# Estimating the contribution of non-genetic factors to gene expression using Gaussian Process Latent Variable Models

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Learning and Inference in Computational Systems Biology

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#### 1 eQTL mapping

#### 2 Dataset

#### 3 The model

#### 4 Experiments

#### 5 Conclusions



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# Expression Quantitative Trait Loci - eQTL

- Transcript abudance is regulated by polymorphisms in the regulatory elements
- Statistical methods can be used to discover which polymorphism affects the expression levels of a gene
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# Project GENEVAR - GENe Expression VARiation

- Gene expression data from EBV-transformed lymphoblastoid cell lines (Stranger et al., Nature Genetics 2007)
- 270 individuals from Hapmap phase I and II
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# Confounding factors

Several studies have shown that non-genetic factors can obfuscate associations:

- Known Factors: age, sex, ethnicity, ...
- Batch effects: optical effects
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# Modelling non-genetic factors

- Our model is inspired by *Stegle et al, Lecture notes in Computer Science (2006)*.
- We model non-genetic factors as unobserved latent variables.
- Gene expression levels are described as a linear function of SNP data and non-genetic factors

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# dual Probabilistic Principal Component Analysis

We learn the parameters by:

- Marginalizing  $\mathbf{W}, \mathbf{V}, \mu, \epsilon$
- Maximizing the log-likelihood with respect to the latent variables (X)

For a particular choice of priors over  ${\bf W}$  and  ${\bf V}$  this approach is equivalent to probabilistic Principal Component Analysis

### dual Probabilistic Principal Component Analysis

We put Gaussian priors over **W**, **V** and  $\mu$ :

$$P(\mathbf{W}) = \prod_{i=1}^{D} N(\mathbf{w}_i | \mathbf{0}, \alpha_w \mathbf{I})$$
$$P(\mathbf{V}) = \prod_{i=1}^{D} N(\mathbf{v}_i | \mathbf{0}, \alpha_v \mathbf{I})$$

 $P(\mu) = N(\mu | \mathbf{0}, \alpha_{\mu} \mathbf{I})$ 

### dual Probabilistic Principal Component Analysis



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The likelihood of  ${\bf Y}$  can be then written as

$$P(\mathbf{Y}|\mathbf{W},\mathbf{X},\mathbf{S},\mu) = \prod_{j=1}^{D} N(\mathbf{y}_{j}|\mathbf{W}\mathbf{x}_{j} + \mathbf{V}\mathbf{s}_{j} + \mu,\sigma^{2}\mathbf{I})$$

Marginalizing  $\mathbf{W},\mathbf{V},\mu,\epsilon$  we obtain the marginal likelihood

$$P(\mathbf{Y}|\mathbf{X}) = \prod_{j=1}^{D} N(\mathbf{y}_j|\mathbf{0}, \alpha_w \mathbf{X} \mathbf{X}^\top + \alpha_v \mathbf{S} \mathbf{S}^\top + \alpha_\mu + \sigma^2 \mathbf{I})$$

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### Population structure



### Accounting for population structure

$$C = \alpha_{w} \mathbf{X} \mathbf{X}^{\top} + \alpha_{v} \mathbf{S} \mathbf{S}^{\top} + \alpha_{\mu} + \sigma^{2} \mathbf{I}$$

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$$\mathcal{C} = \alpha_{w} \mathbf{X} \mathbf{X}^{\top} + \alpha_{v} \mathbf{S} \mathbf{S}^{\top} + \alpha_{\rho} \mathbf{P} \mathbf{P}^{\top} + \alpha_{g} \mathbf{G} \mathbf{G}^{\top} + \alpha_{\mu} + \sigma^{2} \mathbf{I}$$



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### eQTL scan using data from Hapmap and GENEVAR

At each locus we compute the log-odds score:

$$L_{i} = \log_{10} \left\{ \prod_{n} \frac{P(Y_{m}|s_{n,j}, \theta_{i,n})}{P(Y_{m}|\theta_{bkg})} \right\}$$
(1)

The significance of an association is evaluated via permutation testing.

### Traditional eQTL scan



### eQTL scan accounting for non-genetic factors



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- We presented a model that explicitly accounts for non-genetic factors
- Using this model we can detect an higher number of significant associations
- Many extensions are possible (future work)