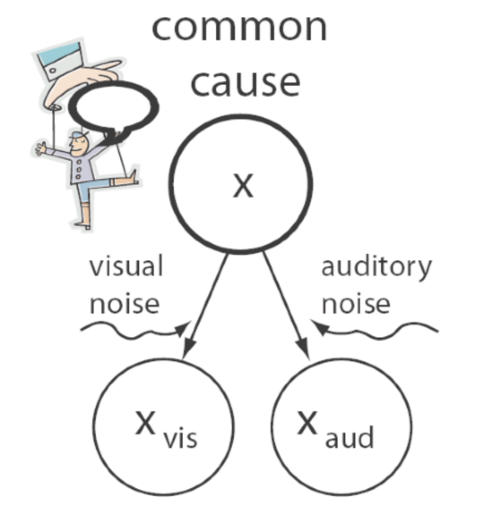
Bayesian modeling of action and perception and some other stuff

Konrad Kording

Part I: Cue Combination

Modeling: Where do cues come from?

Generate



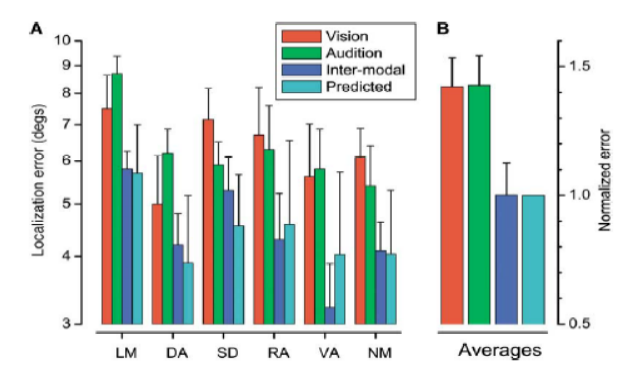
Optimal behavior

$$p(x|V, A) \propto p(x)p(V|x)p(A|x)$$

Assume Gaussian noise on both:

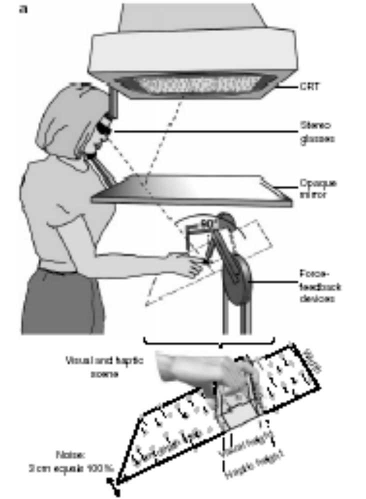
$$w_{\text{vis}} = \frac{\sigma^2_{\text{aud}}}{\sigma^2_{\text{aud}} + \sigma^2_{\text{vis}}} \qquad w_{\text{aud}} = 1 - w_{\text{vis}} = \frac{\sigma^2_{\text{vis}}}{\sigma^2_{\text{aud}} + \sigma^2_{\text{vis}}}.$$

Comparison to predicted behavior



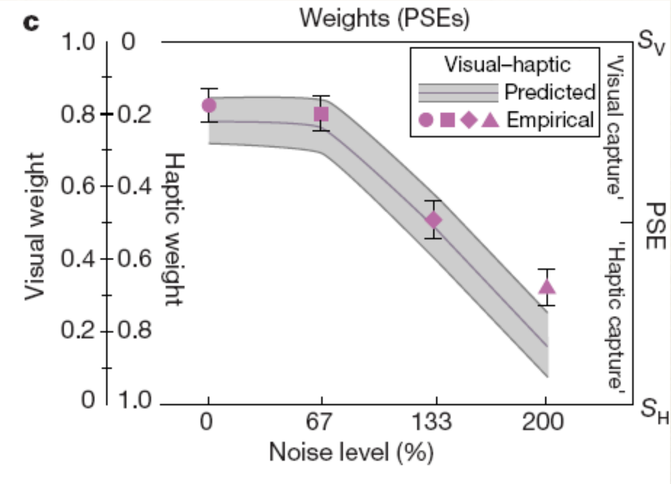
Alais and Burr 2004

Experimental test



Ernst and Banks 2002

Results

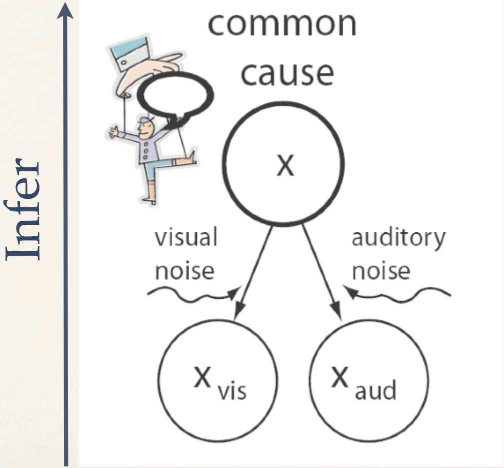


c
Weights (PSEs)
1.0
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1.0
Visual weight
Haptic weight
Visual–haptic
Predicted
Empirical
'Visual capture'
'Haptic capture'
PSE
S_V
S_H
0
67
133
200
Noise level (%)

