

Warped Geometry

Consequences

and

Signatures

Lisa Randall, Harvard University

Entering New Era

- Many challenges as LHC approaches
- Critical questions:
 - Are we optimizing existing searches?
 - Are we doing all the searches possible?
 - Models
 - Lower-Scale: Supersymmetry, Little Higgs
 - Higher-Scale: Strongly Interacting theories, Extra dimensions
- Focus today on extra dimensions
 - Bonus: Way to learn about quantum gravity and strongly interacting physics

OUTLINE OF TALKS

- Lecture 1: Introduction to RS1
 - Motivation
 - Original Model
 - KK signatures of original model
- Lecture 1-2 Bulk RS
 - KK Signatures when bulk gauge bosons, fermions
- Lecture 2
 - Black Holes from Extra Dimensions
- Lecture 3
 - Flavor from Warped Extra Dimensions

General->Expts->New Theories

Mini Extra Dimensions

Motivations for New Dimensions

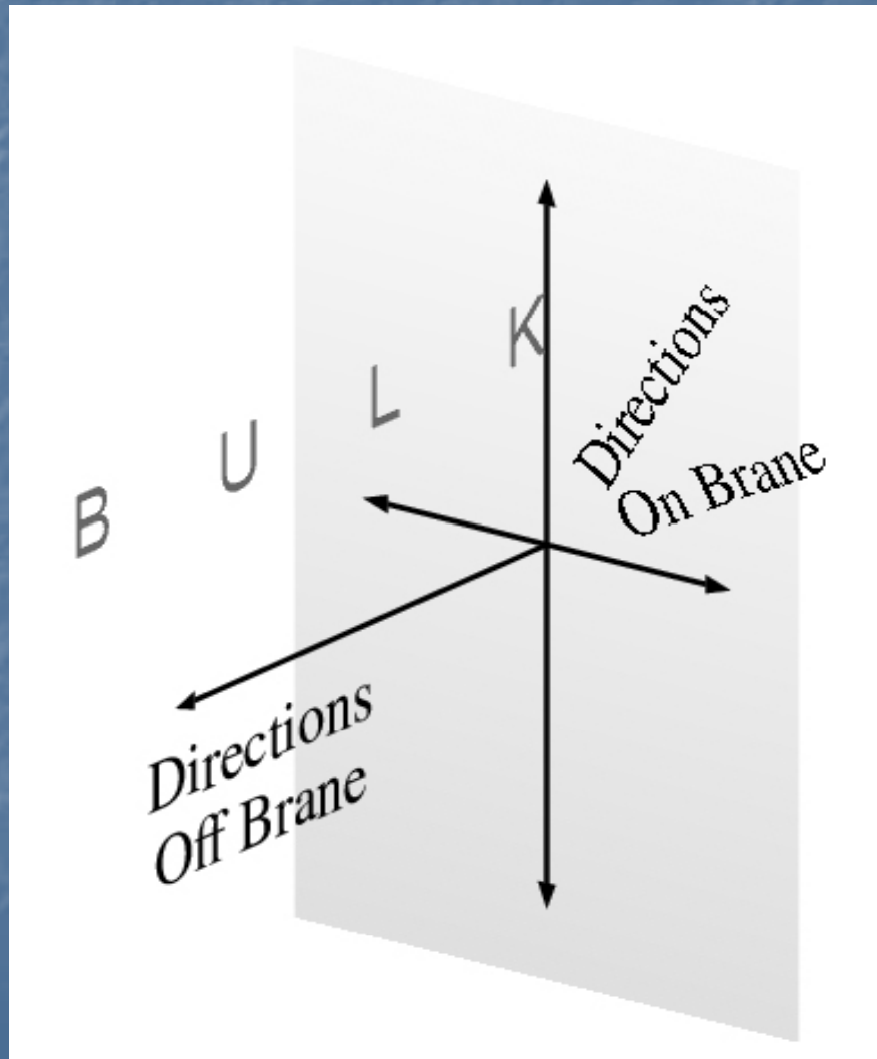
- String theory: extra dimensions essential
- Lower energies: Extra dimensions can illuminate connections among observable phenomena in new ways
- General Relativity works for any number of dimensions
- Plus:
- Bonus: Even shed light on purely four-dimensional physics

A New View of Weak Scale Physics

- Many new results in physics in last ten years
- Some of the most exciting involve extra dimensions of space
- Intriguing possibilities for our universe: both theoretical and experimental
- Warped geometry plays a central role in string theory today
- Warped geometry has particularly interesting signatures: resonances!
- Critical to explore theoretical possibilities and experimental consequences

Lesson? Higher energies?

What's New? Branes



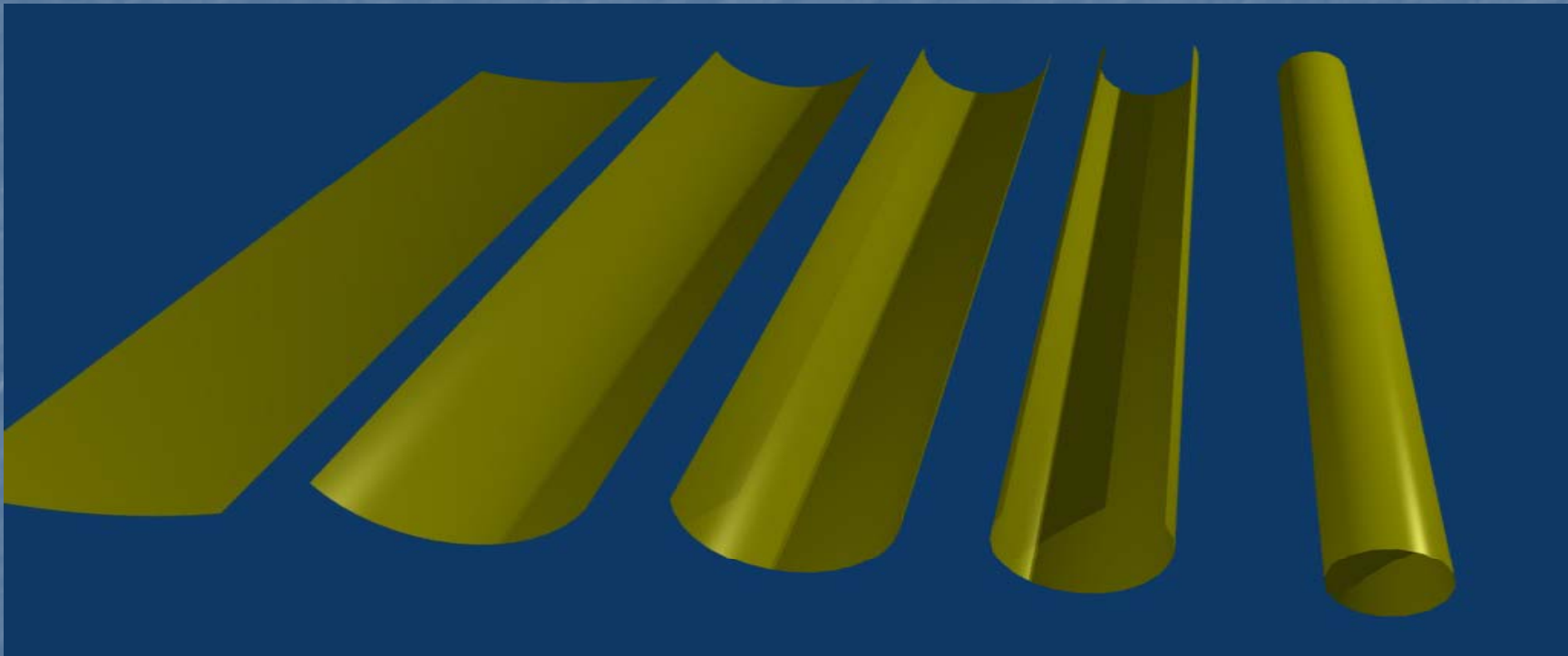
Branes:

Distinguish
dimensions along
a brane from
those
perpendicular to it

Important Phenomenological/Model-Building Implications

- 1) Don't need curled up dimensions to get small extra dimension
 - 1) Can be bounded by branes

Before Branes, Curled-up Dimension only Way to Hide Dimension



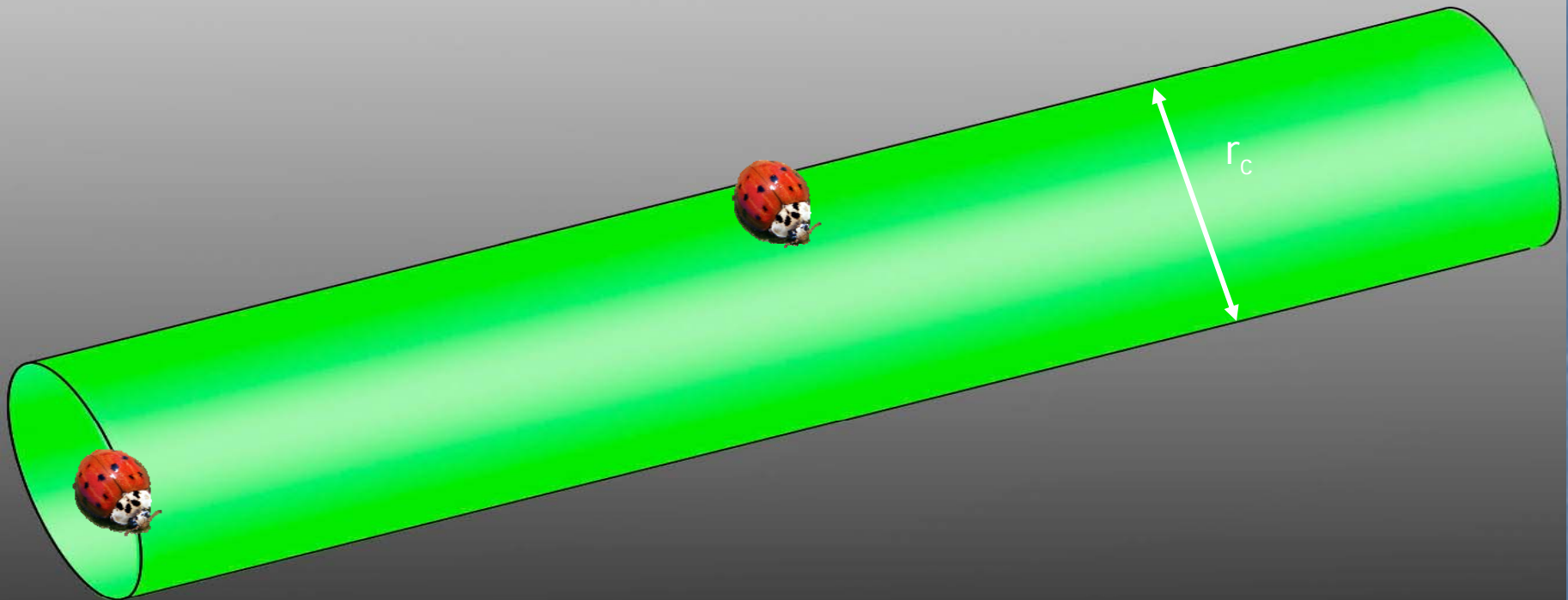
Idea is that something small is invisible

Here the small thing is a dimension of space

Entire universe is curled up!

