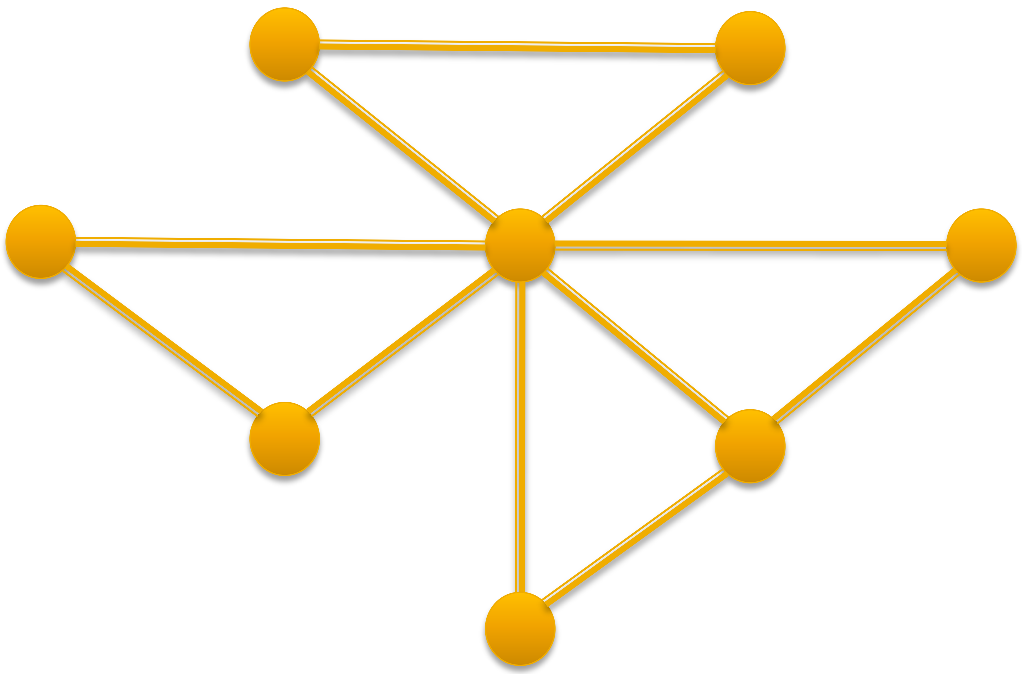
Andreas Björklund

The Path Taken for k-Path

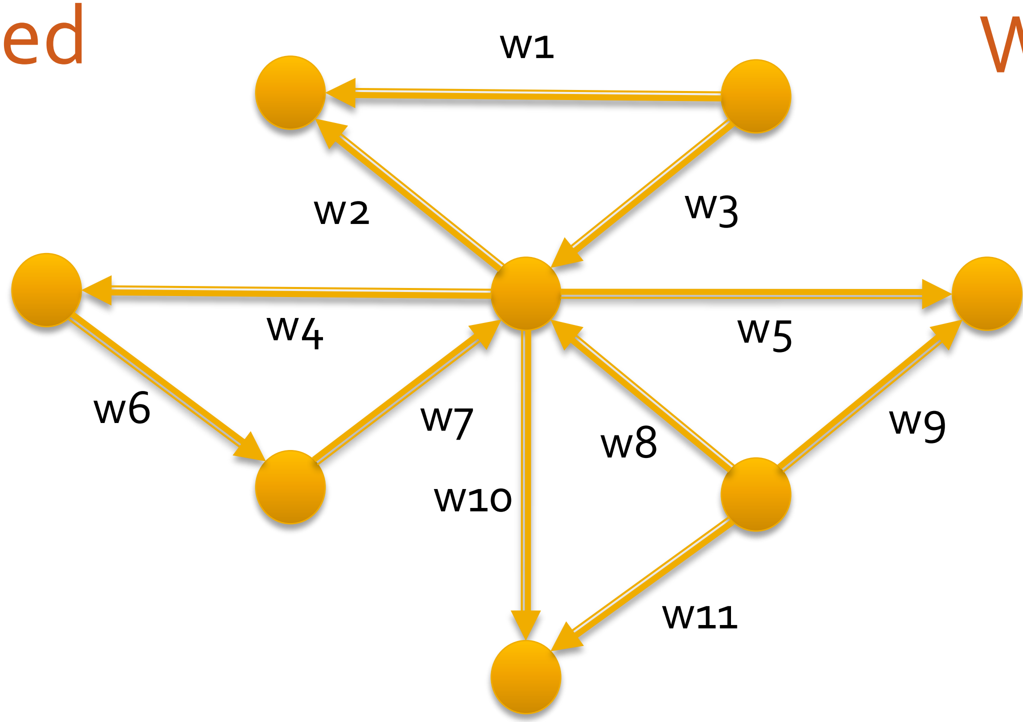
The k-Path Problem



The k-Path Problem

Directed

Weighted



Complexity

- Recursively build paths:

$$O(n^k)$$

- Can there be algorithms with run times on the form

$$f(k)n^{O(1)}?$$

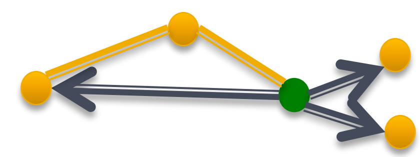
… and if so, how small can f(k) be?

Intuition for FPT

Consider regular graphs of degree d:

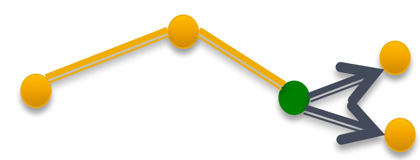
Either d>k:

There must be k-path.

