Multi-view Structure Computation without explicitly estimating camera motion

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Fine print:: Some of the figures/slides used in this talk are downloaded from the internet, which may without proper acknowledgement. They are used here purely for concept illustration and explanation purpose.

The 3D-Reconstruction Problem

- In this talk, we consider how to compute the 3D structure of a static scene from its multiple images captured by a calibrated camera.

Two related sub-tasks:

1. camera motion estimation.

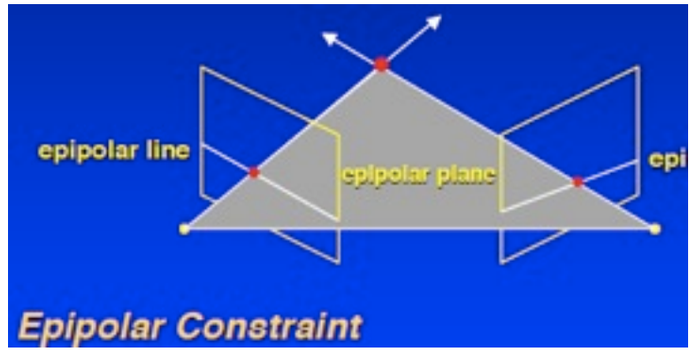
2. 3D structure computation.

The standard approach for 3D-Reconstruction

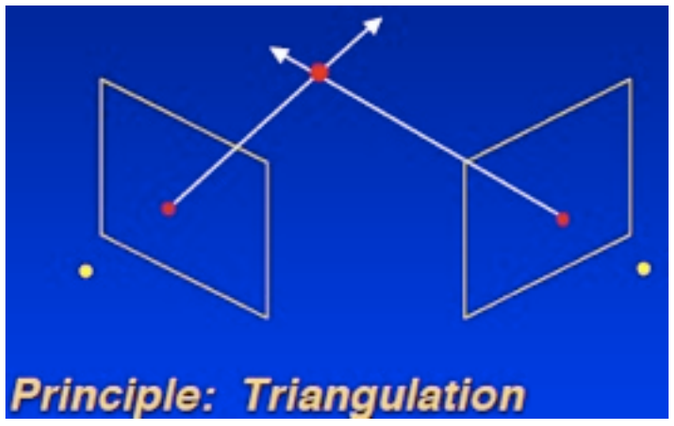
- The standard approach to 3D reconstruction is...

  - the "**structure-FROM-motion**" in its literal sense, meaning that,...

    - First, estimate camera motion via e.g. epipolar geometry or fundamental matrix.



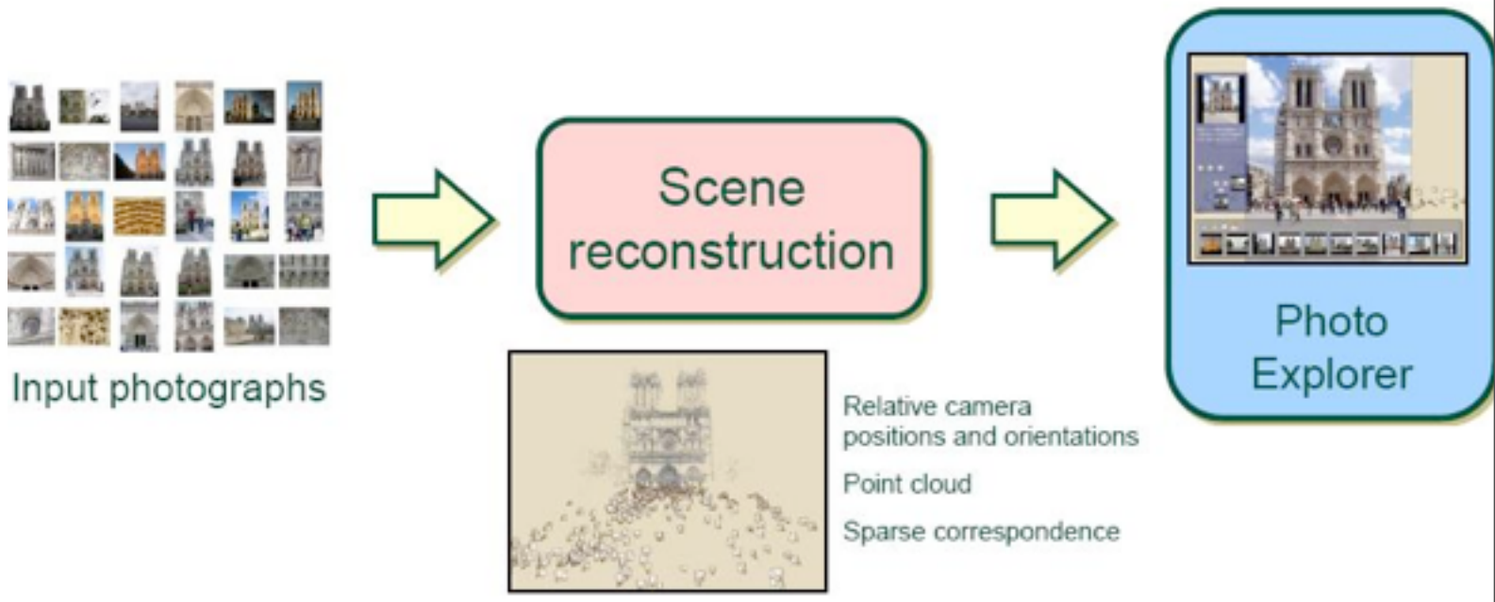
    - Then, compute 3D structure via intersection and resectioning.



    - Finally, bundle adjustment.

This approach has been quite successful in practice

- Represents the state-of-the-art 3D reconstruction technique.
- Two representative systems.
  - **PhotoTourism** (Snavely, et. al.); Build Rome in a day (Agarwal, et al.)
  - **3D Urban Modeling** (Pollefeys et.al.)



Motion-Estimation is almost always the first step in the processing pipeline...

- While most existing approaches follow *the* "structure-from-motion" paradigm, ... , in this work,

Video by Sameer Agarwal et.al.

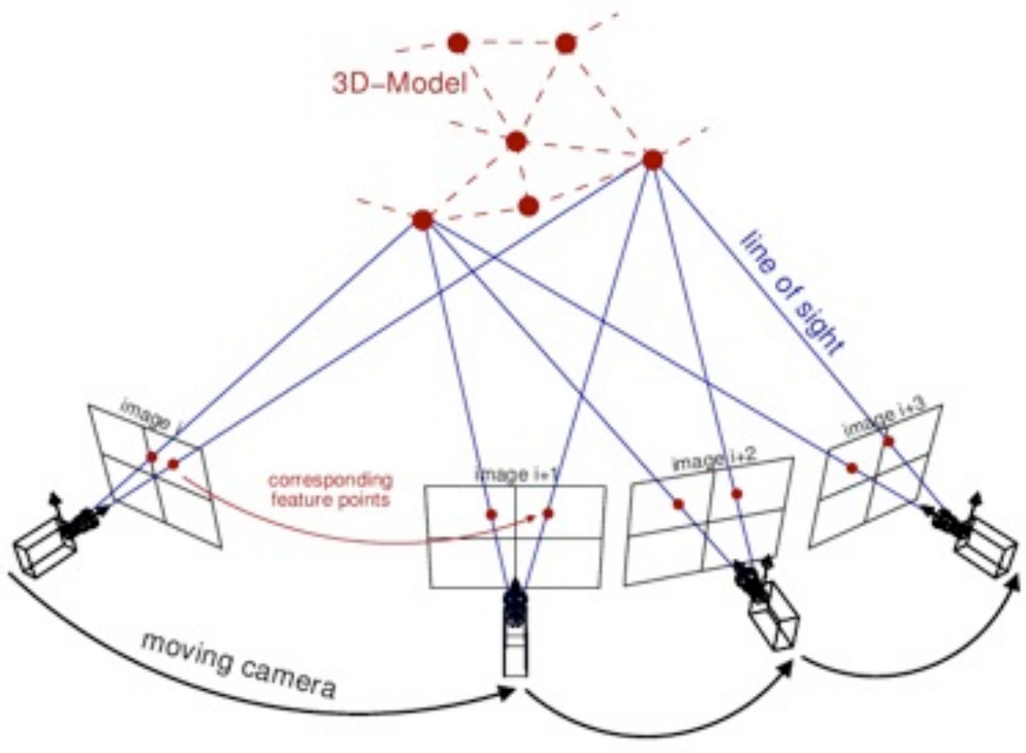


Figure taken from http://www.tnt.uni-hannover.de/print/project/motionestimation/index.php

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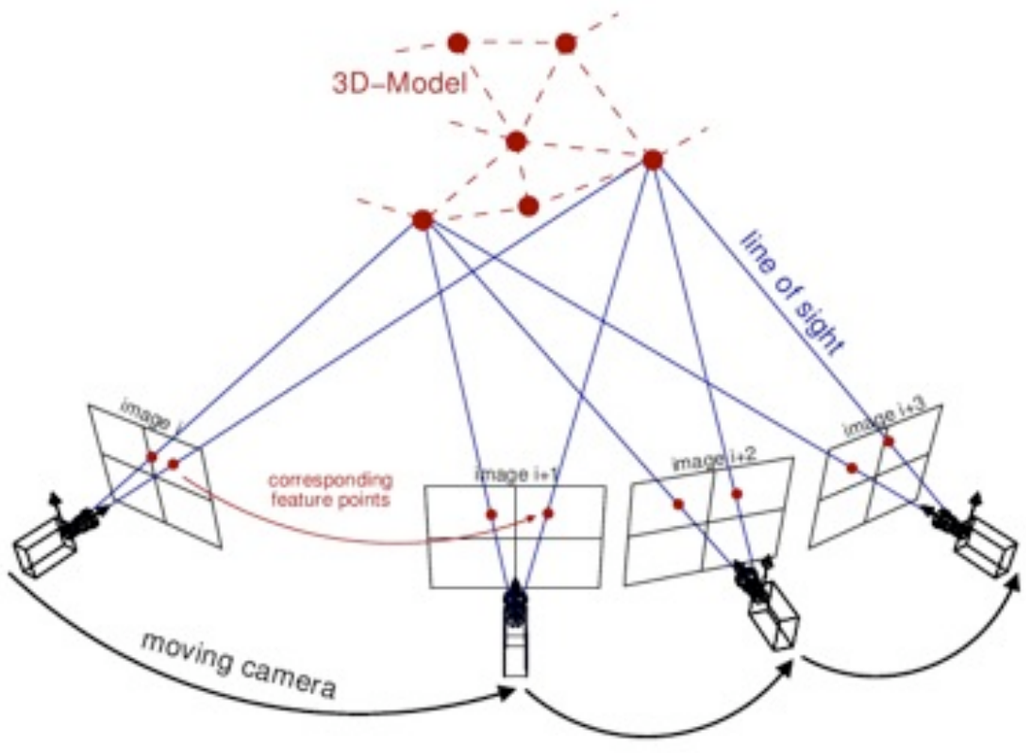

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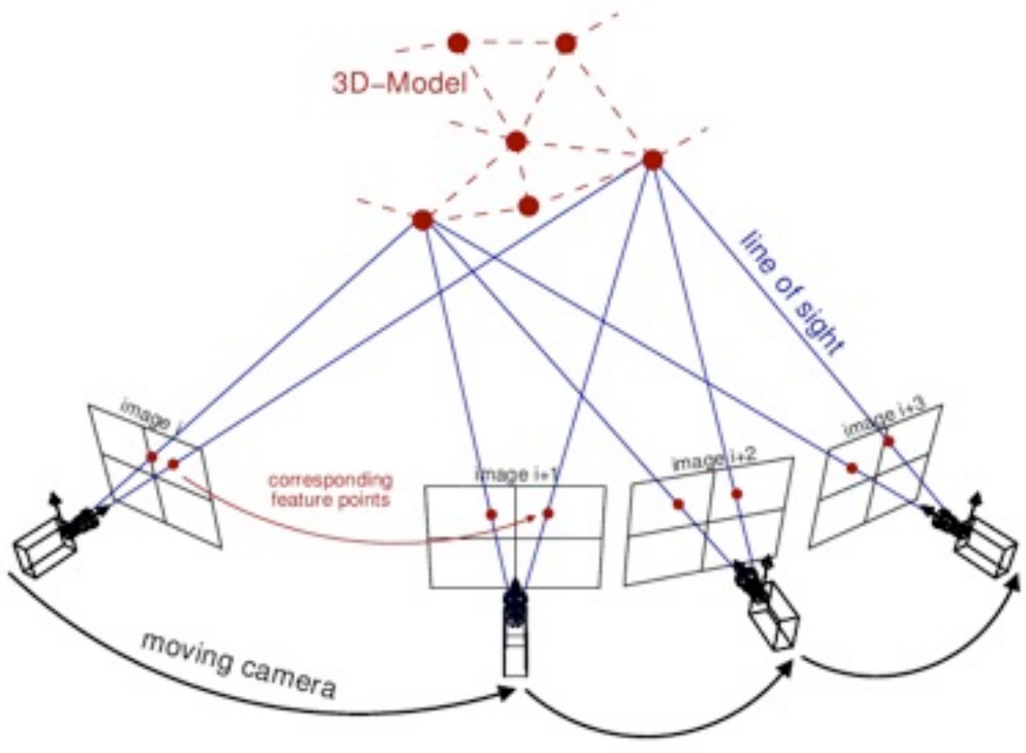

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  - Can we bypass the Motion-Estimation stage, and go directly to structure computation ?

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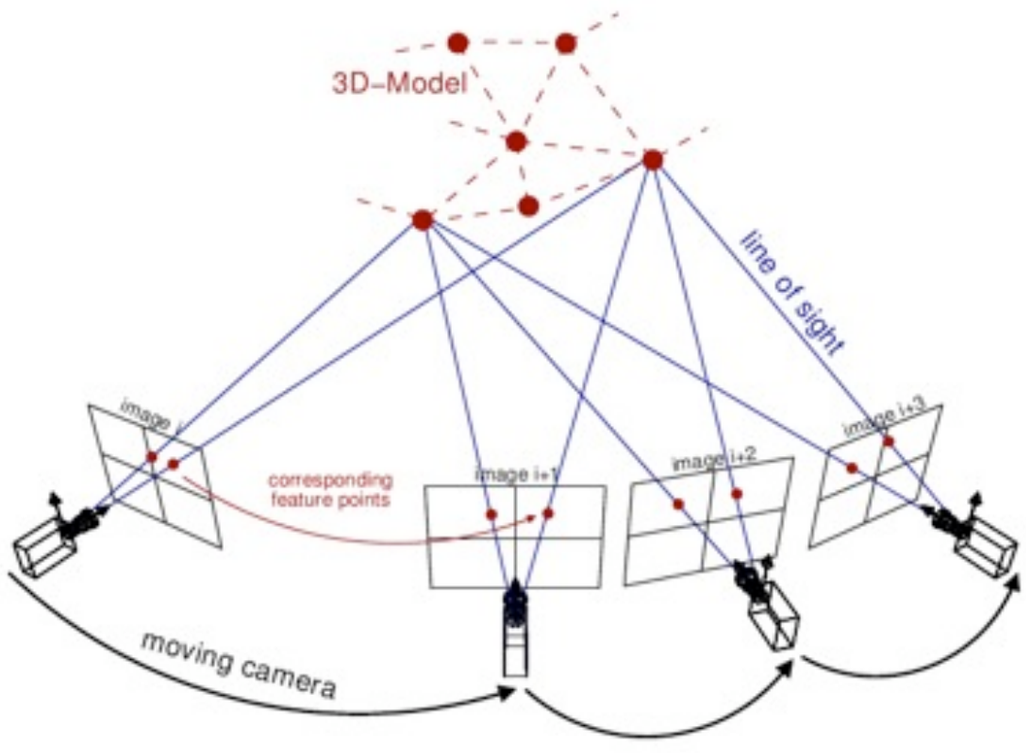

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- **We intend to do it differently ...**
  - Can we bypass the Motion-Estimation stage, and go directly to structure computation ?
  - Can we do it as "**motion FROM structure**" ?

