Modeling Diffusion in Social Networks using Network Properties

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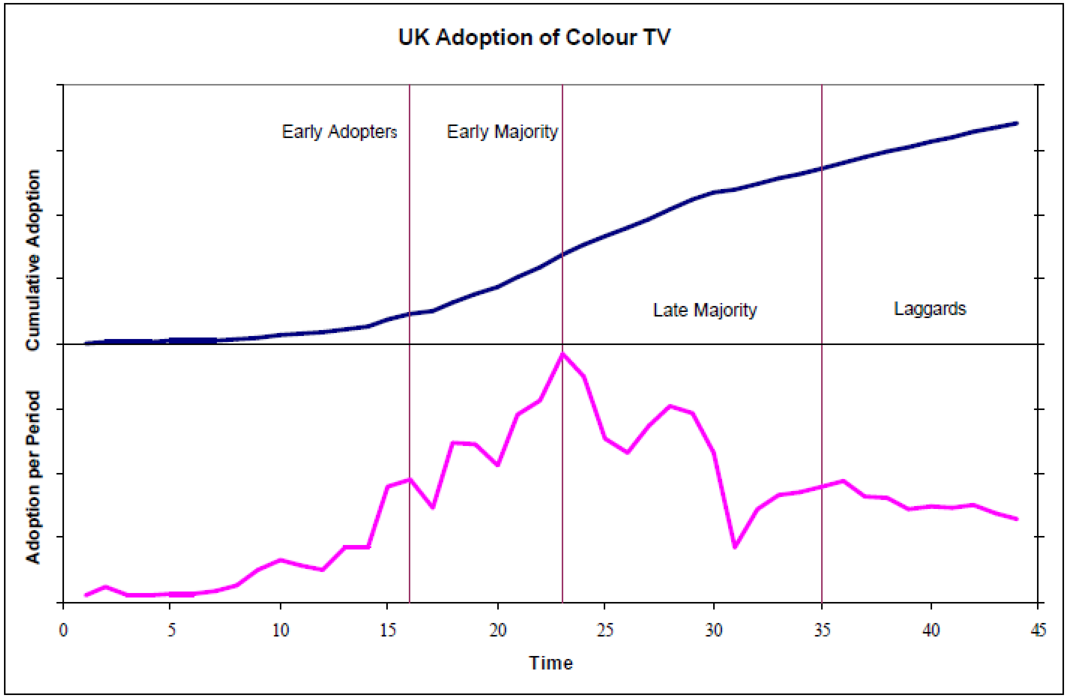
Why Diffusion in Social Networks?

Diffusion of items (videos, news, photos, etc) is important and ubiquitous in social networks



Proper models of diffusion can predict:

- Rate of adoption at a particular time
- The time of peak demand
- The magnitude of peak demand

Applications of Diffusion Models in Telecommunications, Nigel Meade
http://www.itu.int/ITU-D/finance/work-cost-tariffs/events/expert-dialogues/forecasting/meade-presentation.pdf

Why Modeling Diffusion using Network Properties?

For item diffusion we have micro and macro models.

	Micro models	**Macro** models
Work at	**Individual** level	**Network** level
Representatives	Independent Cascade (**IC**)[1], Linear Threshold (**LT**)[2]	**Bass Model**[3] and its extensions
Parameters	**Local**, each user has his own parameters	**Global**, for the whole network
Network properties	Exploit (**+**)	Do NOT exploit (**-**)
No. of parameters	So many (**-**)	Just a few (**+**)

[1] Goldenberg et al. (2001)*Talk of the Network: A Complex Systems Look at the Underlying Process of Word-of-Mouth*

[2] Granovetter, M. (1978) *Threshold Models of Collective Behavior*

[3] Bass, F. M. (1969) *A new Product Growth Model for Consumer Durables*

Research Questions

How can macro models exploit network properties (e.g. degree distribution)?

In this work:

Q1) How to model diffusion in a network given its degree distribution?

Q2) How to combine parameters of diffusion and of degree distribution to give a better model?

Concepts & Notations

- *N:* network size.
- *f(t)*: instantaneous fraction of adopters at time *t*
- *F(t)*: cumulative fraction of adopters at time *t*

$$f(t) = F'(t)$$

- a_t : adoption at *t* , A_t : adoption before *t*.

Observe that:

$$\underbrace{f(t)}_{\substack{\text{fract of}\\ \text{new adopters}}} = \underbrace{(1 - F(t))}_{\text{nonadopters at } t} \cdot \underbrace{\mathrm{E}\left[P\left(a_t \mid \overline{A_t}\right)\right]}_{\text{avg. adopting prob}}$$

Ordinary differential equation (ODE) linking *F(.), P(.)*:

$$F'(t) = (1 - F(t)) \cdot P\left(a_t \mid \overline{A_t}\right)$$

Goal: estimate the adoption probability *P(.)* as a function of *F(.)*

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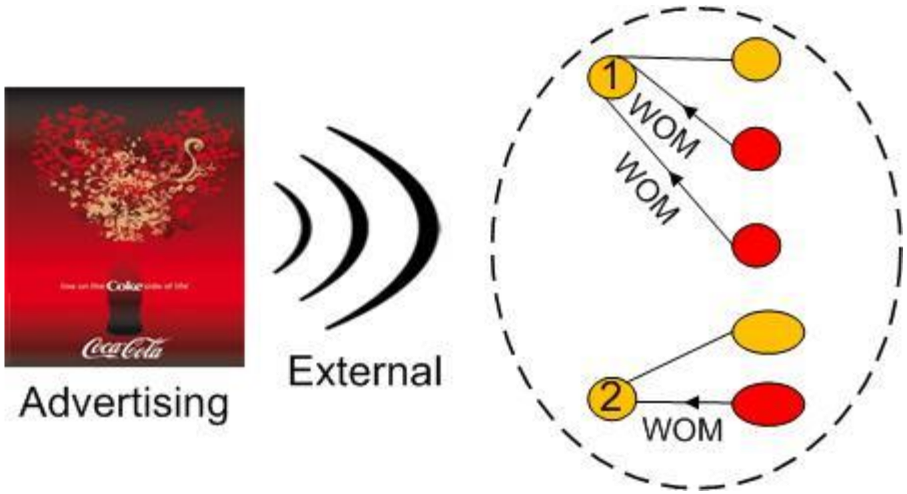
Estimation of Adoption Probability General Case



