Identifying interactions in the time and frequency domains in local and global networks – A Granger causality approach

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Outline

- Systematic Description on Granger Causality
- Local Network Result
- Global Network Result
- Recent Results on Sheep Data

Pair-wise Granger Causality (1)

- One Time Series X

$$\begin{cases} X_t = \sum_{i=1}^{\infty} a_{1i} X_{t-i} + \varepsilon_{1t} \\ Y_t = \sum_{i=1}^{\infty} b_{1i} Y_{t-i} + \varepsilon_{2t} \end{cases}$$

- Two Time Series X and Y

$$\begin{cases} X_t = \sum_{i=1}^{\infty} c_{1i} X_{t-i} + \sum_{i=1}^{\infty} d_{1i} Y_{t-i} + \varepsilon_{3t} \\ Y_t = \sum_{i=1}^{\infty} e_{1i} Y_{t-i} + \sum_{i=1}^{\infty} f_{1i} X_{t-i} + \varepsilon_{4t} \end{cases}$$

Pair-wise Granger Causality (2)

- The conditional Granger causality from Y to X conditional on Z is defined as

$$F_{Y \to X} = \ln\left(\frac{\left|\mathrm{var}(\varepsilon_{1t})\right|}{\left|\mathrm{var}(\varepsilon_{3t})\right|}\right)$$

Pair-wise Granger Causality (3)

- Pair-wise Granger Causality Problem

Conditional Granger Causality(1)

- Two time series X and Z

$$\begin{cases} X_t = \sum_{i=1}^{\infty} a_{1i} X_{t-i} + \sum_{i=1}^{\infty} c_{1i} Z_{t-i} + \varepsilon_{1t} \\ Z_t = \sum_{i=1}^{\infty} b_{1i} Z_{t-i} + \sum_{i=1}^{\infty} d_{1i} X_{t-i} + \varepsilon_{2t} \end{cases}$$

- The noise covariance matrix for the system can be represented as

$$S = \begin{bmatrix} \mathrm{var}(\varepsilon_{1t}) & \mathrm{cov}(\varepsilon_{1t}, \varepsilon_{2t}) \\ \mathrm{cov}(\varepsilon_{2t}, \varepsilon_{1t}) & \mathrm{var}(\varepsilon_{2t}) \end{bmatrix}$$

Conditional Granger Causality(2)

- Three time series X, Y and Z

$$\begin{cases} X_t = \sum_{i=1}^{\infty} a_{2i} X_{t-i} + \sum_{i=1}^{\infty} b_{2i} Y_{t-i} + \sum_{i=1}^{\infty} c_{2i} Z_{t-i} + \varepsilon_{3t} \\ Y_t = \sum_{i=1}^{\infty} d_{2i} X_{t-i} + \sum_{i=1}^{\infty} e_{2i} Y_{t-i} + \sum_{i=1}^{\infty} f_{2i} Z_{t-i} + \varepsilon_{4t} \\ Z_t = \sum_{i=1}^{\infty} g_{2i} X_{t-i} + \sum_{i=1}^{\infty} h_{2i} Y_{t-i} + \sum_{i=1}^{\infty} k_{2i} Z_{t-i} + \varepsilon_{2t} \end{cases}$$

- The noise covariance matrix for the system can be represented as

$$\Sigma = \begin{bmatrix} \mathrm{var}(\varepsilon_{3t}) & \mathrm{cov}(\varepsilon_{3t}, \varepsilon_{4t}) & \mathrm{cov}(\varepsilon_{3t}, \varepsilon_{5t}) \\ \mathrm{var}(\varepsilon_{4t}, \varepsilon_{3t}) & \mathrm{var}(\varepsilon_{4t}) & \mathrm{cov}(\varepsilon_{4t}, \varepsilon_{5t}) \\ \mathrm{var}(\varepsilon_{5t}, \varepsilon_{3t}) & \mathrm{var}(\varepsilon_{5t}, \varepsilon_{4t}) & \mathrm{var}(\varepsilon_{5t}) \end{bmatrix}$$

Conditional Granger Causality(3)

- The conditional Granger causality from Y to X conditional on Z is defined as

$$F_{Y \to X|Z} = \ln\left(\frac{\left|\mathrm{var}(\varepsilon_{1t})\right|}{\left|\mathrm{var}(\varepsilon_{3t})\right|}\right)$$

Conditional Granger Causality(3)

- Conditional Granger Causality Problem

Partial Granger Causality(1)

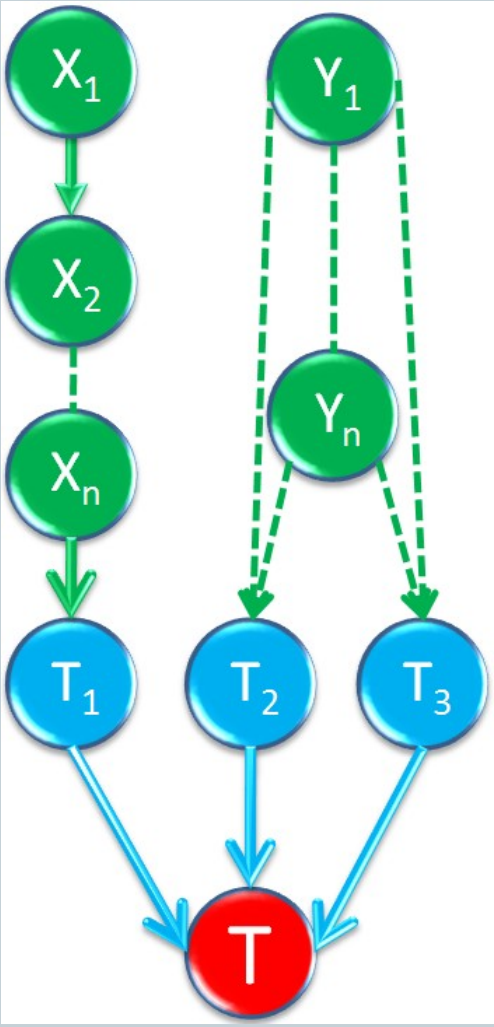
- The partial Granger causality from Y to X conditional on Z is defined as

$$F_{Y\to X|Z} = \ln\left(\frac{\left|\mathrm{var}(\varepsilon_{1t}) - \mathrm{cov}(\varepsilon_{1t},\varepsilon_{2t})\,\mathrm{var}(\varepsilon_{2t})^{-1}\,\mathrm{cov}(\varepsilon_{2t},\varepsilon_{1t})\right|}{\left|\mathrm{var}(\varepsilon_{3t}) - \mathrm{cov}(\varepsilon_{3t},\varepsilon_{5t})\,\mathrm{var}(\varepsilon_{5t})^{-1}\,\mathrm{cov}(\varepsilon_{5t},\varepsilon_{3t})\right|}\right)$$