Dimensions you observe depend
on size:

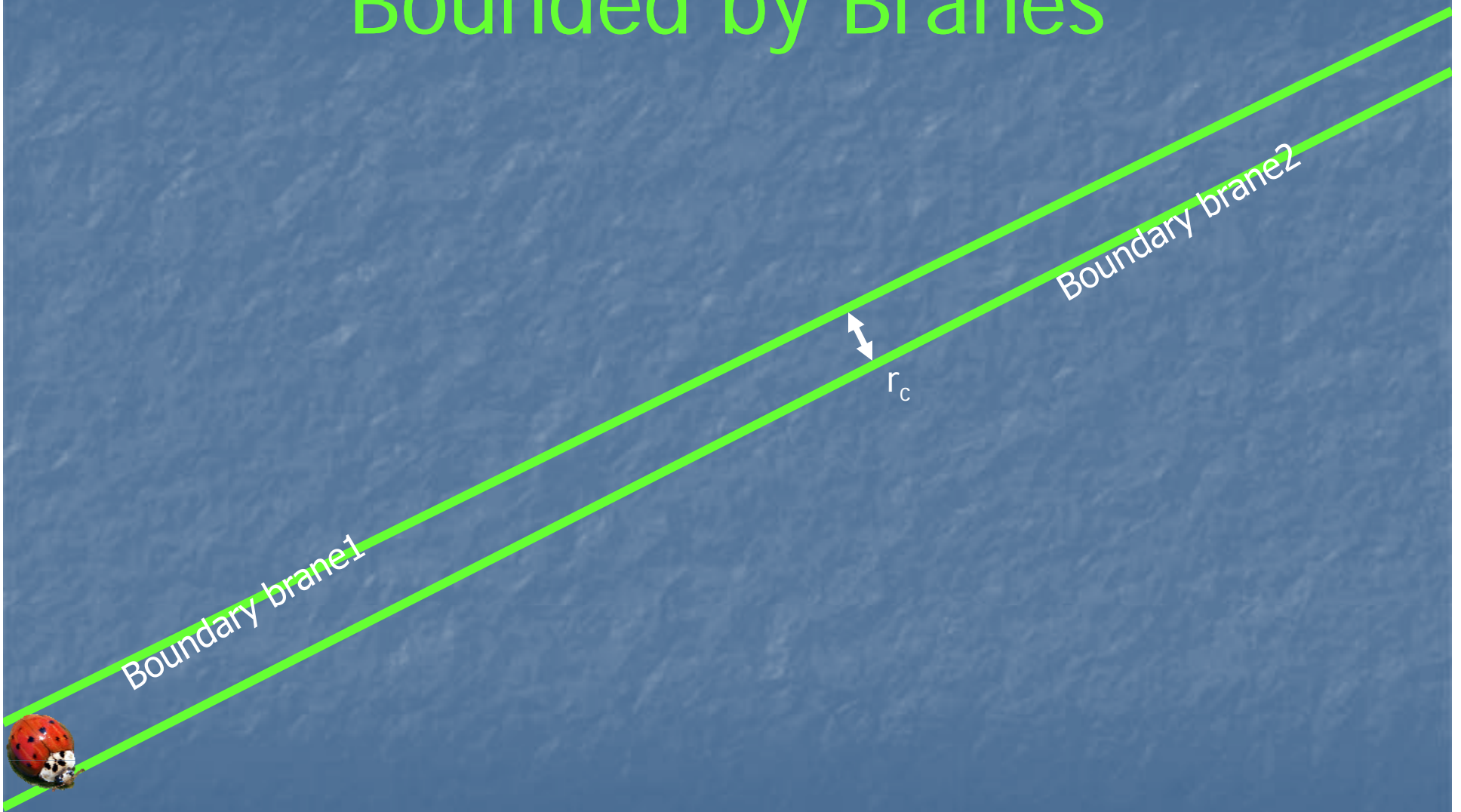
Two dimensions for a small bug



One dimension for a bigger (relative to rolled-up dimension) bug



Bounded by Branes



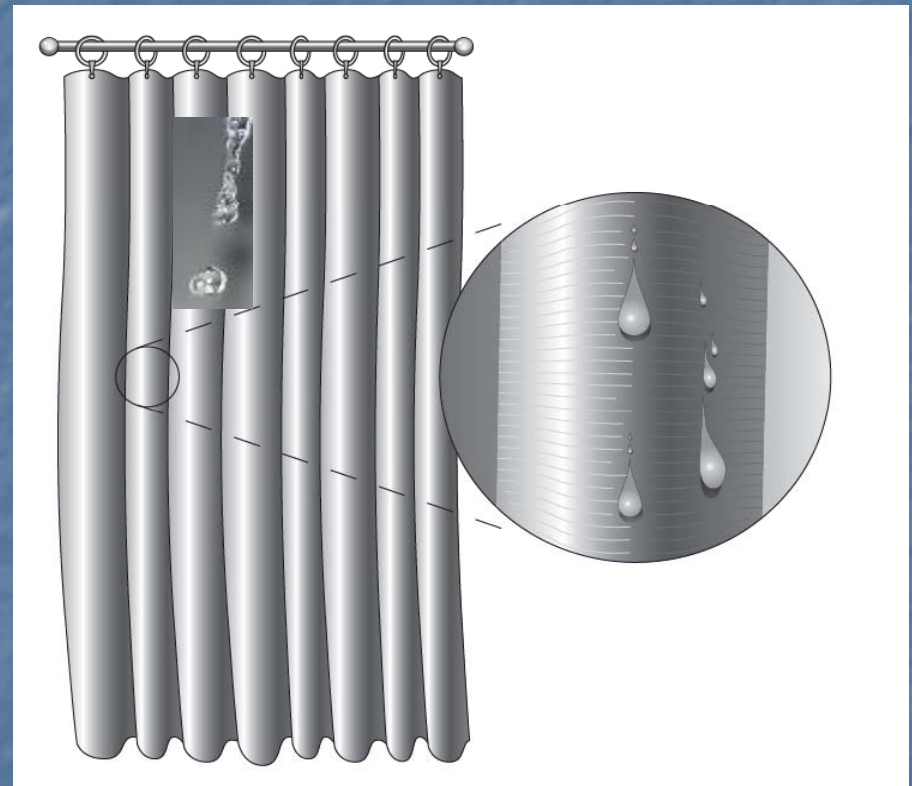
Important Phenomenological/Model-Building Implications

- 1) Don't need curled up dimensions to get small extra dimension
 - 1) Can be bounded by branes

- 2) Not all matter travels in bulk; BRANEWORLD
 - 1) Not necessarily KK mode of electron, photon
 - 2) Extra Dimensions can be much bigger than thought

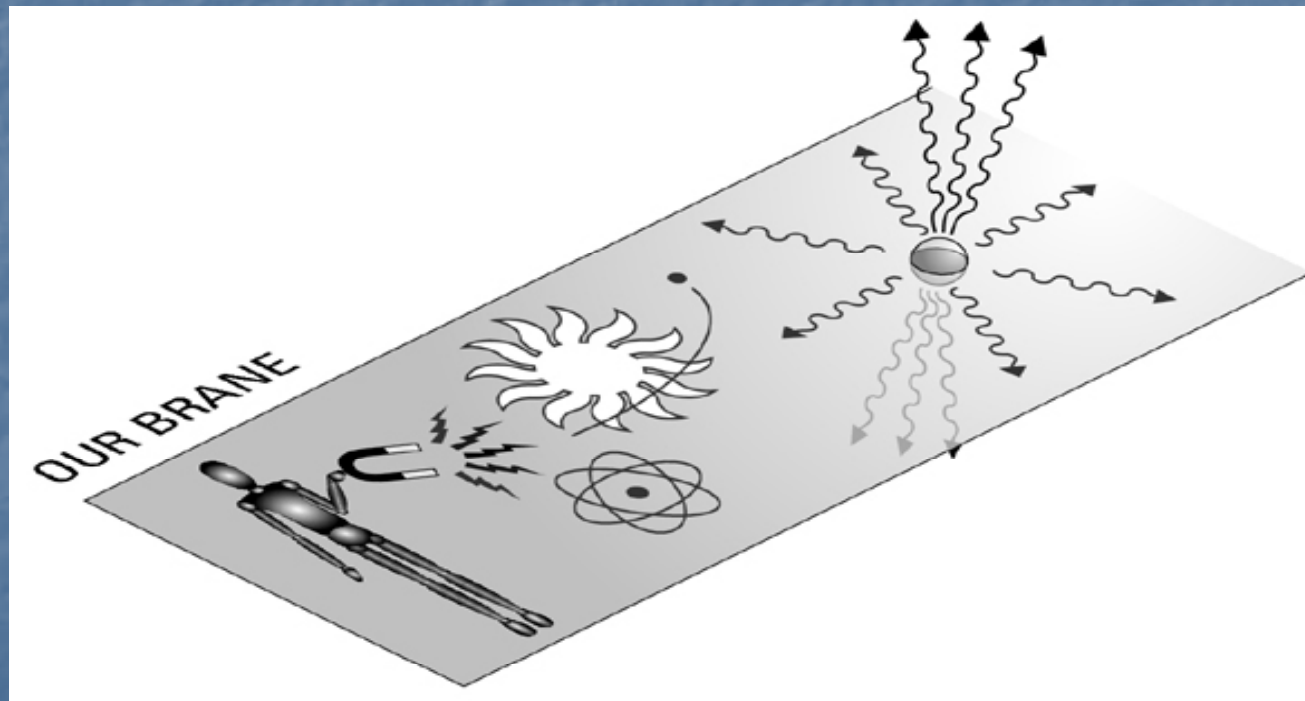
Particles can be stuck on a brane Like Water droplets on "shower brane"

- Lower-dimensional surface in higher-dimensional space
- Shower curtains trap things (water droplets) on lower-dimensional surfaces.
- Similarly, particles can be confined to a three-dimensional brane



Braneworld

Higher-dimensional world in which particles and matter are stuck on a brane



Drawn as
2D
Really 3D
And really
infinite

The boundary brane might house matter, particles; or could be brane inside bulk

Important Phenomenological/Model-Building Implications

- 1) Don't need curled up dimensions to get small extra dimension
 - 1) Can be bounded by branes

- 2) Not all matter travels in bulk; BRANEWORLD
 - 1) Not necessarily KK mode of electron, photon
 - 2) Extra Dimensions can be much bigger than thought

- 3) Warping permits even infinite dimension
 - 1) Gravity gets localized

With branes, we've found warped geometry solutions that provide:

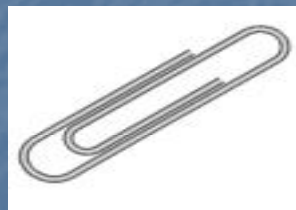
- New way to hide dimensions
- New concept of our place in the universe
- New way to explain weakness of gravity

Connection to Physics

Mapping and Hierarchy (RS1)

Hierarchy Problem:

- Why is gravity so weak compared to the other elementary forces?