Internal comes from WOM (word of mouth)

Contributions from external & internal influences are weighted with w_e and $1\text{-}w_e$ respectively.

$$P\left(a_t \mid \overline{A_t}\right) = w_e \cdot P_{ext}\left(a_t \mid \overline{A_t}\right) + \left(1 - w_e\right) \cdot P_{int}\left(a_t \mid \overline{A_t}\right)$$
$$= w_e \cdot p_e + \left(1 - w_e\right) \cdot P_{int}\left(a_t \mid \overline{A_t}\right)$$

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Bass Model (BM)

Assumptions of BM:

B1) Each user can influence every other user.

$$|u\text{'s adopted neighbors}| = N \cdot F(t), \quad \forall u, \forall t$$

B2) Internal influence is proportional to No. of adopted neighbors: $P_{int}\left(a_t \mid \overline{A_t}\right) = q_1 \cdot N \cdot F(t)$

$$\Rightarrow P\left(a_t \mid \overline{A_t}\right) = p + q \cdot F(t) \qquad (*)$$

where $p = w_e \cdot p_e$ and $q = (1 - w_e) \cdot q_1 \cdot N$

(*) combines with the ODE
→ Bass Model (1969)

$$F(t) = \frac{e^{[(p+q)t]} - 1}{e^{[(p+q)t]} + q/p}$$

$$f(t) = F'(t) = \frac{(p+q)^2}{p} \cdot \frac{e^{[(p+q)t]}}{\left\{e^{[(p+q)t]} + q/p\right\}^2}$$

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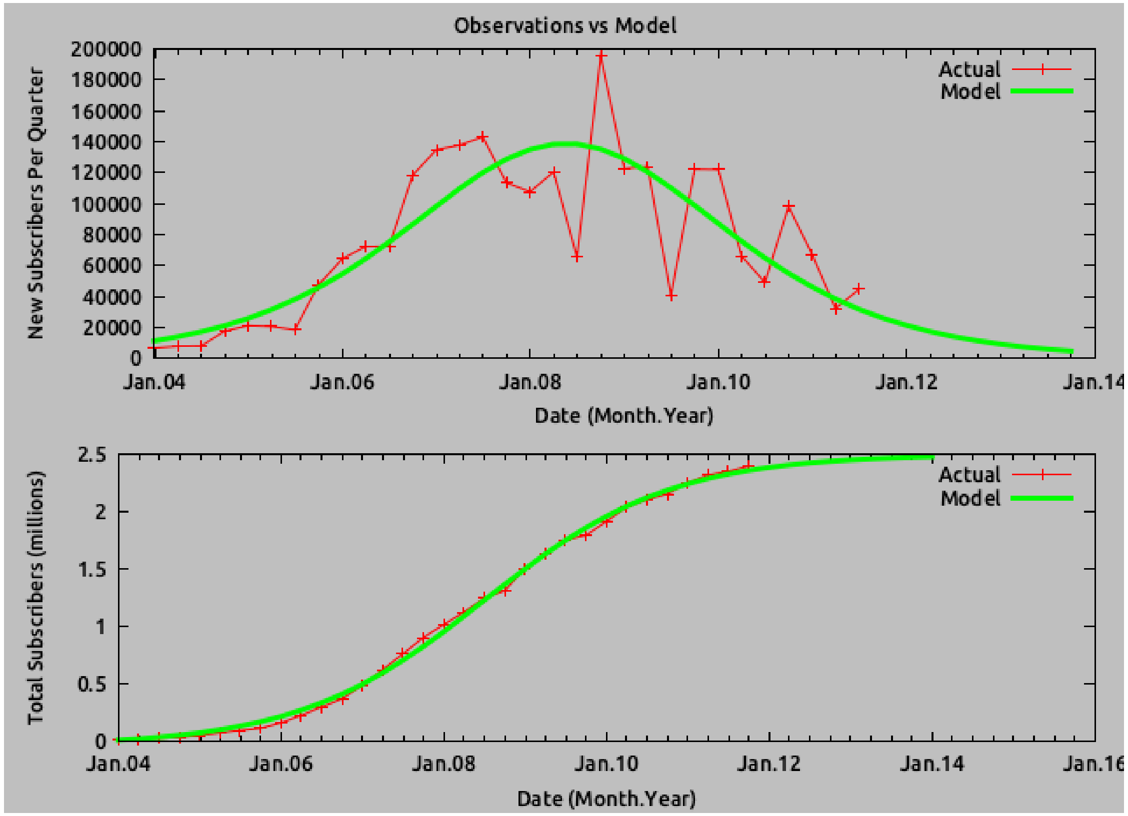
Bass Model (cont.)