Ancestor-Finding Partial Granger Causality

- Suppose a subset network of whole network is:

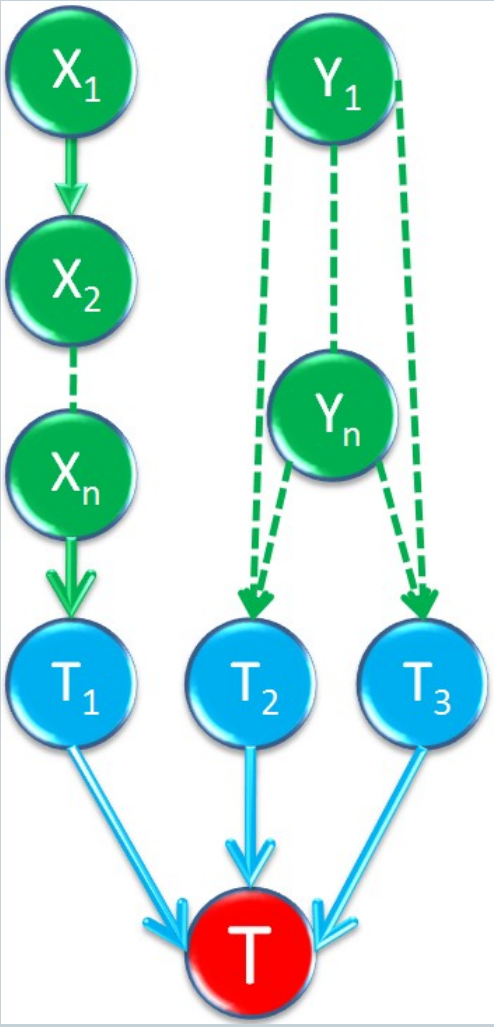
- Task is to find the Direct ancestor (parents) T_1 T_2 T_3



Ancestor-Finding Partial Granger Causality

- Step one: Pair-Wise

$A_0(T)=$
$\{T_1,T_2,T_3, X_1,\ldots X_n ,Y_1 ,\ldots,Y_n\}.$



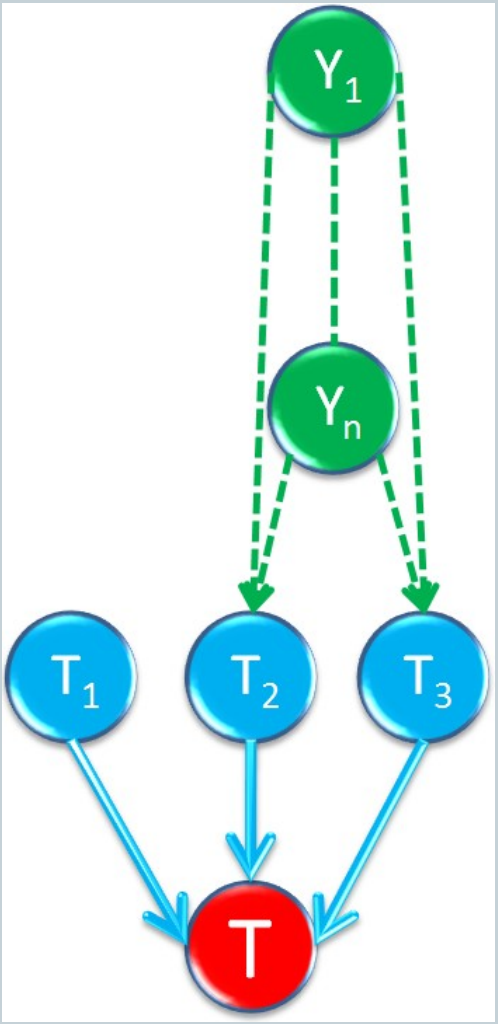
Ancestor-Finding Partial Granger Causality

- Step two:

Partial Granger Causality

On single nodes.

$A_1(T) = \{T_1, T_2, T_3, Y_1, \ldots, Y_n\}$



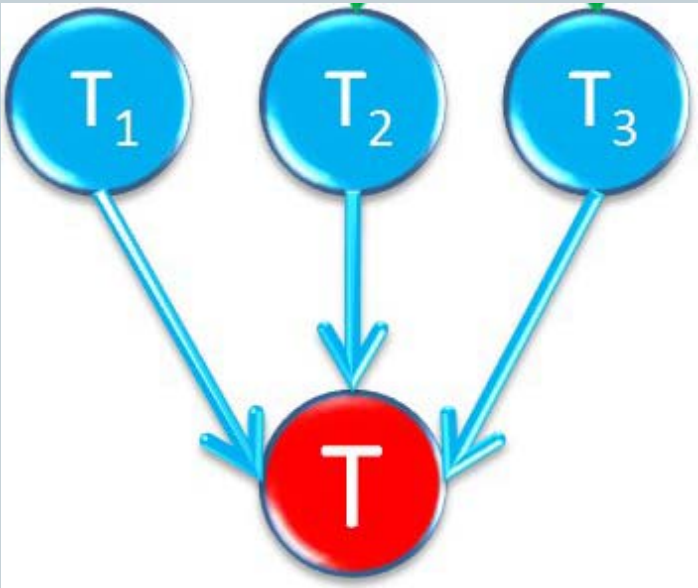
Ancestor-Finding Partial Granger Causality