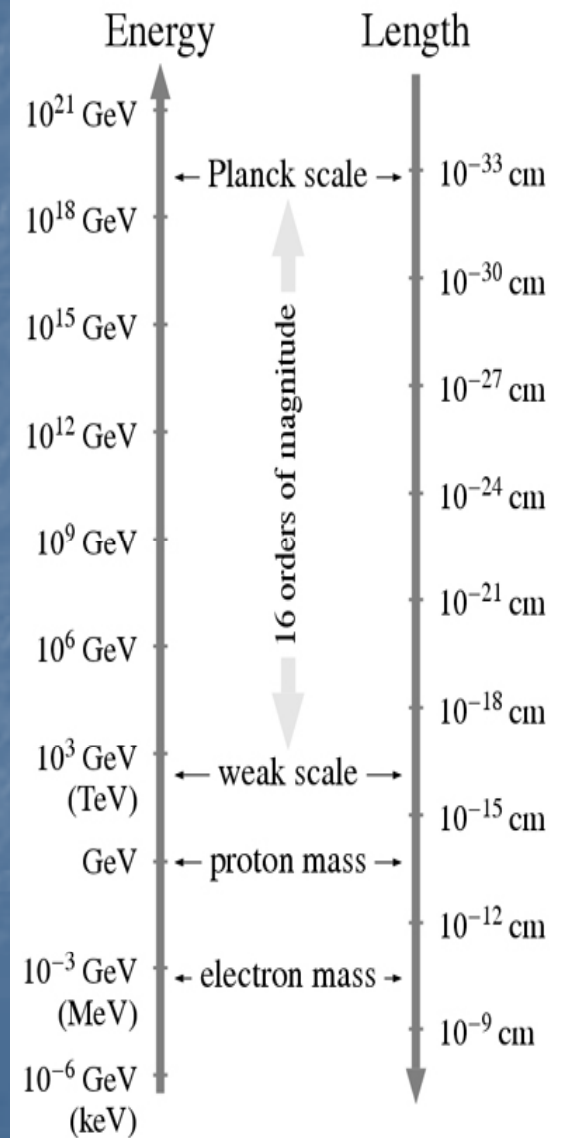
**Might not seem weak but
magnet can take on the entire
Earth-**

Hierarchy Problem

Need “fine-tuning” to get very different masses

Key issue in particle physics today

One that will be resolved at LHC



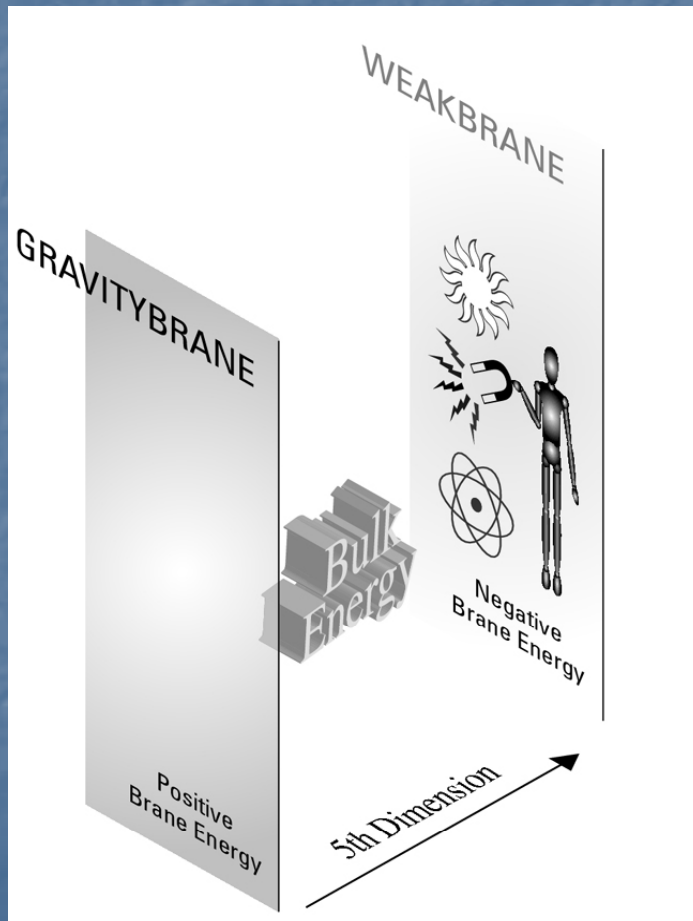
“Fine-tuning” is unlikely:



Barnett Newman: *Broken Obelisk*

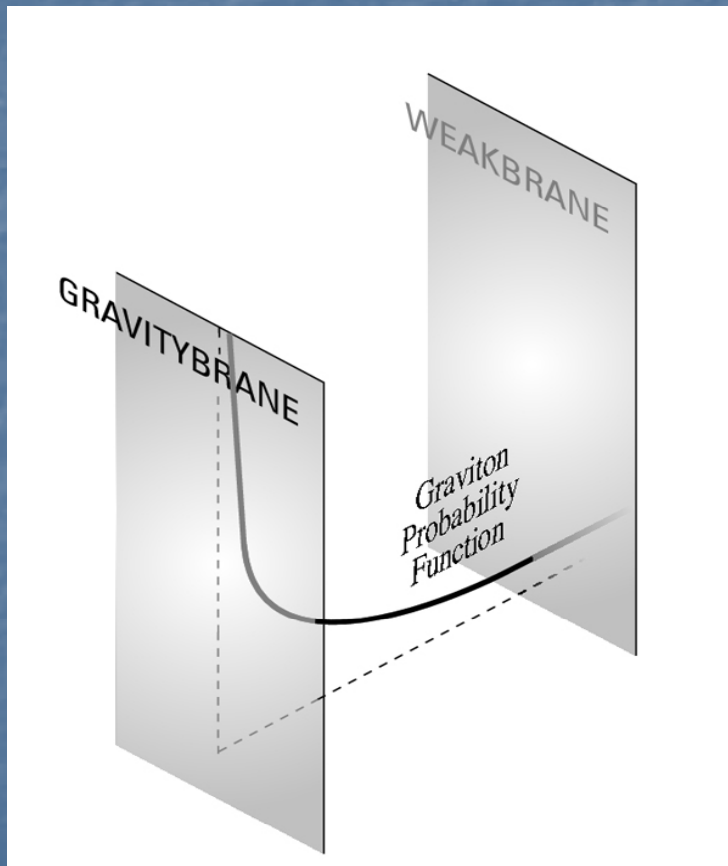
RS1 "Multiverse:" Warped Spacetime Geometry

With Raman
Sundrum



- Two branes
- Gravity will be concentrated on Gravitybrane
- But we live on a second brane:
- The Weakbrane/TeV Brane

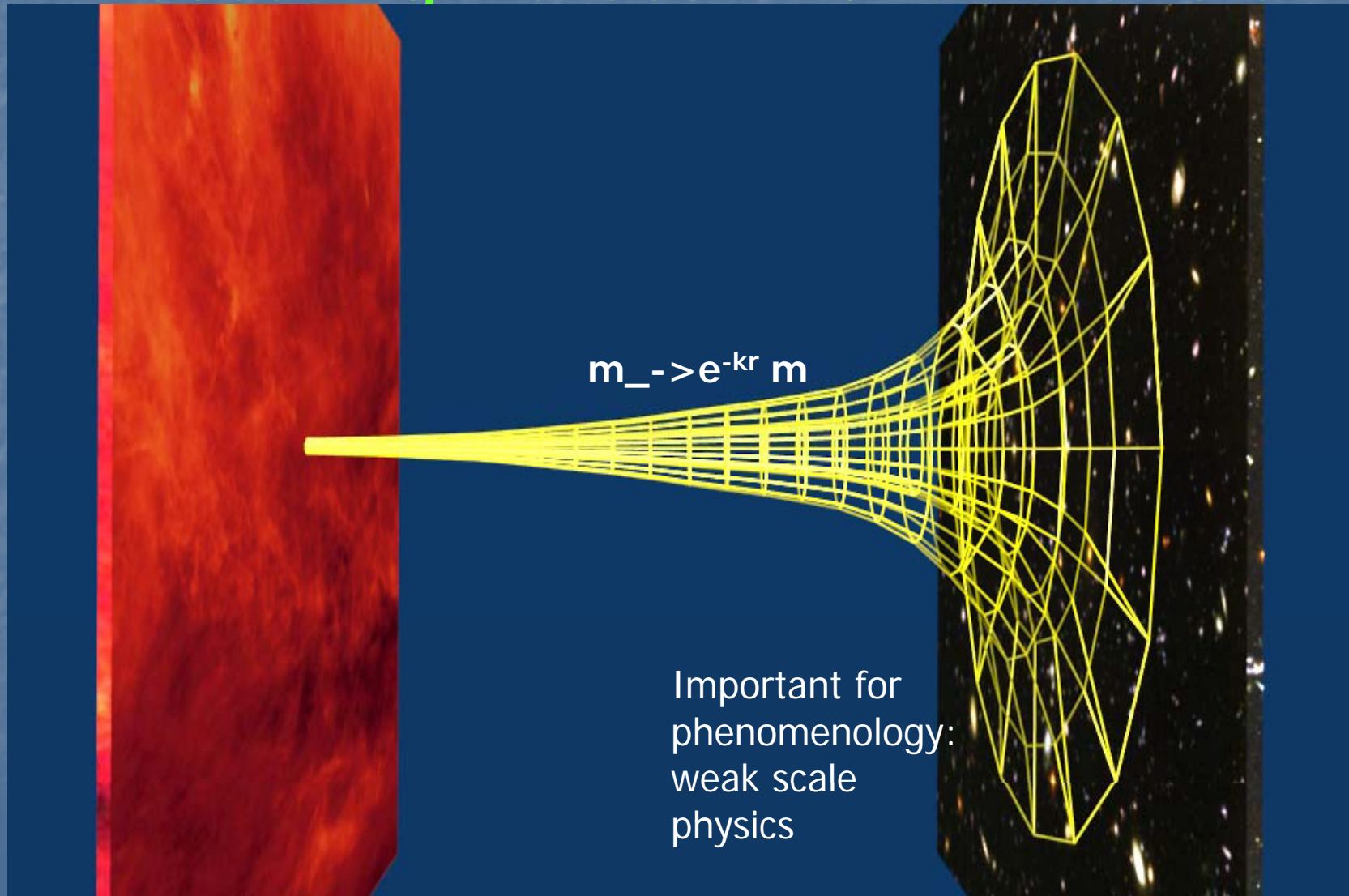
Natural for gravity to be weak!



- Small probability for graviton to be near the Weakbrane
- If we live anywhere but the Gravitybrane, gravity will seem weak
- Natural consequence of warped geometry

$$ds^2 = g_{MN} dx^M dx^N = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2,$$

Rescaling Across Fifth Dimension



Higgs mass rescaled

- Kinetic term has 2 powers of warp factor
 - Mass term has 4
 - Result is mass rescaled
 - In fact warp factor is like conformal factor
-
- So...if modes localized near TeV brane
 - Expect TeV mass
 - Turns out to be true for KK modes!

$$S_{vis} \supset \int d^4x \sqrt{-g_{vis}} \{g_{vis}^{\mu\nu} D_\mu H^\dagger D_\nu H - \lambda(|H|^2 - v_0^2)^2\}, \quad (17)$$

which contains one mass parameter v_0 . Substituting Eq. (3) into this action yields

$$S_{vis} \supset \int d^4x \sqrt{-\bar{g}} e^{-4kr_c\pi} \{\bar{g}^{\mu\nu} e^{2kr_c\pi} D_\mu H^\dagger D_\nu H - \lambda(|H|^2 - v_0^2)^2\}, \quad (18)$$

After wave-function renormalization, $H \rightarrow e^{kr_c\pi} H$, we obtain

$$S_{eff} \supset \int d^4x \sqrt{-\bar{g}} \{\bar{g}^{\mu\nu} D_\mu H^\dagger D_\nu H - \lambda(|H|^2 - e^{-2kr_c\pi} v_0^2)^2\}. \quad (19)$$