Motivations

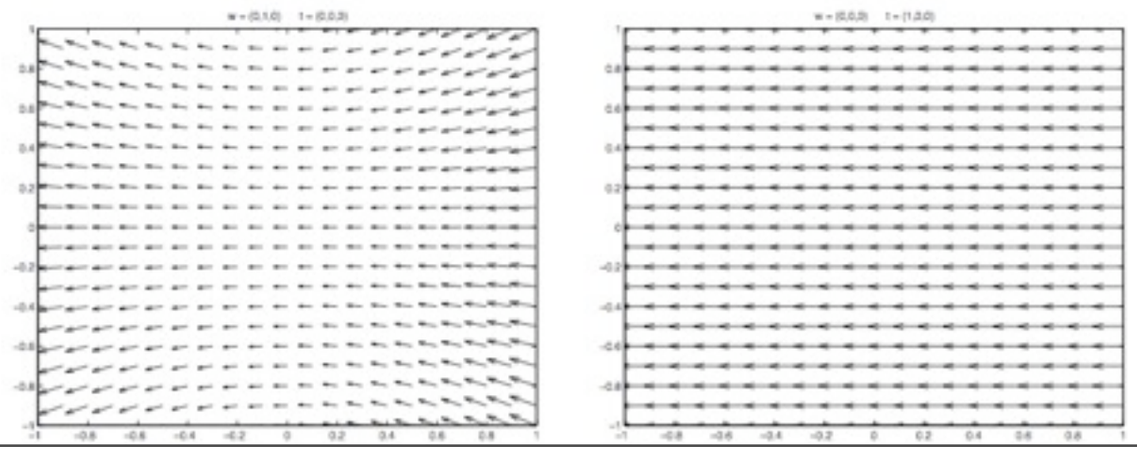
1. Apart from intellectual curiosity, it may provide other benefits as well, e.g.,

2. By avoiding explicit motion computation, we may be able to avoid the well-known **noise sensitivity/inherent ambiguity** in motion estimation.

- **Rotation-Translation** (Bas-Relief) ambiguity;

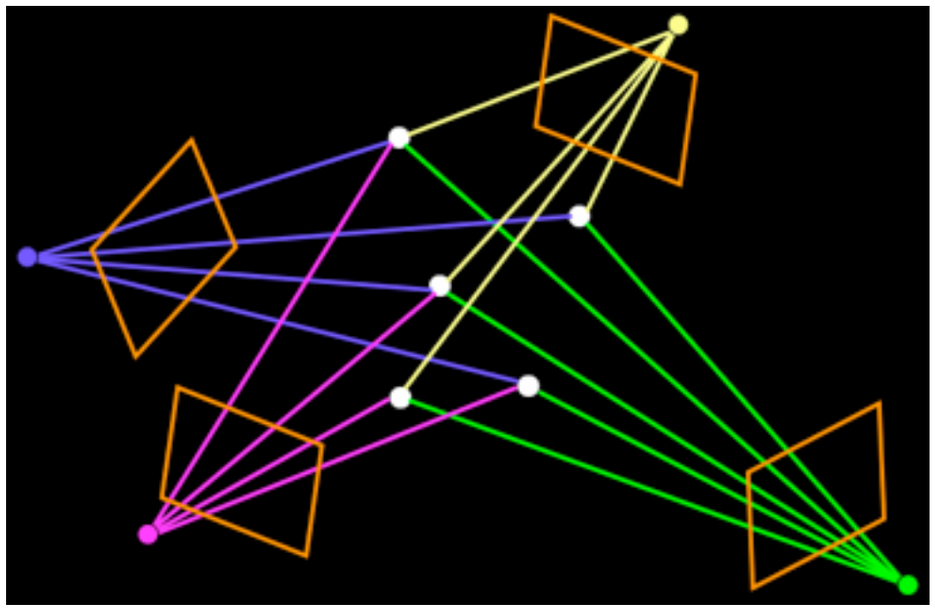
Daniilidis, K. and Spetsakis, (1996). Understanding noise sensitivity in structure from motion.

Fermüller, C. and Aloimonos, Y. 1998. Ambiguity in Structure from Motion: Sphere versus Plane. *Int. J. Comput. Vision* 28, 2 (Jun. 1998)

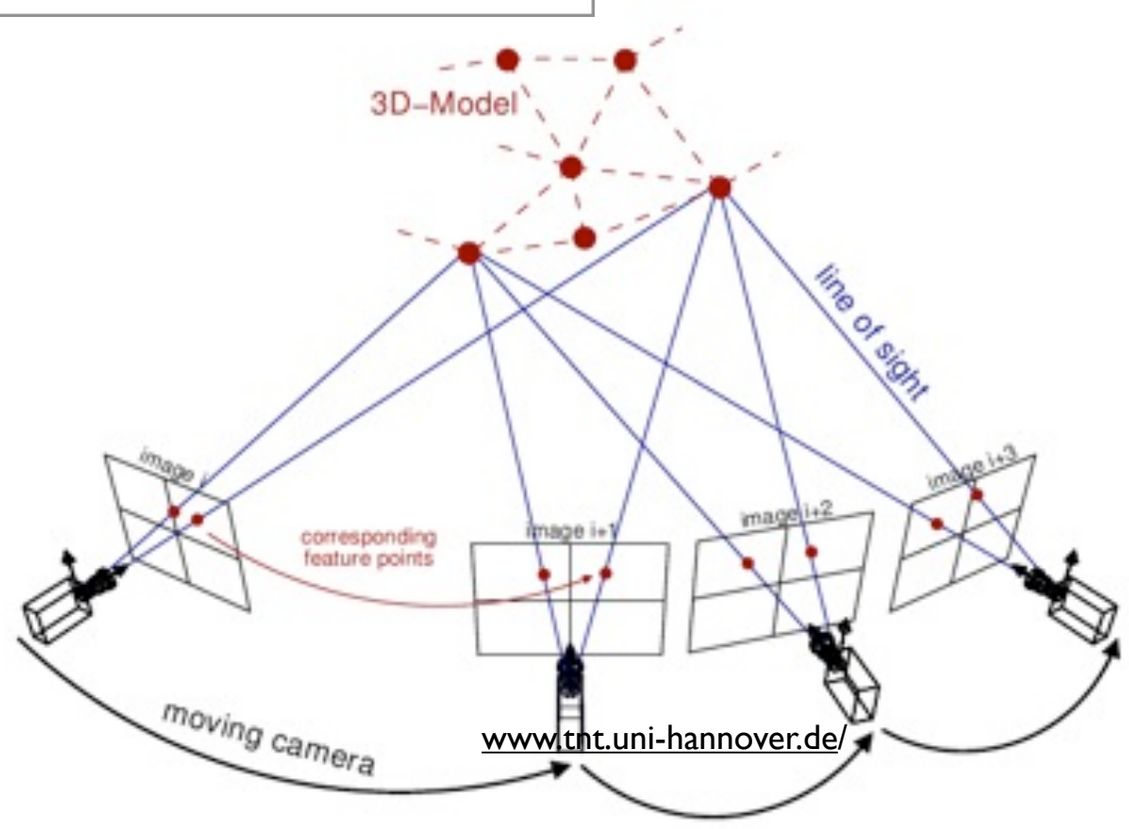


Our goal

- To reconstruct (solely) the 3D Structure (3D Shape) of a set of N point clouds from M calibrated camera measurements, but not the cameras' motion (rotation and translation).

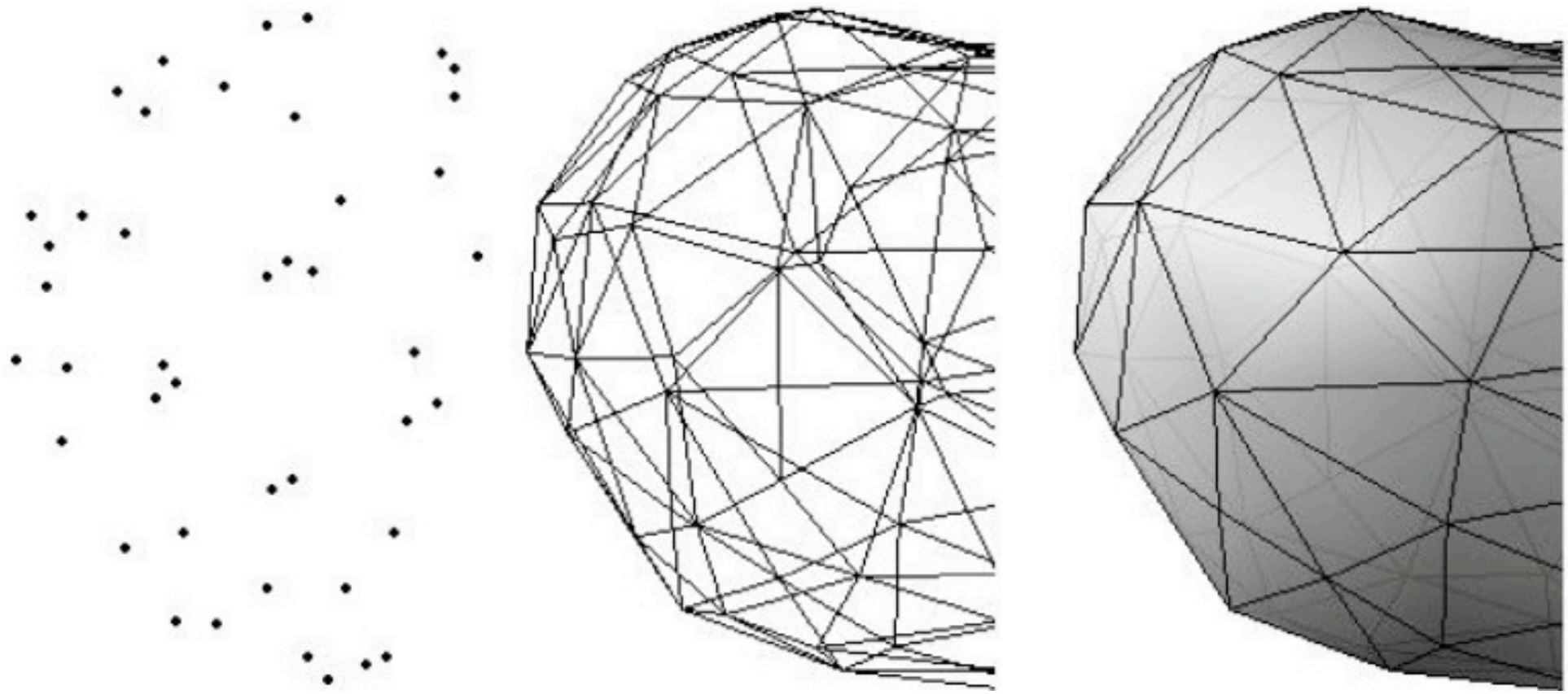


(C) Manmohan Chandraker, vision.ucsd.edu/~manu/research.html



www.tnt.uni-hannover.de/

How do we describe the 3D shape of point clouds ?



- X-Y-Z coordinates representation

- (3N-7) independent parameters (modulo rotation translation, scale).

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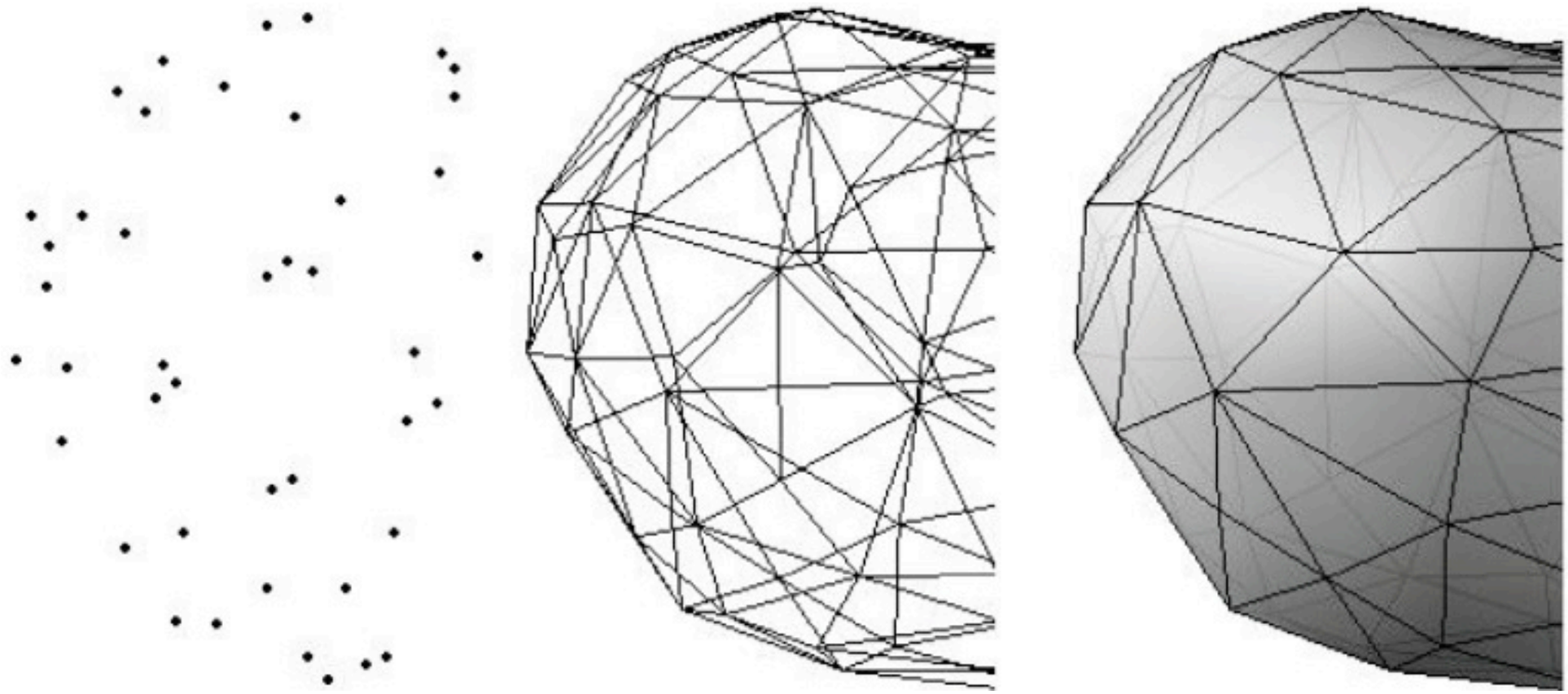


- X-Y-Z coordinates representation
- (3N-7) independent parameters (modulo rotation translation, scale).

