Exploiting Feature Covariance in High-Dimensional Online Learning

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Presentation for AISTATS

May 14, 2010

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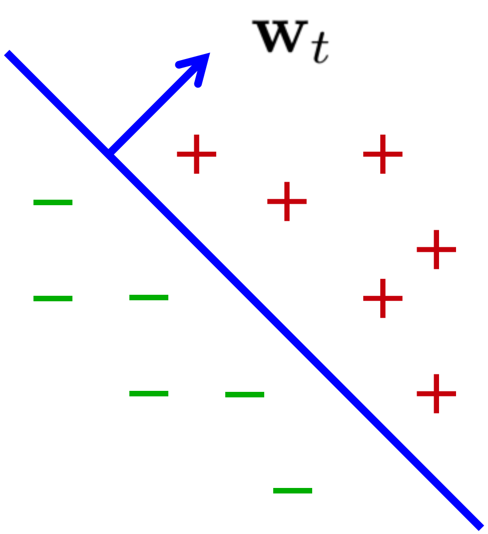
Online Learning of Linear Classifiers

- Input $\mathbf{x}_t$
- Predict $\hat{y}_t = \mathrm{sign}(\mathbf{w}_t \cdot \mathbf{x}_t)$
- Receive label $y_t \in \{-1, +1\}$
- Record error if $y_t \neq \hat{y}_t$
- Modify $\mathbf{w}_t$



Real-World Motivations

- Industrial-scale applications
  - Large data sets ($\sim 10^6$—10^9 examples)
  - Nonstationary data, drifting concepts

- Attractions of online learning
  - Single pass over data
  - Incremental update
  - Low overhead in storage & compute

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High-Dimensional Applications

- Ex: sentiment classification, malicious URL detection, Web spam
- Bag-of-words $\sim 10^6$ features
- New features introduced over time

Sentiment classification

Malicious URLs

Which online algorithm?

Perceptron [Rosenblatt, 1958]

Stochastic gradient [generalization in Bottou, 1998]
Bayesian logistic regression [MacKay, 1992] [Jaakkola & Jordan, 2000]

Online convex programming [Zinkevich, 2003]
Second-order perceptron [Cesa-Bianchi et al, 2005]
Passive-aggressive [Crammer et al, 2006]

Confidence-weighted [Dredze et al, 2008]
Online ellipsoid method [Yang et al, 2009]
AROW [Crammer et al, 2009]
AdaGrad [Duchi et al, 2010]

Trend toward more complex updates

…(e.g., using **2nd-order** information)

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Which online algorithm?

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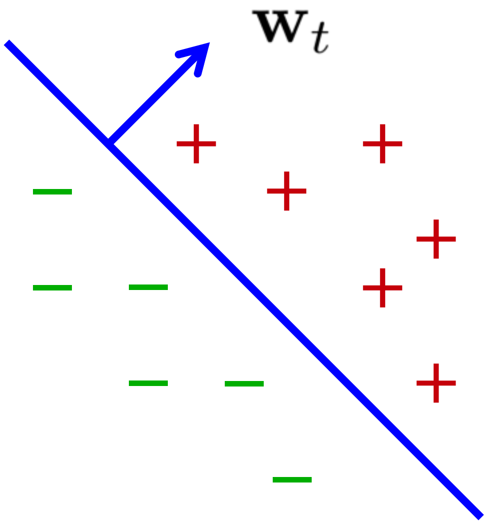
Perceptron

[Rosenblatt, 1958]



- Per-mistake update:

$$\mathbf{w}_{t+1} \leftarrow \mathbf{w}_t + y_t \mathbf{x}_t$$

- Convergence in finite rounds for separable data

Passive-Aggressive (PA) Algorithm

[Crammer et al., 2006]

- Constrained optimization

$$\mathbf{w}_{t+1} \leftarrow \operatorname*{argmin}_{\mathbf{w}} \quad \frac{1}{2}\|\mathbf{w}_t - \mathbf{w}\|^2$$
$$\text{s.t.} \quad y_t(\mathbf{w} \cdot \mathbf{x}_t) \geq 1$$

- Closed-form update

Amount of error

Proportional

$$\boxed{\alpha_t = \max\left\{\frac{1 - y_t(\mathbf{w}_t \cdot \mathbf{x}_t)}{\|\mathbf{x}_t\|^2}, 0\right\}}$$

$$\mathbf{w}_{t+1} \leftarrow \mathbf{w}_t + \boxed{\alpha_t} y_t \mathbf{x}_t$$

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Confidence-Weighted (CW) Learning

[Dredze et al., 2008] [Crammer et al., 2009]

- Gaussian distribution over weight vector:

$$\mathbf{w}_t \sim \mathcal{N}(\boldsymbol{\mu}_t, \boxed{\boldsymbol{\Sigma}_t^{-1}})$$

How to represent this matrix?

