Stochastic Methodology for Prognosis Under Time-Varying Environmental Future Profiles

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Agenda

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  - Problem Definition
  - Motivation and Objectives
  - Relevant Literature
- Part II: Model and Methodology
  - Estimate the Residual Life Distribution
  - Update the Degradation Model in Real Time
- Part III: Numerical Studies
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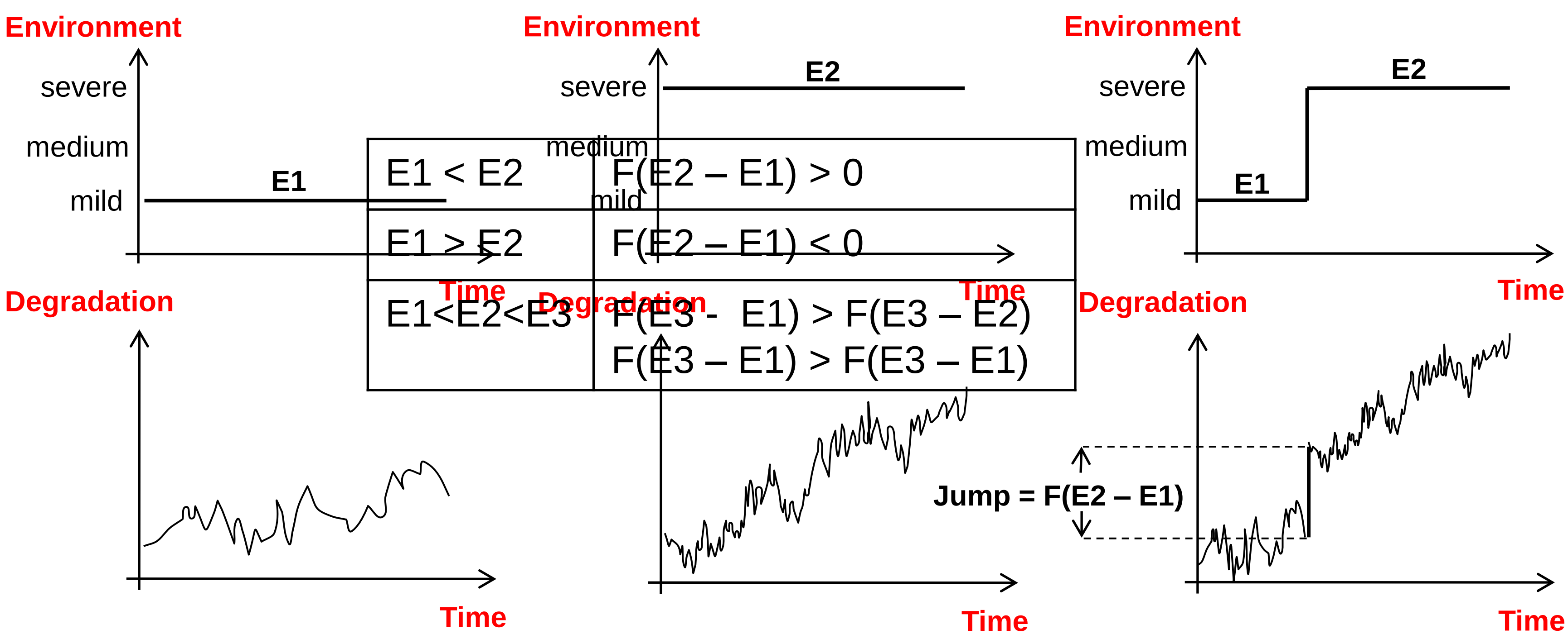
  - Case Study: Bearing Implementation

Problem Definition

- Most failures of engineering systems result from the gradual and irreversible accumulation of damage:
  - A process known as **Degradation**
- We examine degradation processes that can be monitored using sensor technology and that result in
  - **Degradation Signals**
- We focus on
  - Modeling degradation signals under time-varying environmental conditions.
  - Computing the distribution of residual life under deterministic time-varying future environments.