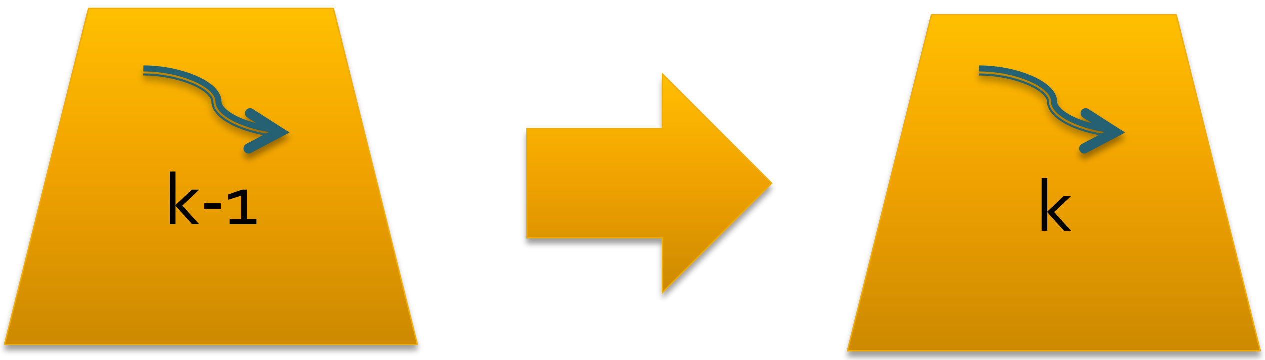


Or d<=k:

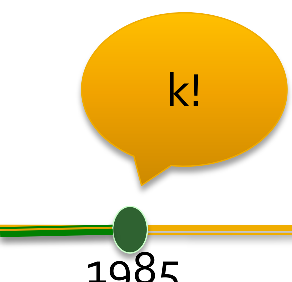
We can list all potential k-paths in $nd^k <= nk^k$ time.



Monien 1985

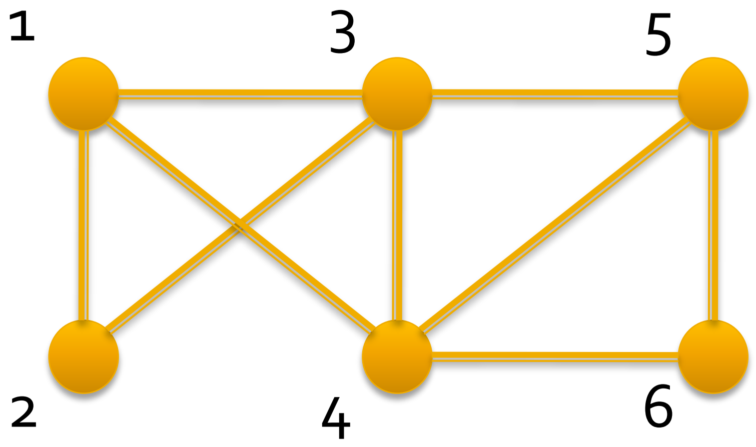


$F^{k-1}_1, F^{k-1}_2, \ldots, F^{k-1}_n$ $\quad$ $F^{k}_1, F^{k}_2, \ldots, F^{k}_n$



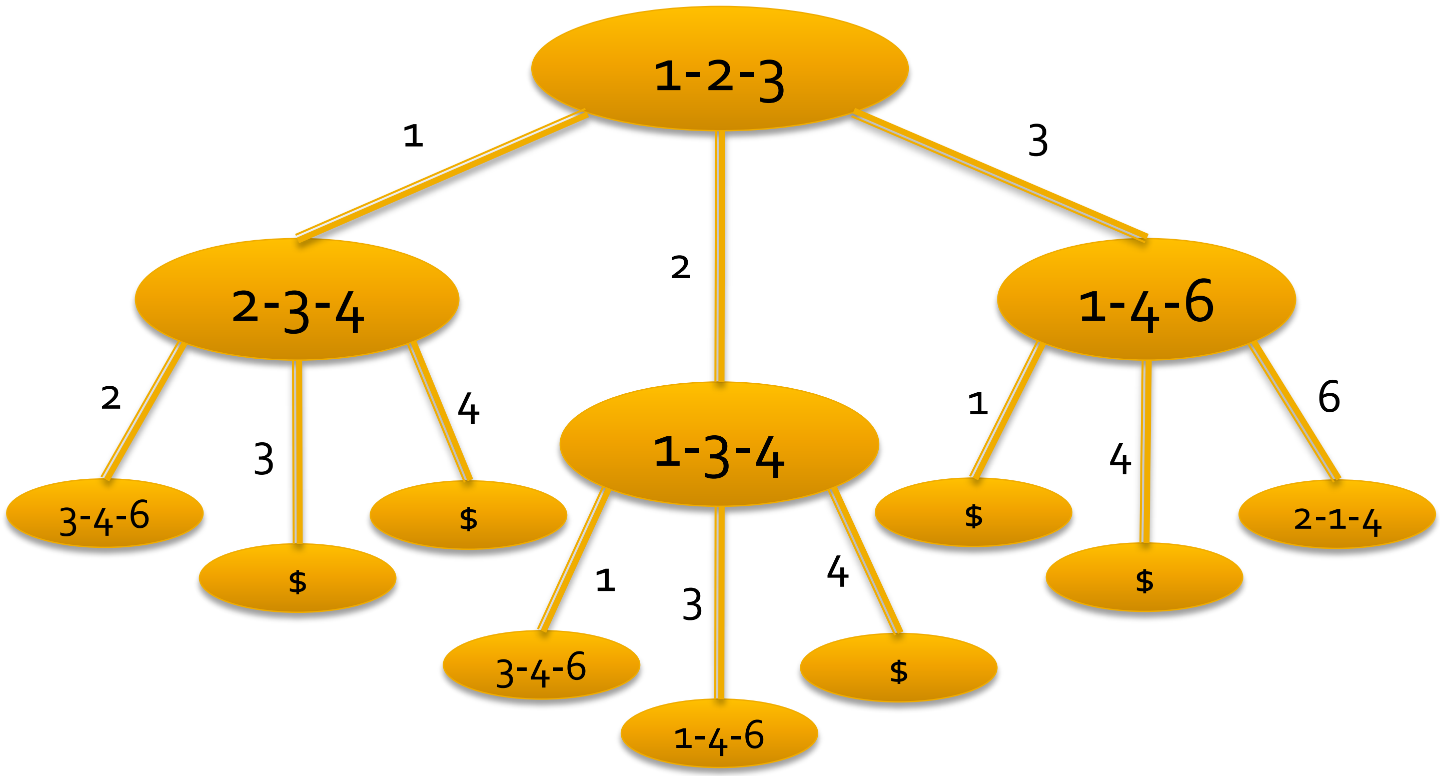
Monien 1985

- 1-2-3-5
- 1-3-4-5
- 1-4-3-5
- 1-4-6-5
- 2-1-3-5
- 2-1-4-5
- 2-3-4-5
- 3-1-4-5
- 3-4-6-5
- 4-1-3-5