- Constrained problem:

$$(\boldsymbol{\mu}_{t+1}, \boldsymbol{\Sigma}_{t+1}^{-1}) \leftarrow \underset{\boldsymbol{\mu}, \boldsymbol{\Sigma}^{-1}}{\operatorname{argmin}} \ \mathrm{KL}(\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}^{-1}) \| \mathcal{N}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t^{-1}))$$
$$\text{s.t. } \Pr[y_t(\mathbf{w} \cdot \mathbf{x}_t) \geq 0] \geq \eta$$

- Closed-form update:

$$\boldsymbol{\mu}_{t+1} \leftarrow \boldsymbol{\mu}_t + \alpha_t y_t \boxed{\boldsymbol{\Sigma}_t \mathbf{x}_t}$$
$$\boldsymbol{\Sigma}_{t+1}^{-1} \leftarrow \boldsymbol{\Sigma}_t^{-1} + \frac{\alpha_t \phi}{\sqrt{u_t}} \mathbf{x}_t \mathbf{x}_t^\top$$

Update features at different rates

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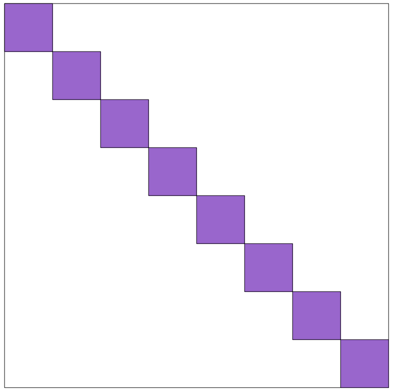
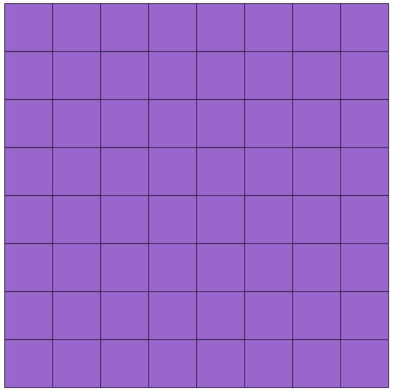
Representing Correlations