"…Bass model ignores the network structure…"
[Xiaodan S. et al, WWW 07]

B1) Each user can influence/influenced every other user

Each user can influence/influenced only his friends
→his adoption prob. depends on his degree

Bass Model (cont.)



http://serial.innovatiasystems.eu/cms/

Adoption Probability for Specific Degree Distributions

- Given any degree distribution *P(k)*, we obtained the formula:

(**Internal** adoption probability)

$$P_{\text{int}}\left(a_t \mid \overline{A_t}\right) = \sum_{k=1}^{N-1} P(k) \sum_{j=0}^{k} \left[\binom{k}{j} F_t^{\,j} \left(1 - F_t\right)^{k-j} P\left(a_t \mid \overline{A_t}, j\right) \right]$$

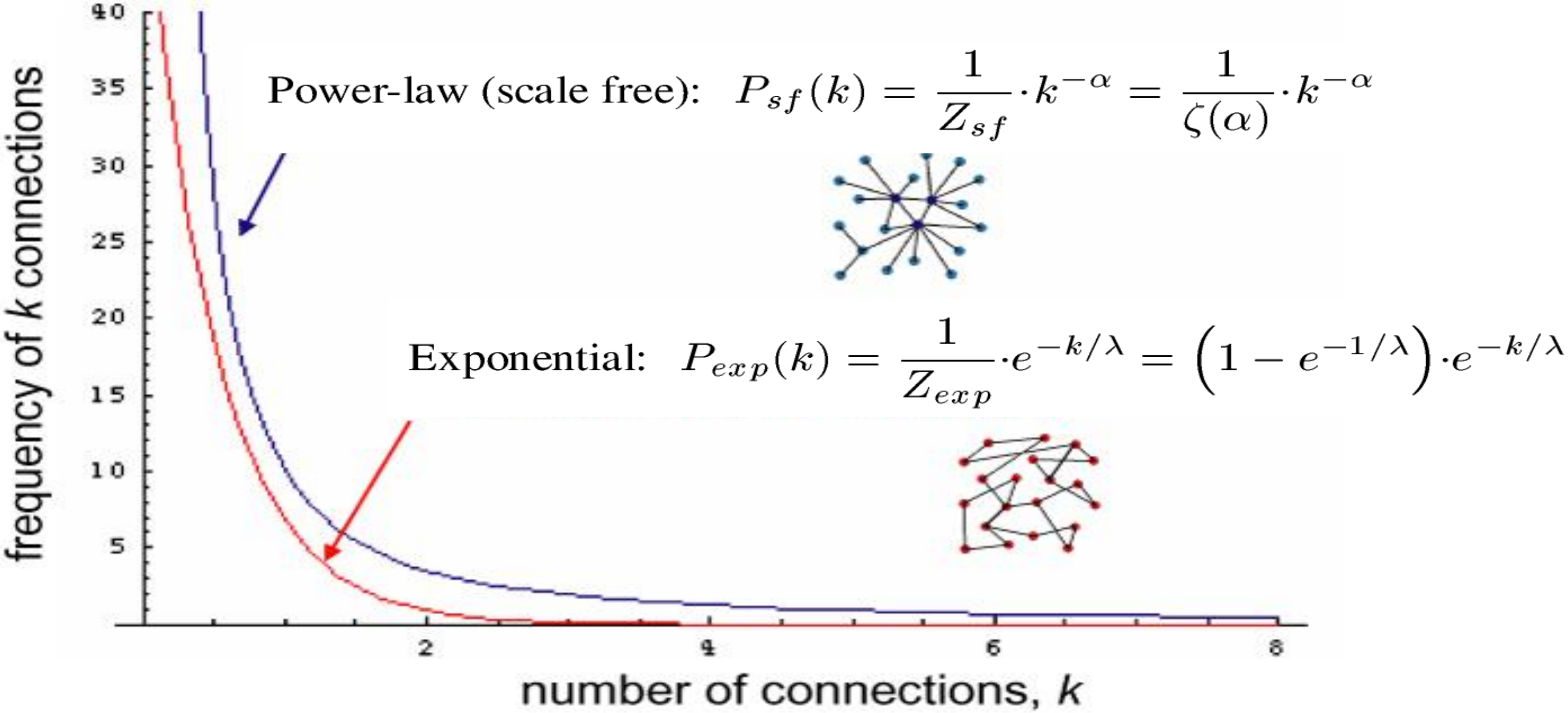
where $F_t \equiv F(t)$ and $P\left(a_t \mid \overline{A_t}, j\right)$ is the prob of adopting given that a user has j adopted neighbors

- Still keep B2), linear influence: $P\left(a_t \mid \overline{A_t}, j\right) = c \cdot j$

where c is a constant

To complete estimation, needs specific degree distributions!

Specific Degree Distributions



Pagel *et al. BMC Evolutionary Biology*
2007 **7**(Suppl 1):S16

Parameter of degree distribution: α (power law) or λ (exponential)

Estimation of Internal Adoption Probability

Linear assumption & specific degree distribution provide estimations:

1) Scale free network:

$$P_{int}^{sf}(a_t|\overline{A_t}) = \frac{\zeta(\alpha-1)}{\zeta(\alpha)} \cdot c \cdot F_t$$

where $\zeta(\alpha) = \sum_{k=1}^{\infty} k^{-\alpha}$ is the Riemann Zeta function

2) Exponential network:

$$P_{int}^{exp}(a_t|\overline{A_t}) = \frac{e^{-1/\lambda}}{1-e^{-1/\lambda}} \cdot c \cdot F_t$$

These estimations → two models in our work.