Motivation and Challenge

- Future environmental profiles can affect the performance (degradation or failure time) of an engineering system
  - Environments can affect the degradation rate of a system.
  - Changes in environments cause jumps in degradation signals.

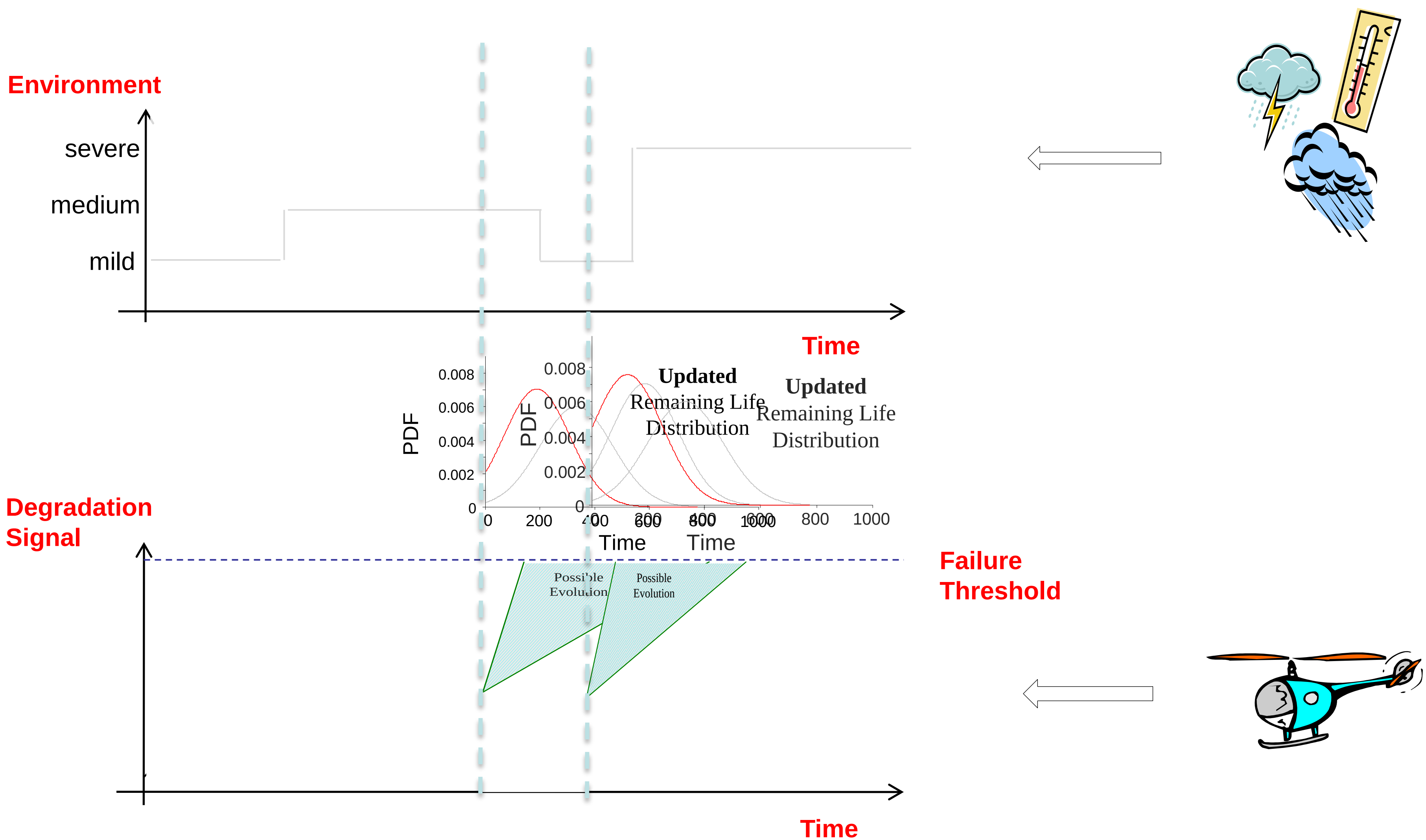


E1 < E2	F(E2 – E1) > 0
E1 > E2	F(E2 – E1) < 0
E1<E2<E3	F(E3 - E1) > F(E3 – E2) F(E3 – E1) > F(E3 – E1)