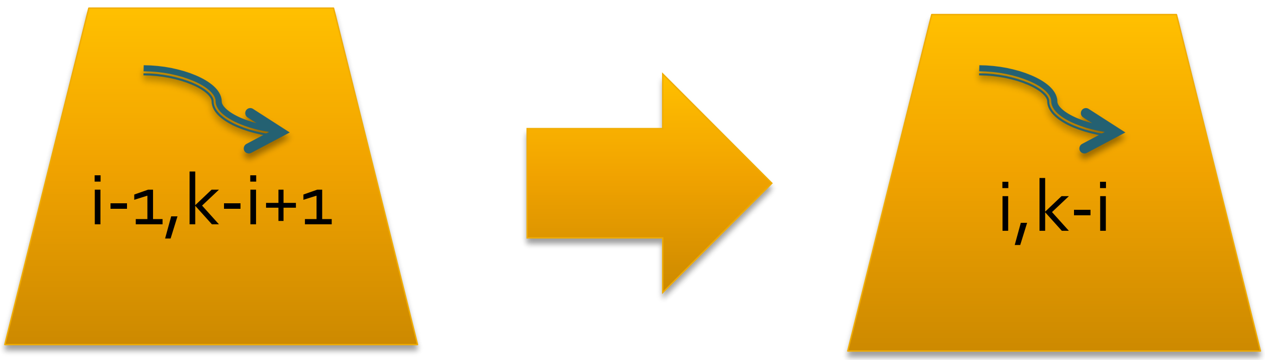


Monien 1985 Path Representatives

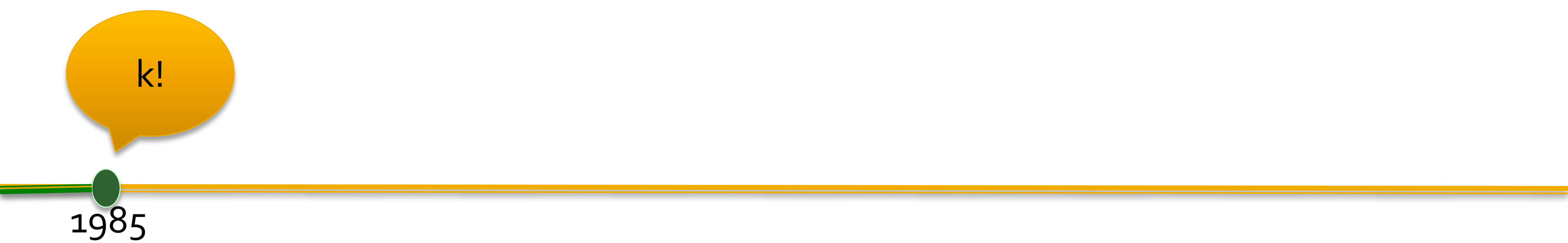
- 1-2-3-5
- 1-3-4-5
- 1-4-3-5
- 1-4-6-5
- 2-1-3-5
- 2-1-4-5
- 2-3-4-5
- 3-1-4-5
- 3-4-6-5
- 4-1-3-5