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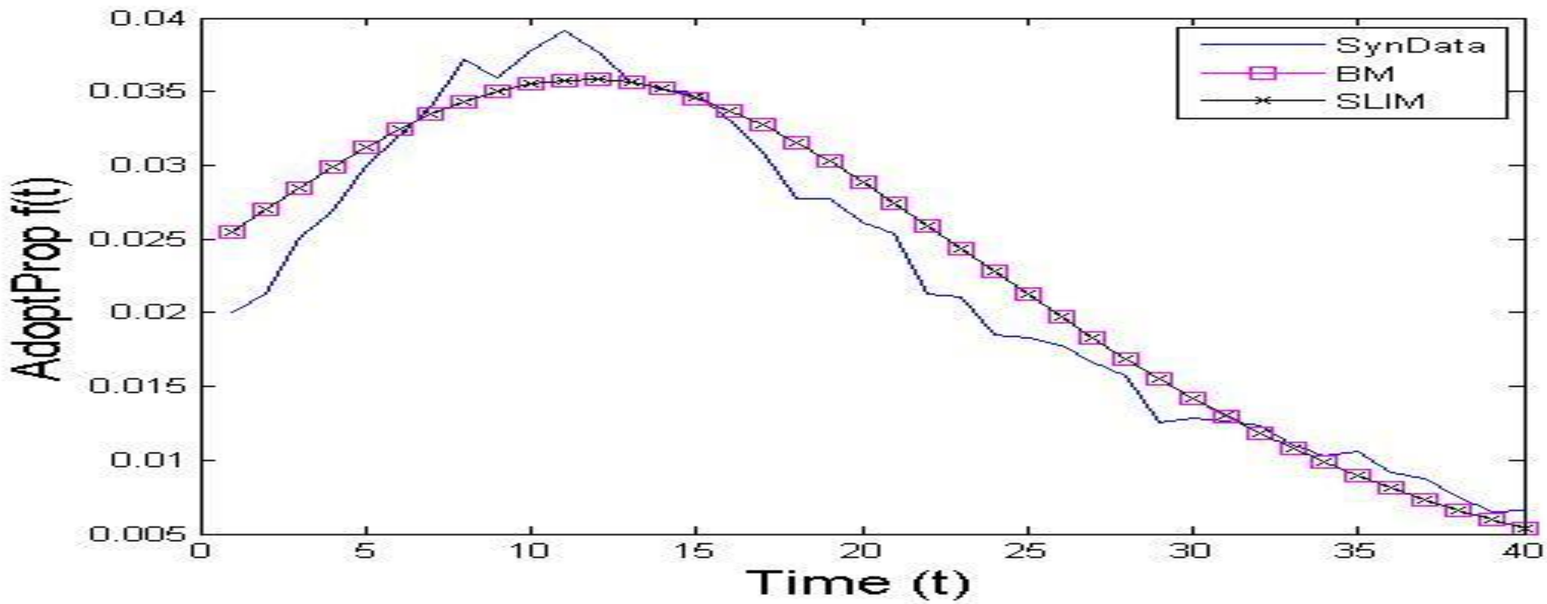
Proposed Models

1. SLIM (Scale-free Linear Influence Model): Scale-free network.
2. ELIM (Exponential Linear Influence Model): Exponential network

Remarks:

- Give more rigorous estimation of adoption probability by combining parameters of diffusion and of degree distribution.
- Give the **same** fitting error as BM though!



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What is the problem?

$$\underbrace{f(t)}_{\text{fract of new adopters}} = \underbrace{(1-F(t))}_{\text{nonadopters at } t} \cdot \underbrace{\mathrm{E}\left[P\left(a_t \mid \overline{A_t}\right)\right]}_{\text{avg. adopting prob}}$$

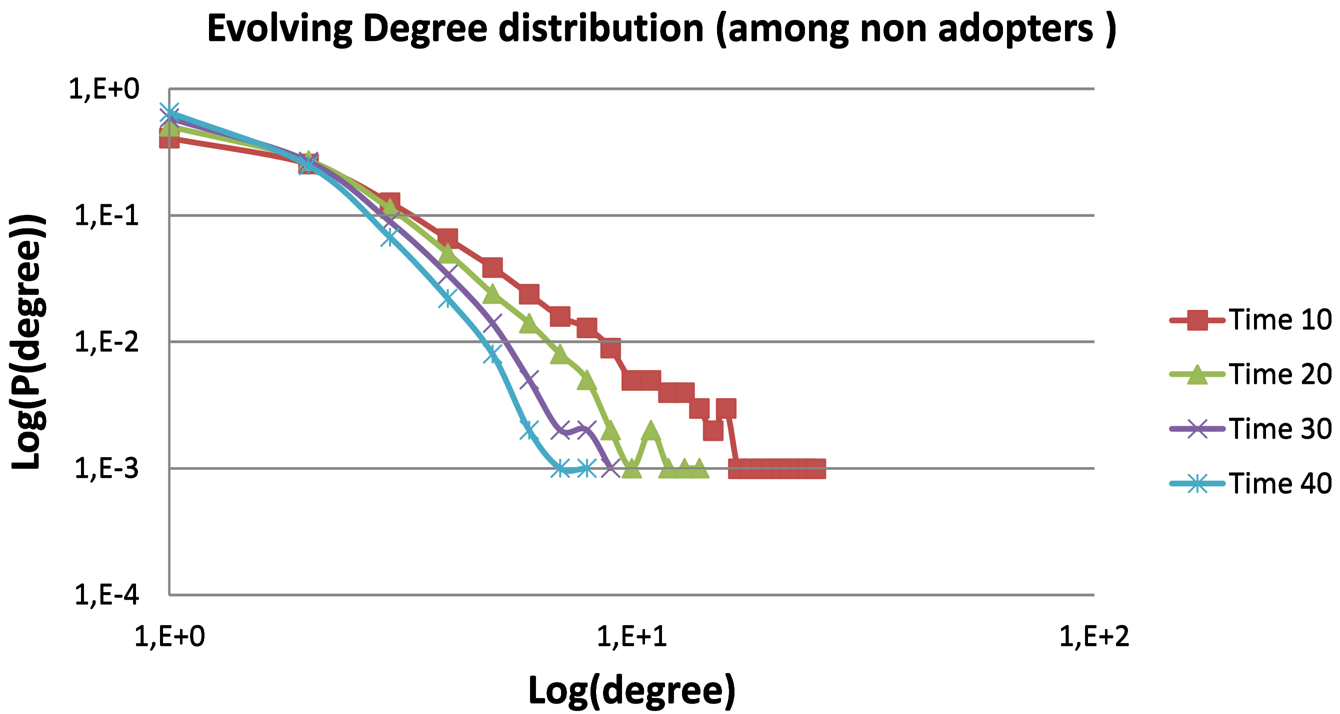
- Is it correct to use degree distribution of the whole network for *P(k)*?