- Inter-point distance representation.
- Point -> Graph vertex
- Distance ->Graph edges

Coordinate Representation VERSUS Distance Representation

- From x-y-z coordinate representation, to compute distances (edge length) is trivial:

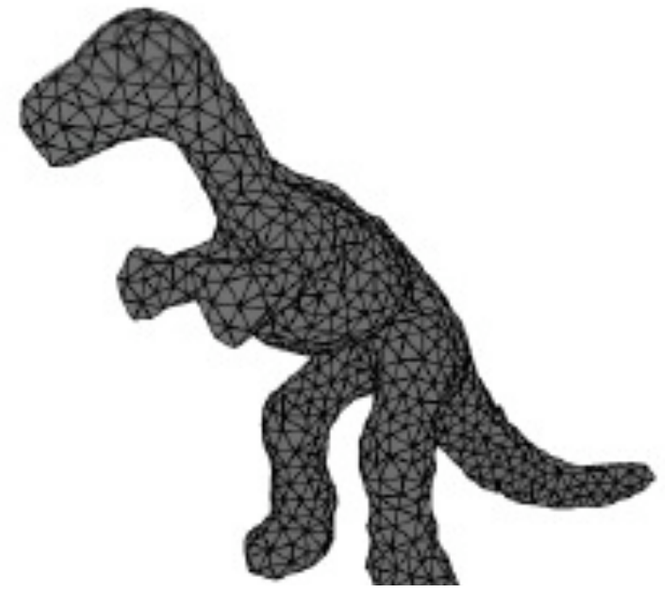
$$l_{ij} = \sqrt{((x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2)}$$

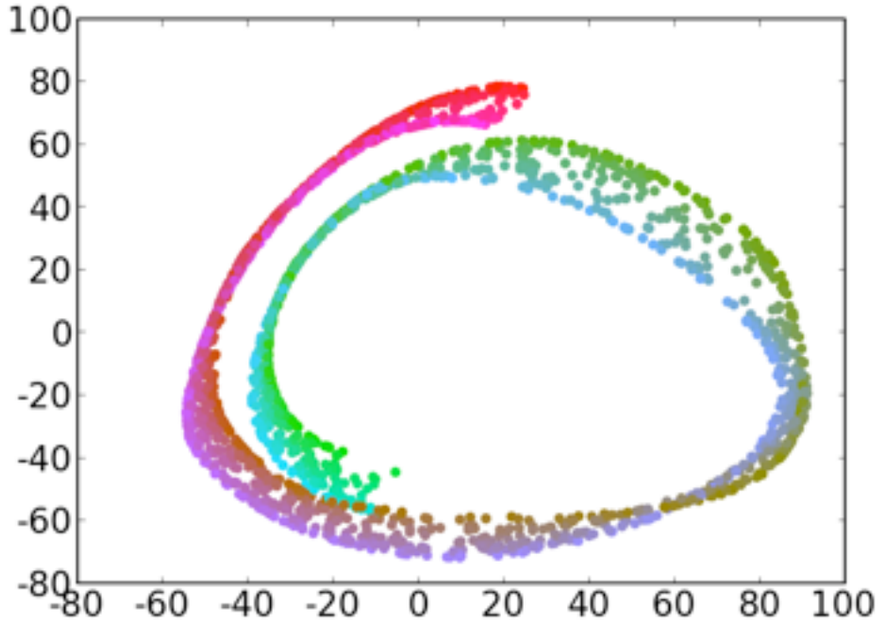
- From distances to recover coordinates (up to Euclidean motion) is also possible:
- known as: graph embedding problem.

Graph-Embedding Problem

- Problem: find $\mathbf{p}_i, i = 1..|V|$, such that $\|\mathbf{p}_i - \mathbf{p}_j\|_2^2 = l_{ij}^2, \forall (i,j) \in E$.

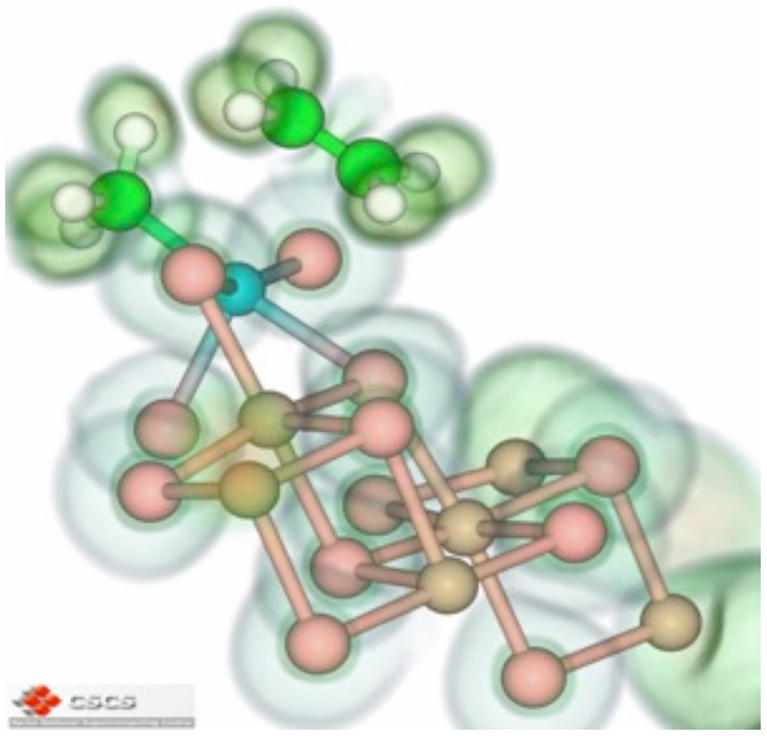
- a.k.a.
  - Graph Realization
  - Graph Drawing
- Closely relates to
  - MDS (multi-dimensional scaling)
  - Dimen-Reduction
  - IsoMap/LLE
- Has also been studied in **Structural Chemistry/Biology.**

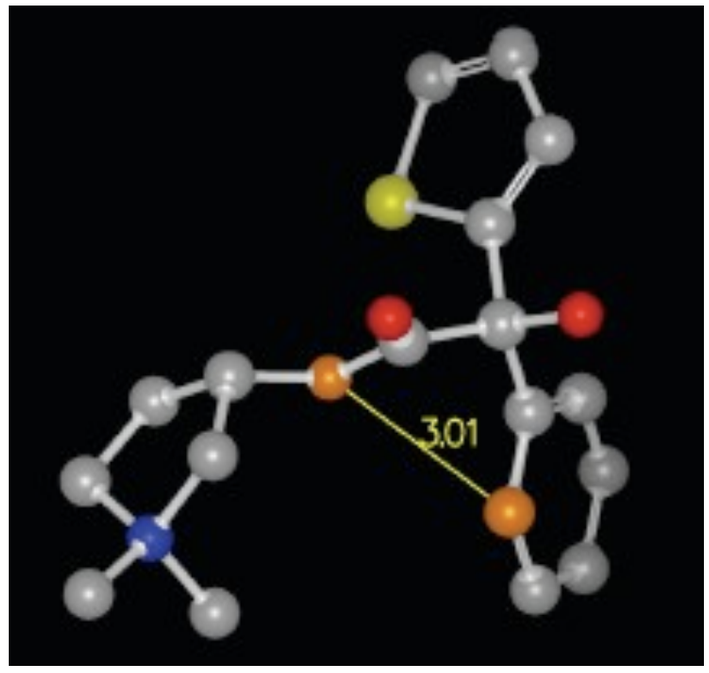




The Molecule Problem

– Suppose that you given distance constraints on the inter-atom distances, determine a 3D shape of a molecule that satisfies those distance constraints.



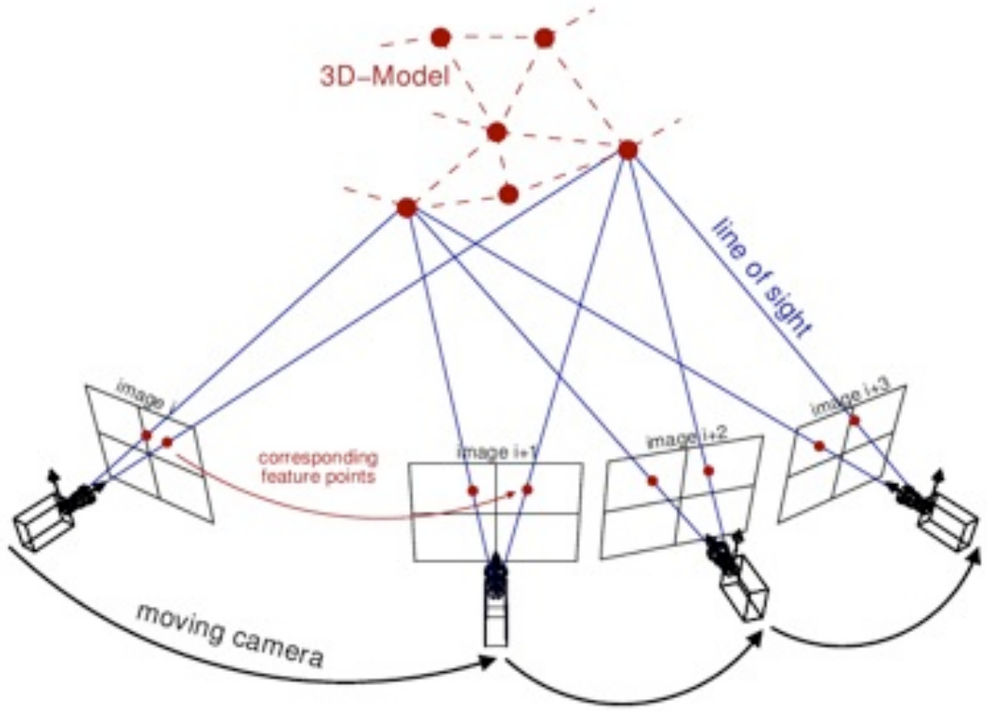


Key message: *the original 3D shape/structure can be faithfully recovered from a sufficient set of inter-point distances.*

Our unique idea

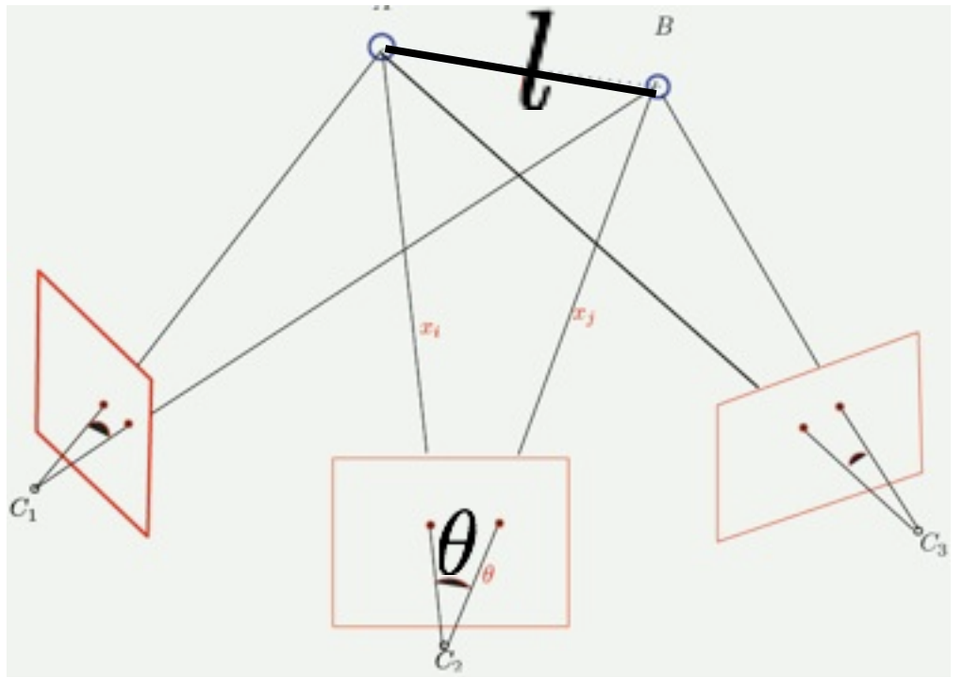
- In doing 3D structure reconstruction,
- instead of directly recovering the 3D coordinates of the point clouds,
- we compute the inter-point distances between pairs of 3D points.
- *why this is relevant to the proposed "motion from structure", we shall see later ...*

How to measure distance in 3-space ?



l

θ



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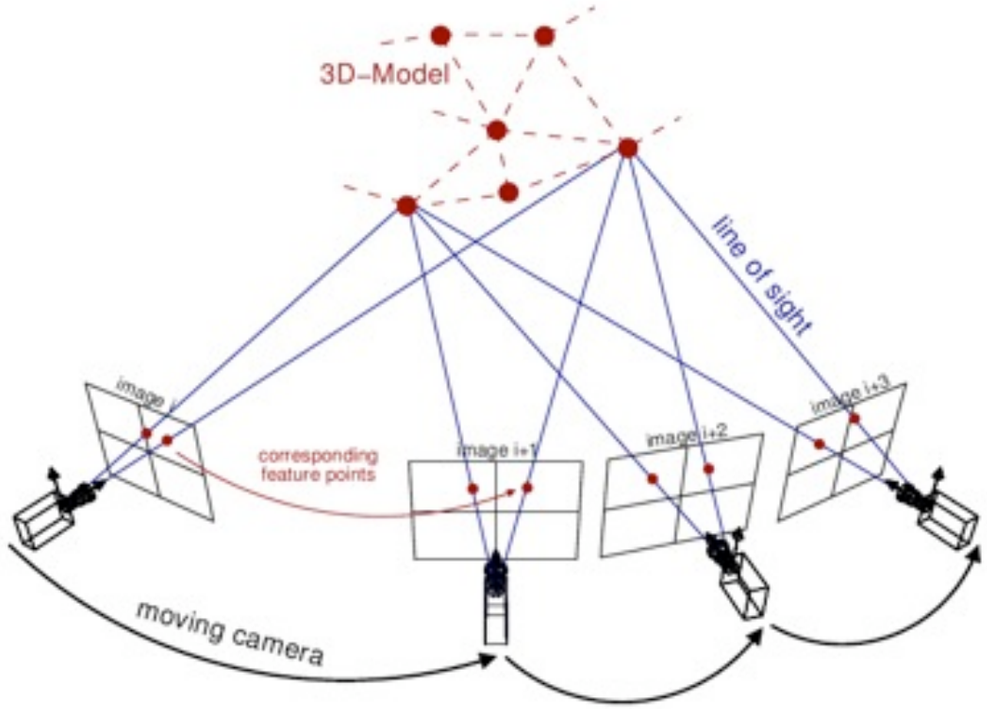
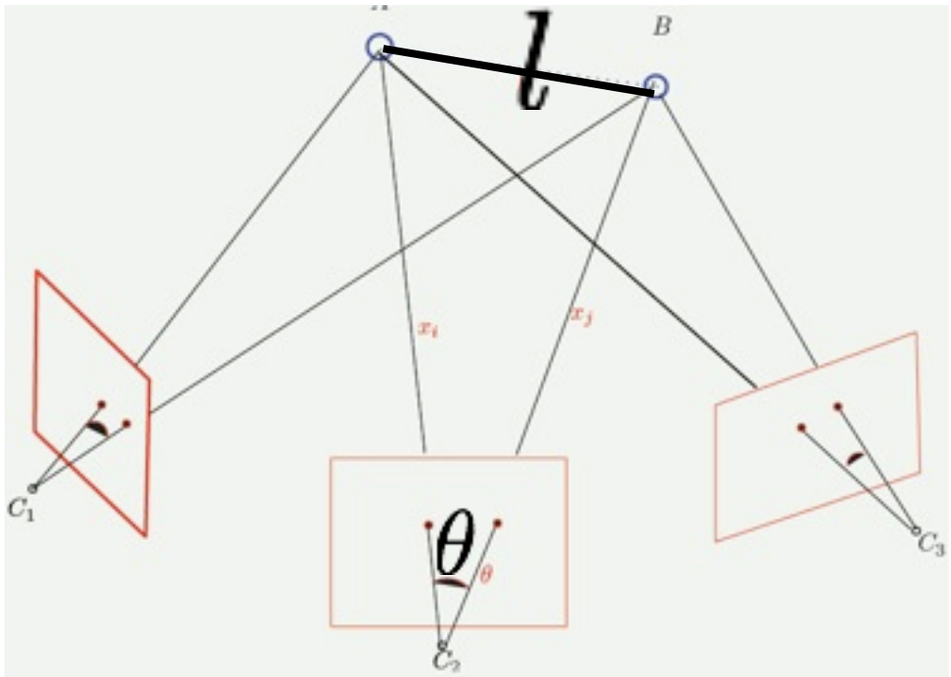