Visual Auditory combination (Ventriloquist effect)

Both cues 

Traditional Bayesian model



Some groups: Angelaki, Banks, Burr, Deneve, Ernst, Ghahramani, Kersten, Knill, Landy, Mamassian, Maloney, Schrater, Yuille, van Beers, Wolpert

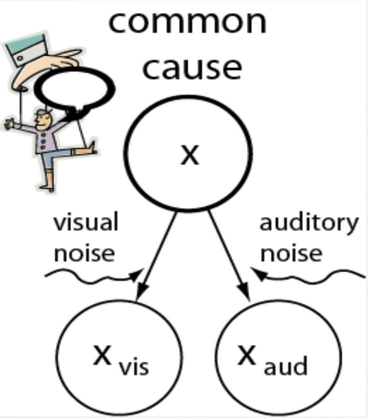
Visual Auditory combination (Ventriloquist effect)

What would happen now?

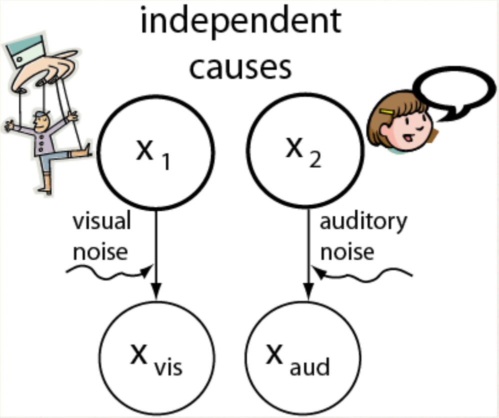
Obviously there may be more than one source.

Joint work with Beierholm, Ma, Tenenbaum, Quartz, Shams

Mixture model



or



e.g. Kording, et al 2007, Sato et al 2007, Ernst and coworkers, Stein and coworkers, Knill and coworkers, Stocker and coworkers

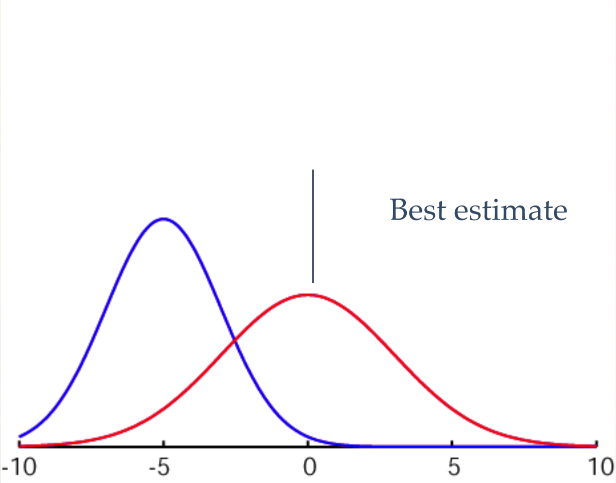
p(causal model)

- Using Bayes rule:

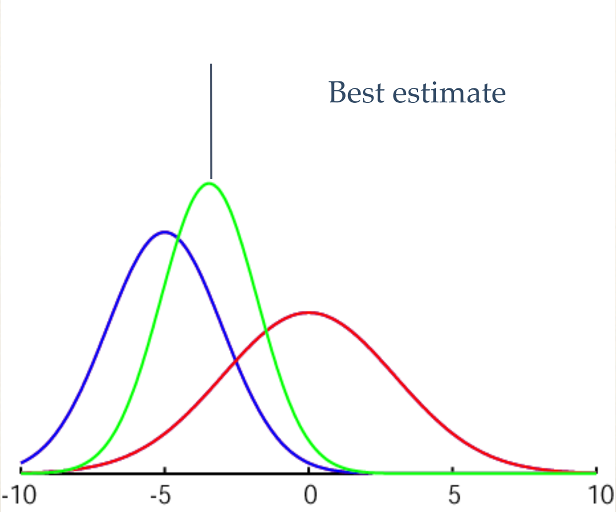
$$p(common|x_{vis}, x_{aud}) = \frac{p_{common}p(x_{vis}, x_{aud}|common)}{p_{common}p(x_{vis}, x_{aud}|common) + (1 - p_{common})p(x_{vis}, x_{aud}|\neg common)}$$

-

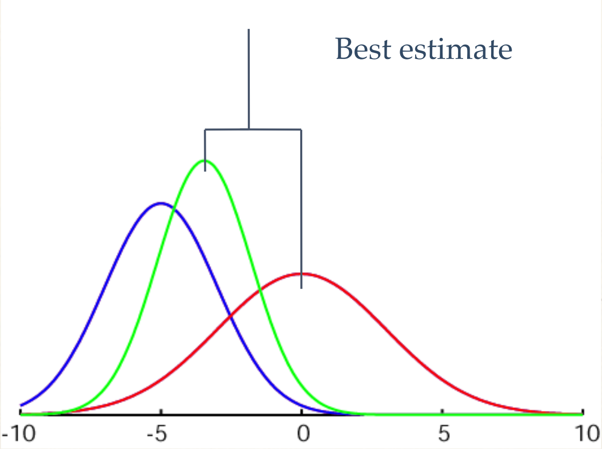
Independent causes: where is the auditory stimulus