A remarkable thing has happened. We see that the physical mass scales are set by a symmetry-breaking scale,

$$v \equiv e^{-kr_c\pi} v_0. \quad (20)$$

This result is completely general: any mass parameter m_0 on the visible 3-brane in the fundamental higher-dimensional theory will correspond to a physical mass

$$m \equiv e^{-kr_c\pi} m_0 \quad (21)$$

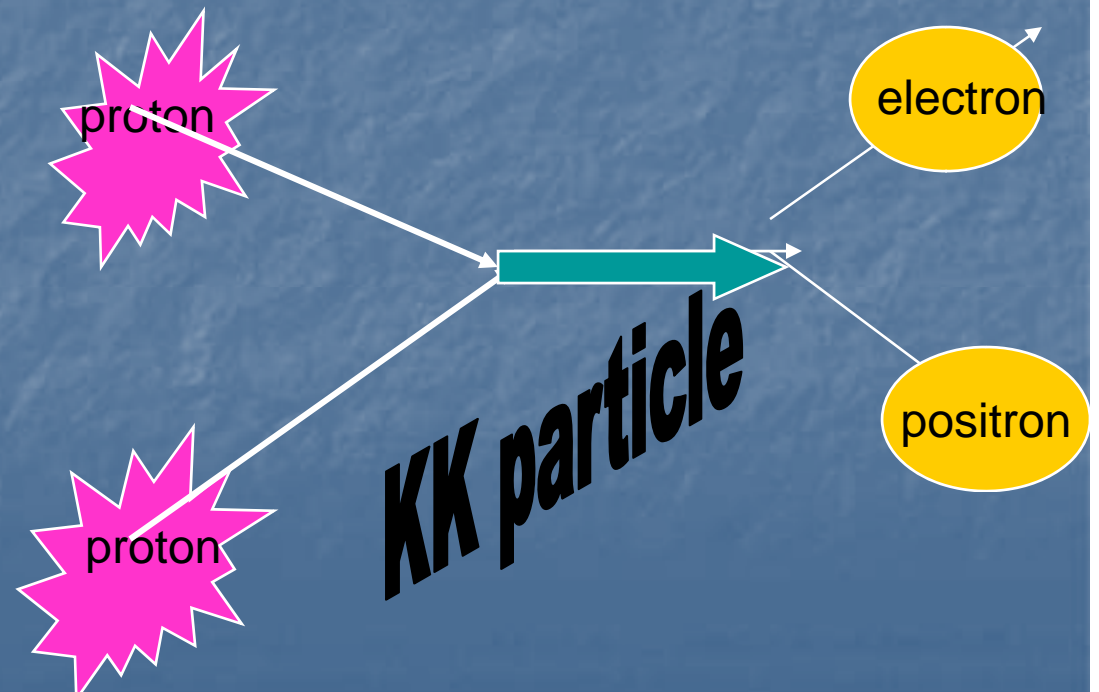
How to test?

- Search for new particles!
- ✦ Everything near weak brane should have TeV mass
- Kaluza-Klein (KK) particles
- Carry momentum in extra dimensions
- Looks like mass in 4 dimensions
- Connection to mass and weakness of gravity relative to other known forces tells us
- LHC will have the right energy to search for consequences of this theory

Experimental Signal: TeV-scale Graviton Resonances!

- Protons collide
- Produce a Kaluza-Klein particle
- Which Decays
- Definite mass spectrum and "spin"-2

LHC Collision



How to find KK masses and interactions

- Solve equations of motion
- Bessel function solutions
- Impose two boundary conditions at branes
- One gives correct linear combination
- Second gives quantization condition

Wavefunction Equation

We therefore perform a Kaluza-Klein reduction down to four-dimensions. To do this, we need to do a separation of variables; we write $h(x, y) = \psi(y)e^{ip \cdot x}$, where $p^2 = m^2$ and m^2 permits a solution to the linearized equation of motion for tensor fluctuations following from Eq. (3) expanded about Eq. (4):

$$\left[\frac{-m^2}{2} e^{2k|y|} - \frac{1}{2} \partial_y^2 - 2k\delta(y) + 2k^2 \right] \psi(y) = 0, \quad (8)$$

where the assumed orbifold boundary conditions tell us to consider only even functions of y . The effect of the regulator brane will be considered later; here it has been taken to infinity. The $\mu\nu$ indices are the same in all terms if we work in the gauge where $\partial^\mu h_{\mu\nu} = h^\mu_\mu = 0$, so they are omitted. Here m is the four-dimensional mass of the KK excitation.

Change variables

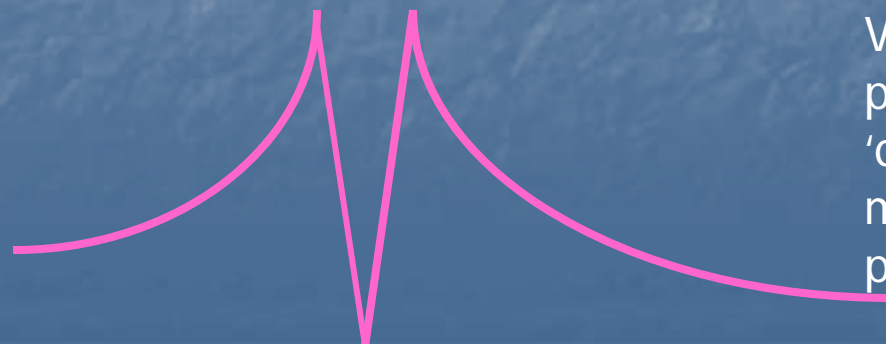
$$z \equiv \operatorname{sgn}(y) \left(e^{k|y|} - 1 \right) / k,$$

$\hat{\psi}(z) \equiv \psi(y)e^{k|y|/2}$, $\hat{h}(x, z) \equiv h(x, y)e^{k|y|/2}$. Eq. (8) then reads

$$\left[-\frac{1}{2}\partial_z^2 + V(z) \right] \hat{\psi}(z) = m^2 \hat{\psi},$$

where

$$V(z) = \frac{15k^2}{8(k|z| + 1)^2} - \frac{3k}{2}\delta(z).$$



Volcano
potential:
'quantum
mechanics'
potential

Find solution

$$J_2(m(|z| + 1/k)) \sim \frac{m^2(|z| + 1/k)^2}{8}, \quad Y_2(m(|z| + 1/k)) \sim -\frac{4}{\pi m^2(|z| + 1/k)^2} - \frac{1}{\pi} \quad (11)$$

Therefore to satisfy the boundary condition implied by the δ -function potential on the brane at $z = 0$, for small m (relevant at long distances) we must choose the linear combination,

$$\hat{\psi}_m \sim N_m(|z| + 1/k)^{1/2} \left[Y_2(m(|z| + 1/k)) + \frac{4k^2}{\pi m^2} J_2(m(|z| + 1/k)) \right]. \quad (12)$$

Here N_m is a normalization constant. For large mz ,

$$\sqrt{z} J_2(mz) \sim \sqrt{\frac{2}{\pi m}} \cos(mz - \frac{5}{4}\pi), \quad \sqrt{z} Y_2(mz) \sim \sqrt{\frac{2}{\pi m}} \sin(mz - \frac{5}{4}\pi). \quad (13)$$

Impose Boundary Conditions

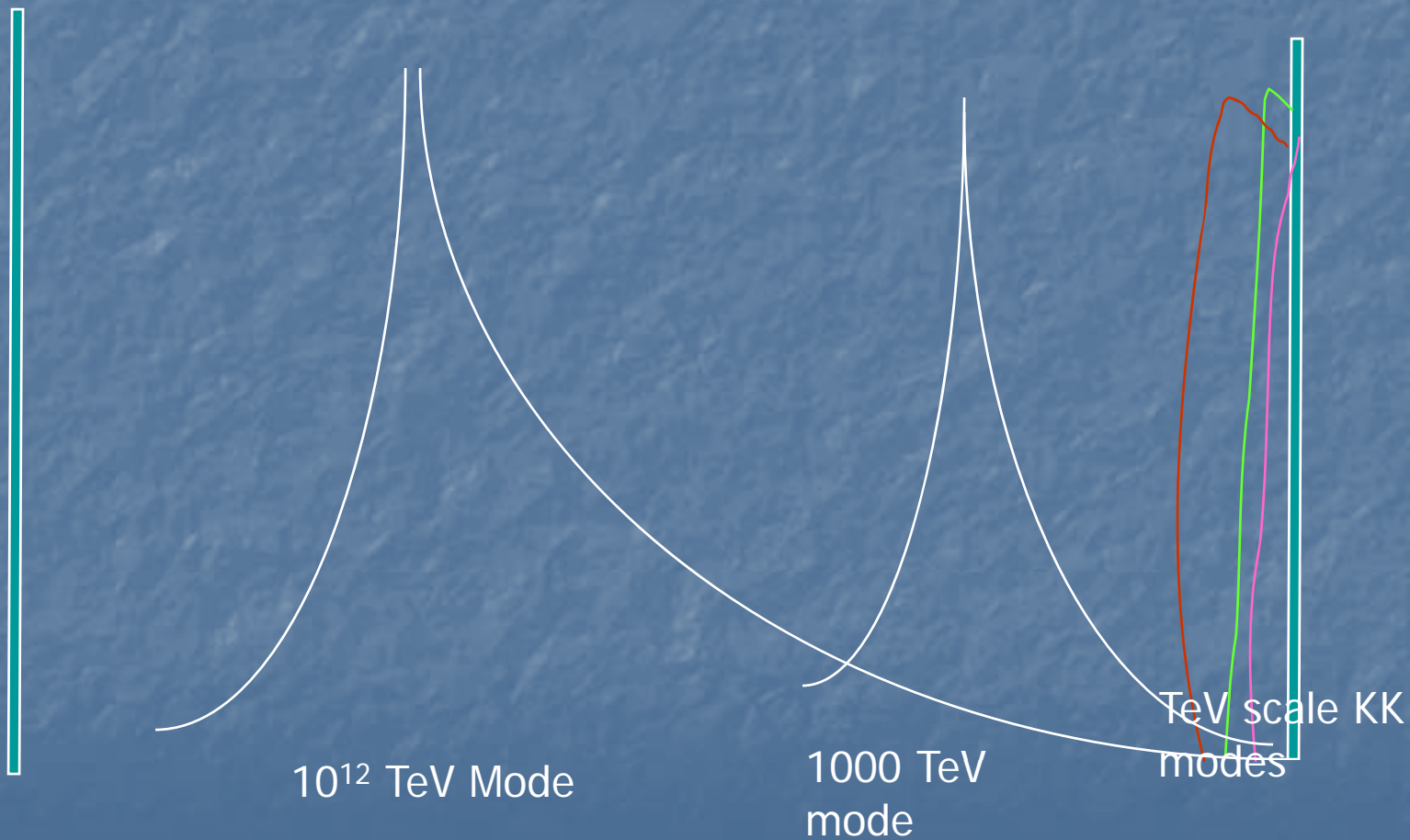
Let us now consider what happens when we reintroduce the regulator brane at $y_c \equiv \pi r_c$, that is $z_c \equiv (e^{k\pi r_c} - 1)/k$. It simply corresponds to a new boundary condition at z_c ,

$$\partial_z \hat{\psi}(z_c) = -\frac{3k}{2(kz_c + 1)} \hat{\psi}(z_c). \quad (14)$$

m. For large z_c they are all in the plane-wave asymptotic regime of Eq. (13) when they satisfy the new condition. Therefore their masses are approximately quantized in units of $1/z_c$. Furthermore their normalization constants are predominantly those of plane waves, in particular, $N_m \sim \pi m^{5/2}/(4k^2 \sqrt{z_c})$.