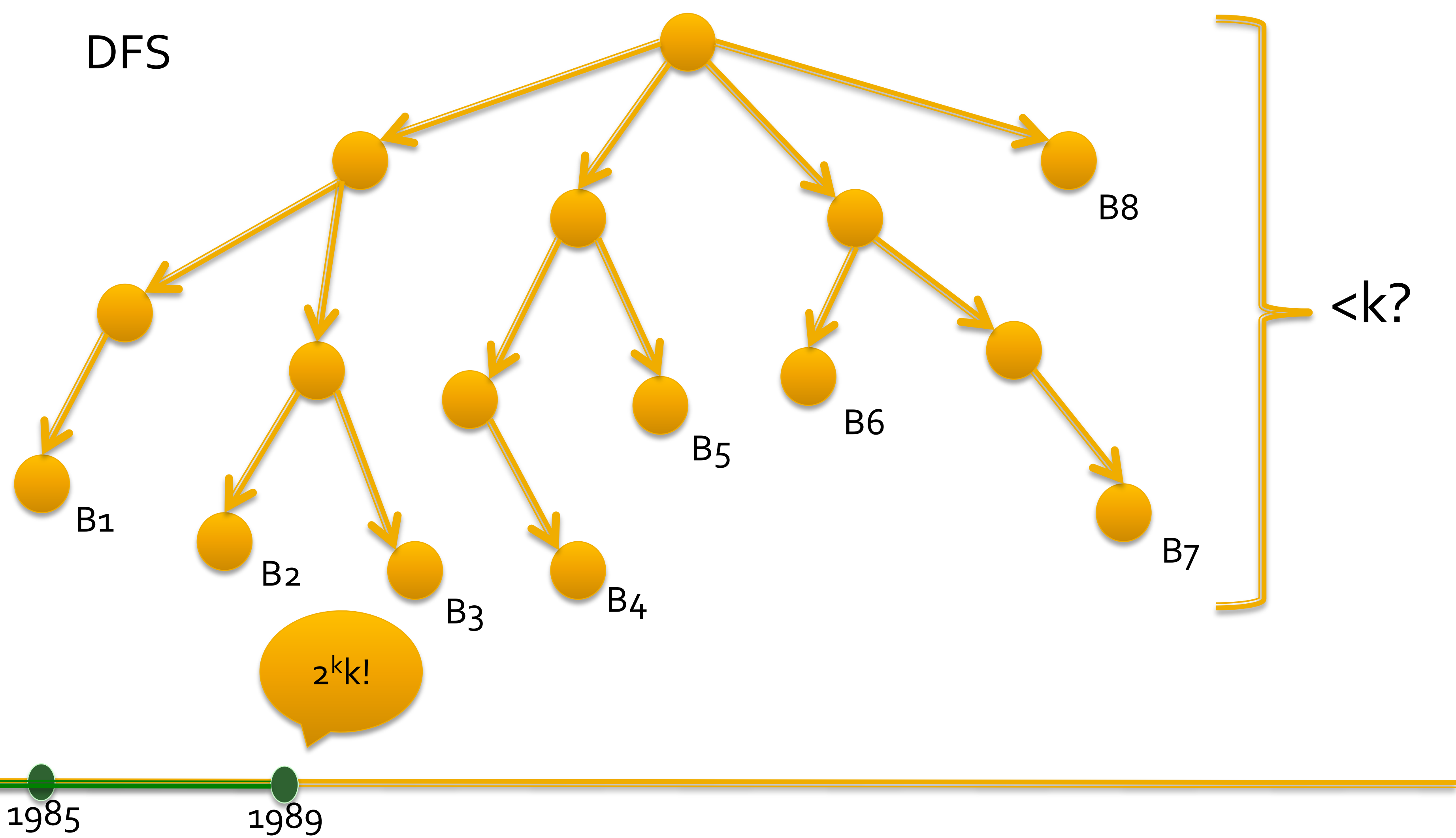
Monien 1985



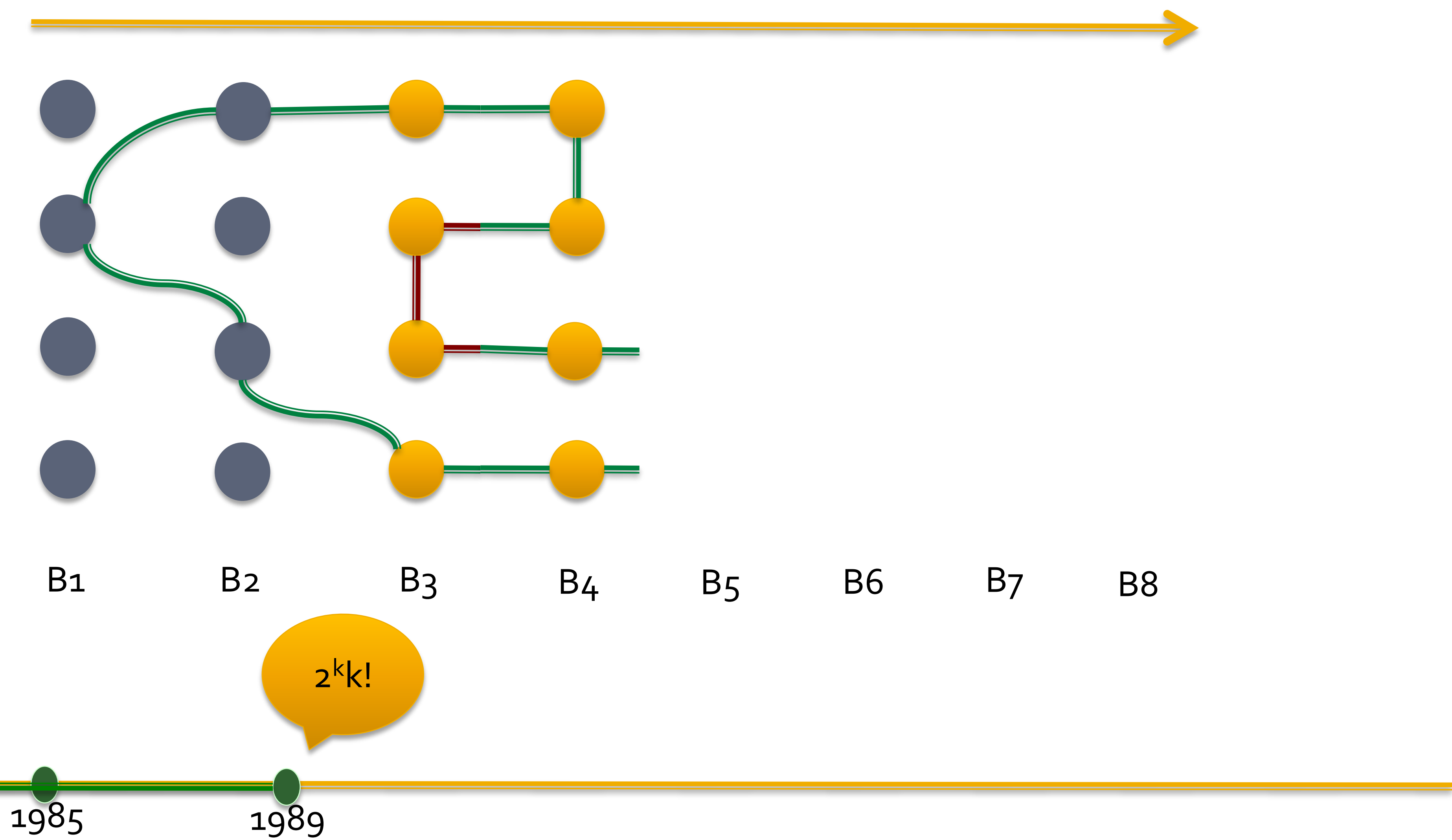
$R^{i-1,k-i+1}_{1}, R^{i-1,k-i+1}_{2}, \ldots, R^{i-1,k-i-1}_{n}$ $R^{i,k-i}_{1}, R^{i,k-i}_{2}, \ldots, R^{i,k-i}_{n}$