$\Sigma^{-1} =$ 

$\Sigma^{-1} =$ 

Diagonal

- ✔ $O(n)$ storage
- ✔ Low compute time

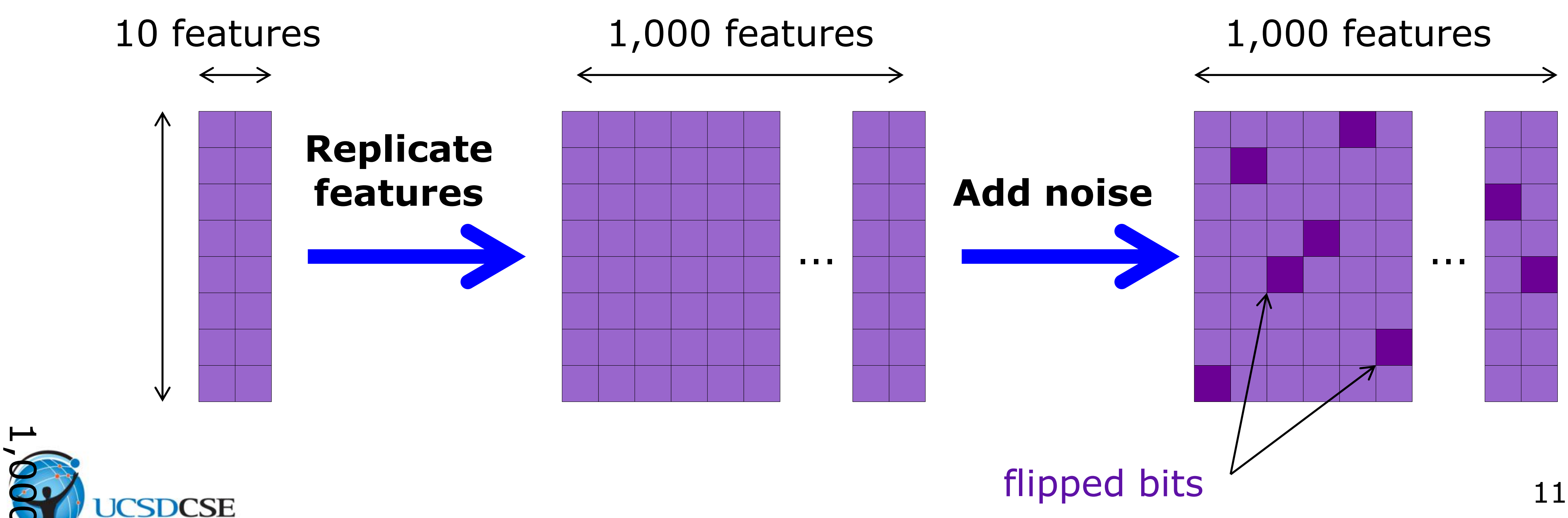
Full

- ✗ $O(n^2)$ storage
- ✗ High compute time

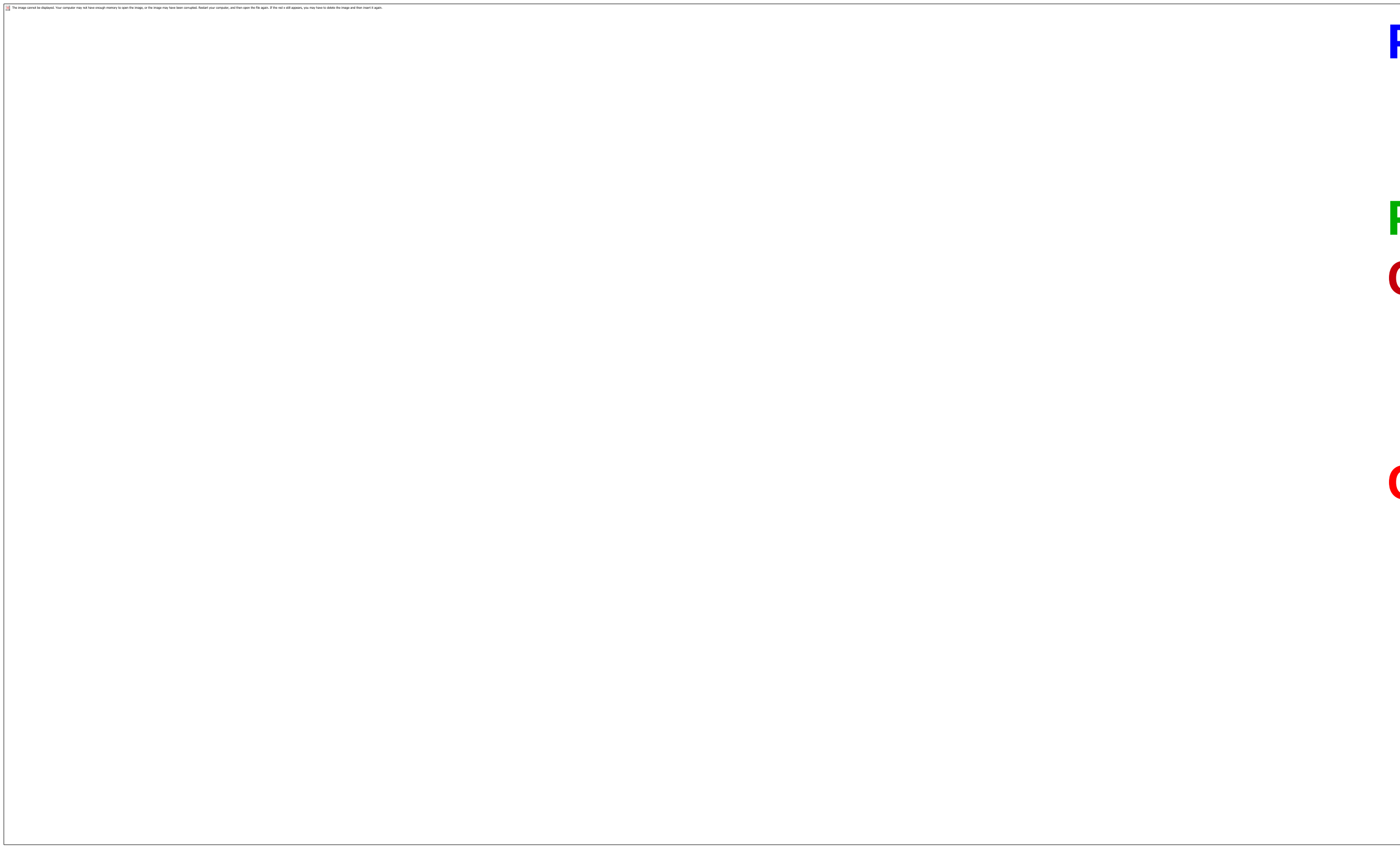
Why bother with full?