KK modes

- Find modes at every mass scale
- However essentially linearly quantized in multiples of ke^{-kr_c} within mass range of order m (cutoff)
- That is (m/k) modes in our vicinity
- To us theory looks strongly interacting at TeV
- (not true for full 5d theory)



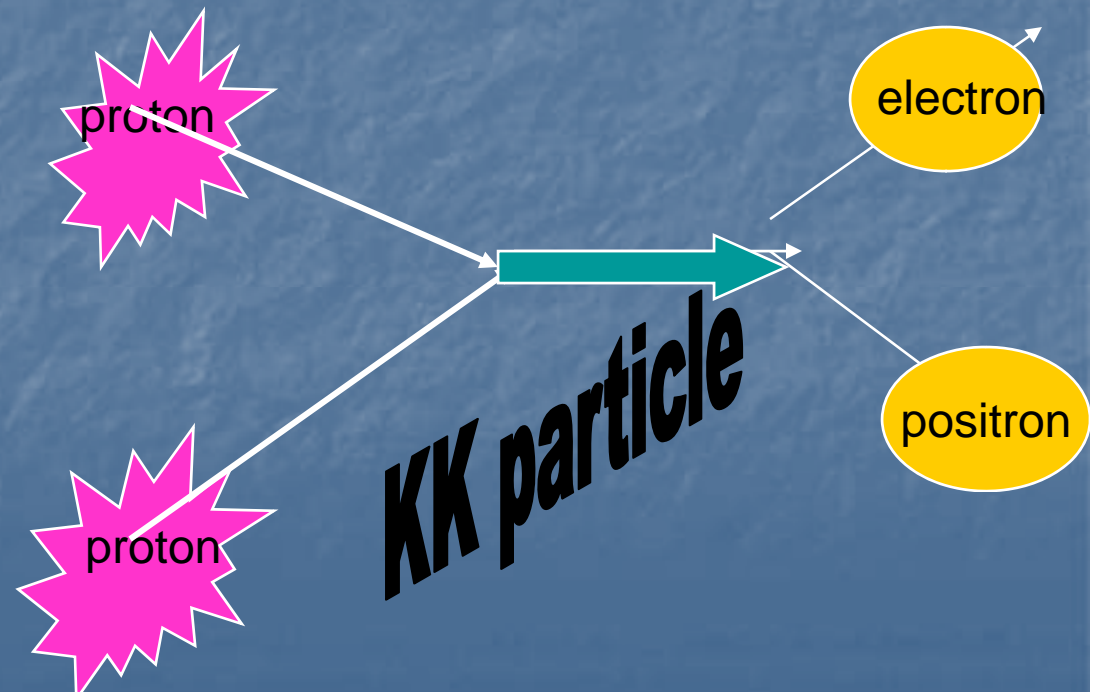
Observable KK modes

- In RS1, KK spectrum very distinctive
- TeV, 2 TeV, 3 TEV (rough) spectrum
- TeV scale modes localized on weak brane
- Have “warped” gravitational interaction
- So 1/TeV scale graviton interaction with Standard Model particles (not $1/M_{\text{pl}}$)
- Much stronger than gravitational!

Experimental Signal: Can search for extra dimensions!

- Protons collide
- Produce a Kaluza-Klein particle
- Which Decays
- Definite mass spectrum and "spin"-2

LHC Collision



- If you produce a KK mode of the graviton
- Not just missing energy!
- Mode decays inside detector—just like most other heavy particles we hope to discover
- ✓ Means we can reconstruct mass, spin (we hope!)
- ✓ Would be first genuine signature of quantum gravity
- ✓ Graviton itself too weakly interacting to detect directly
- ✓ Not true for its KK modes!

Search for Resonances

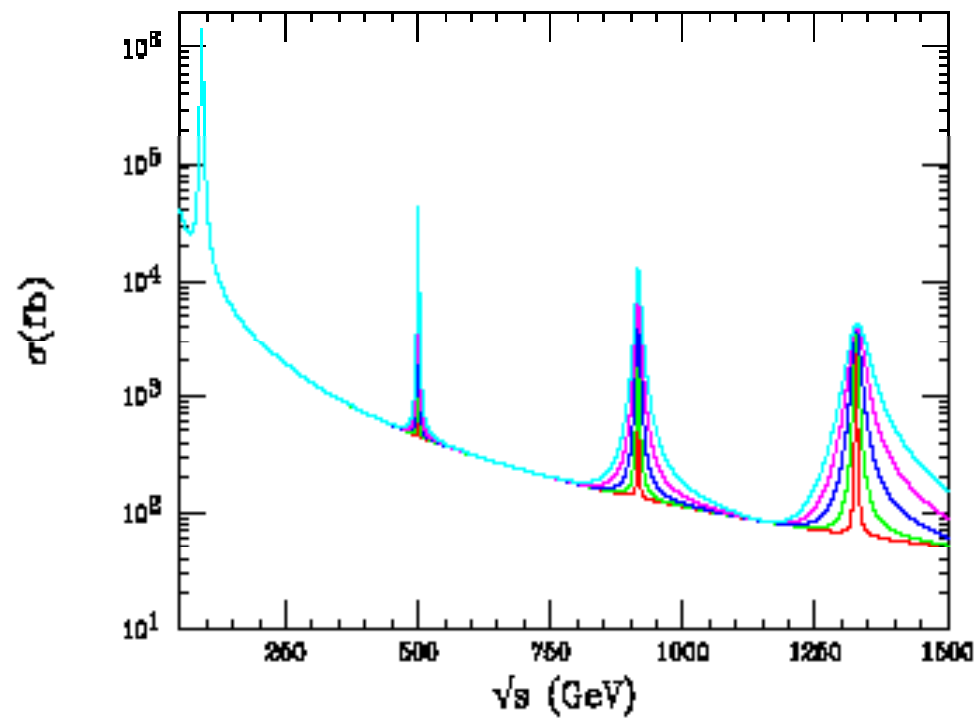
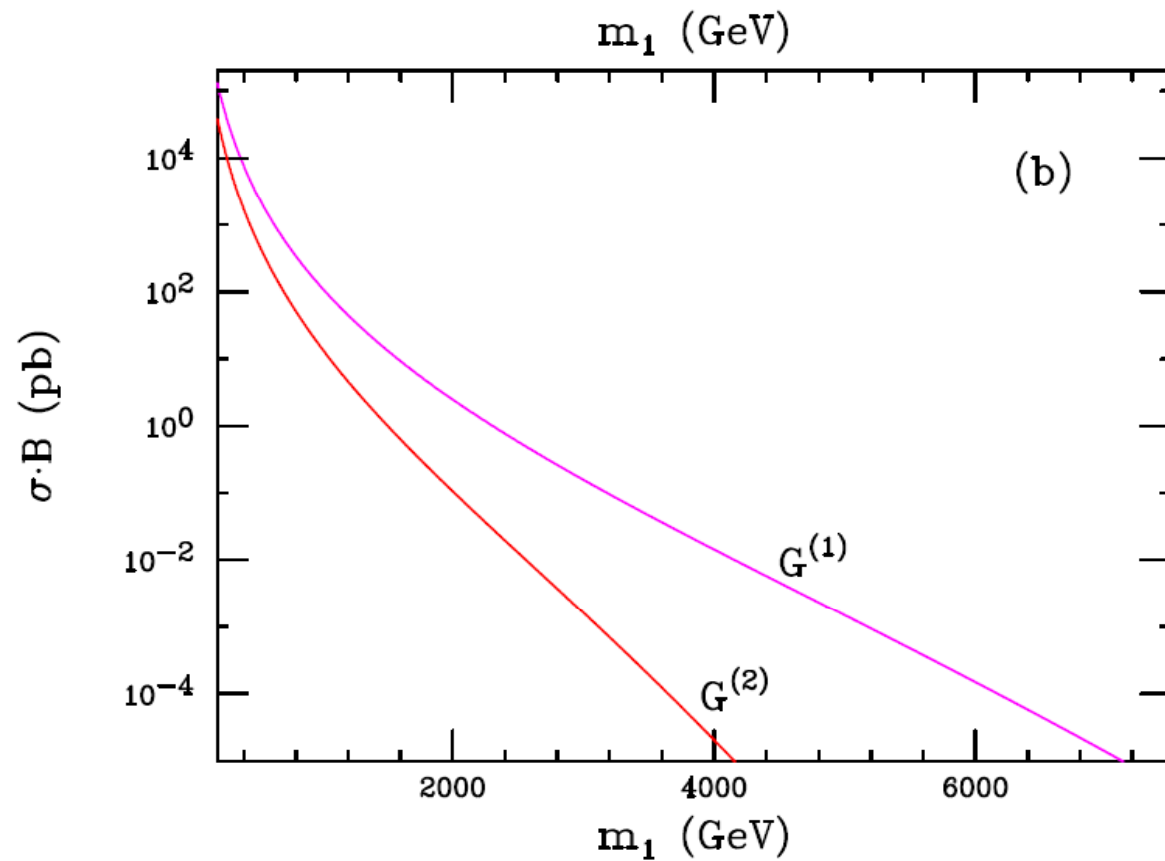


Figure 4: The cross section for $e^+e^- \rightarrow \mu^+\mu^-$ including the exchange of a KK tower of gravitons in the Randall-Sundrum model with $m_1 = 500$ GeV. The curves correspond to $k/\overline{M}_{\text{Pl}}$ in the range 0.01 – 0.05.

H. Davoudiasl, J. Hewett, T. Rizzo

Reach for Graviton KK Modes

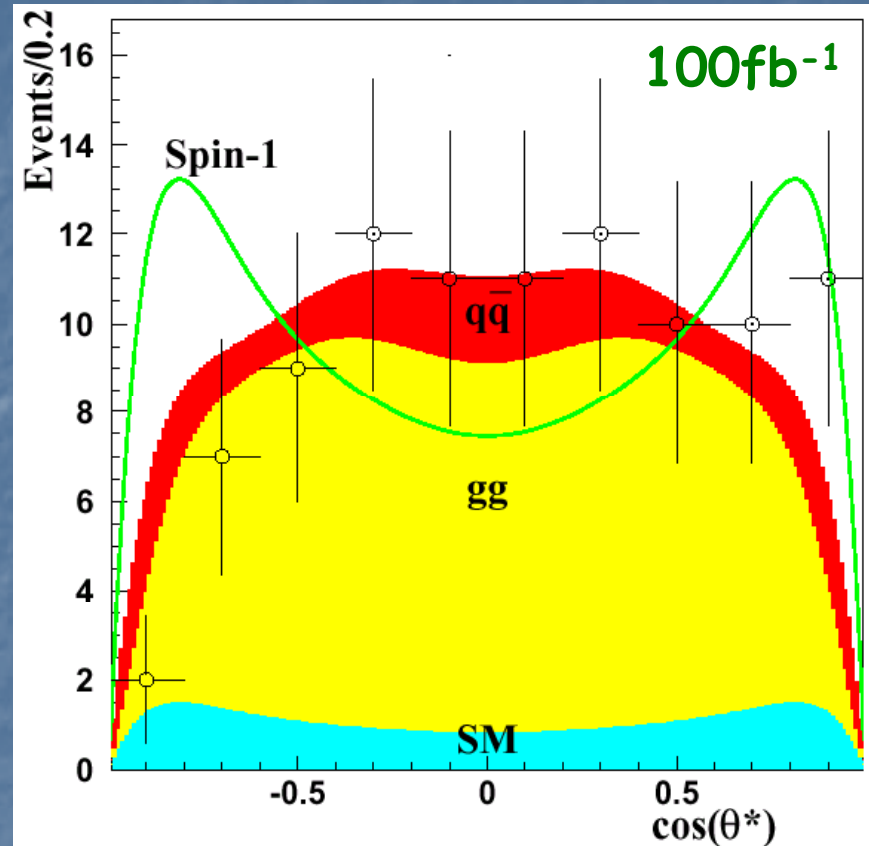


Davoudiasl, Hewett, Rizzo

graviton has spin 2

Angular distributions

- $qq \rightarrow G \rightarrow ff: 1 - 3 \cos^2 \theta + 4 \cos^4 \theta$
- $gg \rightarrow G \rightarrow ff: 1 - \cos^4 \theta$
- $qq \rightarrow G \rightarrow VV: 1 - \cos^4 \theta$
- $gg \rightarrow G \rightarrow VV: 1 + 6 \cos^2 \theta + \cos^4 \theta$
- DY background: $1 + \cos^2 \theta$