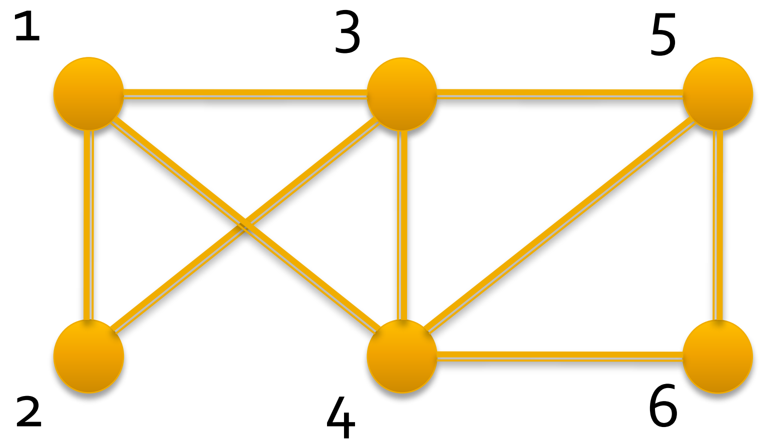
Bodlaender 1989



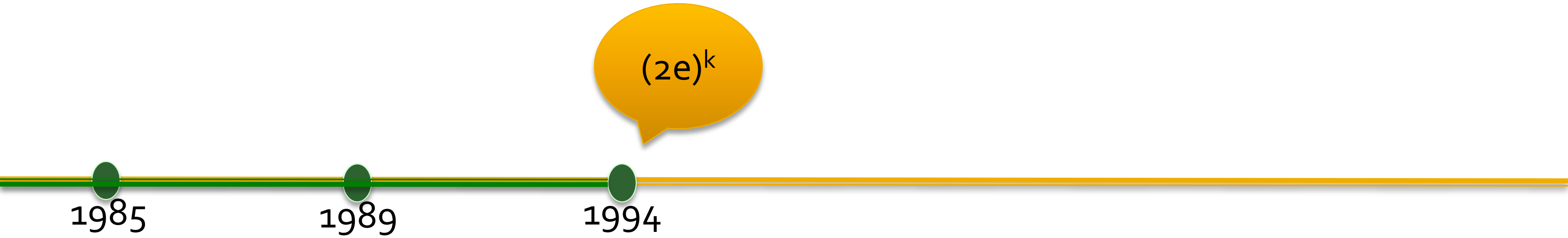
Bodlaender 1989



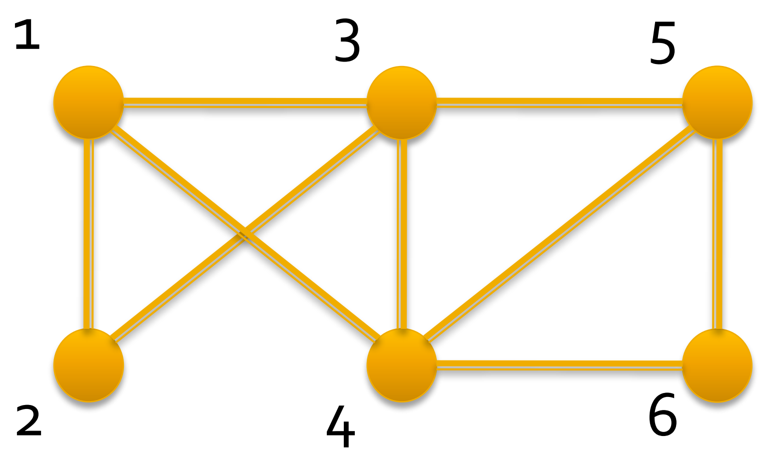
Alon, Yuster, and Zwick 1994



Prob[rainbow k-path] >= $k!/k^k \sim e^{-k}$

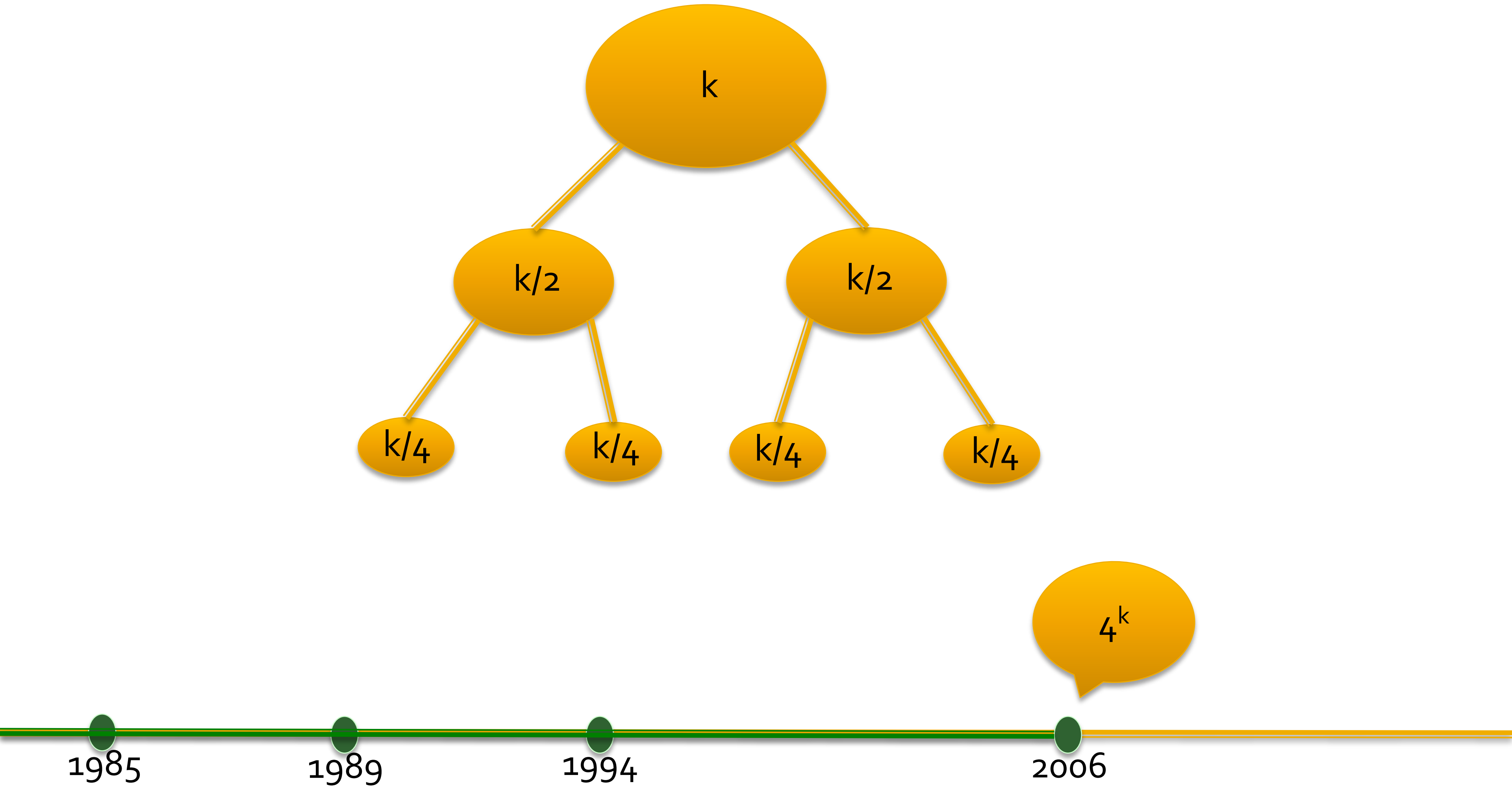


Kneis et al. & Chen et al. 2006



Prob[k-path split] >= 2^{-k}

Kneis et al. & Chen et al. 2006



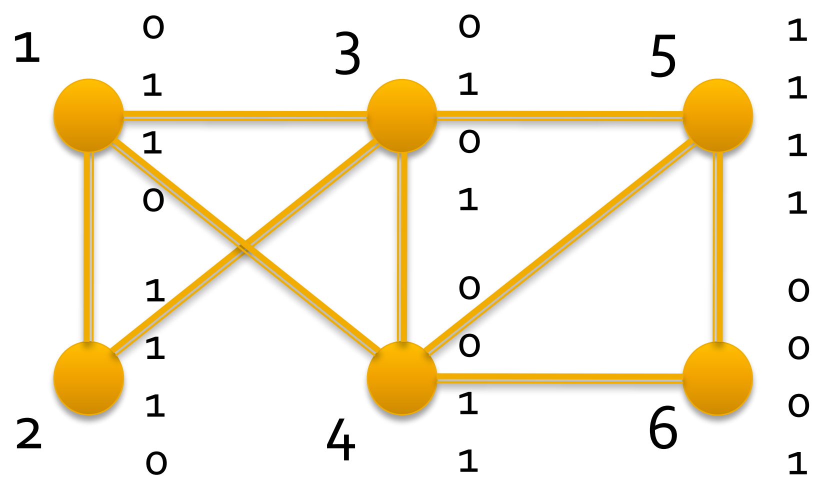
Koutis 2008

Combinatoric methods: Tries to construct the object explicitly piece-by-piece.