- Step two:

Partial Granger Causality

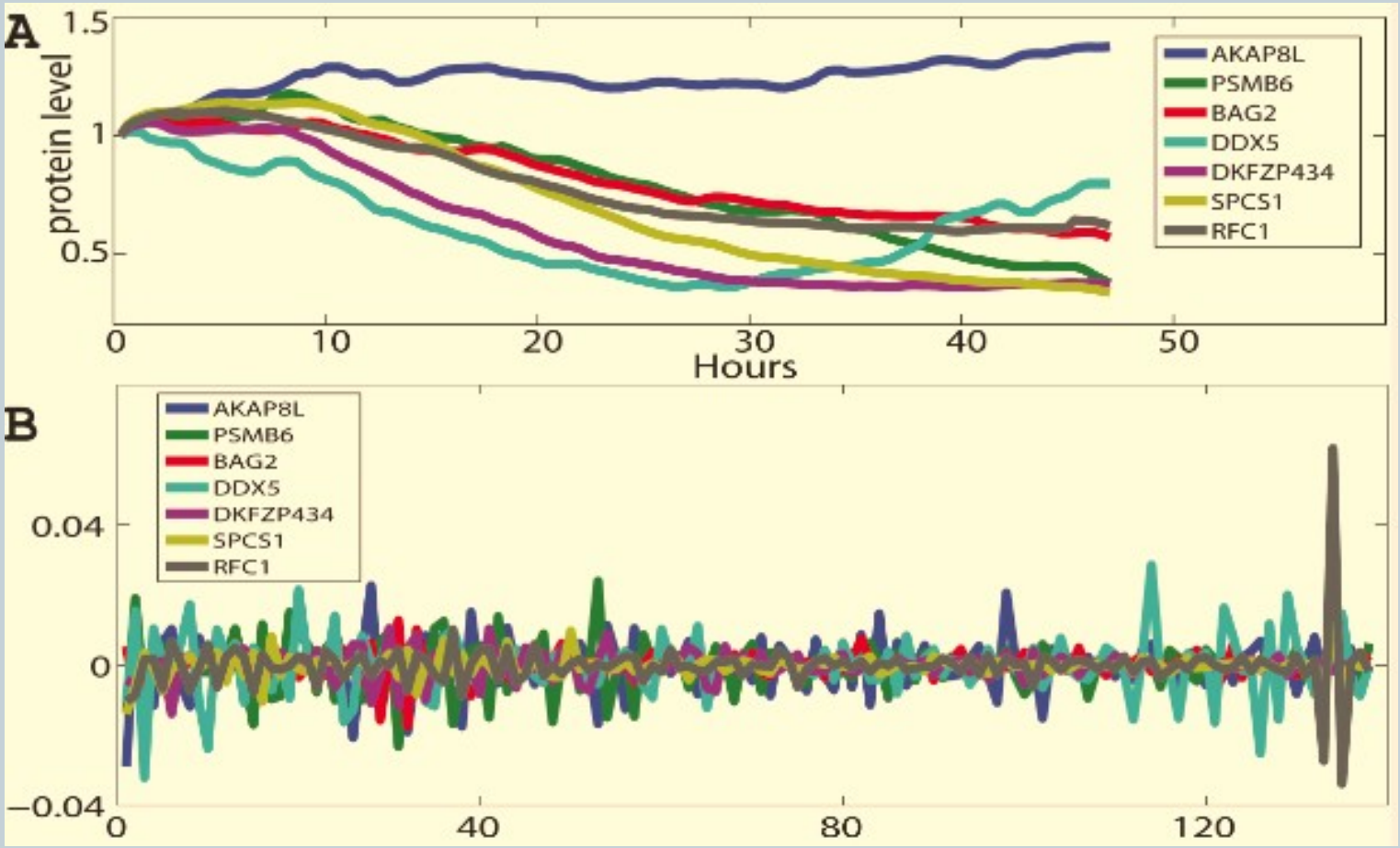
On two nodes.