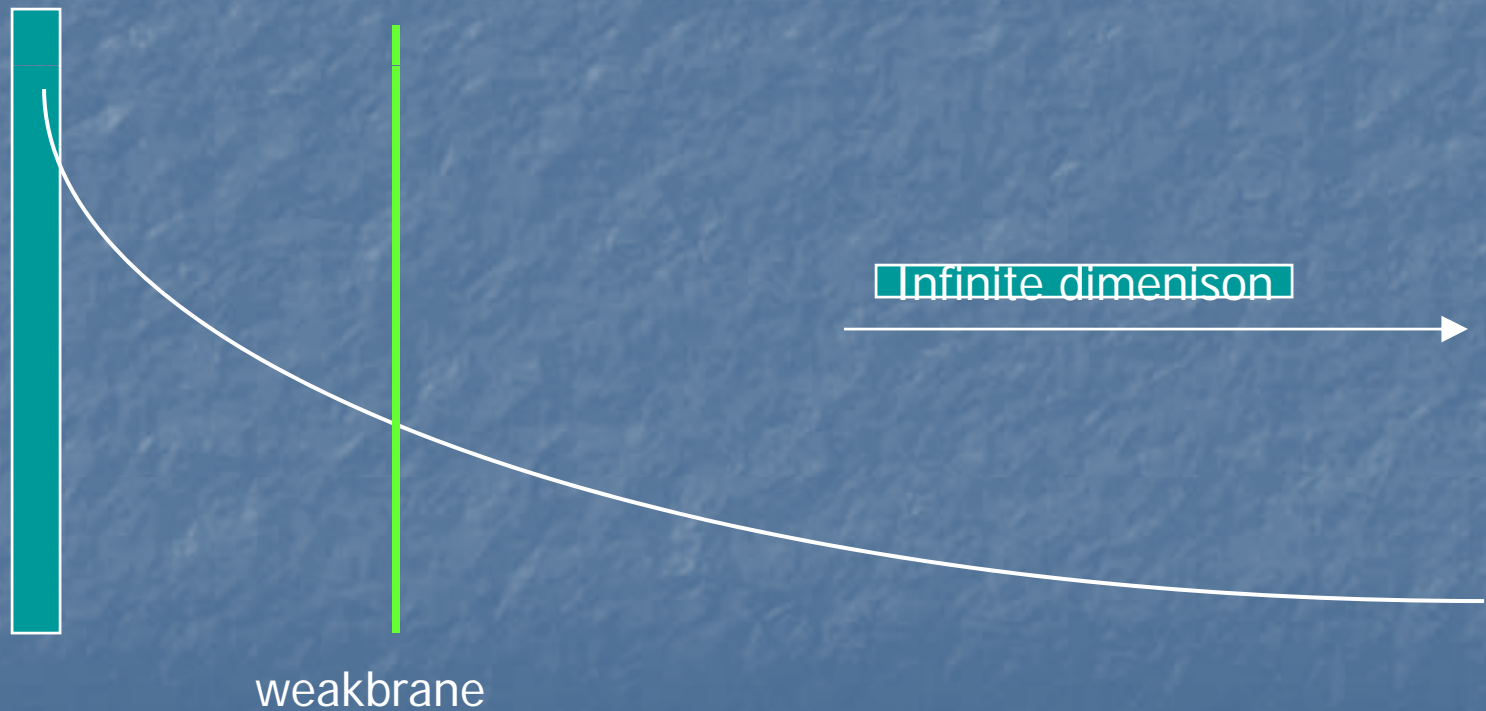


- RS1 gives clean TeV-KK-graviton signal
- One of first things LHC could find
- Spin-2 and gap in spectrum definite indication of warped extra-dimensional geometry
- Could also exist strongly interacting TeV-scale physics to complement this measurement
- Even low scale quantum gravity or light string states could appear (we'll return to this)
- **But not the only implementation of RS1 mechanism**
- **What does this imply about search strategies?**

Other Warped Models

Addressing the Hierarchy?

Variations on RS1: Infinite extra dimension



Missing Energy Signal

- Looks like 6 large dimensions
- In this case KK mode decays to lighter KK modes
- KK energy goes to missing energy

■ Variations of RS1:

matter in bulk or on brane??

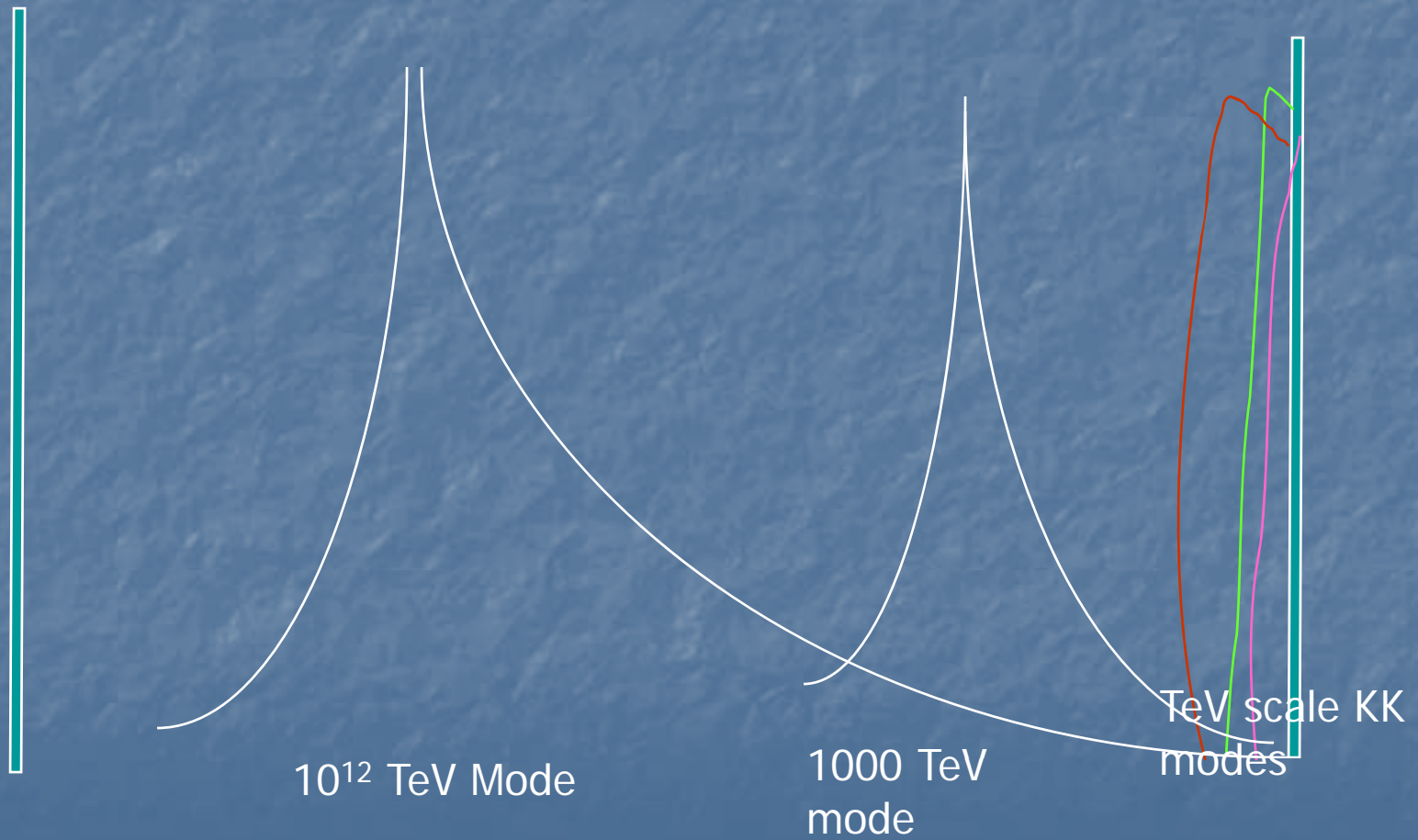
■ Two key features that make bulk matter possible

- Size of fifth dimension extremely small (only about 30 times fundamental scale—exponential hierarchy)
 - Means coupling won't be too diluted/weak
- You only need Higgs on the Weakbrane to address the hierarchy
 - ❖ Problem only for the Higgs scalar: gauge boson and fermion masses are protected

Merits of bulk fermions and gauge fields

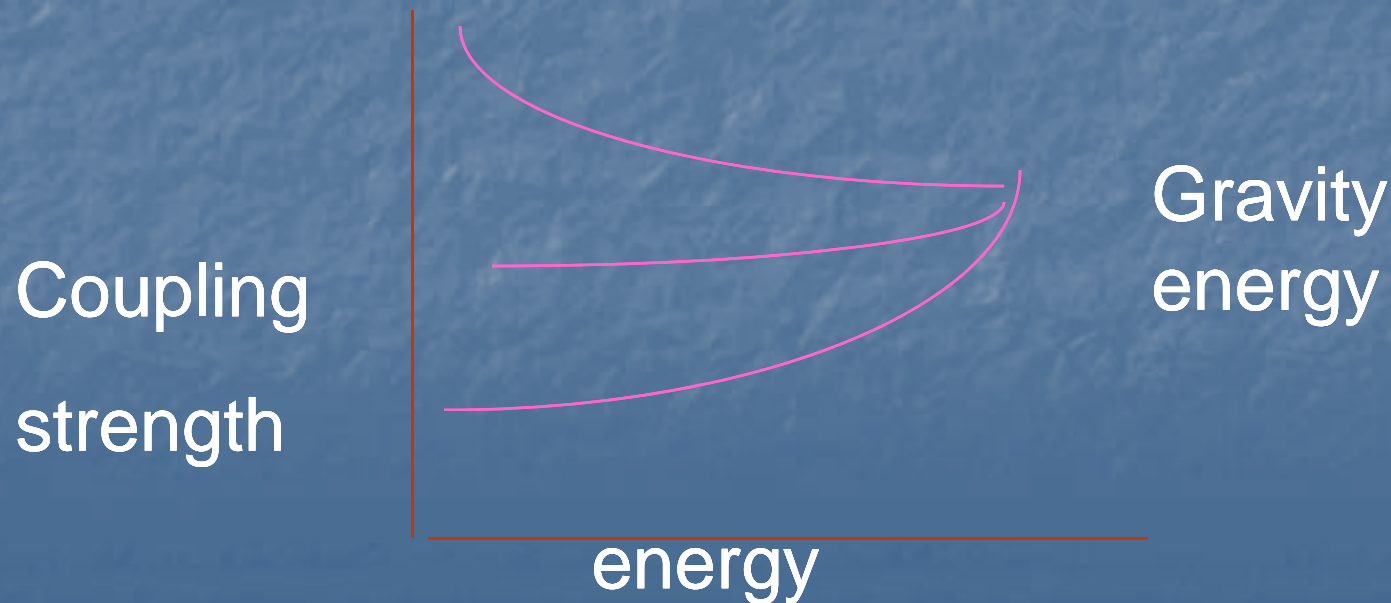
- **5D** cut-off is Planck scale
 - ✓ Allows for unification!
- Also interesting model-building:
 - ✓ Fermion masses from wavefunction overlap with Higgs field (on Weakbrane)
- We'll see that bulk scenarios have distinctive signatures