Algebraic methods: Implicitly sieves for the object by evaluating a sum.

Example: Triangle detection in a graph via fast matrix multiplication.

Koutis 2008



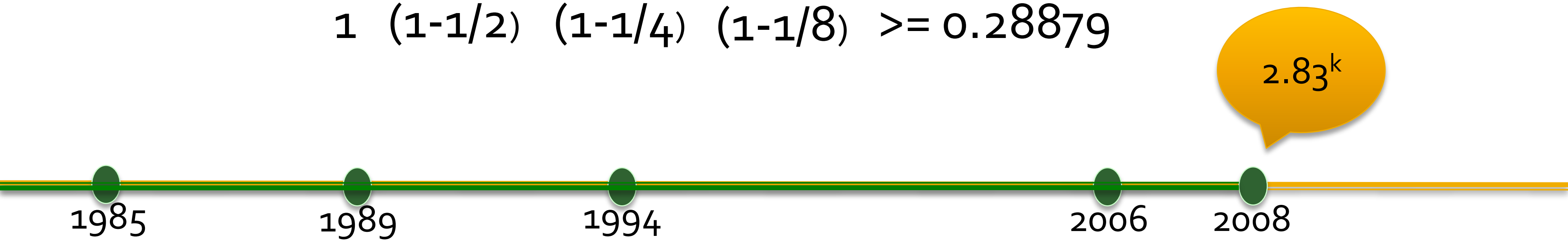
Prob[k-path linear independent] >= 1/4

Koutis 2008

1	0	0	1
1	1	1	1
1	1	0	1
0	0	1	1

Prob[k-path linear independent] >=

1 (1-1/2) (1-1/4) (1-1/8) >= 0.28879



Koutis 2008

- For each vector s in $\{0,1\}^k$, look at the subgraph induced by V_s: the vertices whose vectors v_i obey $<v_i,s>=0$.

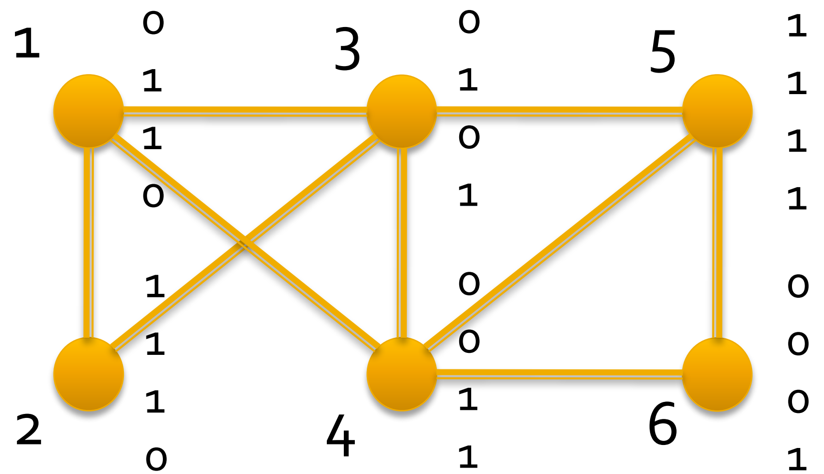
- Count the k-walks in $G[V_s]$, call the result w_s.

- Sum over all w_s, if sum odd report existence of k-path.

Koutis 2008

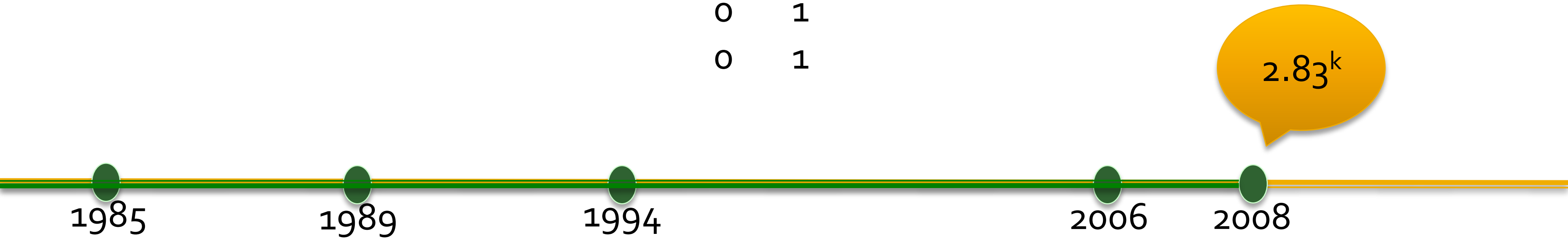
No False Positives!

k-walk: 1,2,3,1



s=

0	0
0	1
0	1
0	1



Koutis 2008

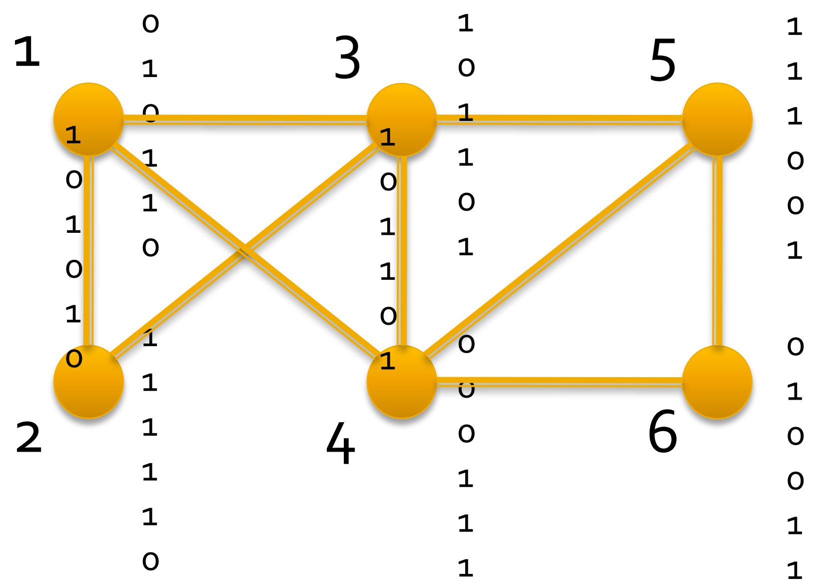
Problem: Often False Negatives!