Common cause: where is the auditory stimulus



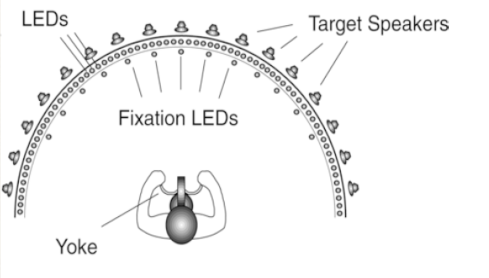
Mean squared error estimate



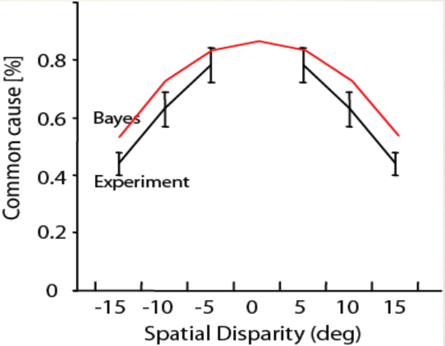
$$\hat{X} = P(C|X_{vis}, X_{aud})\hat{X}_c + P(\neg C|X_{vis}, X_{aud})\hat{X}\neg c$$

Remark: Knill and Sato use virtually identical math

Experimental test

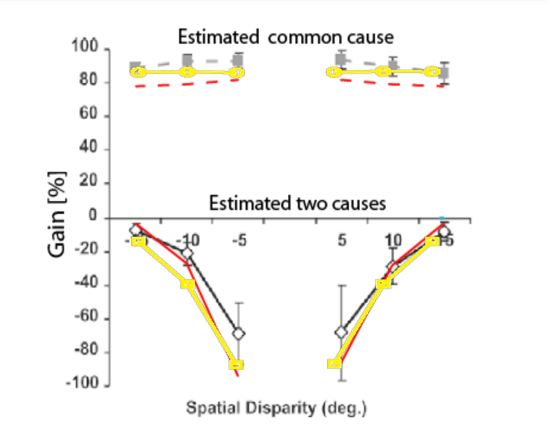


Button: common cause or two



Wallace et al 2005 Hairston et al 2004

Measured gain

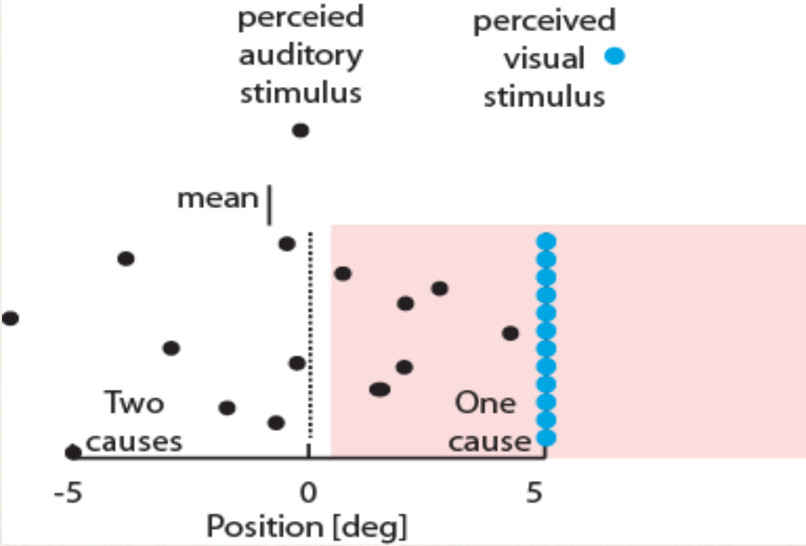


— Data

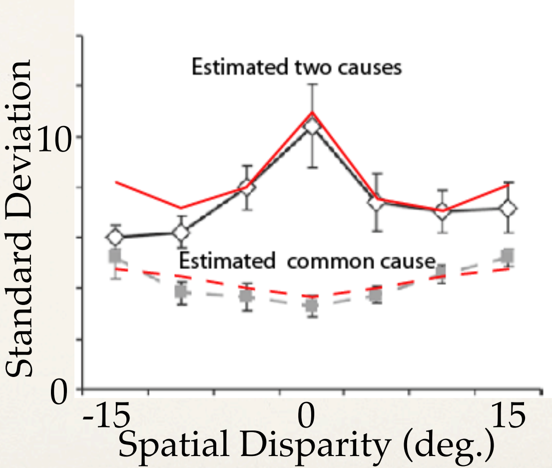
—Kording et al — Sato et al, 2007

Wallace et al 2005 Hairston et al 2004

How can the gain be negative?