$A_2(T) = \{T_1, T_2, T_3\}$

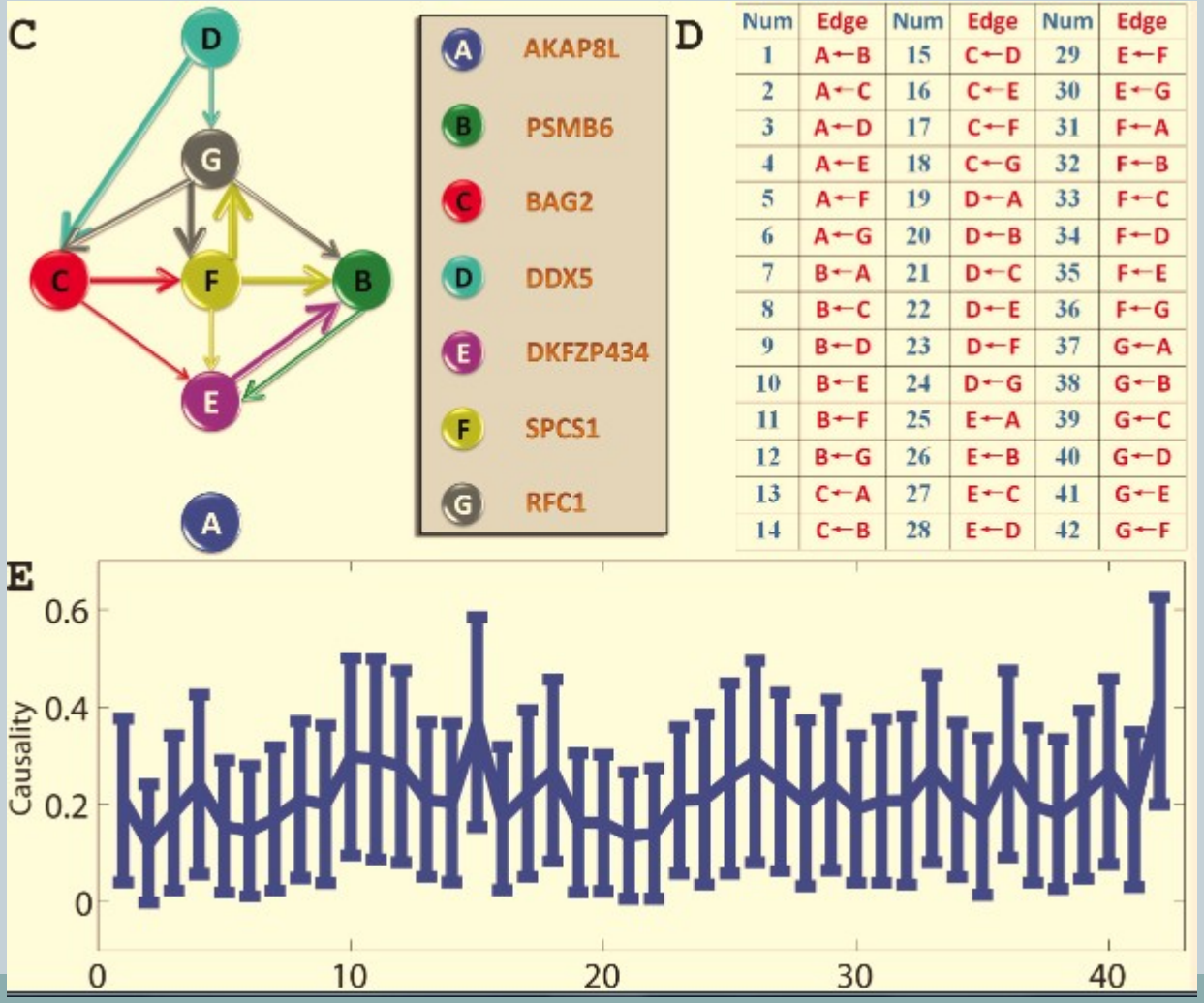


Local Network Result

- **A Local Circuit of Seven Proteins**



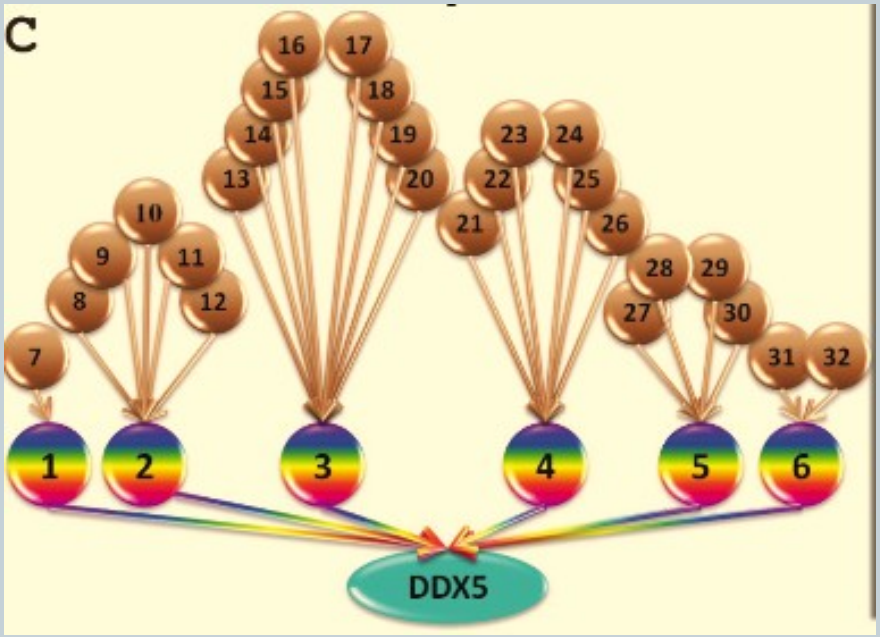
Local Network Result

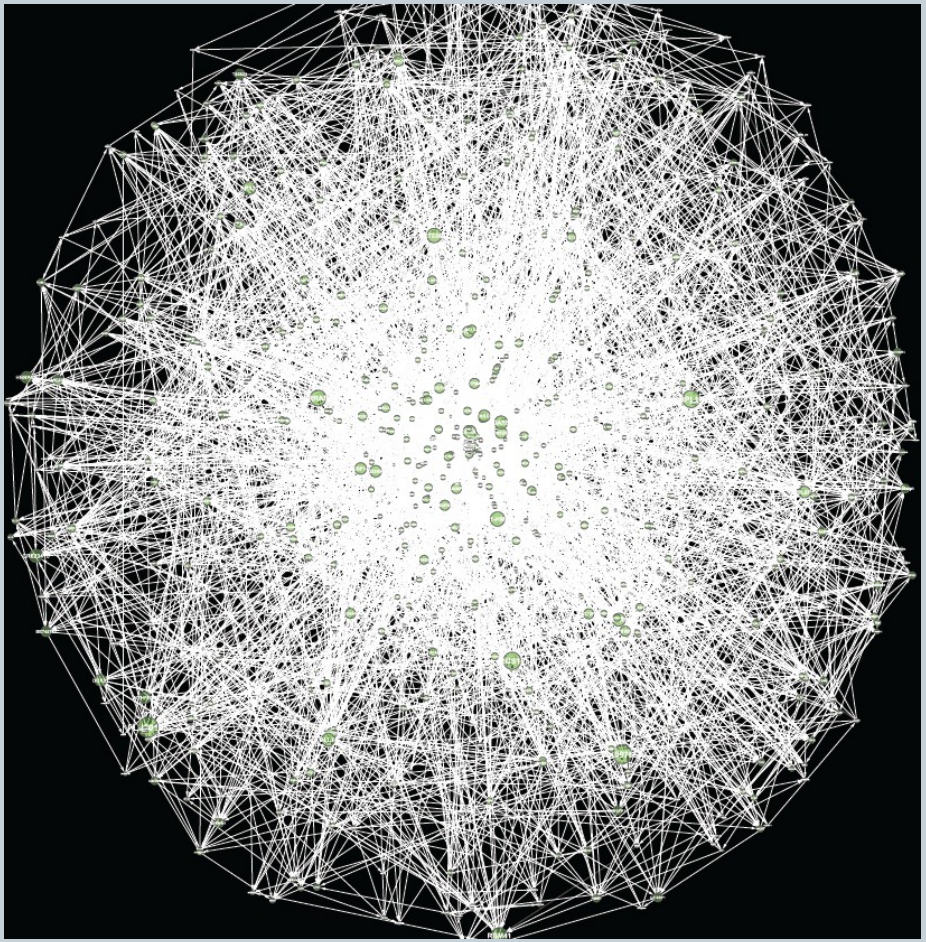


Num	Edge	Num	Edge	Num	Edge
1	A←B	15	C←D	29	E←F
2	A←C	16	C←E	30	E←G
3	A←D	17	C←F	31	F←A
4	A←E	18	C←G	32	F←B
5	A←F	19	D←A	33	F←C