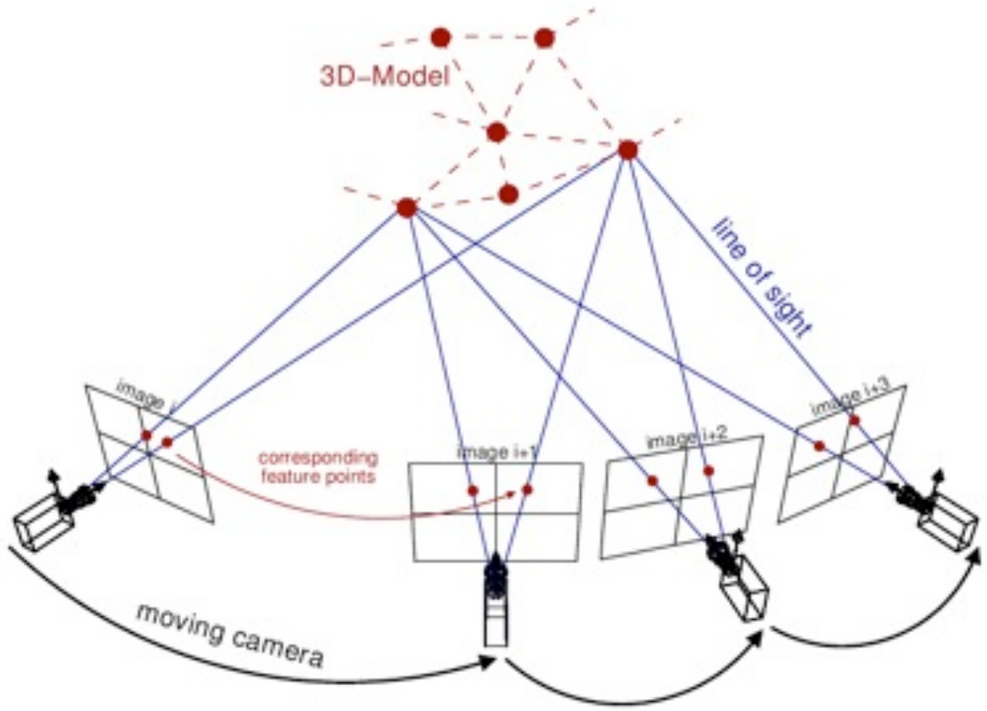
How to measure distance in 3-space ?

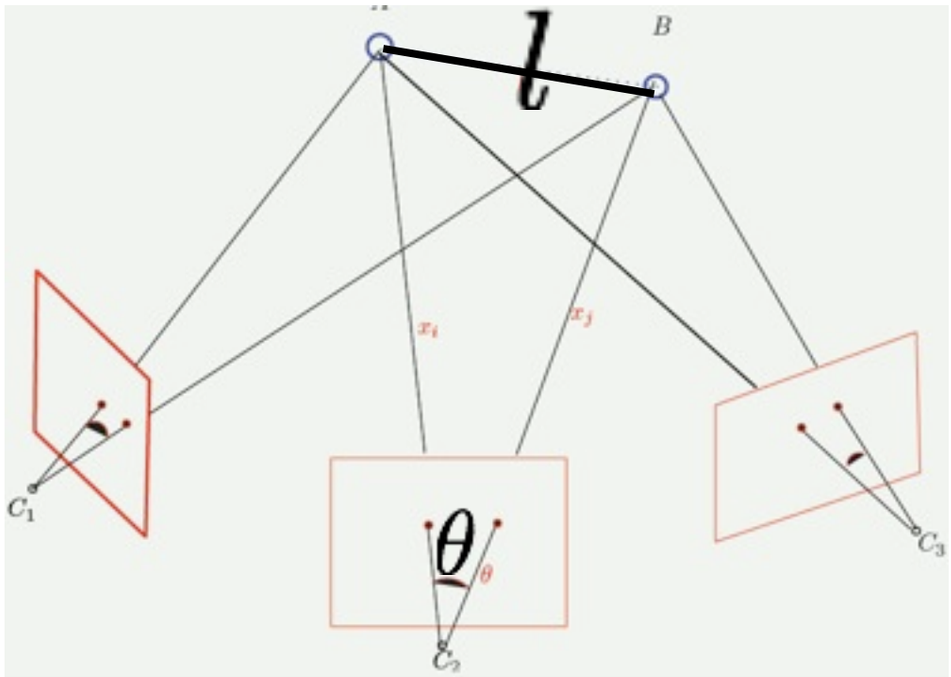
- We cannot directly measure distance in 3D with a camera.

- We however, can measure the **viewing angle,** from a calibrated camera.

l

θ





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- l is the unknown we want to estimate.

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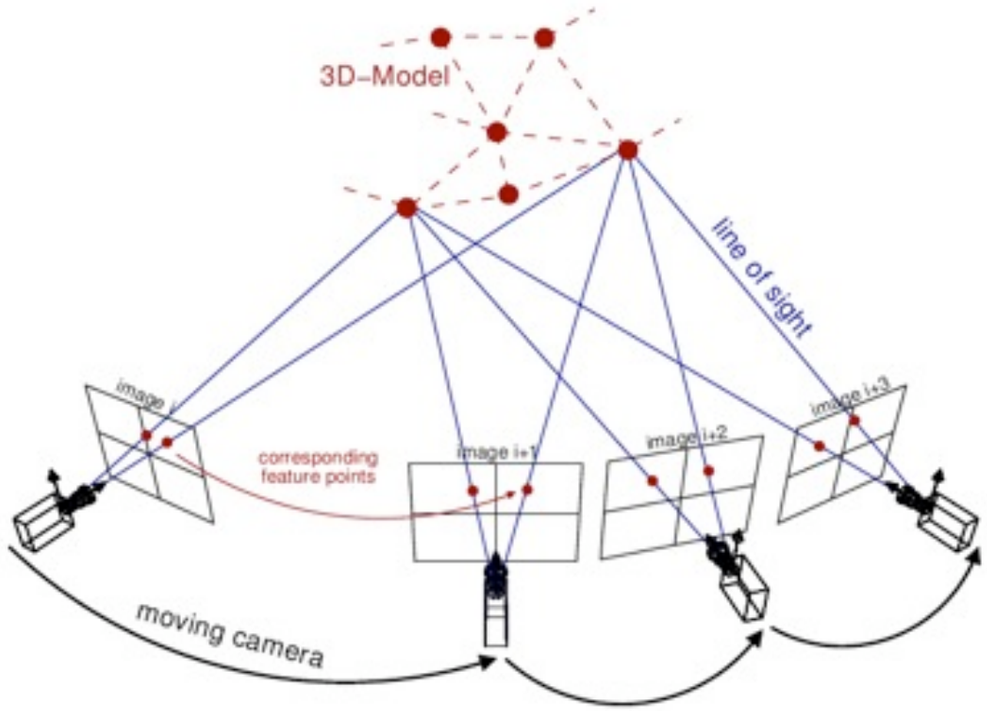

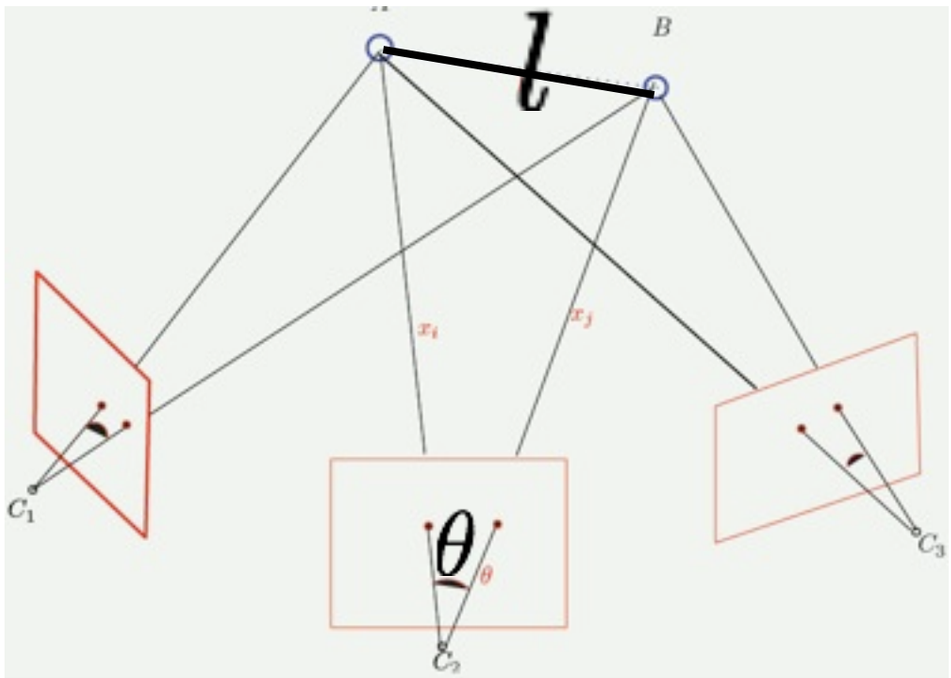

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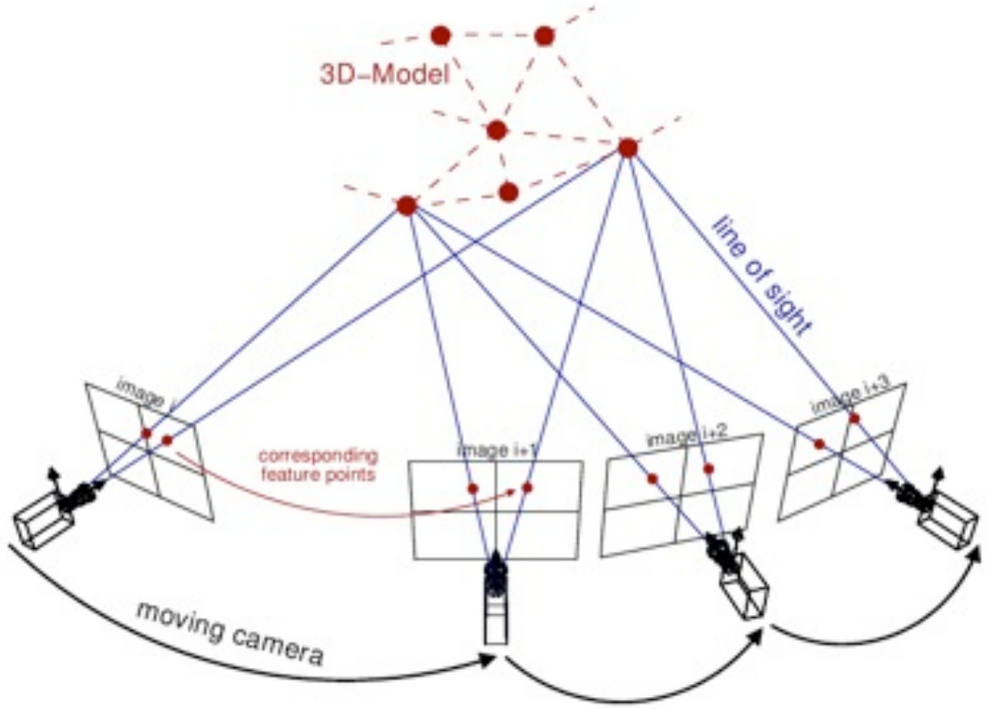

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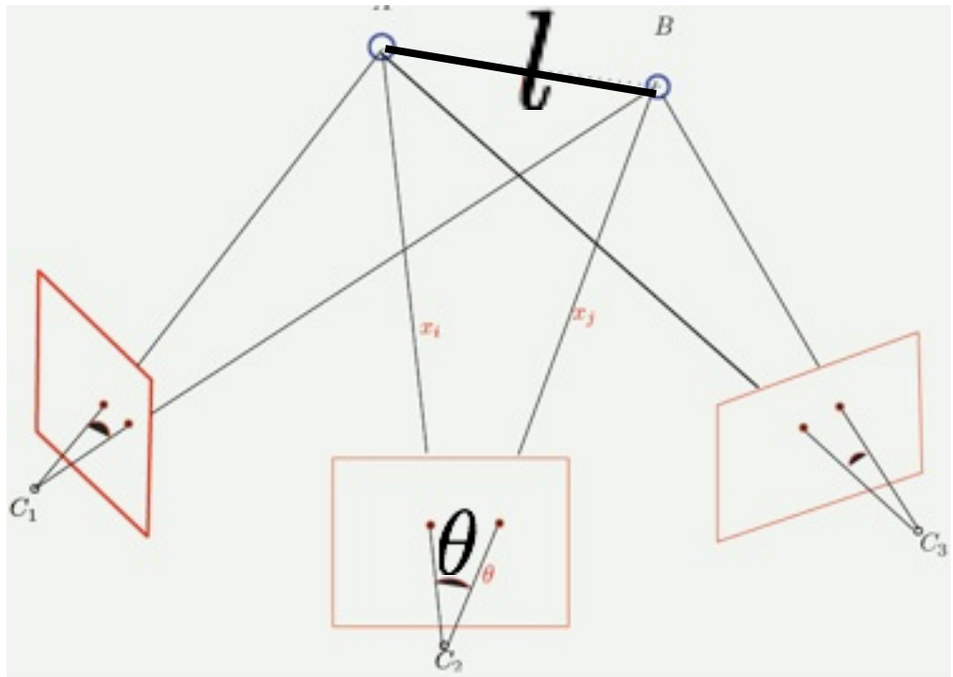
- l is the unknown we want to estimate.
- θ *is what we can measure.*