Objectives

- To estimate and continuously update the residual life distributions of partially degraded components operating under time-varying environmental conditions by incorporating
  - Real-time degradation signals and environments observed from components operating in the field
  - Prior knowledge of the degradation characteristics associated with the entire population
  - Knowledge of past and future environments

Predicting RLD in Deterministic Environments



Relevant Literature

Approaches		Papers	Remarks
Degradation Models Without Considering Environments		Parker and Padgett (2005) Pettit and Young (1999) Gebraeel (2005)	Do not consider the effects of the environment
Degradation Models With Environments	**Environments as shocks**	Esary and Marshall (1973) Gottlieb (1980) Lemoine and Wenocur (1986)	Only consider the effects of shocks
	Environments as Markov processes	Cinlar (1977) Kharoufeh (2003) Kharoufeh, Finkelstein, & Mixon (2006)	Do not consider the shocks caused by environmental transitions
Hazard Models With Covariates		Jardine, Anderson, & Mann (1987) Lee &Whitmore (2004)	Cannot be used to update the RLD in real time.
Accelerated Life Testing		Doksum &Hoyland (1992) Liao (2009)	Only focus on with specific environmental profiles.

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Degradation Model Under Deterministic Environments

S(t): degradation signal at time t

$\psi(v)$: environmental condition at time v.
$\psi(v)$ is completely ordered by severity

$$S(t) = S_0 + \int_0^t R(\psi(v))dv + \int_0^t J(d\psi(v)) + \gamma W(t)$$

Initial value of S(t)

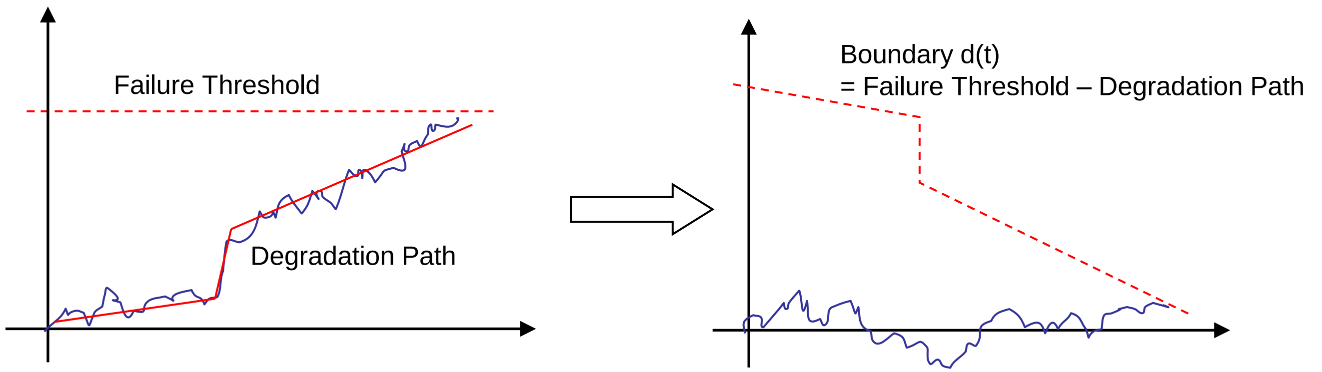
Rate of degradation at environment $\psi(v)$

Jump of degradation signal caused by the environmental change

The stochastic part that cannot be explained by the effect of the environment

Estimating Residual Life Distribution

- We assume that the component fails as its degradation signal S(t) crosses failure threshold D.
- We define the failure time as the first crossing time of S(t) to the pre-specified failure threshold.
- The probability that the component fails by time t is equivalent to the probability that a BM process crosses linear boundary d(t).