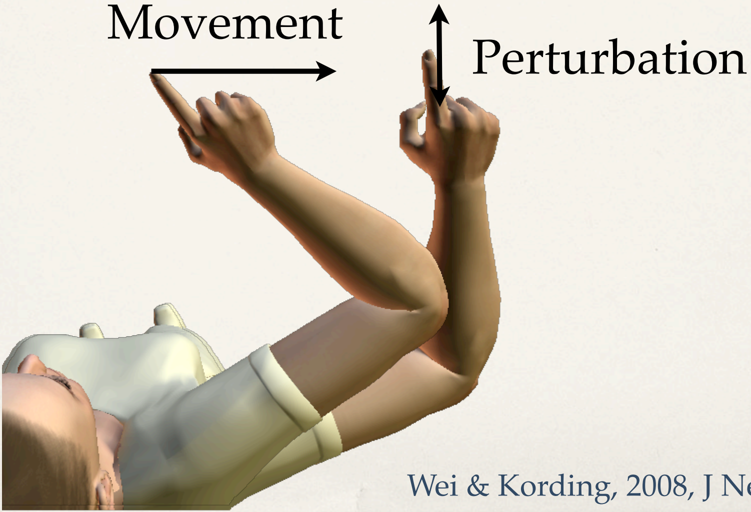


Predicting the variance



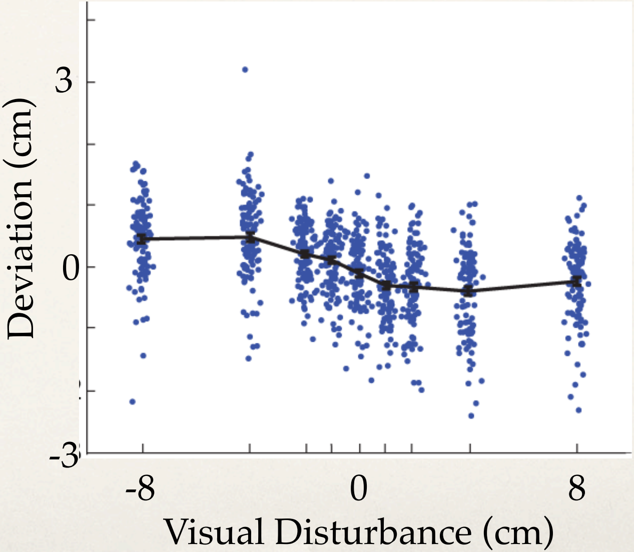
Worse prediction if we assume model selection

Same question in motor learning

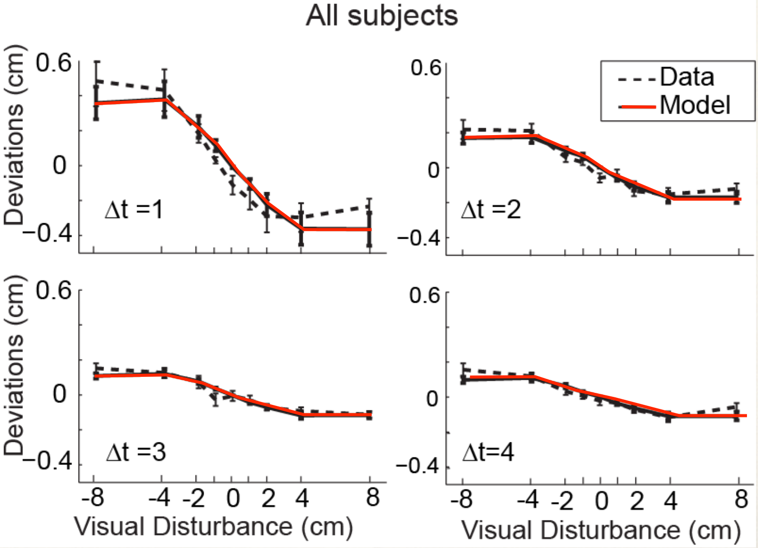


Wei & Kording, 2008, J Neurophys

Typical behavior



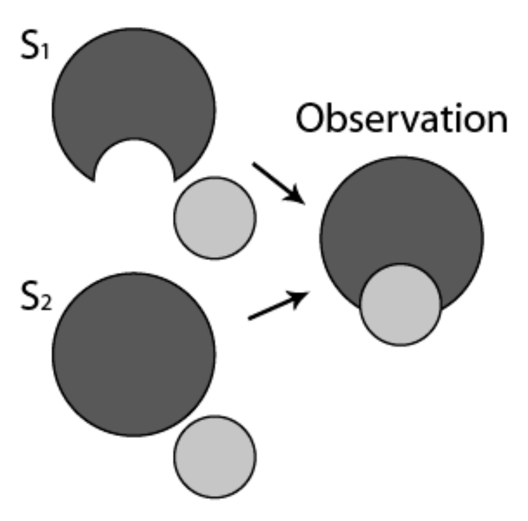
Causal inference in motor adaptation



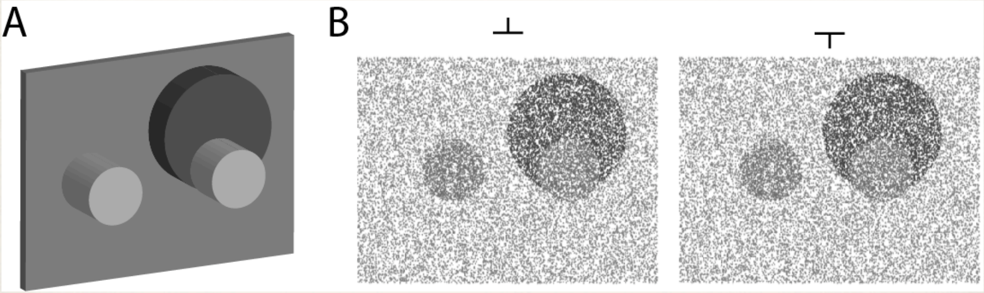
The essence of causal inference

- Noisy identity
- positionsV=positionA with $p=p_{common}$
- independent otherwise