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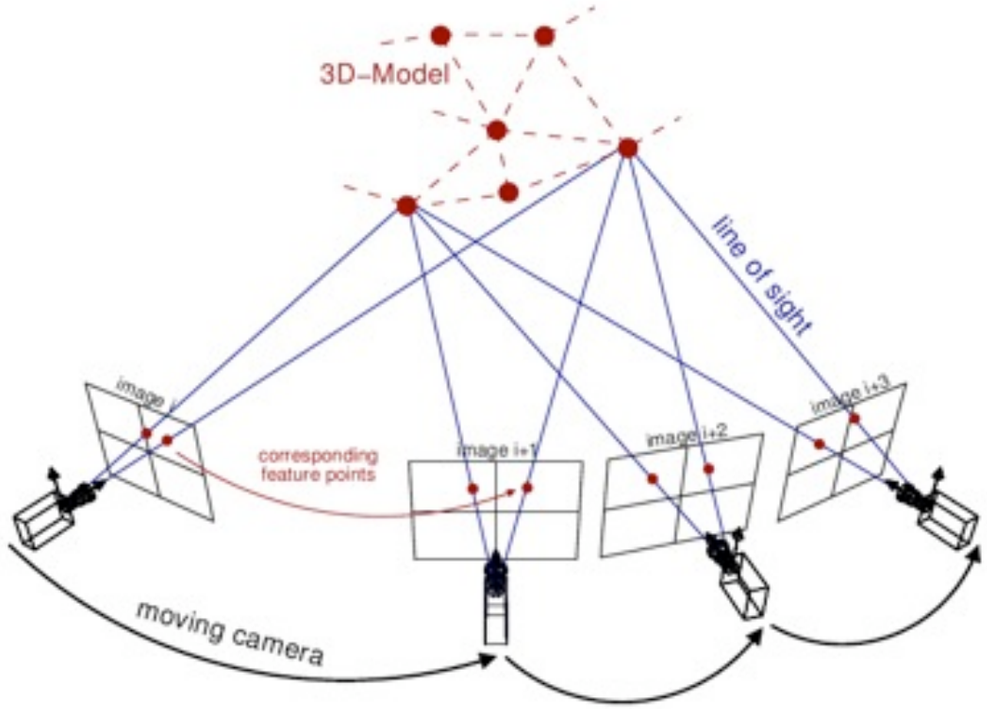
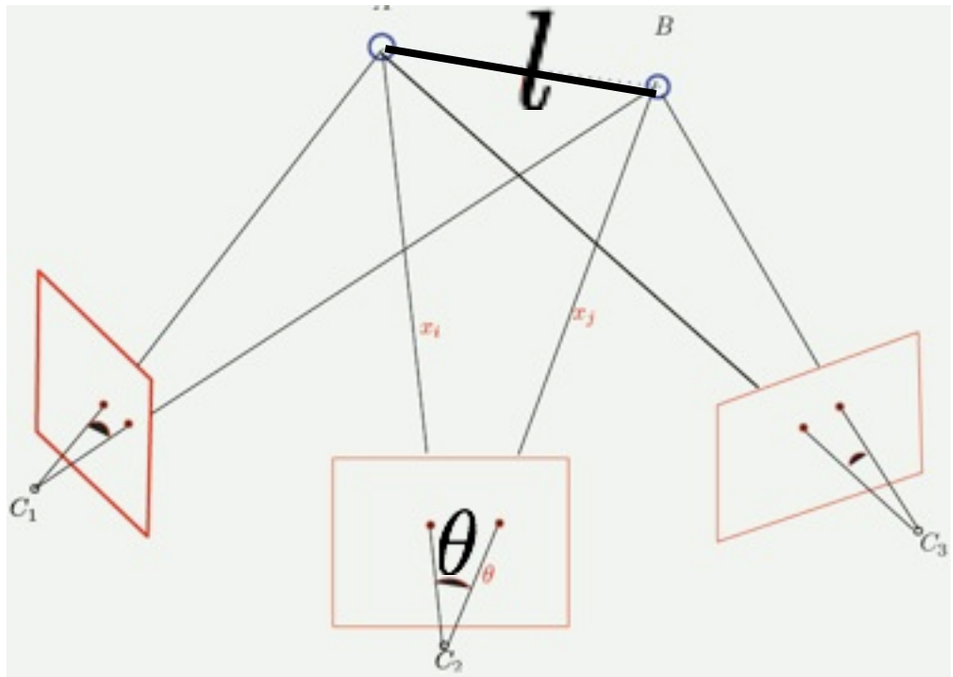
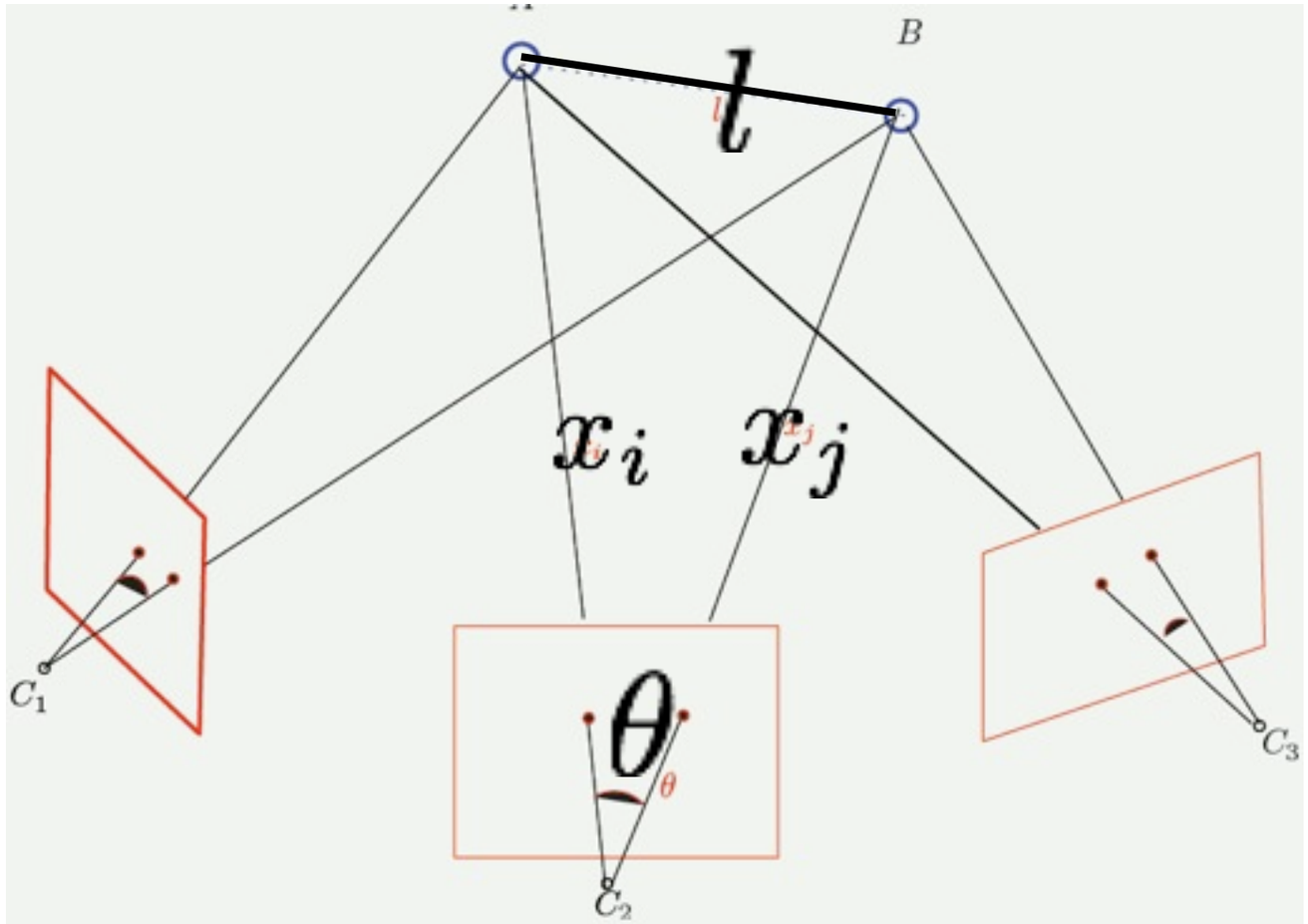


- We however, can measure the **viewing angle,** from a calibrated camera.
- l is the unknown we want to estimate.
- θ *is what we can measure.*
- *they are related by ...*



Basic Equation System



- **Viewing-triangle**
- the **law of cosine**
- polynomial equation
- system of equations

$$x_i^2 + x_j^2 - 2x_i x_j \cos\theta_{ij} = l_{ij}^2$$

$$\begin{cases} x_i^2 + x_j^2 - 2x_i x_j \cos\theta_{ij} = l_{ij}^2 \\ x_k^2 + x_l^2 - 2x_k x_l \cos\theta_{kl} = l_{kl}^2 \\ \cdots\cdots \end{cases}$$

Each viewing-triangle contributes to one basic equation.
Collecting sufficient number of basic equations, and solve this equation system, then we compute the unknown *l*s.

Roadmap

- So far,
  - ✓ we have presented the basic idea, and the basic equation system.
- Next, we study,
  - ✦ [practice] *how to solve the system efficiently ?*
  - ✦ [theory] *when the system will admit unique solution?*

How to solve the system of basic equations efficiently ?

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- a set of second order (quadratic) equations
- in the unknowns of {x's, *l*'s}.
- In theory, one could possibly solve it via, e.g.
  - Gröbner Basis
  - Continuation
  - Global optimization
- In practice, this is an extremely difficult problem, even for moderate size problems.

Strategies for solving it

$$\begin{cases} x_i^2 + x_j^2 - 2x_i x_j \cos\theta_{ij} = l_{ij}^2 \\ x_k^2 + x_l^2 - 2x_k x_l \cos\theta_{kl} = l_{kl}^2 \\ \cdots\cdots \end{cases}$$

- a set of quadratic non-convex equalities
- reduce/relax it ==>
- a set of **linear** equations, subject to **convex** constraints.

Matrix trace form

Define vector $\mathbf{x} = [x_1, x_2, ..., x_k]^T$, and unit basis vector : $\mathbf{e_i} = [0, 0, .., 1, .., 0, 0, 0]^T$, and

$$a_{ij} = (e_i + e_j)^T \mathbf{C} (e_i + e_j)$$

then each $x_i^2 + x_j^2 - 2x_i x_j \cos\theta = l_k^2$ becomes,

$$Tr(a_{ij}\mathbf{x}\mathbf{x}^T) = l_k^2$$

still quadratic.

Further relaxations

- quadratic to linear:

$$\mathbf{Y} = \mathbf{x}\mathbf{x}^T$$

not convex

- non-convex equality to convex inequality:

$$\mathbf{Y} \succeq \mathbf{x}\mathbf{x}^T$$

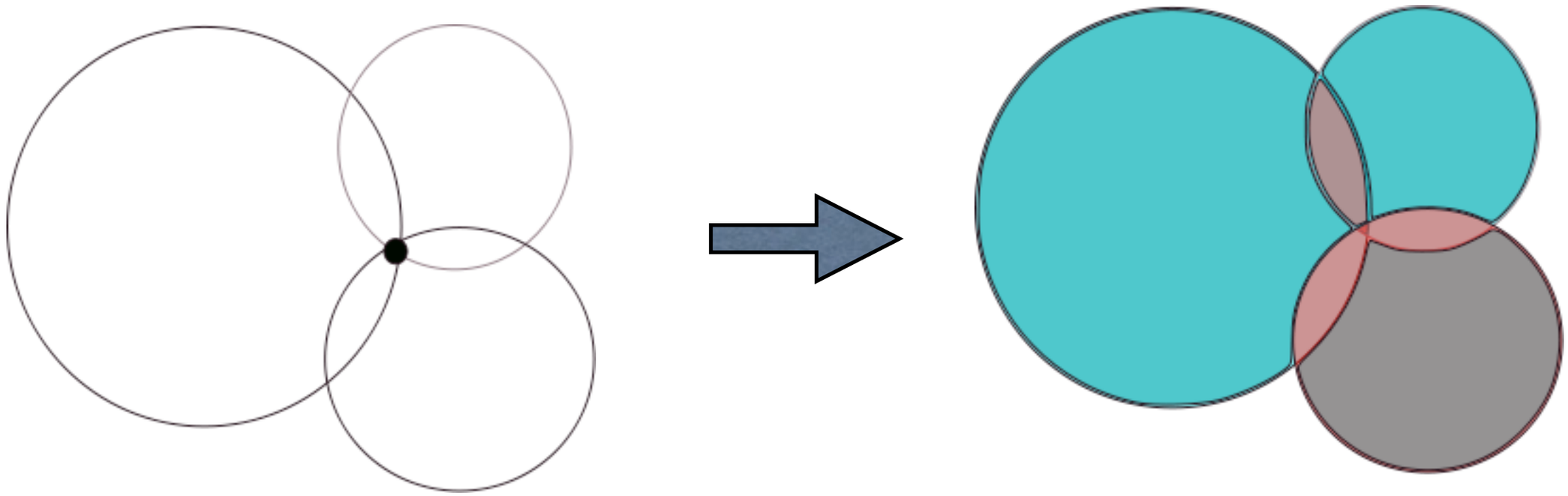
convex

- clearly Y is of rank(1), and we replace this to

$$\min Tr(\mathbf{Y})$$

- add regularization term to avoid all-zero solutions.

The geometry intuition of $\mathbf{Y} = \mathbf{x}\mathbf{x}^T$ → $\mathbf{Y} \succeq \mathbf{x}\mathbf{x}^T$



- Intersecting point of quadratic curves ==>
  Common intersection of convex conical regions.

The final formulation: finding edges

$$\min_{l} \left(\mathrm{Tr}(\mathbf{Y}) - \|\mathbf{x}\|_1\right) \quad \text{such that :}$$

$$\mathrm{Tr}\left(a_{ij}\mathbf{Y}\right) = l_k^2, \quad \forall \triangle_{(i,j,k)},$$

$$\mathbf{Y} \succeq \mathbf{x}\mathbf{x}^T, \quad \|l\|_2^2 = 1, \quad \mathbf{x} \geq \mathbf{0}, \quad l \geq \mathbf{0}.$$

- minimize linear objective function, subject to,
- linear and semi-definite constraints.

The final formulation: finding edges

$$\min_{l} \left(\mathrm{Tr}(\mathbf{Y}) - \|\mathbf{x}\|_1\right) \quad \text{such that :}$$

$$\mathrm{Tr}\left(a_{ij}\mathbf{Y}\right) = l_k^2, \quad \forall \triangle_{(i,j,k)},$$

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- minimize linear objective function, subject to,
- linear and semi-definite constraints.

SDP
(Semi- definite programming)

How to address the solution uniqueness question ?