Computing the Crossing Probability of a BM With a Piecewise Linear Boundary

THEOREM 1. *Let* $0 = v_0 < v_1 < \cdots < v_n = T$ *denote* n *fixed jump times and suppose* $d(v)$ *is linear on* $[v_{j-1}, v_j)$, $j = 1, 2, \ldots, n$ *with* $d(0) > 0$. *Then for each* $v \in [0, T]$, *the complement of the crossing probability of a Brownian motion* $\gamma W(v)$ *with diffusion parameter* γ *for a piecewise linear boundary* $d(v)$ *is given by*

$$\boxed{\mathbb{P}(\gamma W(v) < d(v)) = \mathbb{E}\left[h(W(v_1), W(v_2), \ldots, W(v_n); \boldsymbol{d})\right],}$$

where

$$h(x_1, x_2, \ldots, x_n; \boldsymbol{d}) = \prod_{j=1}^{n} \mathbf{1}(x_j < m_j/\gamma)\Delta(v_j, v_{j-1}),$$

with

$$\Delta(v_j, v_j - 1) = 1 - \exp\left[-\frac{2[d_{j-1}/\gamma - x_{j-1}]\,[d_j^-/\gamma - x_j]}{v_j - v_{j-1}}\right],$$

and $\mathbf{1}(A)$ *is the indicator function for condition* A.

We utilized the result from Wang and Potzelberger (1997), *Boundary Crossing Probability for Brownian Motion and General Boundaries*, J. Appl. Prob.

Updating RLD – A Base Case

- We illustrate the updating of RLD using a base case model with specific forms of the environment process and an environment-dependent jump process.

$$\psi(v) = \begin{cases} x_1, & \text{if } v \in [v_0, v_1) \\ x_2, & \text{if } v \in [v_1, v_2) \\ x_3, & \text{if } v \in [v_2, v_3) \\ \vdots & \vdots \end{cases}$$

$$r(\psi(v)) = \alpha\psi(v) + \beta$$

$$J(\mathrm{d}\psi(v)) = \eta \mathrm{d}\psi(v)$$

$$\alpha \sim N(\mu_1, \sigma_1^2),$$

$$\beta \sim N(\mu_2, \sigma_2^2),$$

$$\eta \sim N(\mu_3, \sigma_3^2),$$

$$\gamma \sim N(\mu_4, \sigma_4^2).$$

- The prior marginal distributions of these parameters are mutually independent and normally distributed.

Updating Model Parameters and RLD – A Base Case

- Given real-time degradation signals and environmental conditions, the posterior distributions of $\alpha, \beta, \eta, \gamma$ are computed using Bayesian formula:

$$\nu_s\left(\alpha, \beta, \eta, \gamma | \boldsymbol{s}_k, \mathcal{G}_{t_k}\right) = \pi_s(\alpha, \beta, \eta, \gamma) \times \prod_{i=1}^{k} \phi_i(s(t_i) - s(t_{i-1}))$$

- The updated RLD can be computed by conditioning on $\alpha, \beta, \eta, \gamma$

$$\begin{aligned}
\mathbb{P}(R_k \leq T | \boldsymbol{s}_k, \mathcal{G}_{t_k}) &= \int\limits_{\alpha,\beta,\eta,\gamma} \mathbb{P}(R_k \leq T | \boldsymbol{s}_k) \times \nu_s\left(\alpha, \beta, \eta, \gamma | \boldsymbol{s}_k, \mathcal{G}_{t_k}\right) \\
&= \int\limits_{\alpha,\beta,\eta,\gamma} \mathbb{P}(R_k \leq T | \boldsymbol{s}_k) \prod_{i=1}^{k} \phi_i(s(t_i) - s(t_{i-1})) \pi_s(\alpha, \beta, \eta, \gamma) \\
&= \mathbb{E}_{\pi_s}\left[\mathbb{P}(R_k \leq T | \boldsymbol{s}_k) \prod_{i=1}^{k} \phi_i(s(t_i) - s(t_{i-1}))\right],
\end{aligned}$$