6	A←G	20	D←B	34	F←D
7	B←A	21	D←C	35	F←E
8	B←C	22	D←E	36	F←G
9	B←D	23	D←F	37	G←A
10	B←E	24	D←G	38	G←B
11	B←F	25	E←A	39	G←C
12	B←G	26	E←B	40	G←D
13	C←A	27	E←C	41	G←E
14	C←B	28	E←D	42	G←F

Global Network Result

- 800 proteins



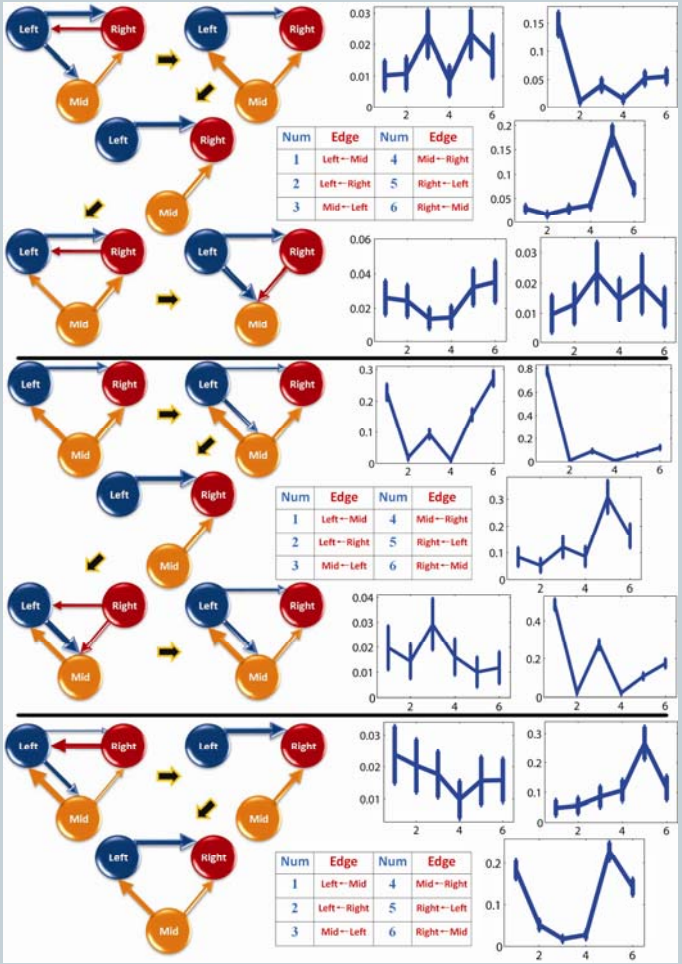


Recent Result

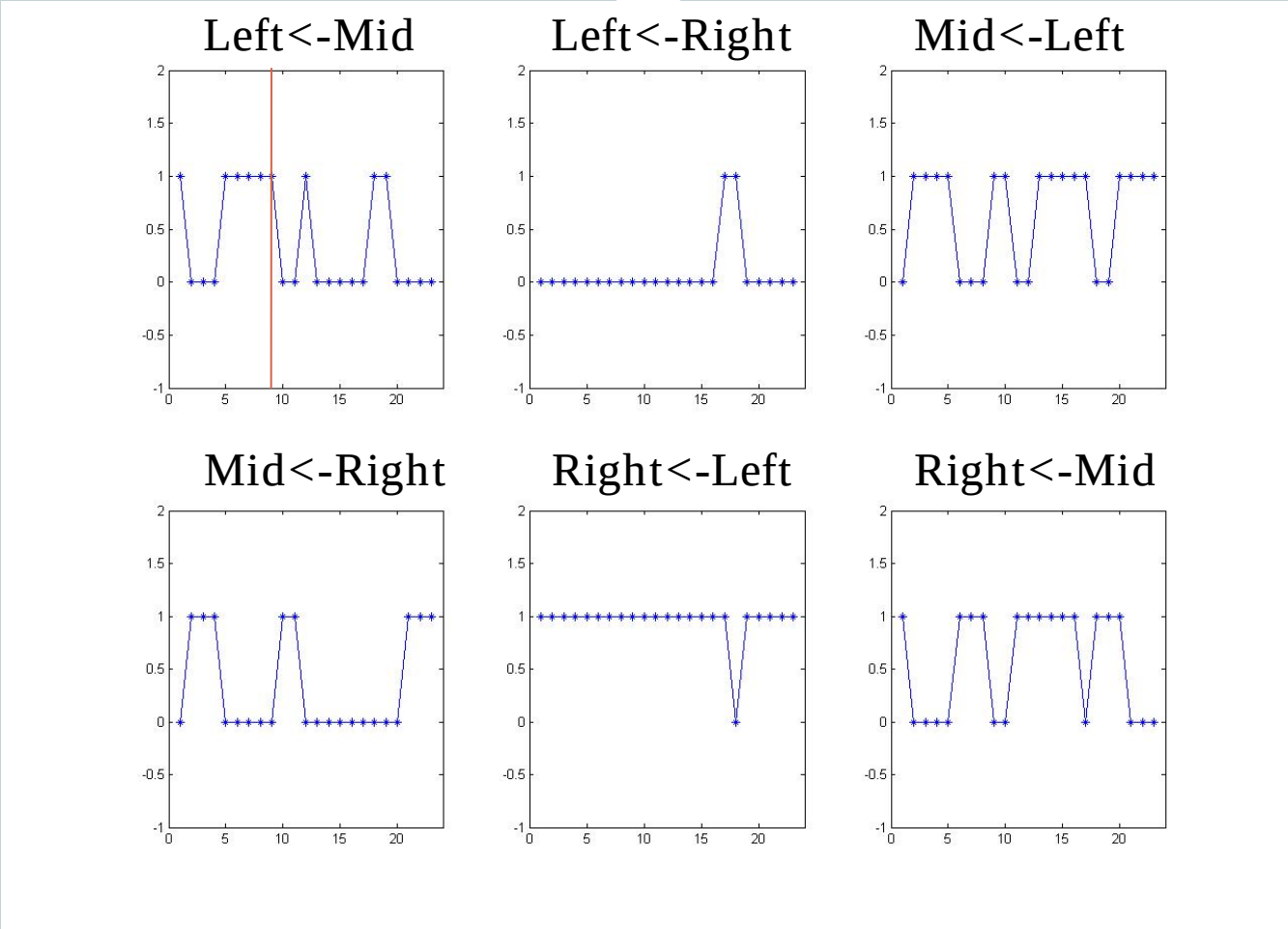
Sheep Data: Local Field Potential Data recorded

