value of $\mathrm{K}_{\mathrm{a}}$, the higher the value of $\mathrm{pK}_{\mathrm{a}}$. The higher the $\mathrm{pK}_{\mathrm{a}}$, the weaker the acid.

## 2. Base in water

$\mathrm{NH}_{3}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \rightleftharpoons \mathrm{NH}_{4}^{+}(\mathrm{aq})+\mathrm{OH}^{-}(\mathrm{aq})$
Base ionization constant $\mathrm{K}_{\mathrm{b}}=\underline{\left[\mathrm{NH}_{4}^{+}\right]\left[\mathrm{OH}^{-}\right]}$
$\left[\mathrm{NH}_{3}\right]$
$\mathrm{K}_{\mathrm{b}}$ is $1.8 \times 10^{-5}$ at $25^{\circ} \mathrm{C}$. This small value tells us that only a small amount of $\mathrm{NH}_{3}$ ionizes to $\mathrm{NH}_{4}{ }^{+}$ and $\mathrm{OH}^{-}$in solution. A strong base reacts essentially completely to give $\mathrm{OH}^{-}(\mathrm{aq})$ when put in water. $\mathrm{NH}_{3}$ is not a strong base. It is a moderately weak base.
$\mathrm{B}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \rightleftharpoons \mathrm{BH}^{+}(\mathrm{aq})+\mathrm{OH}^{-}(\mathrm{aq})$
$\mathrm{A}^{-}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \rightleftharpoons \mathrm{HA}(\mathrm{aq})+\mathrm{OH}^{-}(\mathrm{aq})$
$\mathrm{pK}_{\mathrm{b}}=-\log \mathrm{K}_{\mathrm{b}}$
larger $\mathrm{K}_{\mathrm{b}}$, stronger base
larger $\mathrm{pK}_{\mathrm{b}}$, weaker base

## 3. Conjugate acids and bases

The stronger the acid, the weaker its conjugate base.
The stronger the base, the weaker its conjugate acid.
Consider conjugate acid-base pair $\mathrm{NH}_{3}$ and $\mathrm{NH}_{4}{ }^{+}$:
$\mathrm{NH}_{3}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \rightleftharpoons \mathrm{NH}_{4}^{+}(\mathrm{aq})+\mathrm{OH}^{-}(\mathrm{aq})$
$\mathrm{NH}_{4}^{+}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}(\mathrm{aq})+\mathrm{NH}_{3}(\mathrm{aq})$
Multiply K's together and get:
$\mathrm{K}_{\mathrm{a}} \times \mathrm{K}_{\mathrm{b}}=\frac{\left[\mathrm{NH}_{3}\right]\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]}{\left[\mathrm{NH}_{4}^{+}\right]} \quad \mathrm{X} \quad \frac{\left[\mathrm{NH}_{4}{ }^{+}\right]\left[\mathrm{OH}^{-}\right]}{\left[\mathrm{NH}_{3}\right]}=\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]\left[\mathrm{OH}^{-}\right]$
$\mathrm{K}_{\mathrm{a}} \times \mathrm{K}_{\mathrm{b}}=\mathrm{K}_{\mathrm{w}}$
$\log \mathrm{K}_{\mathrm{a}}+\log \mathrm{K}_{\mathrm{b}}=\log \mathrm{K}_{\mathrm{w}}$ or $\mathrm{pK}_{\mathrm{a}}+\mathrm{pK}_{\mathrm{b}}=\mathrm{pK}_{\mathrm{w}}=14.00$
Strong acid HA (aq) $+\mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \longrightarrow \mathrm{H}_{3} \mathrm{O}^{+}(\mathrm{aq})+\mathrm{A}^{-}(\mathrm{aq})$
Strong base $\mathrm{B}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \longrightarrow \mathrm{BH}^{+}(\mathrm{aq})+\mathrm{OH}^{-}(\mathrm{aq})$

## 4. Relative strengths of acids

Is $\mathrm{HNO}_{3}$ or $\mathrm{NH}_{4}^{+}$a stronger acid? Will the reaction lie far to the right or left?
$\mathrm{HNO}_{3}(\mathrm{aq})+\mathrm{NH}_{3}(\mathrm{aq}) \rightleftharpoons \mathrm{NO}_{3}^{-}(\mathrm{aq})+\mathrm{NH}_{4}^{+}(\mathrm{aq})$

$$
\mathrm{K}=\frac{\left[\mathrm{NO}_{3}^{-}\right]\left[\mathrm{NH}_{4}^{+}\right]}{\left[\mathrm{HNO}_{3}\right]\left[\mathrm{NH}_{3}\right]}
$$

consider each acid separately:

1. $\mathrm{HNO}_{3}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}(\mathrm{aq})+\mathrm{NO}_{3}^{-}(\mathrm{aq})$

$$
\mathrm{K}_{\mathrm{a}}\left(\mathrm{HNO}_{3}\right)=\frac{\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]\left[\mathrm{NO}_{3}^{-}\right]}{\left[\mathrm{HNO}_{3}\right]}=20
$$

2. $\mathrm{NH}_{4}^{+}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}(\mathrm{aq})+\mathrm{NH}_{3}(\mathrm{aq})$

$$
\mathrm{K}_{\mathrm{a}}\left(\mathrm{NH}_{4}^{+}\right)=\frac{\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]\left[\mathrm{NH}_{3}\right]}{\left[\mathrm{NH}_{4}^{+}\right]}=5.6 \times 10^{-10}
$$

Subtract equation 2 from 1 and divide the corresponding equilibrium constants.

$$
\mathrm{K}=\frac{\mathrm{K}_{\mathrm{a}}\left(\mathrm{HNO}_{3}\right)}{\mathrm{K}_{\mathrm{a}}\left(\mathrm{NH}_{4}^{+}\right)}=\frac{\frac{\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]\left[\mathrm{NO}_{3}^{-}\right]}{\left[\mathrm{HNO}_{3}\right]}}{\frac{\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]\left[\mathrm{NH}_{3}\right]}{\left[\mathrm{NH}_{4}{ }^{+}\right]}}=\frac{\left[\mathrm{NO}_{3}^{-}\right]\left[\mathrm{NH}_{4}^{+}\right]}{\left[\mathrm{HNO}_{3}\right]\left[\mathrm{NH}_{3}\right]}=\frac{20 .}{5.6 \times 10^{-10}}=3.6 \times 10^{10}
$$

Reaction lies far to the $\qquad$ . $\mathrm{HNO}_{3}$ is a $\qquad$ than $\mathrm{NH}_{4}^{+}$.

## Types of acid-base problems

1. weak acid in water
2. weak base in water $\leftrightarrows$ salt in water
3. strong acid in water
4. strong base in water
5. buffer

Equilibrium involving weak acids
Example: Vitamin C (ascorbic acid, $\mathrm{HC}_{6} \mathrm{H}_{7} \mathrm{O}_{6}$ ) has a $\mathrm{K}_{\mathrm{a}}$ of $8.0 \times 10^{-5}$. Calculate the pH of a solution made by dissolving 500 mg in $100 . \mathrm{mL}$ of water.
$0.500 \mathrm{~g} \mathrm{x} 1 \mathrm{~mol} / 176.126 \mathrm{~g}=2.84 \times 10^{-3} \mathrm{~mol}$
$2.84 \times 10^{-3} \mathrm{~mol} / 0.100 \mathrm{~L}=0.0284 \mathrm{M}$
$\mathrm{HC}_{6} \mathrm{H}_{7} \mathrm{O}_{6}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}(\mathrm{aq})+\mathrm{C}_{6} \mathrm{H}_{7} \mathrm{O}_{6}^{-}(\mathrm{aq})$
initial molarity
change in molarity
equilibrium molarity

| $\mathrm{HC}_{6} \mathrm{H}_{7} \mathrm{O}_{6}$ | $\mathrm{H}_{3} \mathrm{O}^{+}$ | $\mathrm{C}_{6} \mathrm{H}_{7} \mathrm{O}_{6}{ }^{-}$ |
| :--- | :--- | ---: |
| 0.0284 | 0 | 0 |
| -x | +x | +x |
| $0.0284-\mathrm{x}$ | +x | +x |

$$
\mathrm{K}_{\mathrm{a}}=8.0 \times 10^{-5}=\frac{\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]\left[\mathrm{C}_{6} \mathrm{H}_{7} \mathrm{O}_{6}^{-}\right]}{\left[\mathrm{HC}_{6} \mathrm{H}_{7} \mathrm{O}_{6}\right]}=\frac{\mathrm{x}^{2}}{0.0284-\mathrm{x}}
$$

If $\mathrm{x} \ll 0.0284$, then $(0.0284-\mathrm{x}) \sim=0.0284$.

$$
\mathrm{K}_{\mathrm{a}}=8.0 \times 10^{-5}=\frac{\mathrm{x}^{2}}{0.0284}
$$

$x=0.00151$ (really 2 sf, but carry extra)

Check assumption. Is $0.0284-0.00151 \sim=0.0284$ ?
You can use assumption if x is less than $5 \%$ of the value in question.
Here $(0.00151 / 0.0284) \times 100 \%=5.3 \%$ (more than $5 \%$ ), so must use the quadratic equation.
Using quadratic eq, $\mathrm{x}=0.00147$ (really 2 sf )
$\mathrm{pH}=-\log \left[1.47 \times 10^{-3}\right]=2.83$