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Benchmarks

- Gebraeel & Pan (2008)
  - The future profile of the environment is not considered when estimating the RLD.
  - The RLD is an approximation and does not represent the first-passage time.
- Doksum & Hóyland (1992)
  - Lifetime distribution cannot be updated with real-time signals.
  - Environmental conditions are assumed to increase only in severity.
  - Shocks caused by environmental changes are not considered.

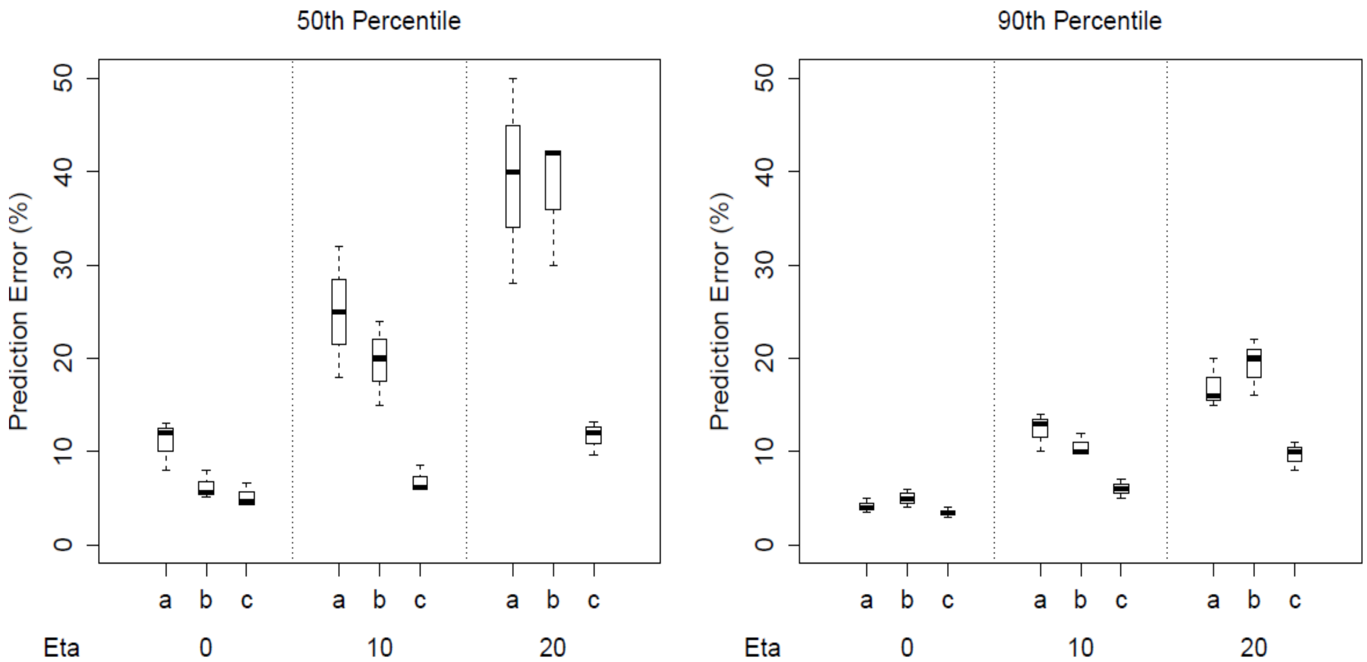
Simulation Study – Experimental Setups

- We conducted 3 groups of simulation studies for various values of the jump parameter:

$$\mu_3 = 0, 10, 20 \text{ with } \sigma_3^2 = 0.1\mu_3$$

  - In Doksum & Hóyland (1992), the jump parameter is equal to zero.
  - In Gebraeel & Pan (2008), future jumps are not considered.
- For each group of simulation study, we simulated 1,000 degradation signals up to failure.
  - Signals 1-500 are used to estimate the prior distributions.
  - Signals 501-1000 are used for online validation.