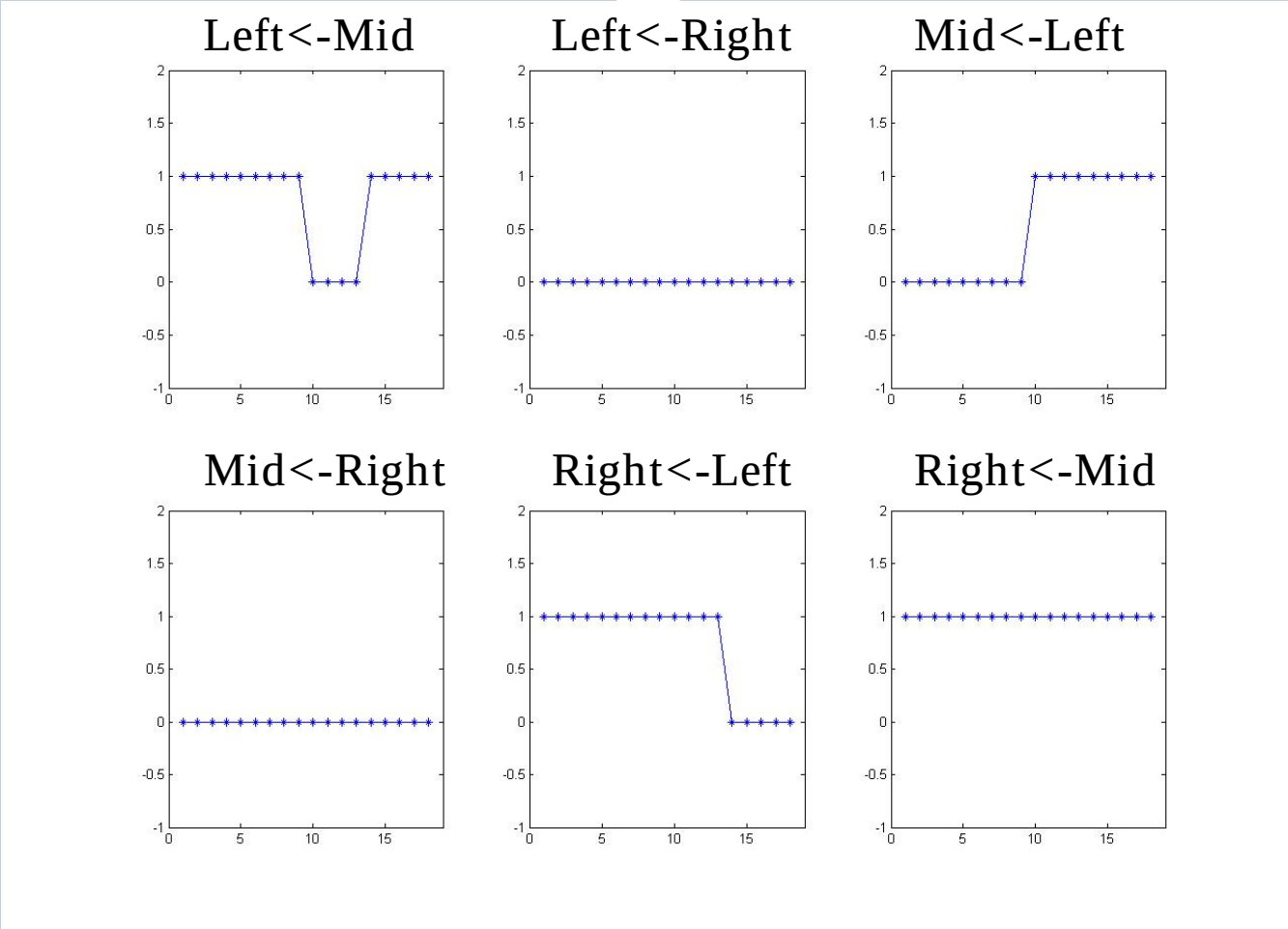
From three areas: Left IT, Right IT

and Mid cingulate

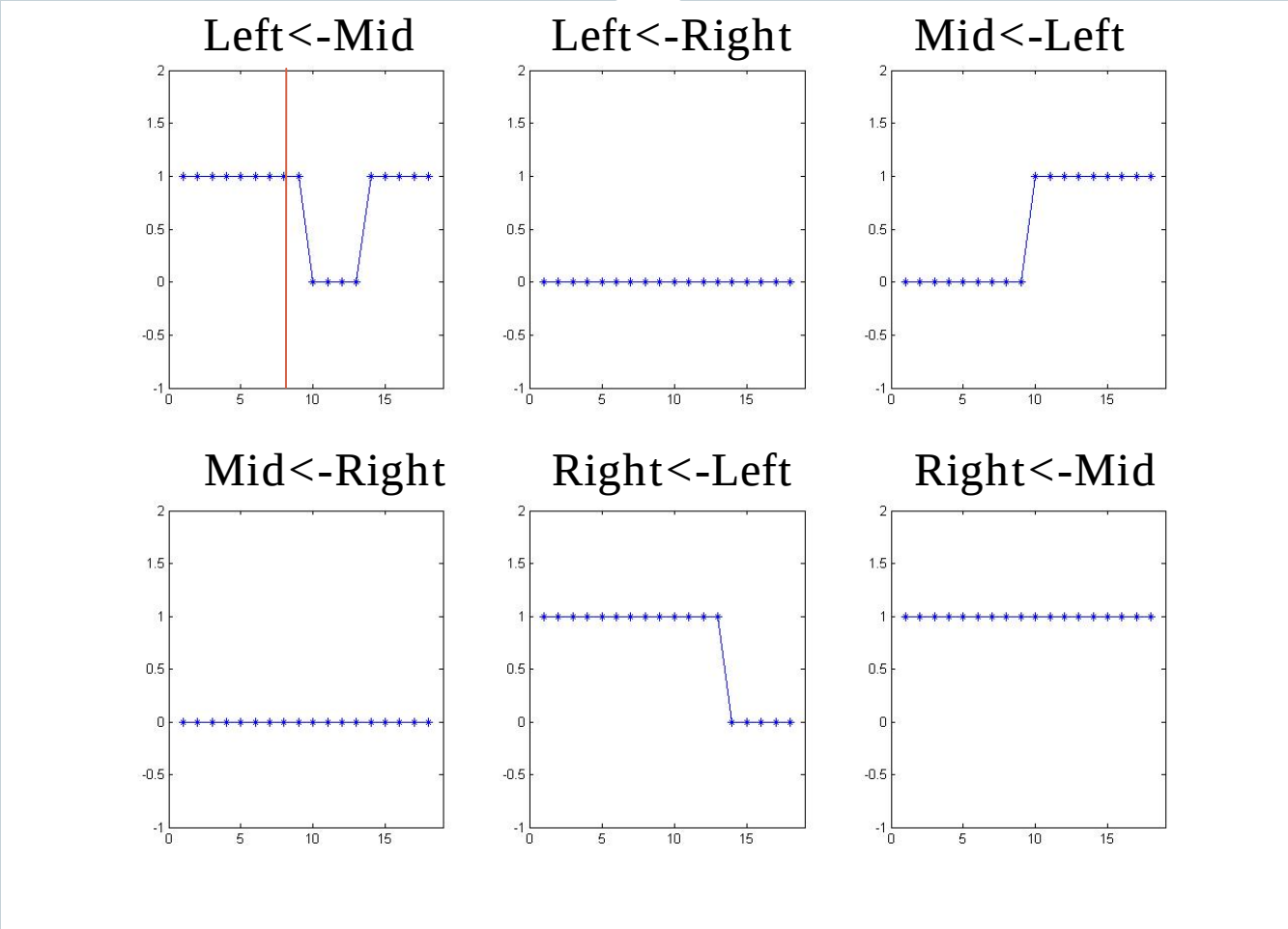