It doesn’t work when all k-sized vertex sets induce a graph with an even number of Hamiltonian cycles!

Koutis 2008

Fix: Increase dimension and label edges as well...

k-walk: 1,2,3,4



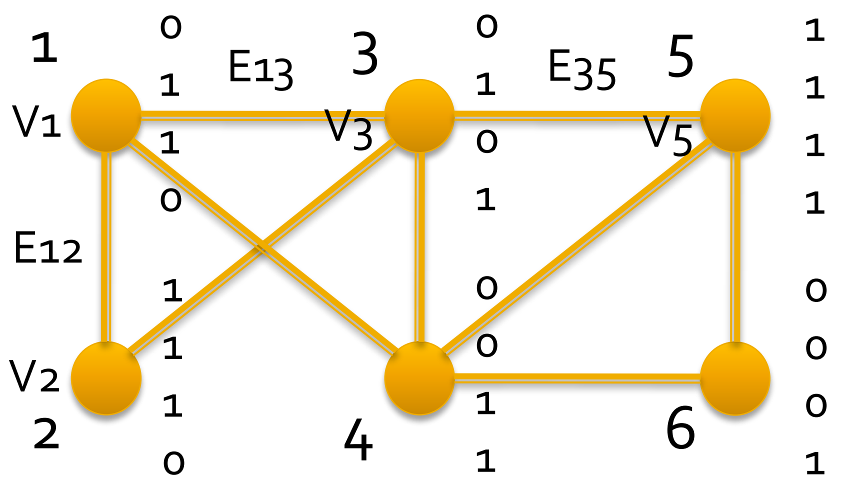
Williams 2009

- Schwartz-Zippel Lemma: Let $P(x_1, x_2, \ldots, x_n)$ be a multivariate non-zero polynomial of total degree d over a finite field F.
  For uniformly and independently sampled values $r_1, r_2, \ldots, r_n$ in F:

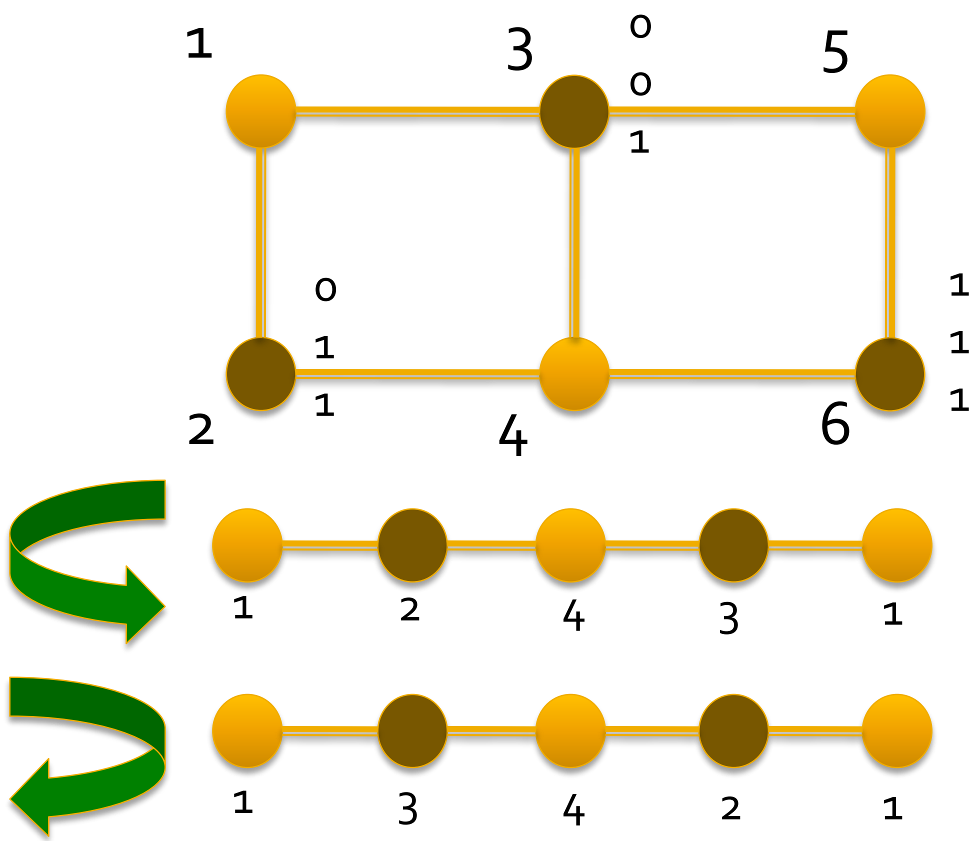
$$\Pr[P(r_1, r_2, \ldots, r_n) = 0] \leq \frac{d}{|F|}$$

Williams 2009

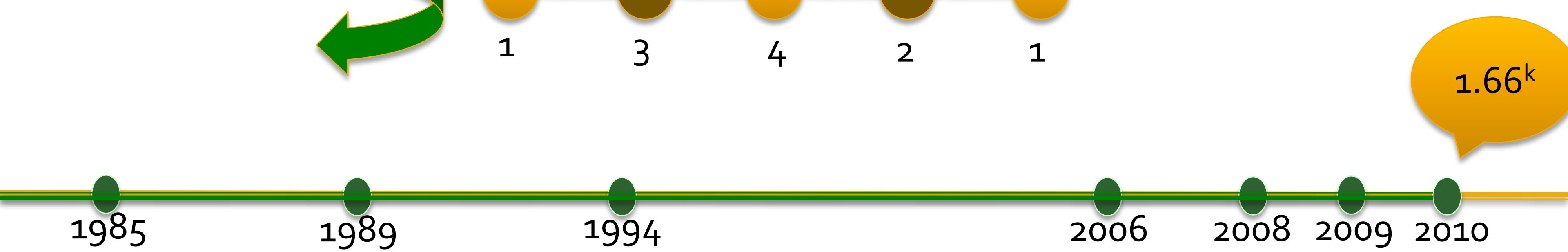
Use larger field of characteristic two!