Simulation Study – Benchmarks



(a) Gebraeel and Pan
(b) Doksum and Hoyland
(c) Proposed Method

Observations

- Bayesian updating improves the prediction accuracy.
- The techniques by (a) and (c) have similar prediction accuracy when no jumps in the degradation signals occur.

Case Study – Bearing Implementation

- Conducted vibration analysis to develop the degradation signals.
- Determined failure thresholds according to industrial standards for machinery vibration.
  - ISO 2372, 2.0 - 2.2 G (G denotes gravitational acceleration)
- Experimental Setup
  - Examined the operational factors: load and speed
  - Ordered the environmental conditions based on the effects on degradation rates

TABLE 2. Definition of ordered environmental states.

Environmental condition	Environmental state
(2,200 rpm, 400 lbs)	1
(2,200 rpm, 500 lbs)	2
(2,600 rpm, 400 lbs)	3
(2,600 rpm, 500 lbs)	4

Table 3. Experiments for prior information and online validation.

Experiment ID	Operating conditions	Number of bearings	
1	(2,200 rpm, 400 lbs)	4	
2	(2,200 rpm, 500 lbs)	4	
3	(2,600 rpm, 400 lbs)	4	
4	(2,600 rpm, 500 lbs)	4	
5	(2,200 rpm, 400 lbs) $\rightarrow$ (2,200 rpm, 500 lbs)	2	
6	(2,200 rpm, 500 lbs) $\rightarrow$ (2,200 rpm, 400 lbs)	2	Experiments for Prior Information
7	(2,600 rpm, 400 lbs) $\rightarrow$ (2,600 rpm, 400 lbs)	2	
8	(2,600 rpm, 400 lbs) $\rightarrow$ (2,600 rpm, 400 lbs)	2	
9	(2,200 rpm, 400 lbs) $\rightarrow$ (2,600 rpm, 400 lbs)	2	
10	(2,600 rpm, 400 lbs) $\rightarrow$ (2,200 rpm, 400 lbs)	2	
11	(2,600 rpm, 400 lbs) $\rightarrow$ (2,200 rpm, 400 lbs)	2	
12	(2,200 rpm, 400 lbs) $\rightarrow$ (2,600 rpm, 400 lbs)	2	
13	(2,200 rpm, 400 lbs) $\rightarrow$ (2,600 rpm, 400 lbs)	1	
14	(2,600 rpm, 400 lbs) $\rightarrow$ (2,200 rpm, 400 lbs)	1	Experiments for Online Validation
15	(2,200 rpm, 400 lbs) $\rightarrow$ (2,200 rpm, 500 lbs)	1	

Case Study – Results

- Predicted residual life distributions using degradation signals from experiments 13 – 15 at the 30th, 60th, and 90th percentiles of lifetimes, respectively.

Table 5 Prediction of lifetime for validation data.

ID	Actual Lifetime	30th Percentile	60th Percentile	90th Percentile
13	283	318.28 (12.5% error)	301.31 (6.5% error)	289.81 (2.4% error)
14	546	489.56 (10.3% error)	575.14 (5.3% error)	563.32 (3.1% error)
15	402	440.24 (9.5% error)	432.21 (7.5% error)	387.86 (3.8% error)

Observation 1: Bayesian updating improves the prediction accuracy of residual life distributions

Observation 2: The prediction error at the 90th percentile of lifetimes is relatively small, partly because the environmental condition does not change after this time.

Conclusions

- We significantly improved the prediction accuracy of the RLD using the real-time degradation signals.
- Our proposed method generalizes the benchmark models by allowing the jump in the amplitude of degradation signals.
- Additional developments are needed when
  - the evolution of the future environmental condition is unknown.
  - the future environmental condition evolves stochastically.