  NO. Should use degree distribution over the set of non-adopters (NA).

- NA changes over time → its degree distribution also changes?

  YES.

Degree Distribution is Dynamic !!



Synthetic scale-free network; 27,289 nodes and 27,031 edges (α_0=3).

As time proceed, users with high/low degs are more/less likely to adopt and leave/stay NA set. Thus later distributions are more biased to low degrees.

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Multi-Stage Model (MLIM)

For different time pts, need to employ different models.
How to decide the proper model?

Heuristic approach: in a **short** duration, degree distribution does NOT significantly change.

- Divide diffusion process into *n* stages. Each has short duration (< 10 time steps).
- For each stage, choose between SLIM and ELIM the one that gives smaller fitting error.

→ Multi-Stage Model

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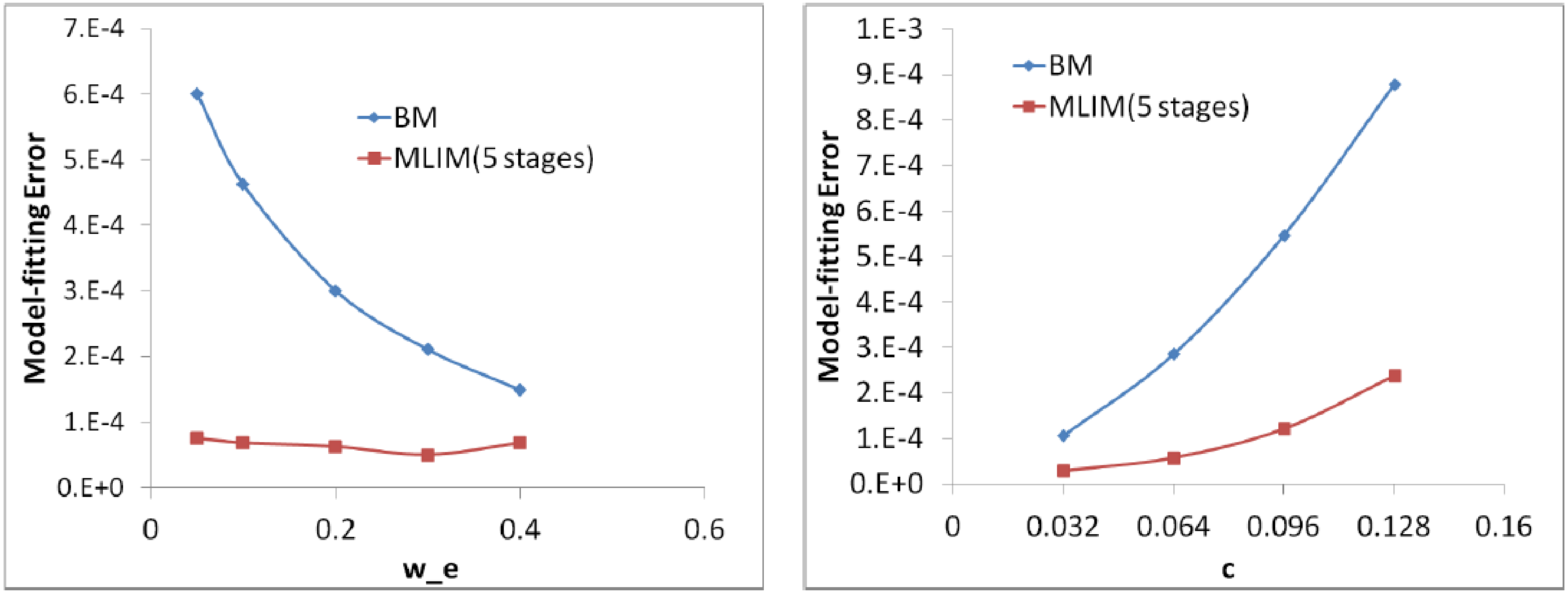
Experiments on Synthetic Data

- Network: 28,172 nodes; 34,578 edges (α=2.5).
- Evaluation metrics: model-fitting error (LSE) & parameter-learning error



(a) w_e in $\{0.05, 0.1, 0.2, 0.3, 0.4\}$ (b) c in $\{0.032, 0.064, 0.096, 0.12\}$