Benefits of Full

- When do we benefit from full covariance?
- Synthetic experiment: noisy correlated features
  - 100 runs, 1,000 examples, 1,000 binary features
  - 5% of features flipped



Noisy Correlated Features



- **Full** exploits correlations

Representing Correlations

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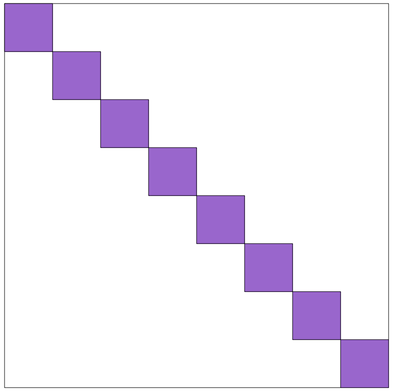
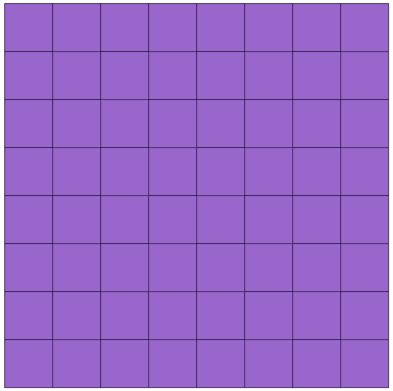


Diagonal

- ✔ O(n) storage
- ✔ Low compute time
- ✔ Ignores correlations

$\Sigma^{-1} =$



Full

- ✗ O(n^2) storage
- ✗ High compute time
- ✗ Exploits correlations

Can obtain benefits of both?

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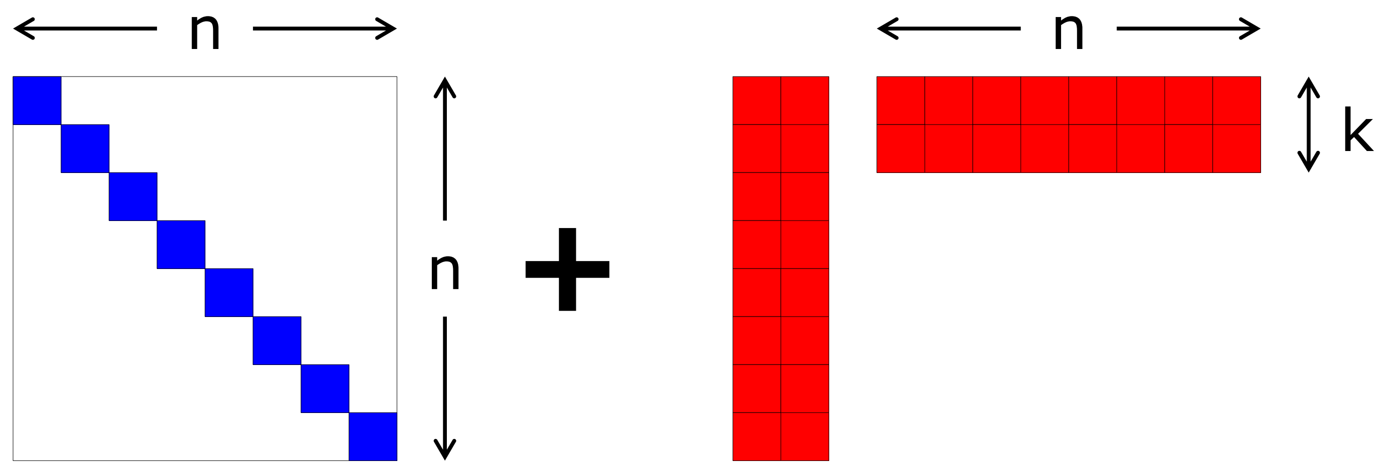
Factored Approximation