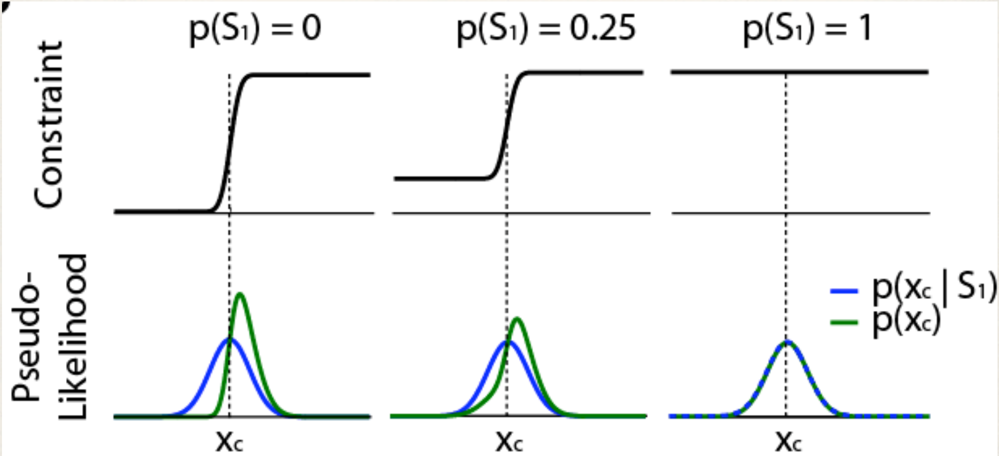
Similar problem in depth perception: structure inference



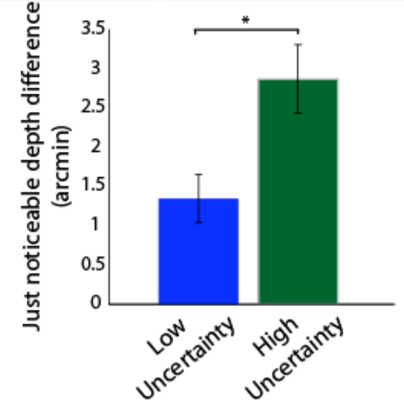
Basic idea



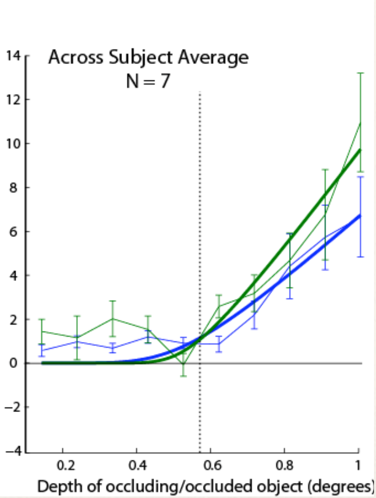
Induced likelihoods



Two levels of uncertainty



Effect of structure knowledge

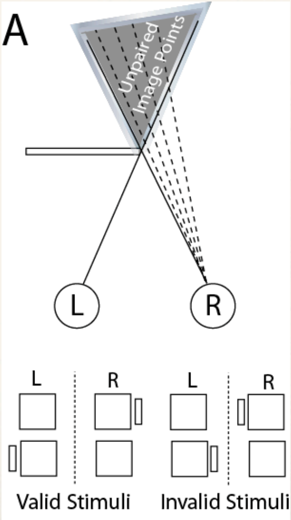


High uncertainty

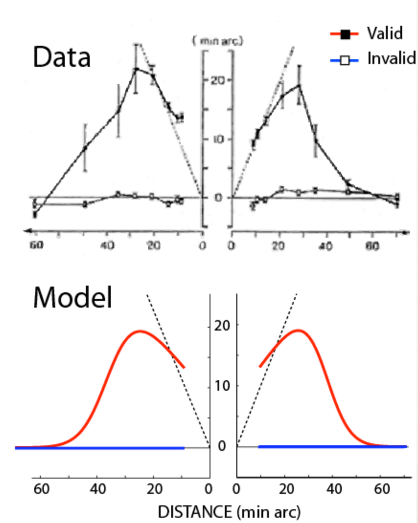
Low uncertainty

No causal breakdown (no holes)