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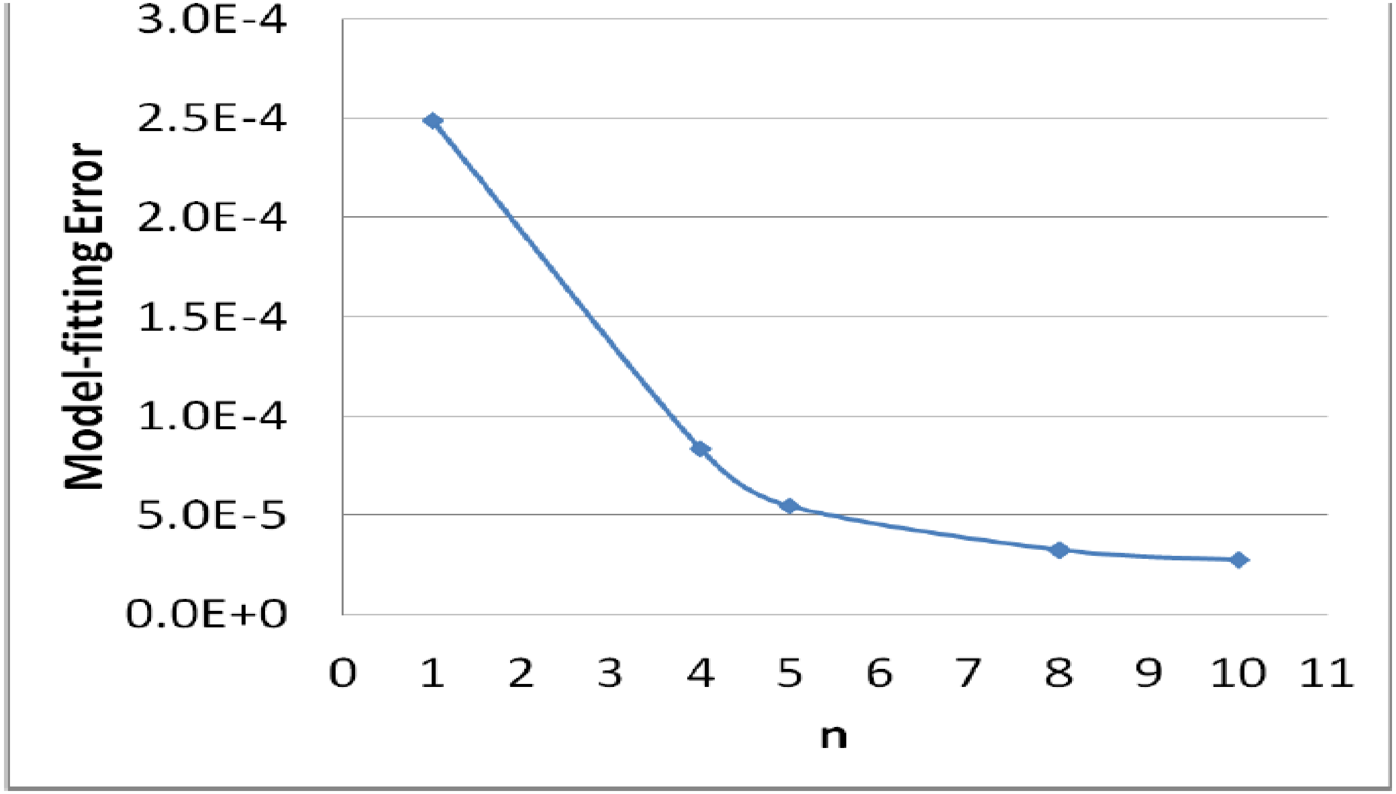
Experiments on Synthetic Data