Modes at All Energies



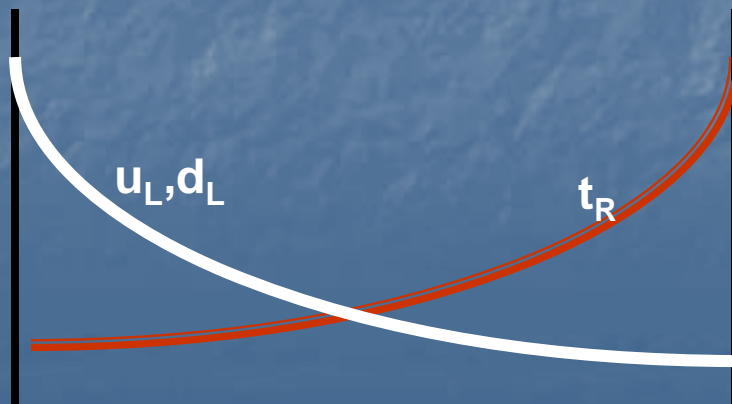
Bulk Unification

- Net contribution from all modes
- Gives *logarithmic* running
- From TeV scale to Planck scale
- LESSON 2: very natural to have unification



Fermion masses from wavefunctions

- Might expect nontrivial profiles
- Masses depend on overlap with Higgs
- Expect light fermions localized near Planck/Gravity brane
- Top near Weakbrane since it's heavy



Flavor hierarchies

- $z \sim e^{kr\phi}$
- $d_z f = (M/k + 1/2) f$
- $f \sim z^\nu$
- $f \sim e^{kr\phi(M/k + 1/2)}$
- $M/k = \nu < -1/2$: light quarks
- $\nu > -1/2$: top quark

$$\begin{aligned} \nu_{t_R} &\approx -0.3, \\ \nu_{Q_{3L}} &\approx -0.4, \\ \nu_{\text{other}} &< -0.5. \end{aligned} \tag{3}$$

That is: the t_R is IR localized, the third generation quark doublet, Q_{3L} is close to flat, and the others are all UV localized. This is important because the coupling of a zero mode fermion

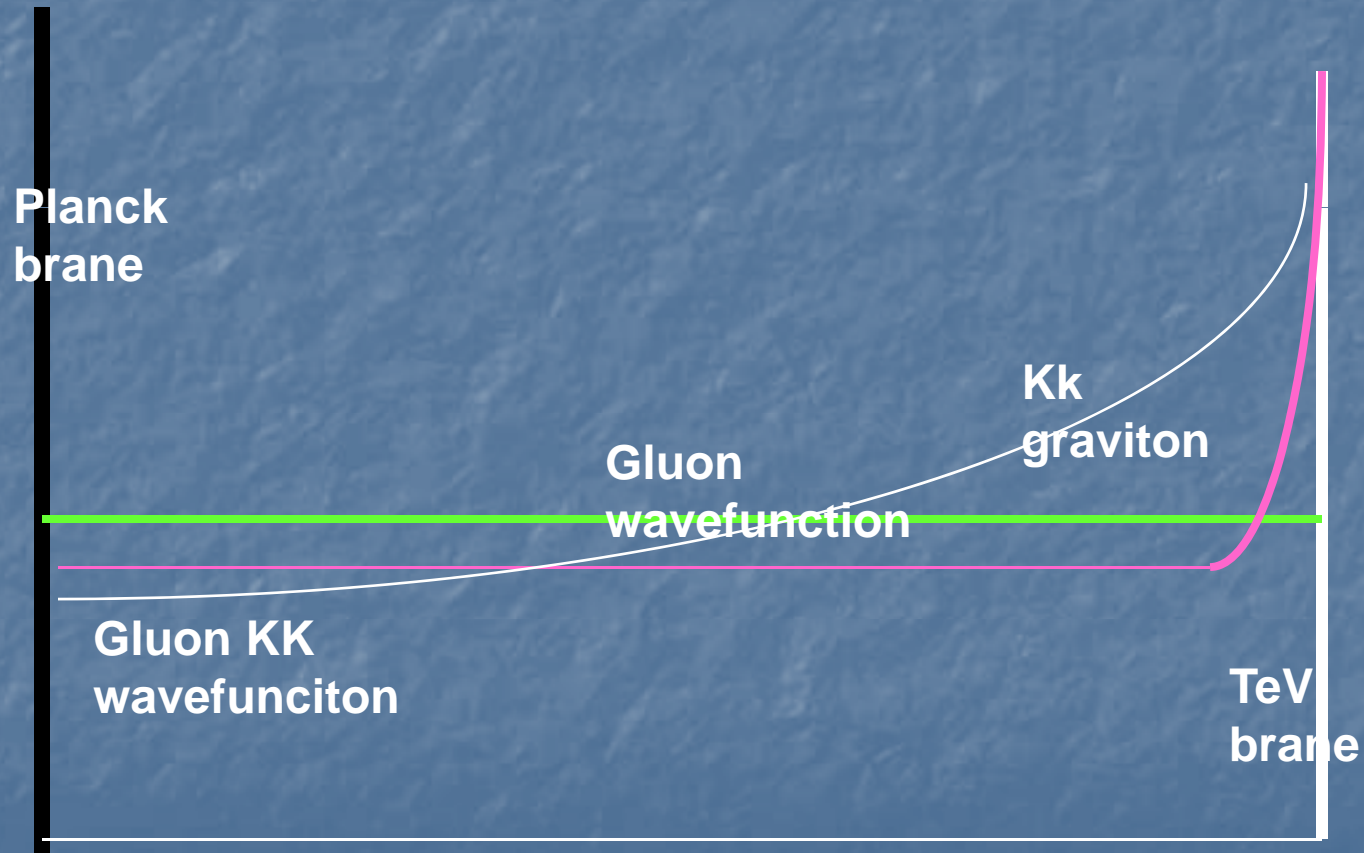
Bulk RS

- Variation of RS1: fermions and gauge bosons in bulk
- Hierarchy just like before
- But flavor, unification possibilities

However: Important Differences in Phenomenology from Brane-Localized Matter

- Richer Spectrum
- KK modes of
 - Weak bosons
 - Gluons
 - Fermions
 - As well as gravitons
- But...lower Production Cross Section for Graviton
- Plus decays primarily into tops
 - Changes search strategies dramatically

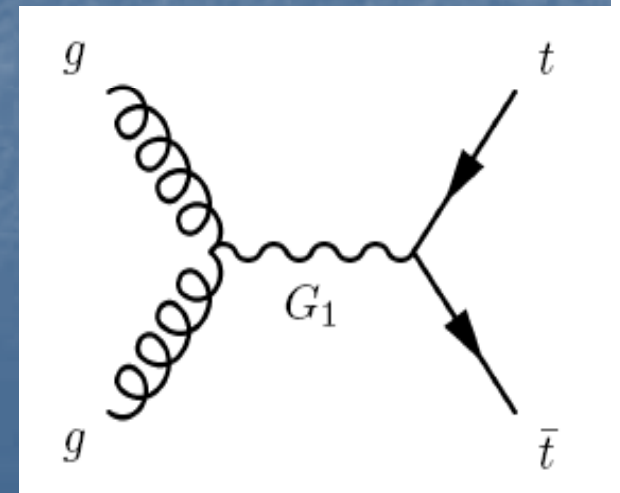
Gluon, Gluon KK WFs



Richer Spectrum...

But Lower Production Cross Section for the Graviton

- Light quarks are localized away from Higgs
 - Hence away from TeV brane
 - No Drell-Yan production from quarks
- Gluons are spread throughout the bulk
 - Hence coupling to graviton is volume suppressed




Graviton Interactions

$$C_{xxG} \int d^4x h_{\mu\nu} T_{xx}^{\mu\nu}$$

$$C_{ggG} = \frac{1}{(\pi k r_c)(M_4 L)\mu_{TeV}} \frac{\int_0^1 dy y J_2(3.83y)}{J_2(3.83)} \approx \frac{0.47}{(\pi k r_c)(M_4 L)\mu_{TeV}}$$

Volume suppressed



$$C_{ffG} = \frac{1}{(M_4 L)\mu_{TeV}} \left(\frac{1+2\nu}{1-e^{-\pi k r_c(1+2\nu)}} \right) \frac{\int_0^1 dy y^{2+2\nu} J_2(3.83y)}{J_2(3.83)}$$

$$C_{ssG} = \frac{2}{(M_4 L)\mu_{TeV}}$$

Graviton interactions

$$C_{XXG} \int d^4x h_{\mu\nu} T_{XX}^{\mu\nu}$$

$$T_{\mu\nu} = 2 \frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} - g_{\mu\nu} \mathcal{L}$$

$$C_{XXG} \int d^4x h_{\mu\nu} T_{XX}^{\mu\nu}$$

XX	$T_{XX}^{\mu\nu}$	c_{XXG}
ss (scalars)	$\frac{1}{2} \partial^\mu \phi \partial^\nu \phi$	$c_{ssG} = \frac{2}{(M_4 L) \mu_{TeV}}$
$f\bar{f}$ (fermions)	$i \psi^\dagger \bar{\sigma}^\mu D^\nu \psi$	$c_{f\bar{f}G} = \frac{1}{(M_4 L) \mu_{TeV}} \left(\frac{1+2\nu}{1-e^{-\pi k r_c (1+2\nu)}} \right) \frac{\int_0^1 dy y^{2+2\nu} J_2(3.83y)}{J_2(3.83)}$
$t\bar{t}_1$ (top+KK-top)	$i \psi^\dagger \bar{\sigma}^\mu D^\nu \psi$	$c_{f\bar{f}G}^{101} = \frac{1}{(M_4 L) \mu_{TeV}} \sqrt{\frac{2(1+2\nu)}{1-e^{1+2\nu}}} \int_0^1 dy y^{\nu+5/2} \frac{J_{\nu-1/2}(x_1^L y)}{J_{\nu-1/2}(x_1^L)} \frac{J_2(3.83y)}{ J_2(3.83) }$
gg (gluons)	$F^{\mu\rho} F^\nu_\rho$	$c_{ggG} = \frac{1}{(\pi k r_c) (M_4 L) \mu_{TeV}} \frac{\int_0^1 dy y J_2(3.83y)}{J_2(3.83)} \approx \frac{0.47}{(\pi k r_c) (M_4 L) \mu_{TeV}}$