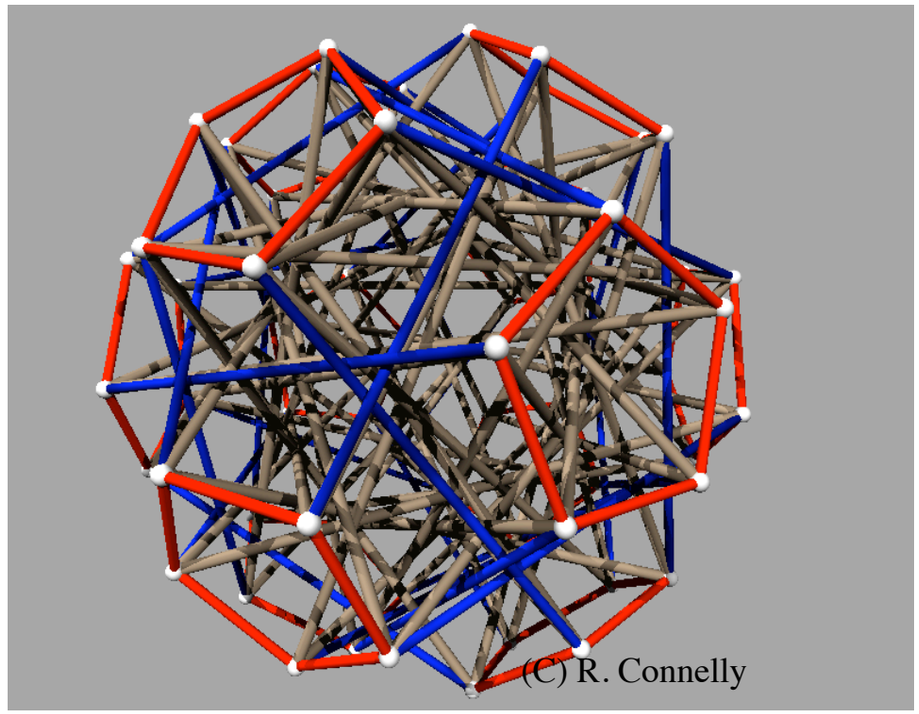
- Motivation:
  - which set of inter-point distances to choose?
  - how many edges (inter-point) distances are necessary, or sufficient ?
  - how to guarantee the established polynomial system (or the relaxed SDP) has unique solution ?

$$\begin{cases} x_i^2 + x_j^2 - 2x_i x_j \cos\theta_{ij} = l_{ij}^2 \\ x_k^2 + x_l^2 - 2x_k x_l \cos\theta_{kl} = l_{kl}^2 \\ \cdots\cdots \end{cases}$$

$$\min_{l} \left(\mathrm{Tr}(\mathbf{Y}) - \|\mathbf{x}\|_1\right) \quad \text{such that :}$$
$$\mathrm{Tr}\left(a_{ij}\mathbf{Y}\right) = l_k^2, \quad \forall \triangle_{(i,j,k)},$$
$$\mathbf{Y} \succeq \mathbf{x}\mathbf{x}^T, \quad \|l\|_2^2 = \mathbf{1}, \quad \mathbf{x} \geq \mathbf{0}, \quad l \geq \mathbf{0}.$$

Resort to **graph rigidity theory**

- To answer this question rigorously, we resort to the graph rigidity theory, in particular the property of "**global rigidity**".



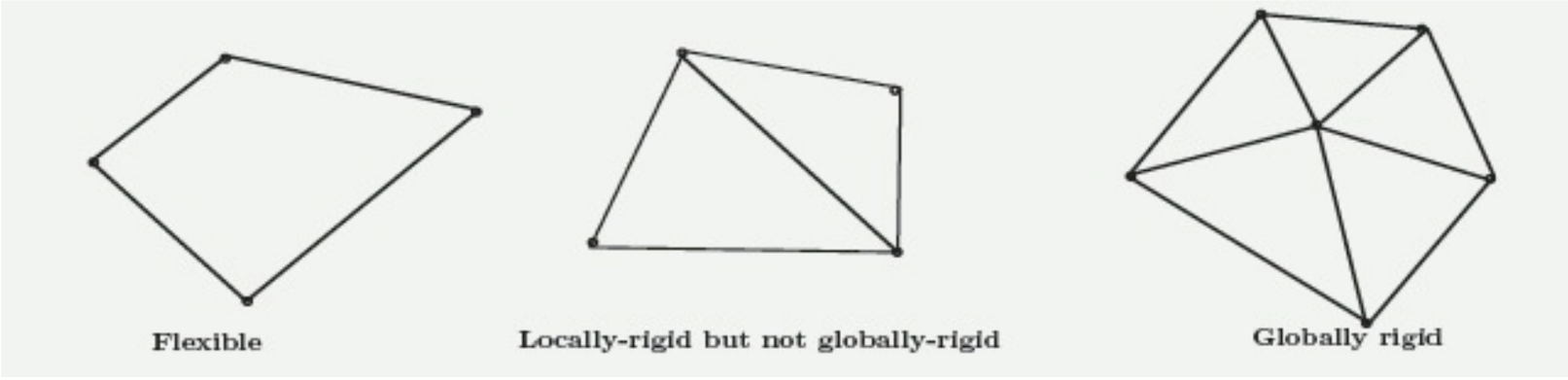
- Graph rigidity theory studies ...
  - ***when is a give framework rigid ?***

Graph Rigidity Theory: some definitions

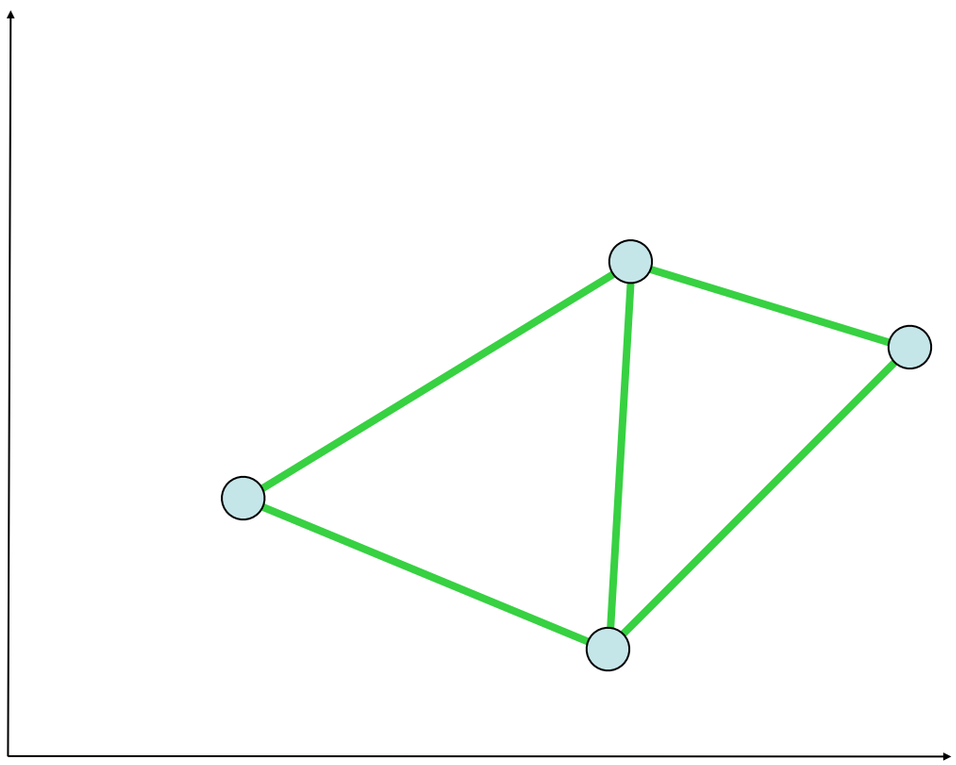
- A (bar-and-joint) **framework** is a graph G together with a finite set of vectors in d-space, denoted by $p = (p_1, p_2, \ldots, p_n)$, where each p_i corresponds to a node of G, and the edges of G correspond to fixed length bars connecting the nodes.

- Consider a straight-line embedding of a graph in Euclidean space. When any other embedding of the graph with the same edge lengths is **congruent** to the original, we say it is **globally rigid**.

- G(V,E) is called **redundantly rigid** if G(V, E−e) is rigid for all e, i.e. the removal of a single edge e from the rigid graph G does not destroy rigidity.

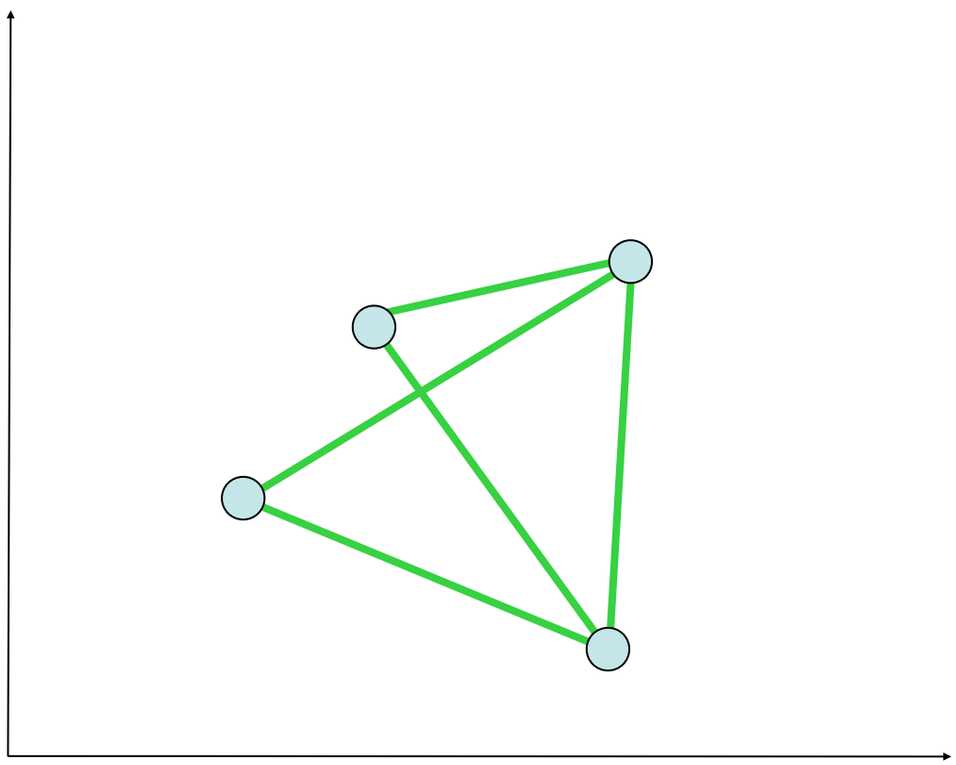


Eg: **Not** globally-rigid graph



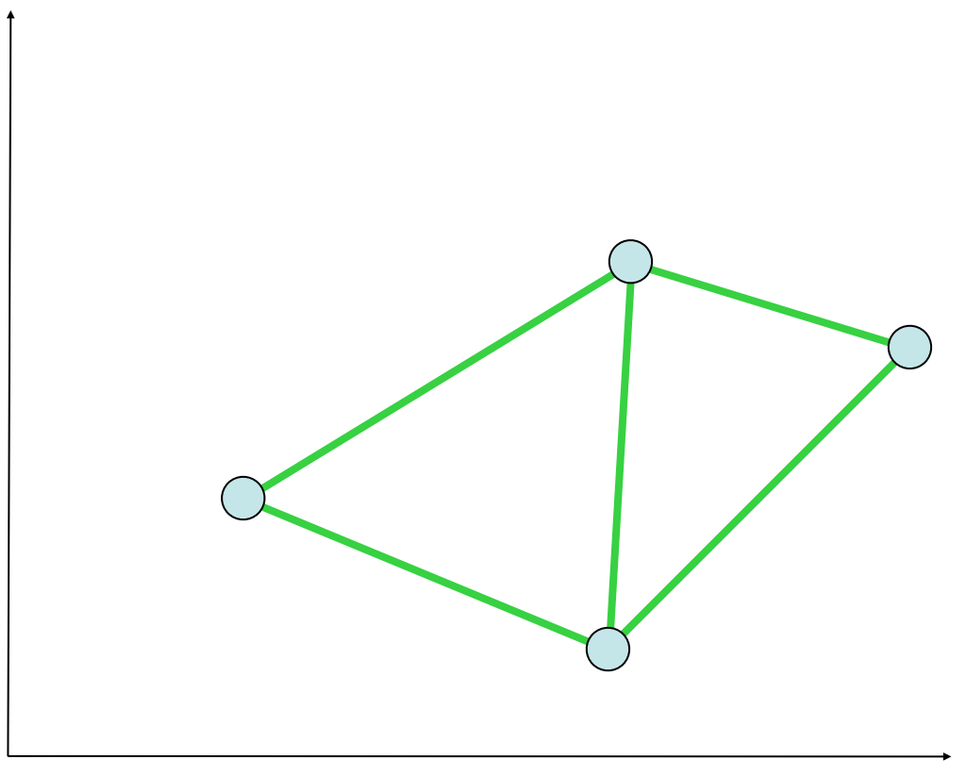
R^2

Eg: **Not** globally-rigid graph



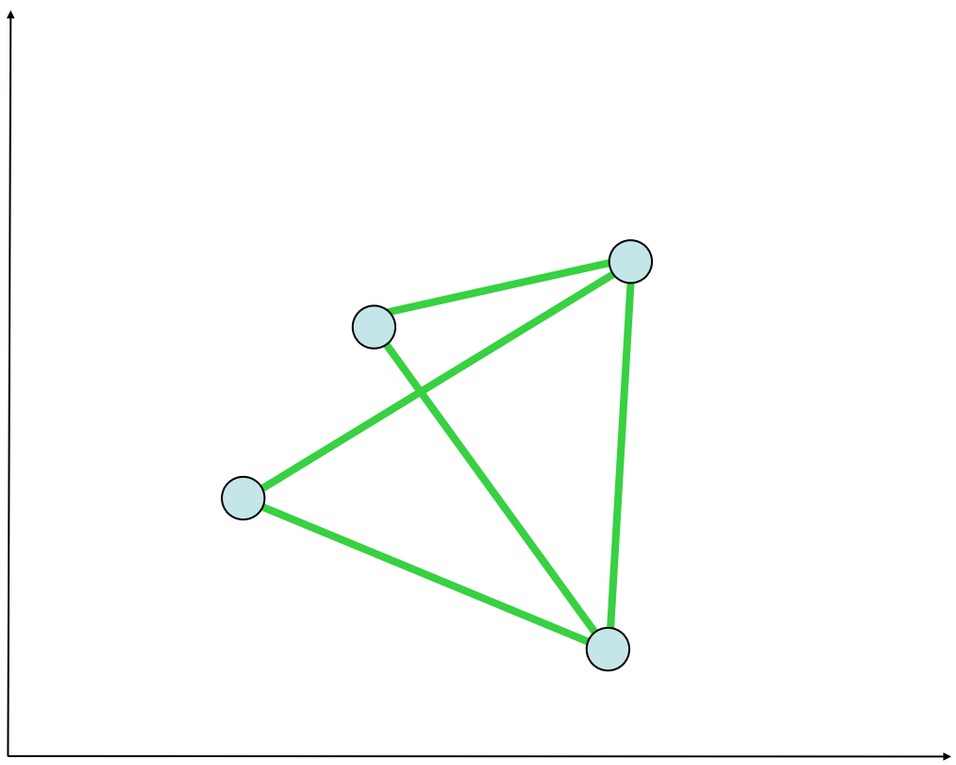
R^2

Eg: **Not** globally-rigid graph



R^2

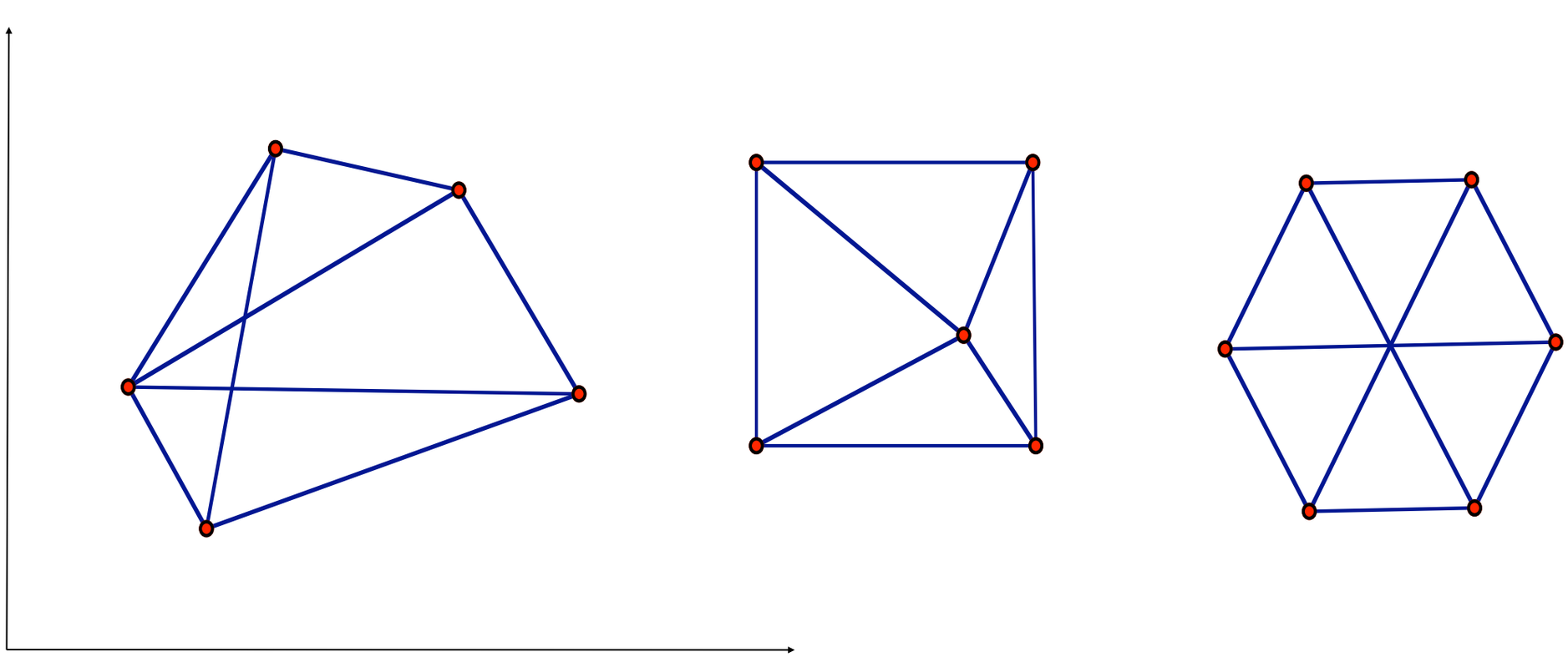
Eg: **Not** globally-rigid graph



R^2

Eg: Globally-**rigid** graph

- *p* is globally-rigid in R^d, if there is no second framework in R^d with the same edge lengths