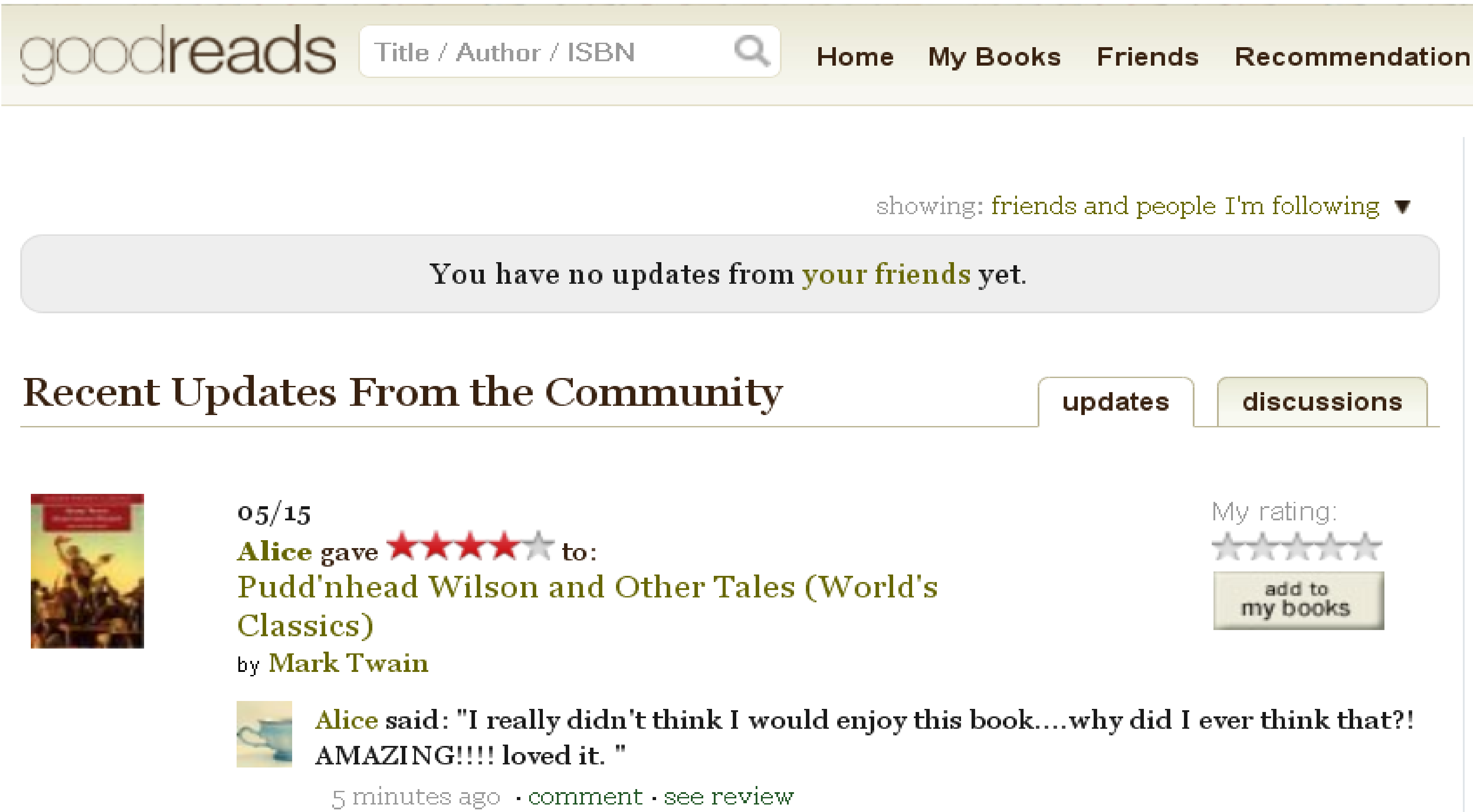


(c) $n \in \{1, 4, 5, 8, 10\}$

n=1 corresponds to BM

Real-world Dataset

From Goodreads network (www.goodreads.com), ≅ 87K users; 159,442 follow links



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Experiment Design

- Adopting a book ≅ writing review on it.
- Review data was collected for 73 popular books.
- Period: 05/2007 to 02/2011 (45 months).
- Filter out books with review data spans < 30 months
  → 20 books remain (Harry Potter 7, Breaking Dawn, …).

- Evaluation metric: ratio of model-fitting errors (LSE of MLIM over LSE of BM)
  → less than 1 shows improvement of our model.

Special thanks to Agus and Anh.T.H

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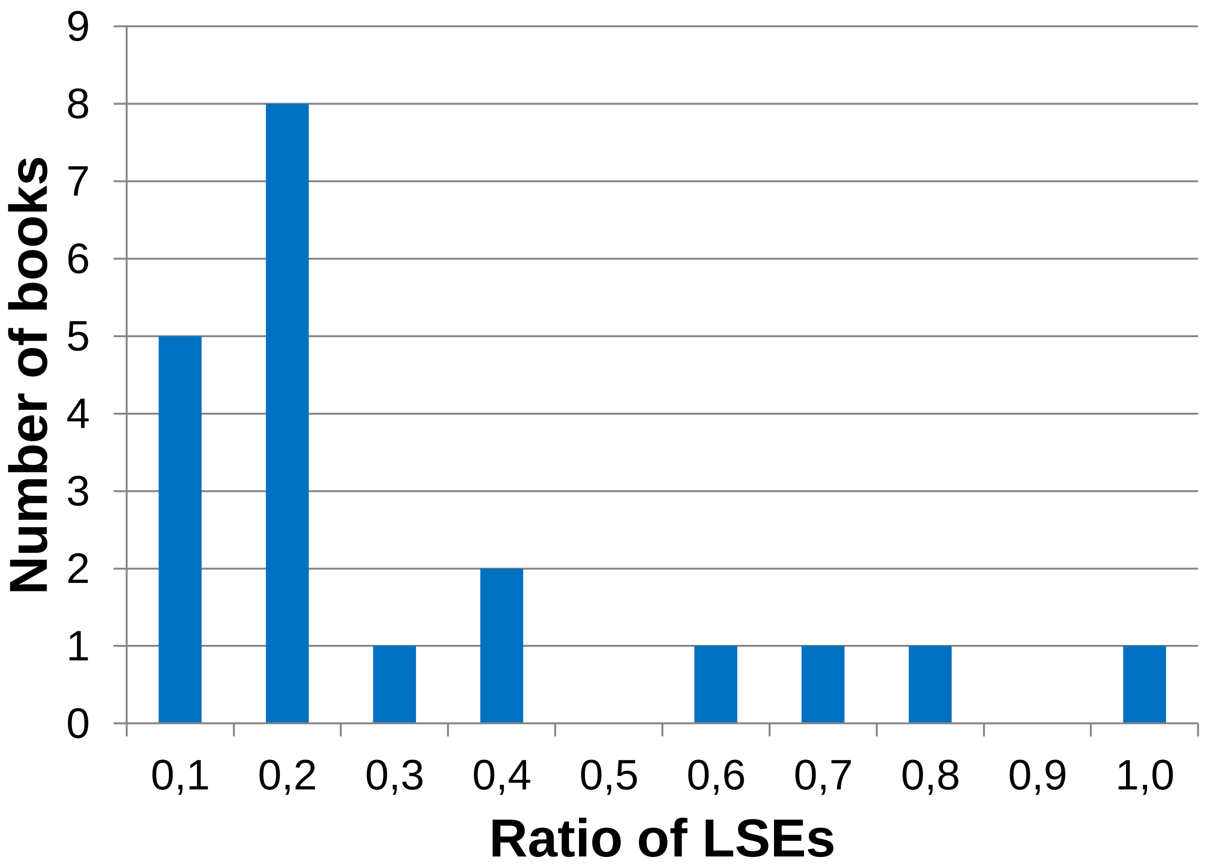
Results for Top-20 Popular Books



- MLIM outperforms BM for all top-20 popular books.
- 75% of books have error ratios less than ½.