Recent Results

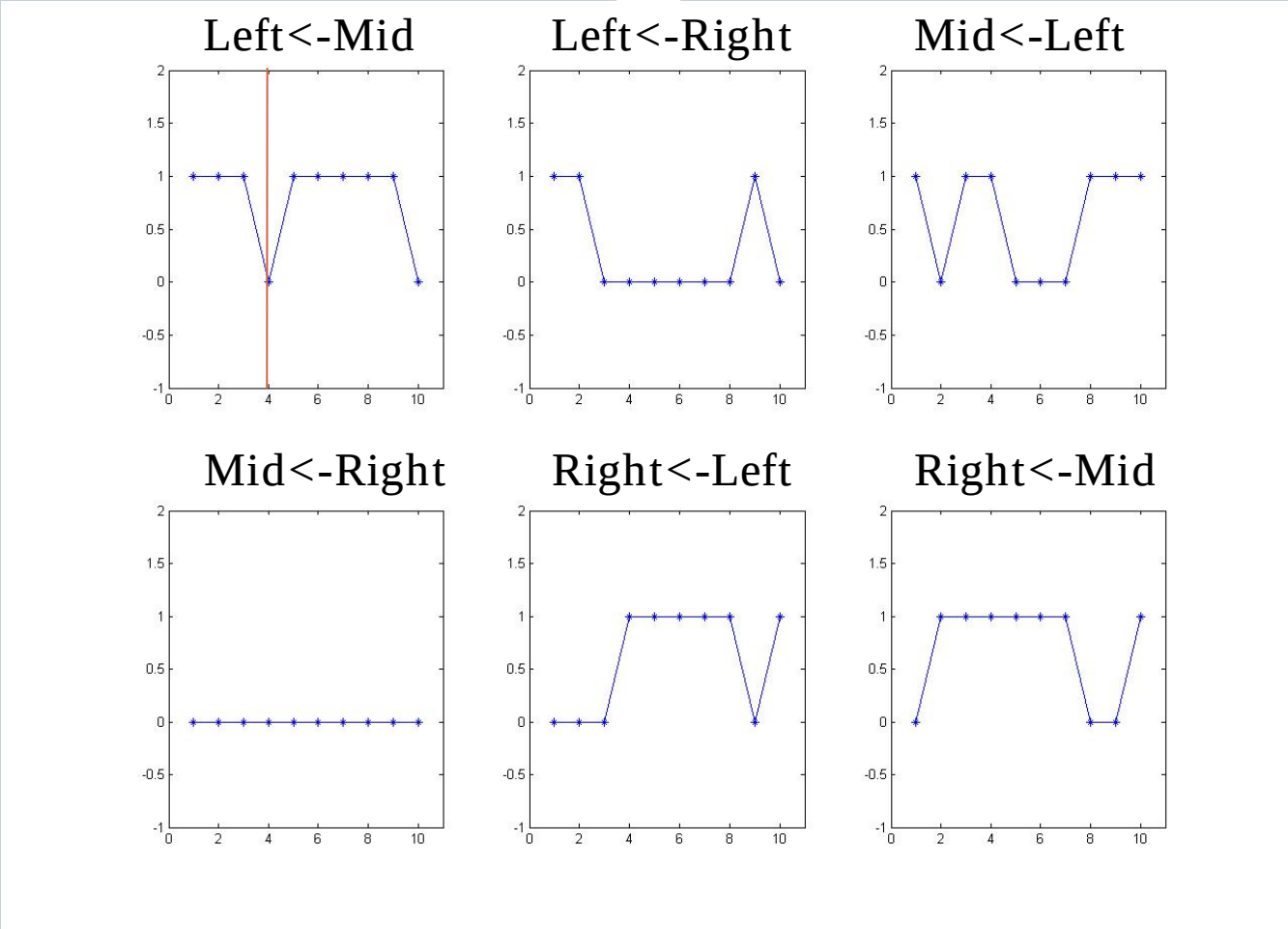



Left<-Mid
Left<-Right
Mid<-Left
Mid<-Right