R^2

globally rigid

In our 3D reconstruction context...

- Our aim is to construct a redundantly **globally-rigid**, spanning graph involving both 3D points and camera centers.

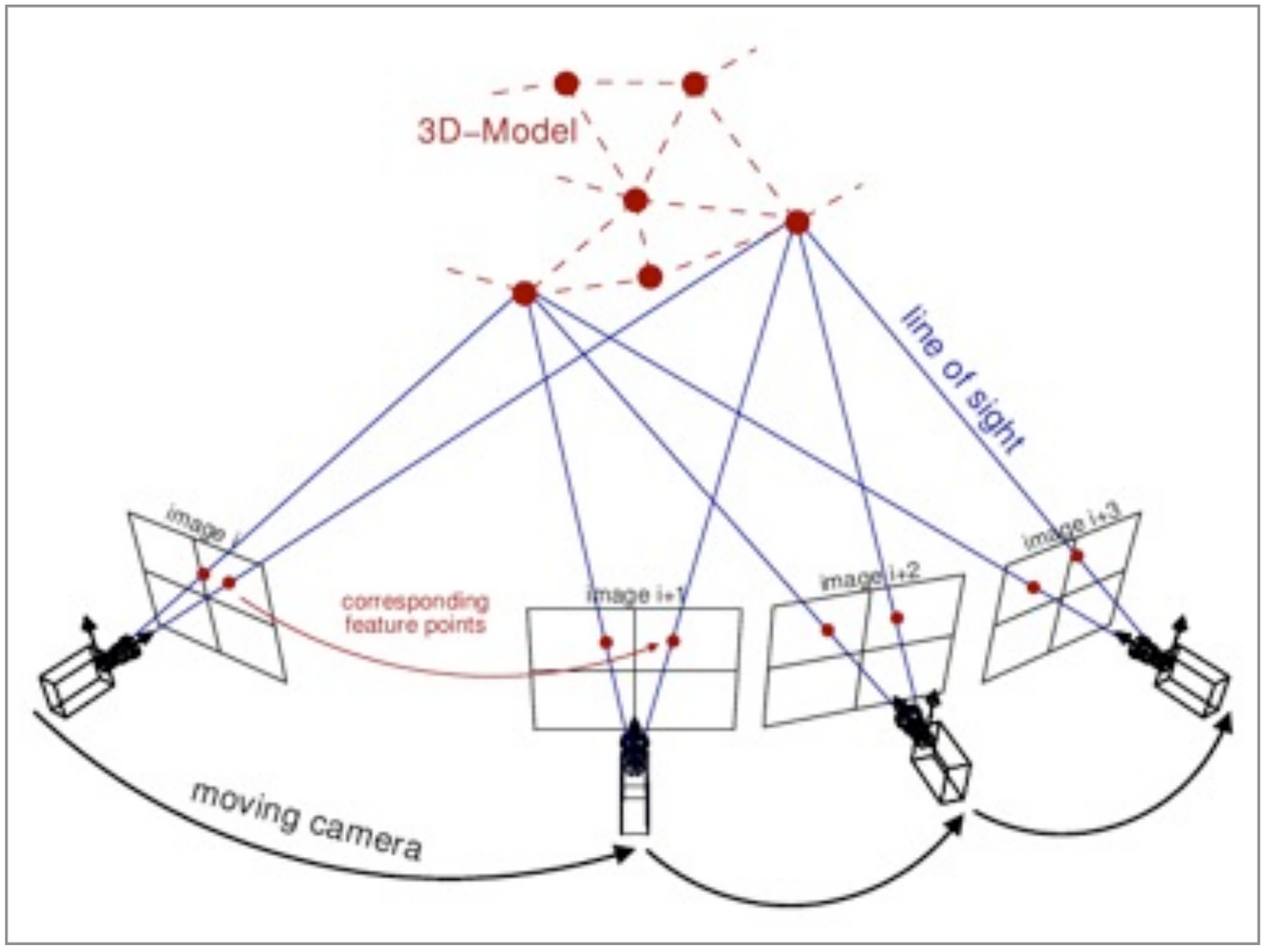


Figure taken from http://www.tnt.uni-hannover.de/print/project/motionestimation/index.php

How to tell when a given framework is globally-rigid?

- In general, to exactly test global rigidity is very hard! (NP-Complete).
- We instead propose using a randomized algorithm to test the generic global rigidity approximately.
- This randomized algorithm is due to Gortler-Healy-Thurston's recent result on characterizing generic global rigidity.

Gortler, Healy, Thurston, Characterizing generic global rigidity, American Journal of Mathematics, 2010.

  - [**Theorem**] A weighted graph is **globally-rigid** if and only if the rank of the associated **stress matrix** is N-d+1.

Algorithm Sketch

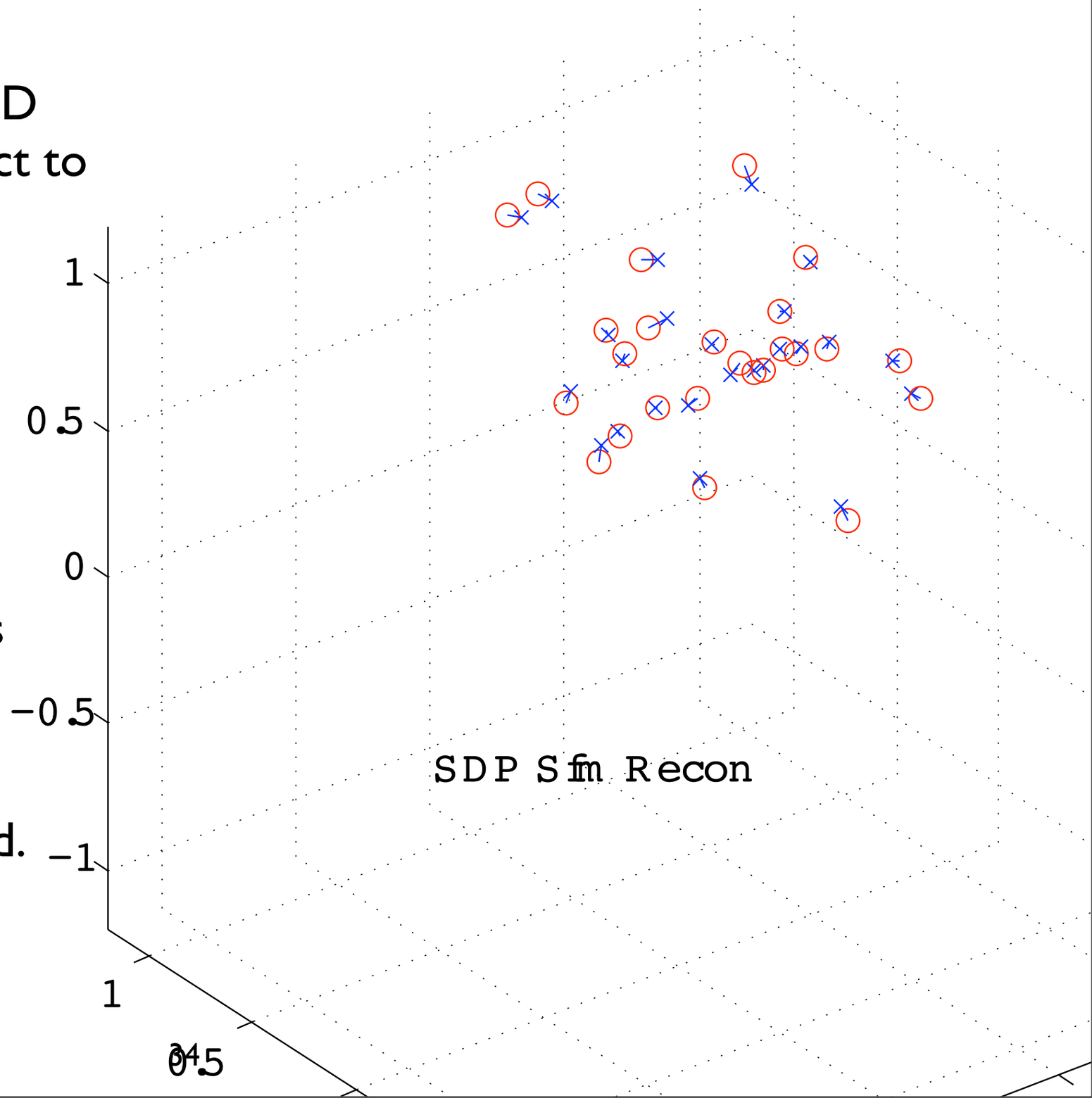
1. From all available point matches, incrementally construct a spanning graph by adding view-triangles, until it is redundantly globally rigid.
    - *To check graph-rigidity, run the randomized algorithm of Gortler et al.*
2. Run the edge-finding SDP to compute the edge distances of the graph.
3. Do a graph embedding to recover 3D coordinates of the point clouds.

Experimental validation

- Purposes:
    - Validate the effectiveness of the SDP relaxation for edge finding.
    - Verify the reconstructed 3D coordinates compared with ground truth.
    - Test performance versus #(points), #(views), noise-levels, and time-complexity, etc.
- Preliminary tests on toy-size problems.

Synthetic data (1)

- Generate 30 random 3D points in [0,1]^3, project to M images, add Gaussian noise with std 0.001.
- Construct a complete graph.
- Solve two SDPs.
- Reconstruction result is shown on the right.
- RMSE coordinate error much less than noise std.



Reconstruction error v/s noise level

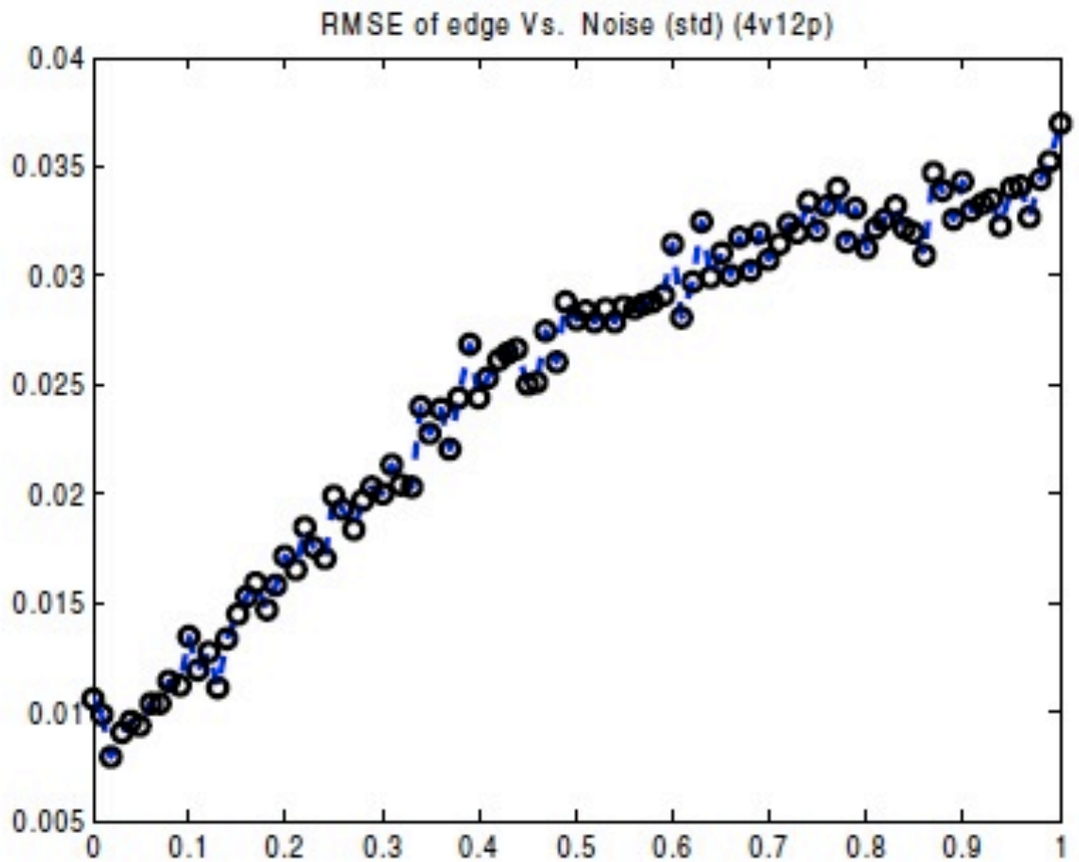


Figure 7. **Error vs. Noise level:** RMS error for estimated edge-length vs. Level of noise.

Worst case time-complexity (running time)

- Test on a modest 2.2G Intel Laptop.
- Using matlab (SEDUMI) as the SDP solver.
- Worst-case (i.e. complete graph): O(N^4), where N= #points.