B., Husfeldt, Kaski, Koivisto 2010



$$\sum = 0$$



Counting k-Paths

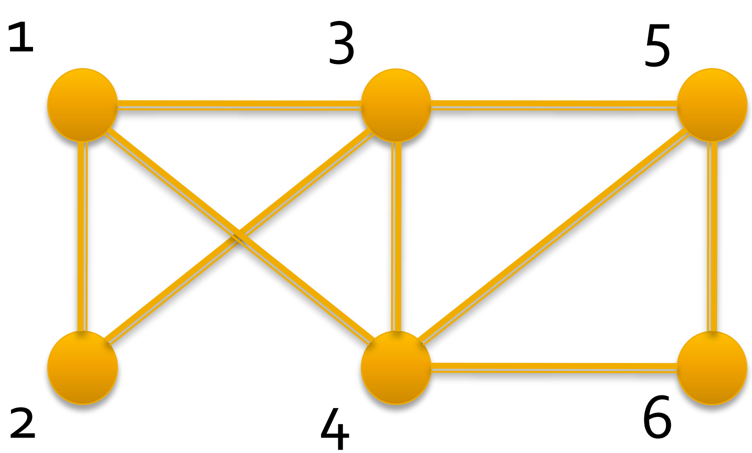
- [Flum and Grohe 2002] Counting the number of k-paths is #W[1]-Hard.
- Approximating the number of k-paths within an arbitrarily small constant using the color-coding technique requires at least

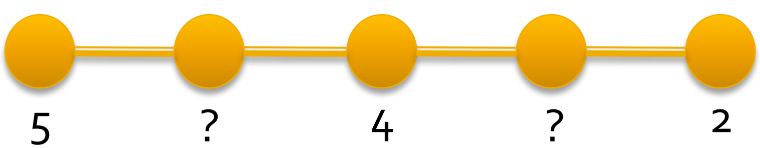
$$\Omega(n^{k/2})$$

time.

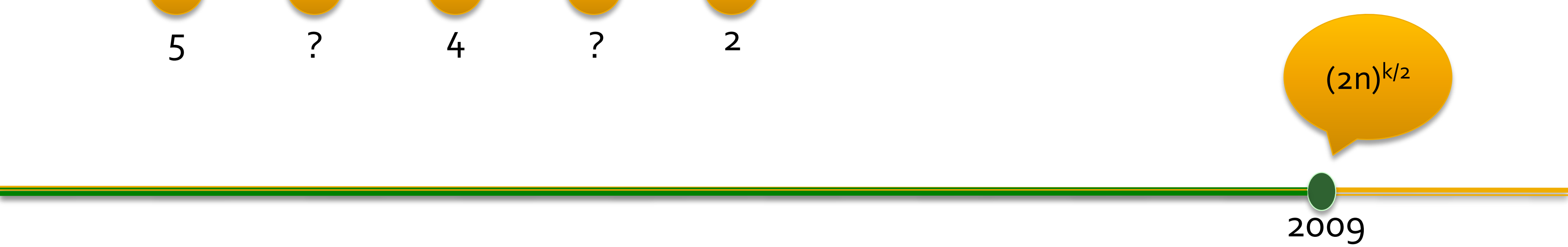
2002 2009

Vassilevska-Williams 2009

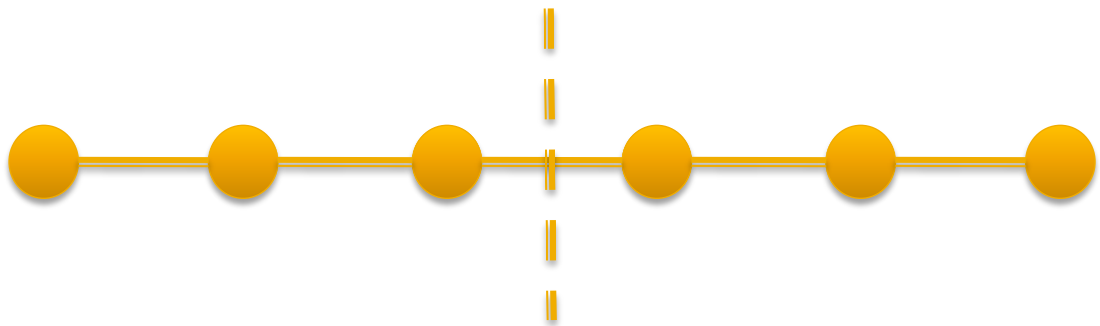




	1	3	6
5-4	0	1	1
4-2	1	1	0

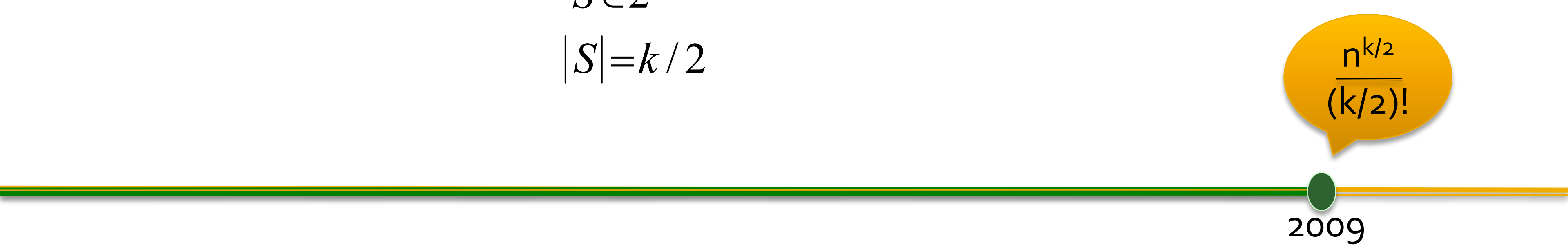


B., Husfeldt, Kaski, and Koivisto 2009



T(S)=Number of k/2-paths containing the vertices S.

$$\sum_{\substack{S \in 2^{[n]} \\ |S| = k/2}} (-1)^{|S|} T(S)^2$$



Thank you for listening!