$$\Sigma^{-1} = D + RR^T$$

- Approx. full inv. covariance (called precision)
- O(kn) storage (k = number of factors)
- Compress matrix updates – factor analysis

Factor Analysis

- Exact and approximate distributions

$$
\begin{aligned}
\rightarrow P(\mathbf{w}_t) &= \mathcal{N}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t^{-1}) \\
\rightarrow \widehat{P}(\mathbf{w}_t) &= \mathcal{N}(\boldsymbol{\mu}_t, \boxed{\boldsymbol{D}_t + \boldsymbol{R}_t \boldsymbol{R}_t^\top})
\end{aligned}
$$

- Minimize KL divergence using EM procedure

$$\min \ \mathrm{KL}(P_t, \widehat{P}_t)$$

- Computation cost: $O(nk^2)$

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Synthetic with CW-fact

Perceptron

PA

CW-diag

CW-fact4

CW-full

Cumulative r

- **Full** exploits correlations
- **Factored** approximates full
- **40x** less memory

Benefits of approximating full in real-world applications?

Detecting Malicious URLs

- Live feature collection of URLs
- Per trial: 200,000 examples, 10^6 features (mostly binary)

Additional mistakes
by CW-diag

Perceptron

PA

CW-diag
CW-fact16

Factored relative improvement: 5%

Web Spam Classification

- Web pages [PASCAL Large-Scale Learning competition]
- Per-run: 175,000 examples, 680,000 features (text 3-grams)

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Perceptron

PA

Additional mistakes by CW-diag

CW-diag

CW-fact16

Factored relative improvement: 18%

Document Classification

[Dredze et al., 2008]

- 2,000—18,000 examples, 10^4—10^6 features

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Improvement by CW-fact8

Reuters 20 Newsgroups Sentiment

Relative

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Conclusion

- Full and factored covariance help when...
  - features are correlated
  - #features > #examples
    (not needed when #examples > #features [Sec. 2])
- Factored improves high-dimensional apps
  - NLP, URL classification, Web spam, others
- Future work
  - Correlation modeling in other online algorithms?
  - Other ways to model correlation structure?

Code is Available

http://sysnet.ucsd.edu/projects/url/

- Coauthors in Sardinia...