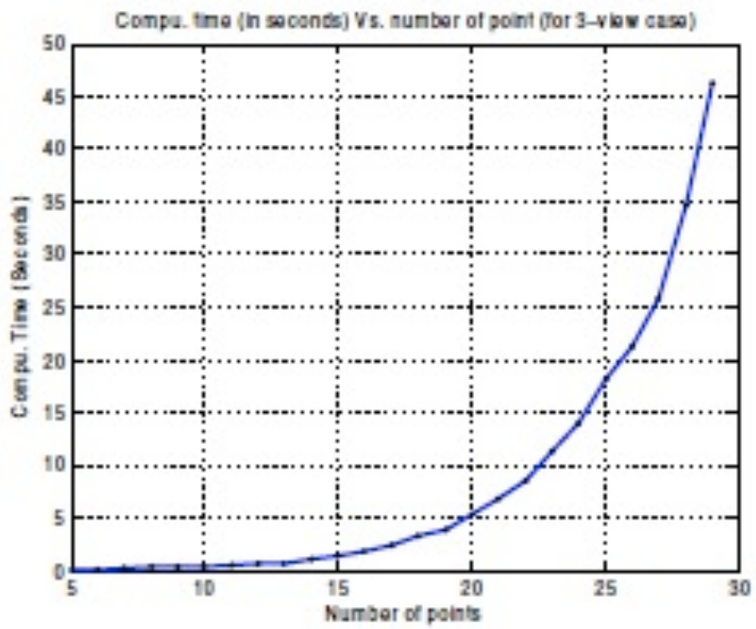


Figure 8. **Timing result:** Computational time vs. Number of points; (Note: this is for a 3-view *complete graph* case, *i.e.* worst-case complexity.)

Error v/s Number of points

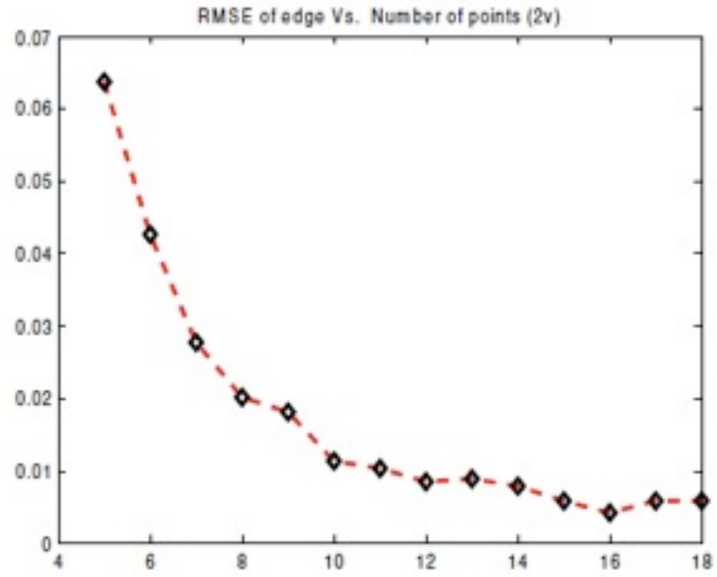


Figure 3. **Error vs. Number of points**: RMS error for the estimated edge lengths Versus number of points (2-view case).

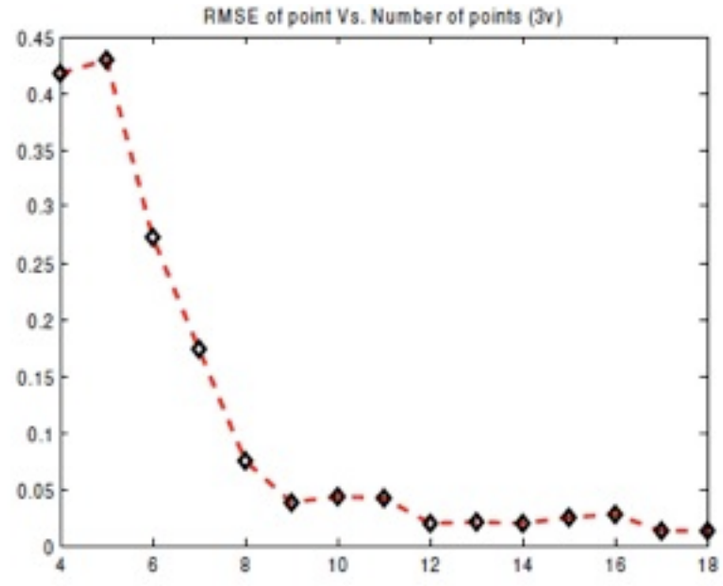


Figure 5. **Error in Coordinates vs. points**: RMS error for estimated 3D point coordinates Versus number of points (3-view case)

- Minimal cases: 2-view 5-point, 3-view 4-points.

Error v/s Number of views

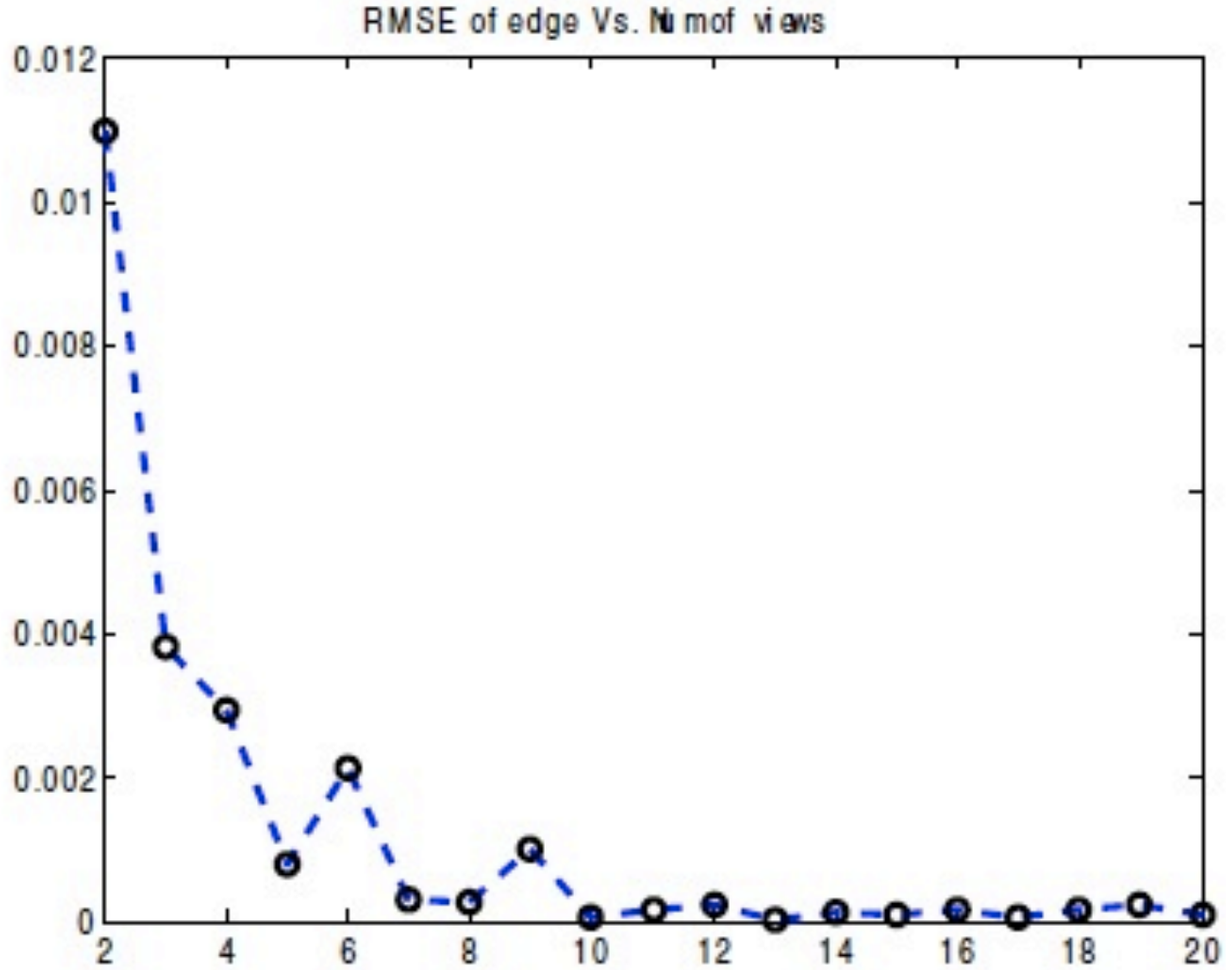


Figure 6. **Error vs. Number of Views:** RMS error for estimated edge-length vs. Number of views (15-point case).

Tests on real images (1): with sparse, non-complete graph

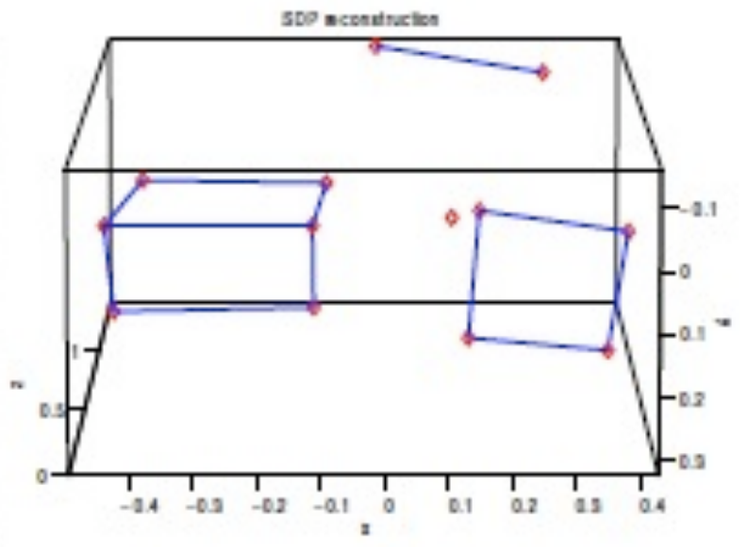


Figure 9. **Box-book**–a toy Sfm problem. Left: images; right: reconstructed 3D points.

Real Image and data from:Yi Ma et al. Springer 2003.

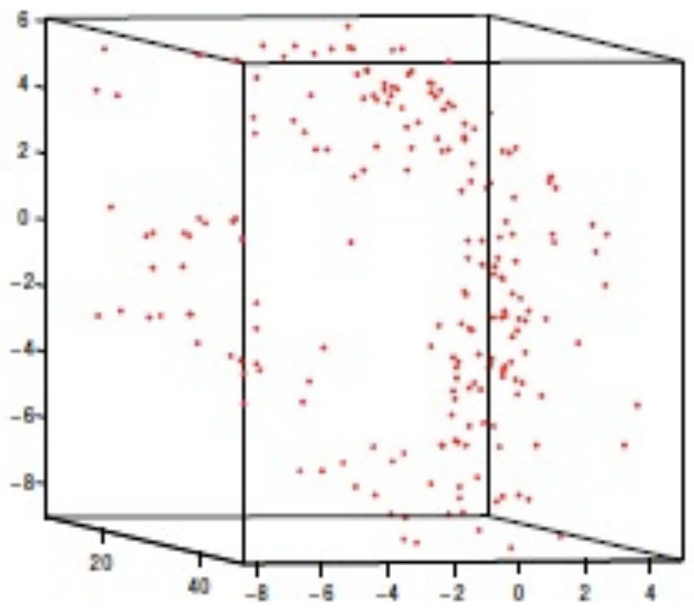
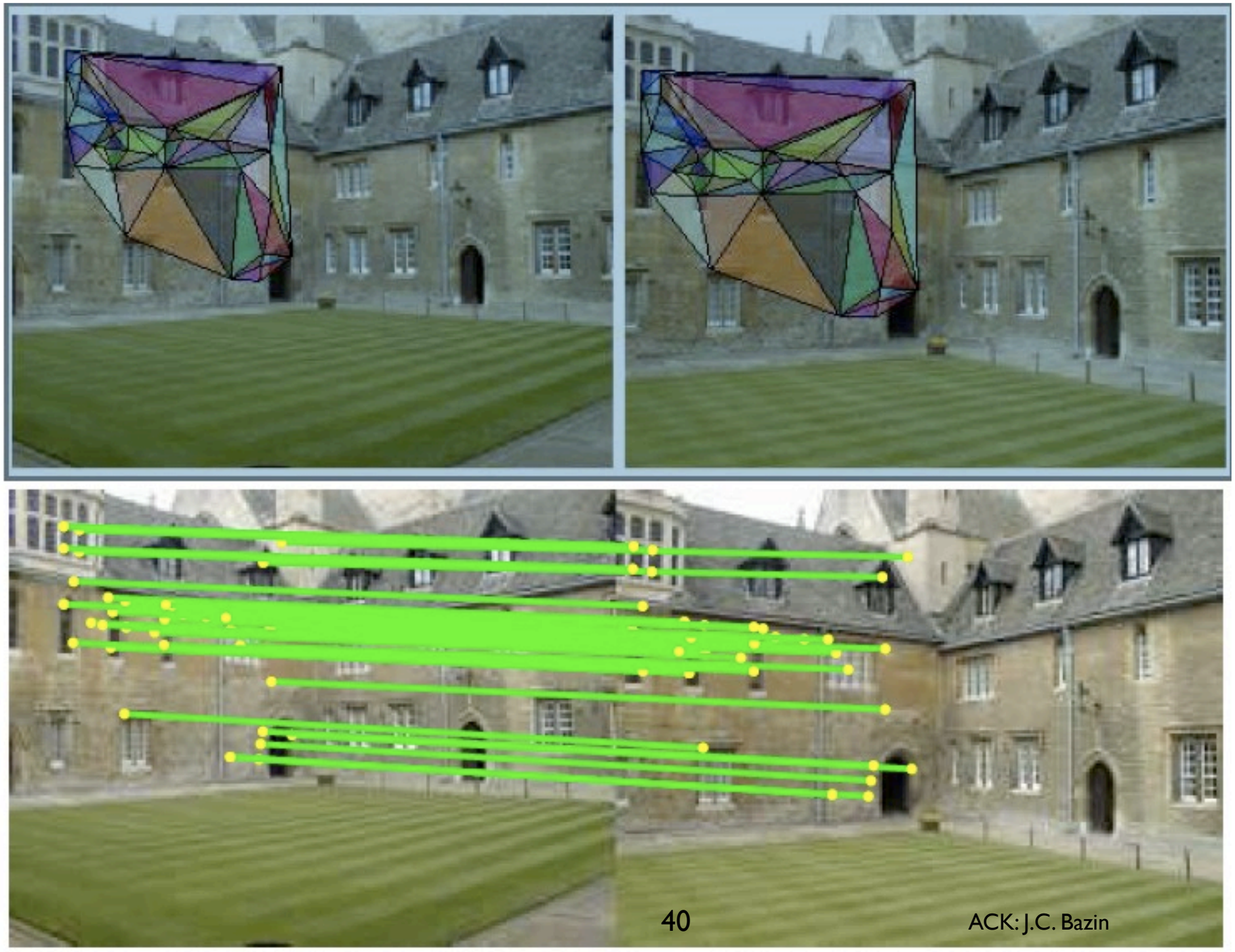


Figure 10. **Oxford Corridor**. Left: one of the input images; right: reconstructed 3D points.

Test on real image (2): with Delaunay mesh



ACK: J.C. Bazin

Conclusions

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- A novel, unconventional 3D reconstruction paradigm.
- No need to learn epipolar geometry, or fundamental matrix.
- Finding viewing-triangle is largely a local operation, making the method potentially suitable for sparse, large scale input.
- **Limitations**:
  - Rely on calibrated camera views.
  - Sensitive to outliers.
  - Computational complexity due to SDP.
  - Lack of quantitative comparisons with conventional methods (i.e. which one is better ? not yet tested).

Thank you !

Welcome to our ECCV 2010 (Greece) paper.