Nakayama et al 1990



Effects of structure inference

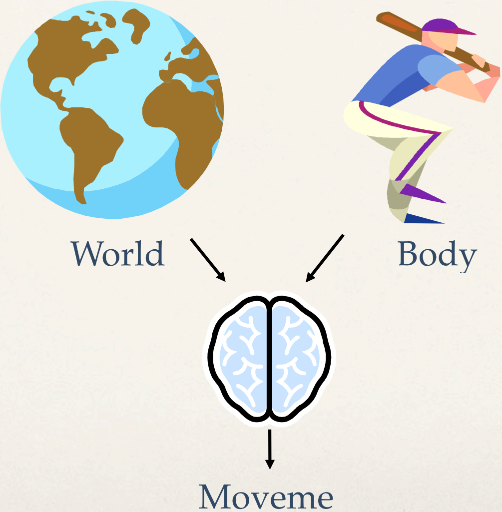


The essence of structure inference

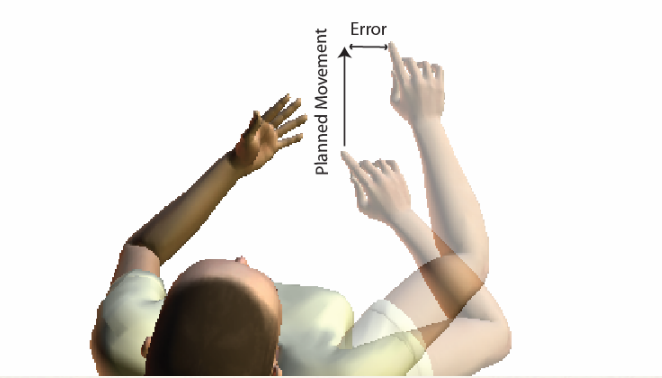
- Noisy inequality
- depth1>depth2 with $p=p_{order}$
- independent otherwise
- Thus this structure inference generalizes causal inference

Part 2: Estimation for movement

Adaptation ~ estimation

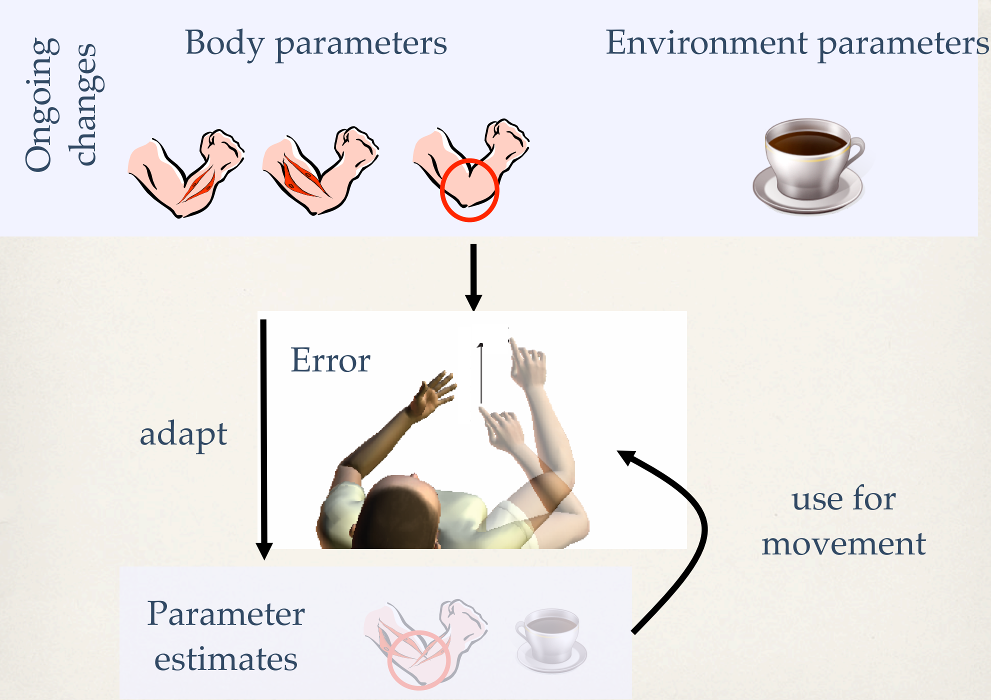


Ambiguity



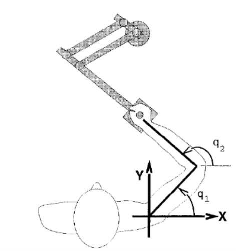
Limb impedance: Viscosity, stiffness, inertia, muscle strength
Environmental affordances: Object inertia, force fields

