Phase transition in the family of *p*-resistances

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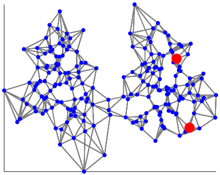
Resistance distance $R(s, t)$

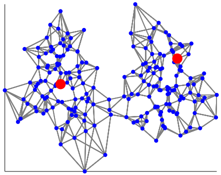
Consider the electrical network corresponding to a graph.

- **R(s,t)**: The effective resistance between s and t.
- $\mathbf{R}(\mathbf{s}, \mathbf{t}) = \min_i \sum_{e \in E} r_e i_e^2 \qquad i = (i_e)_{e \in E}$ is a unit s-t flow.

Pro: In small graphs, it captures the cluster structure!



Small resistance distance



Large resistance distance

Con: (von Luxburg et al. 2010) In large geometric graphs, it converges to the trivial limit

$$R(s, t) \approx \frac{1}{d_s} + \frac{1}{d_t}$$

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p-Resistance

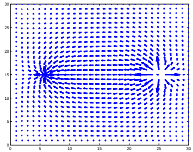
How we can cure this problem?

p-Resistance : For $p \geq 1$, define

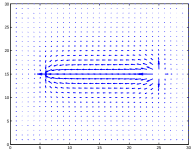
$$R_{\mathbf{p}}(s,t) \ := \ \min_{i} \sum_{e \text{ edge}} r_e \, |i_e|^{\mathbf{p}}$$

Theorem (Special cases of $R_p(s,t)$)

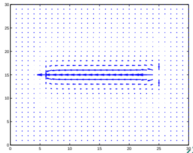
- $p = 1$: Shortest path distance
- $p = 2$: Standard resistance distance
- $p \to \infty$: Related to *s*-*t*-mincut



$p = 2$



$p = 1.33$



$p = 1.1$

Phase transition in the family of p-resistances

Main Theorem:

For **large** random geometric graphs in R^d:

1. If $p < 1 + 1/(d-1)$, then the "global" contribution dominates the "local" one.
   $\leadsto$ meaningful distance ☺
2. If $p > 1 + 1/(d-2)$, then all "global" information vanishes.
   $\leadsto$ useless distance ☹