Right<-Left
Right<-Mid

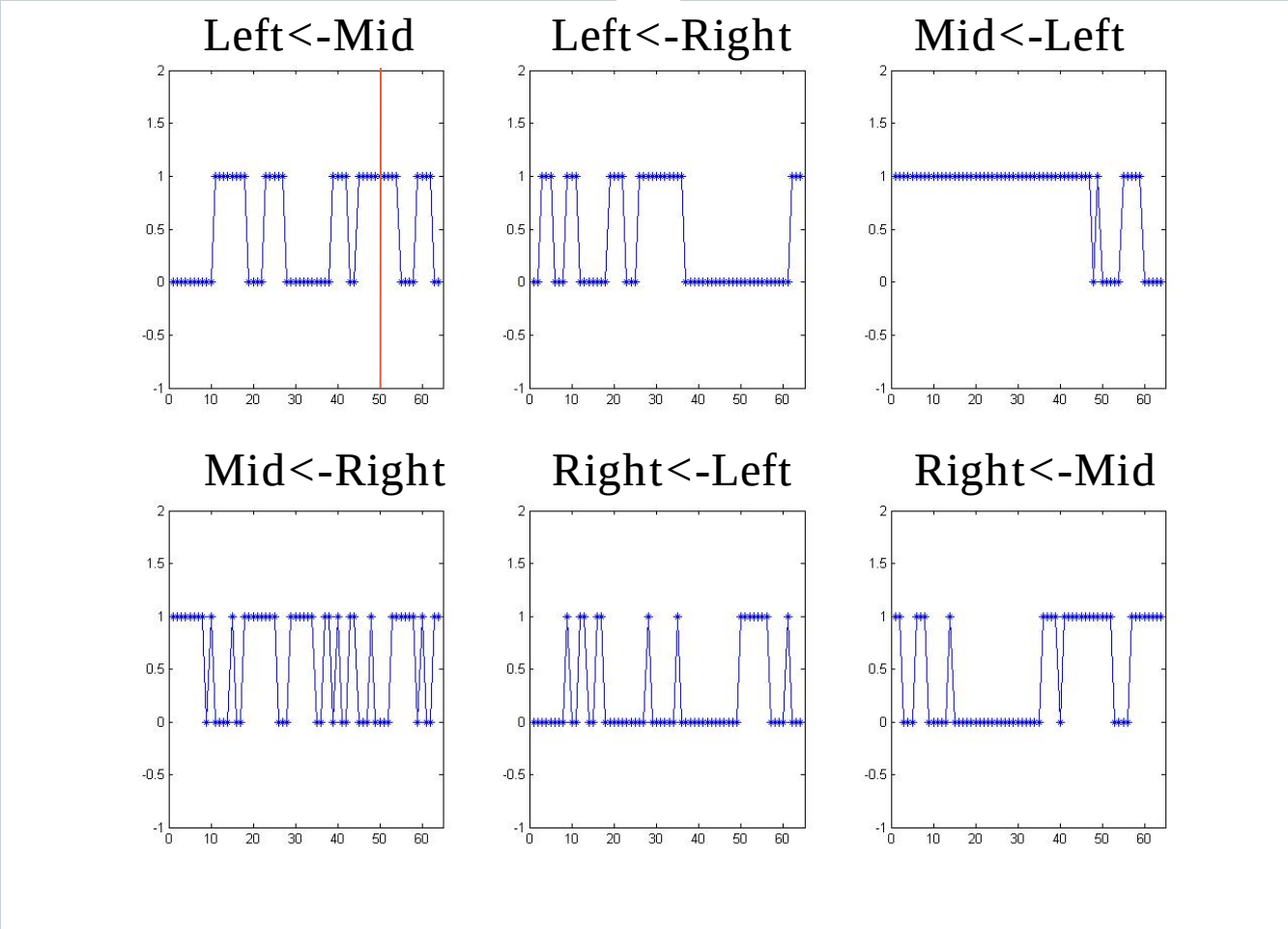
Recent Results



Recent Results



Recent Results



Thank you