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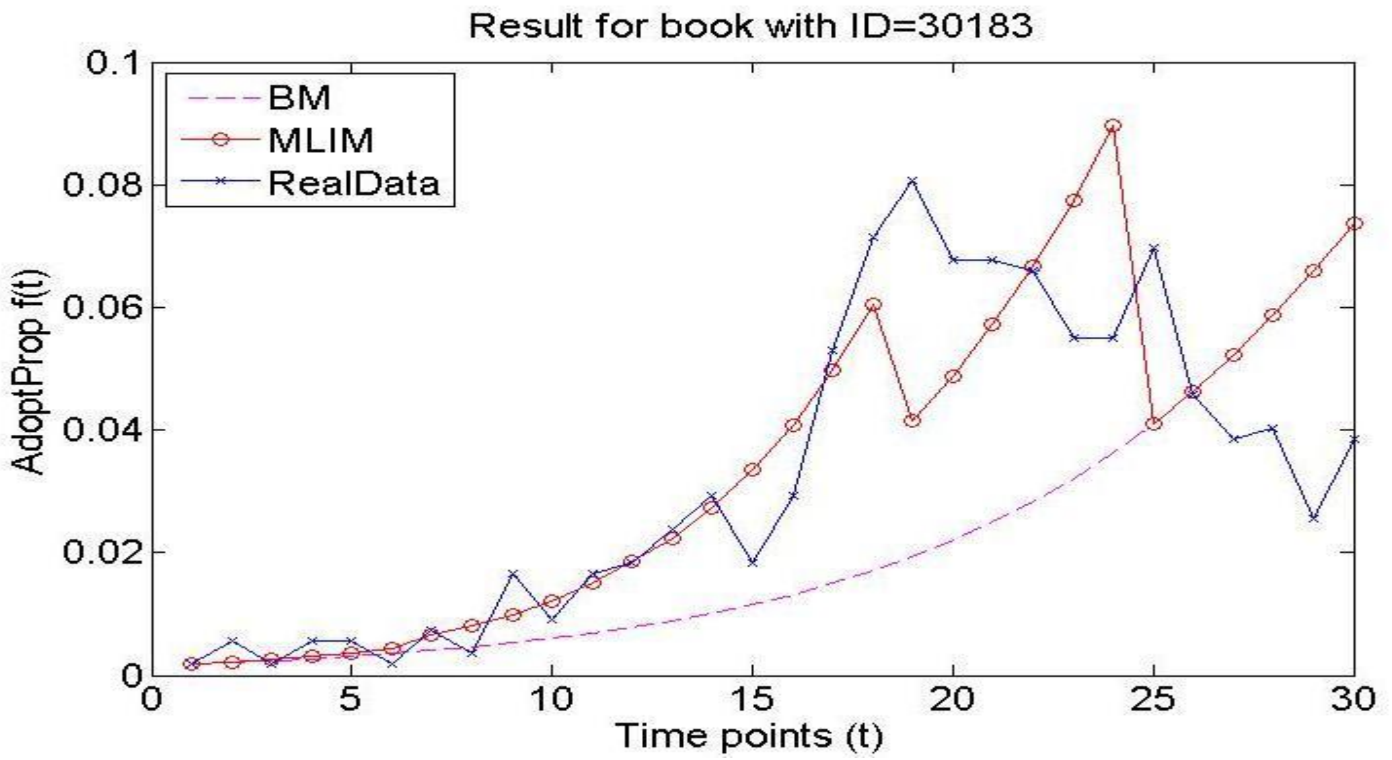
Zoom-in for One Book



Fitting result for *City of Ashes* by Cassandra Clare

MLIM provides significant improvement over BM in terms of fitting data.

Conclusion

- This work:
  - Proposed two models SLIM, ELIM for diffusion in scale-free and exponential networks respectively.
  - Proposed multi-stage model (MLIM) to deal with dynamic degree distribution.
- Future works:
  - Derive a more rigorous way to deal with dynamic degree distribution.
  - Replace linear influence by other (e.g. **quadratic, exponential**) influence?
  - Examine the effect of other network quantities on diffusion.

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More Formulae

Adoption prob. for scale free and exponential network

$$P_{sf}\left(a_t \mid \overline{A_t}\right) = w_e \cdot p_e + \left(1 - w_e\right) \cdot \frac{\zeta(\alpha - 1)}{\zeta(\alpha)} \cdot c \cdot F_t$$

$$P_{exp}\left(a_t \mid \overline{A_t}\right) = w_e \cdot p_e + \left(1 - w_e\right) \cdot \frac{e^{-1/\lambda}}{1 - e^{-1/\lambda}} \cdot c \cdot F_t$$

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Formulae of SLIM, ELIM

$$F_{SLIM}(t) = \frac{\exp\left[\left(p + q_{SLIM}\right)\cdot t\right] - 1}{\exp\left[\left(p + q_{SLIM}\right)\cdot t\right] + \left(q_{SLIM} / p\right)}$$

where $p = p_e \cdot w_e$ and $q_{SLIM} = \left(1 - w_e\right) \cdot \frac{\zeta(\alpha - 1)}{\zeta(\alpha)} \cdot c$

$$F_{ELIM}(t) = \frac{\exp\left[\left(p + q_{ELIM}\right)\cdot t\right] - 1}{\exp\left[\left(p + q_{ELIM}\right)\cdot t\right] + \left(q_{ELIM} / p\right)}$$

where $p = p_e \cdot w_e$ and $q_{ELIM} = \left(1 - w_e\right) \cdot \frac{e^{-1/\lambda}}{1 - e^{-1/\lambda}} \cdot c$

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