Different disturbances may lead to the same error



see Kording et al, Nature Neuroscience 2007

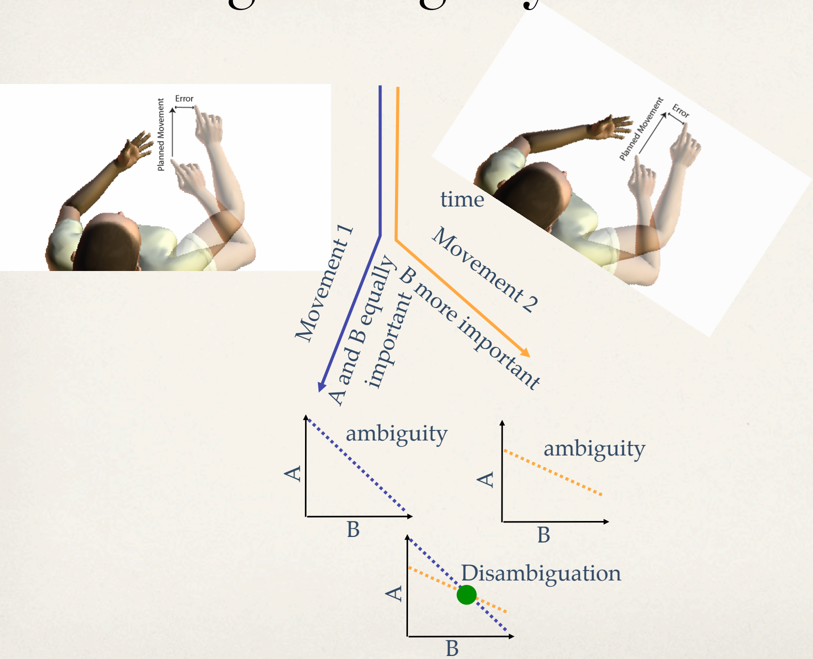
Computational approaches: “Internal models”



Perturb movement property

- Model estimates this property from motor errors

Resolving ambiguity



Simulating trajectories

$$\frac{\ddot{x}}{m} = Output\ \ from\ EP\ controller$$
$$+ F\ \ from\ \ velocity\ dependent\ \ force\text{-}field - Estimated\ F\ \ from\ velocity\ dependent\ \ force\text{-}field$$
$$+ F\ \ from\ position\ dependent\ \ force\text{-}field - Estimated\ F\ \ from\ position\ dependent\ \ force\text{-}field$$
$$+ F\ \ from\ \ joint\ stiffness - F\ \ from\ \ joint\ stiffness$$
$$\ldots$$

Influences on trajectories

Trajectories obtained simulating EP trajectory with corrections and explicit disturbances

$$\text{Error} = f\left(\mathbf{d}, \hat{\mathbf{d}}, movement\right)$$

$$\text{Error} \sim \sum_{i=1}^{N_d} \left(d_i - \hat{d}_i\right) I_i(movement)$$

$$I_i(movement) := \frac{\partial}{\partial d_i} \text{Error}$$

Numerical differentiation ☹

Assuming changes over time

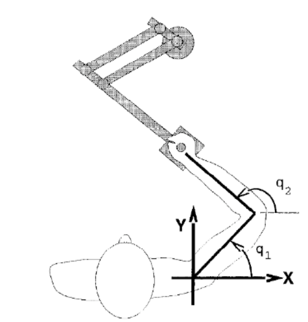
$$d_i(t + \Delta t) = d_i(t) + \varepsilon_i(t)$$

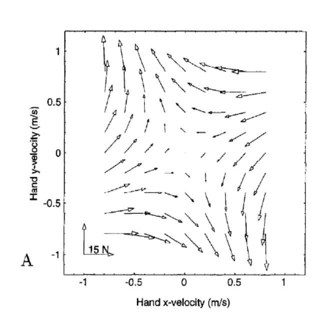
Where $\varepsilon_i(t)$ is drawn from a Normal Distribution of width σ_i

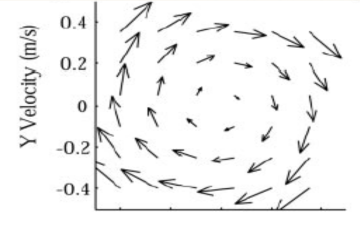
2 Free parameters – all σ_i proportional to nominal values