Alex

Koby

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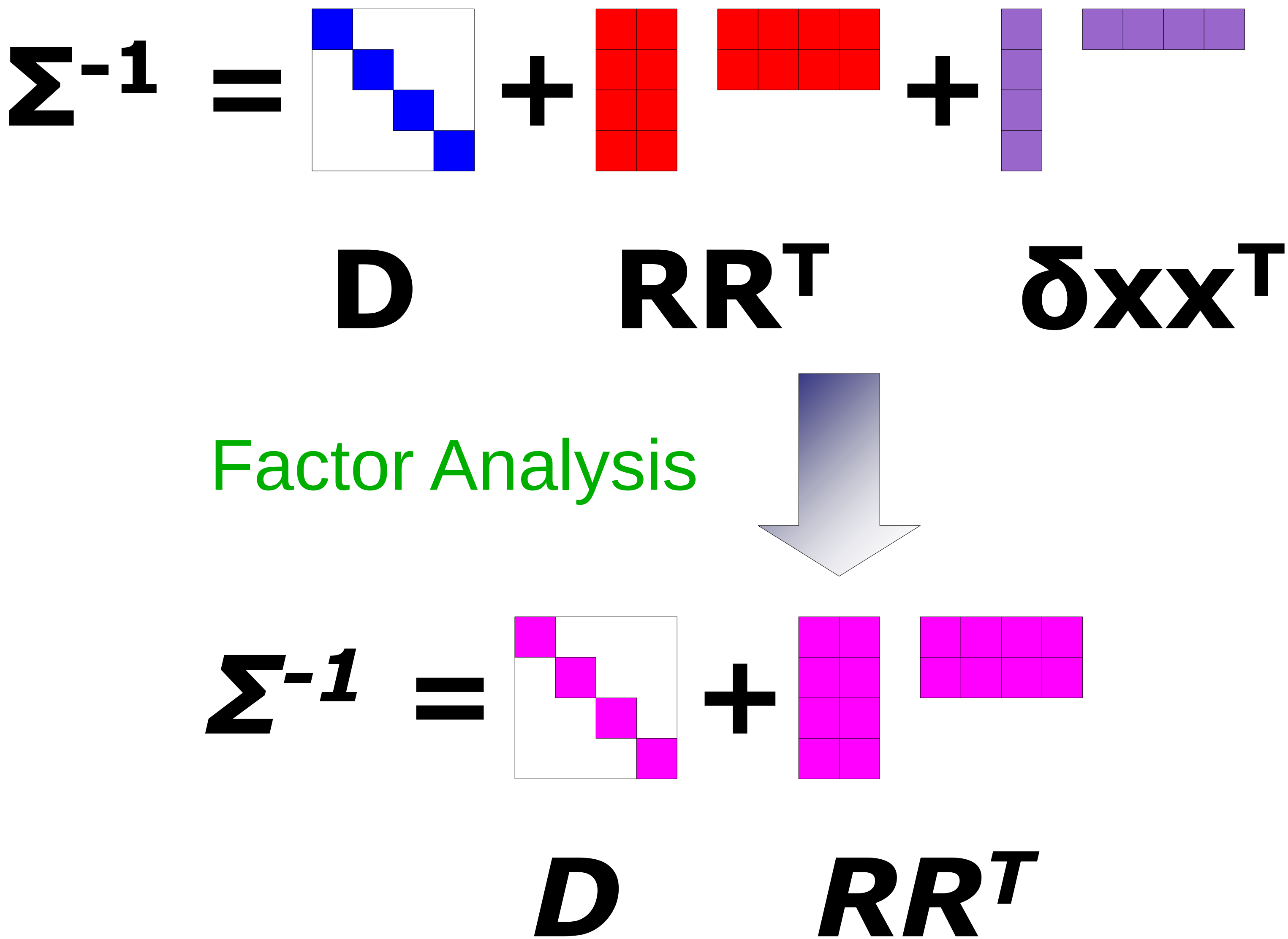
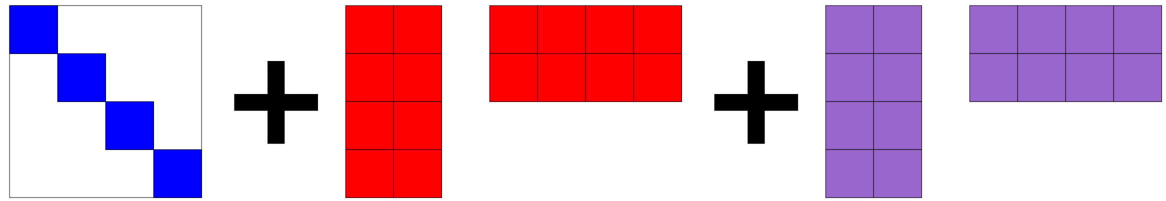
Update Compression



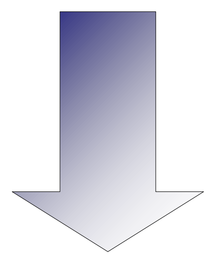
Buffering Updates

$\Sigma^{-1} =$

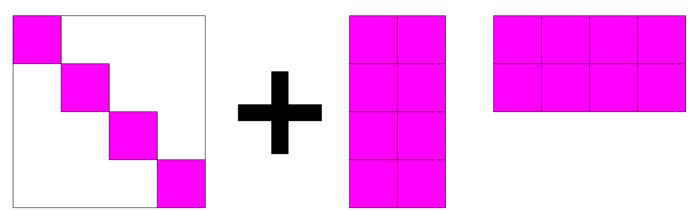


$D \qquad RR^T \qquad BB^T$

Factor Analysis



$\Sigma^{-1} =$



$D \qquad RR^T$