Extended Kalman filter for state estimation

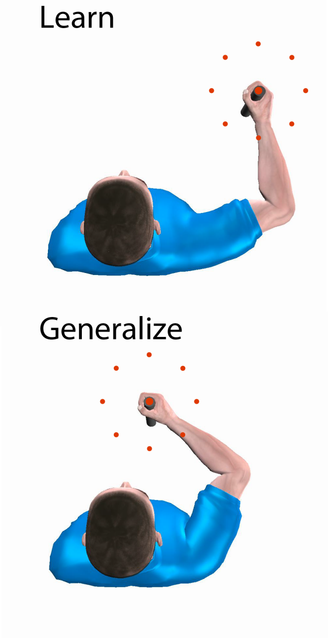
Force field learning

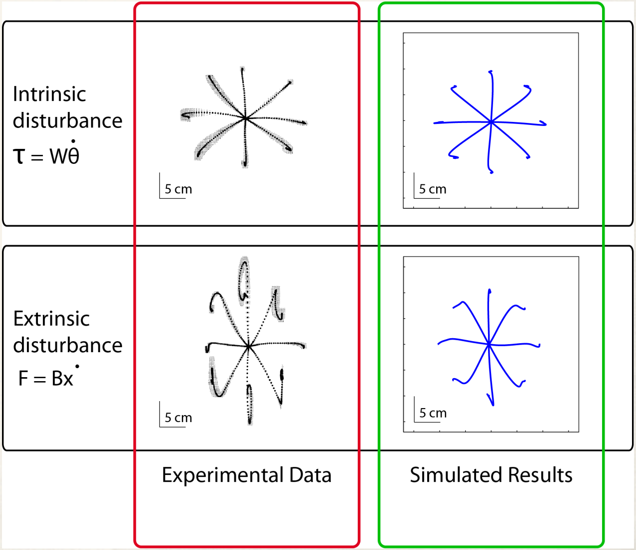




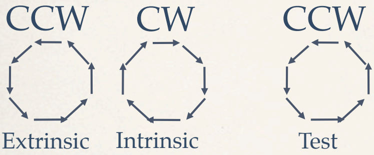


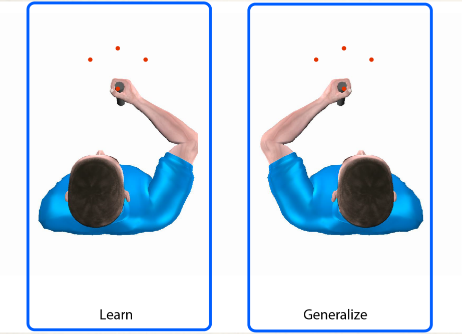
Shadmehr and Mussa Ivaldi 1994

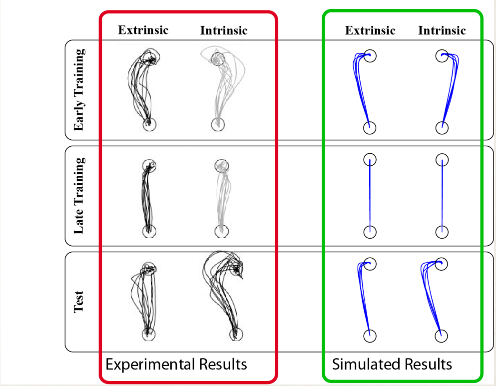




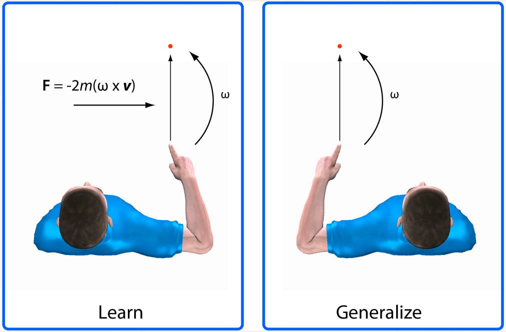
Criscimagna Hemminger et al 2003

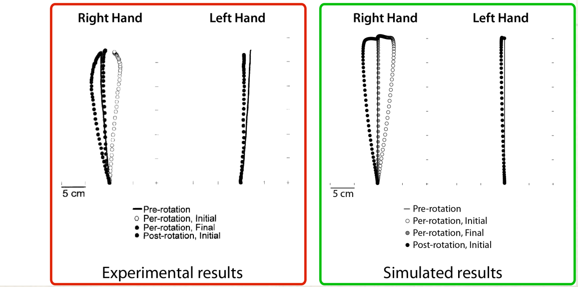




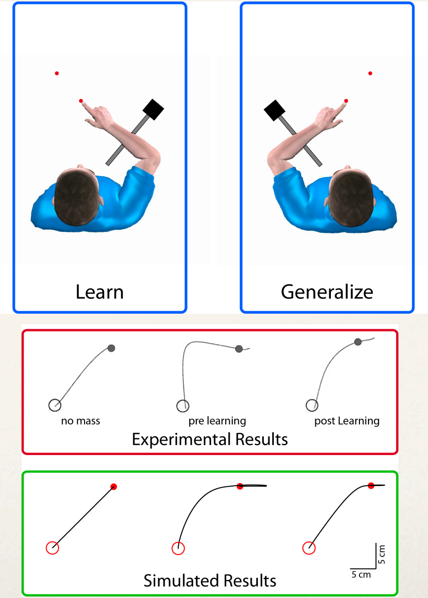


Dizio and Lackner 1995





Wang and Sainburg 2004

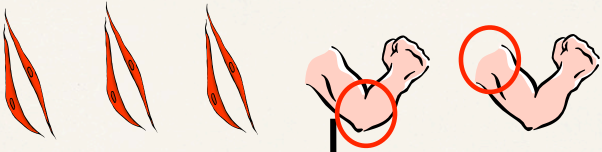


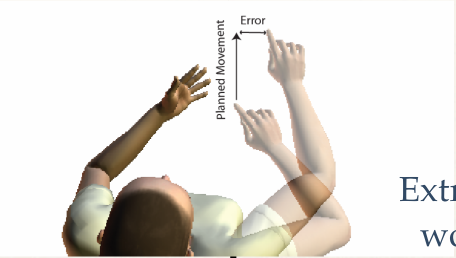
Asymmetry of transfer

- More or only transfer from dominant to non-dominant hand

- Assume more uncertainty about non-dominant hand

Motor Adaptation: Why





Extrinsic world

Error

Take home message

- Uncertainty about causal structure
- Bayesian framework is modular
- Easy to extend
- Causality problems occur in many domains