Features of interactions

- The suppression by M_4L is a factor of $1/N$ from the dual gauge theory perspective
- $1/\mu_{\text{TeV}}$ is local cutoff
- $1/kr_c$ because gluon has flat wave function: volume factor
- Fermion behavior

w/Liam

Fitzpatrick, Jared

Kaplan, Liantao

Wang

KK Graviton Production

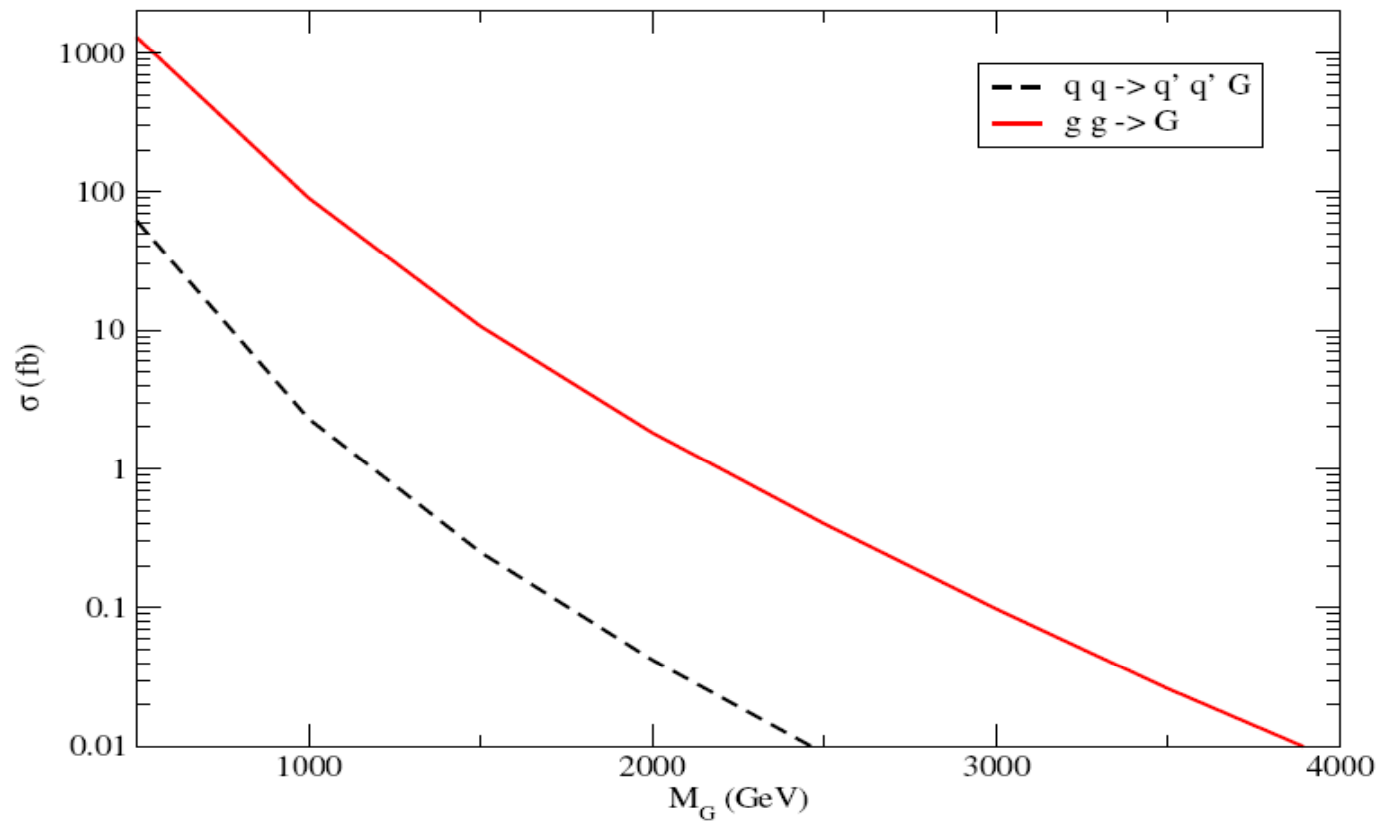
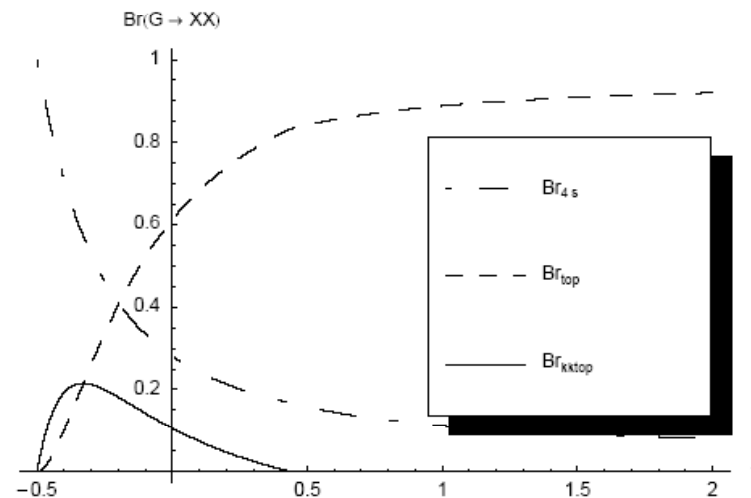


Figure 1: Cross section of KK-graviton production.

Final State? Dominant Decay to right-handed tops



More
narrow

Figure 1: Branching Ratios for graviton decay to scalars and quarks.

$$\Gamma_{top} = \frac{1}{(M_4 L)^2 \mu_{TeV}^2} \left(\frac{1 + 2\nu}{1 - e^{-\pi k r_c (1 + 2\nu)}} \frac{\int_0^1 dy y^{2+2\nu} J_2(3.83y)}{J_2(3.83)} \right)^2 \frac{3m_{grav}^3}{160\pi}$$

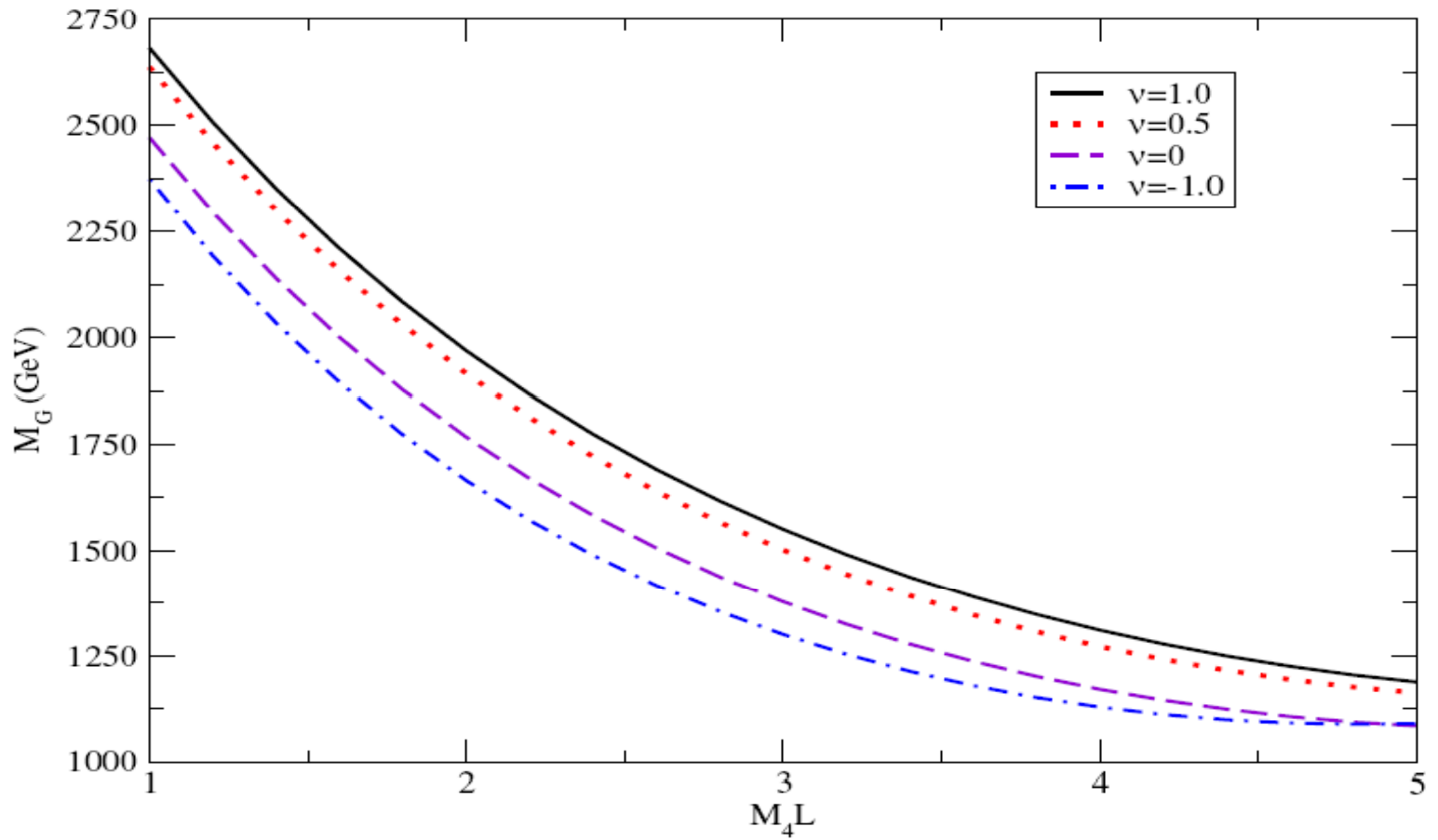
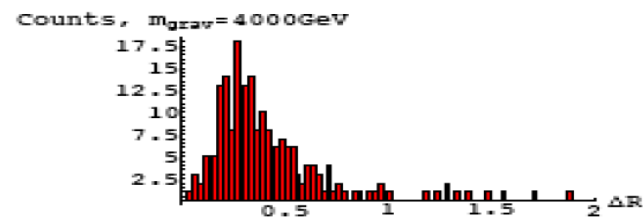
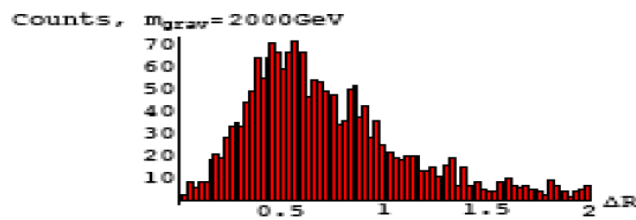
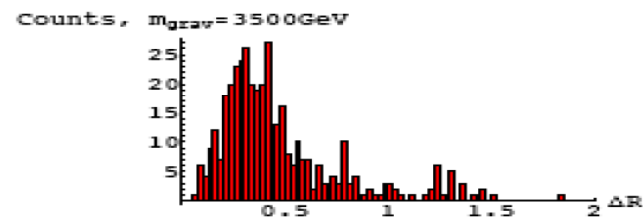
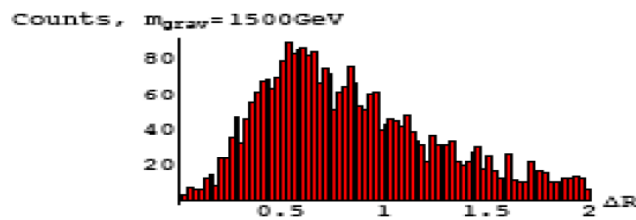
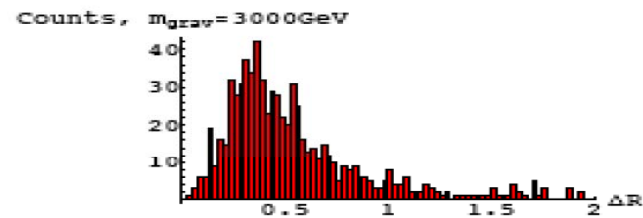
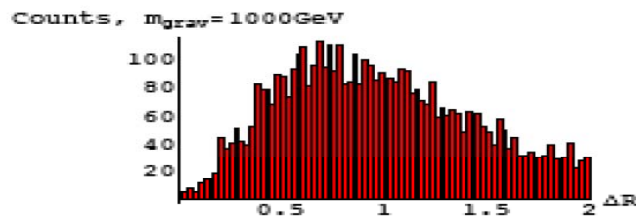
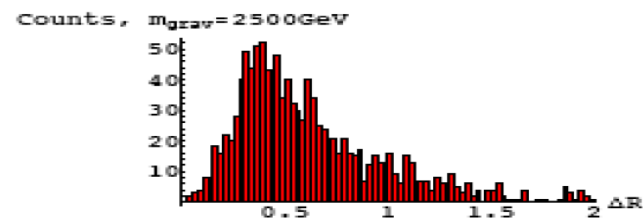
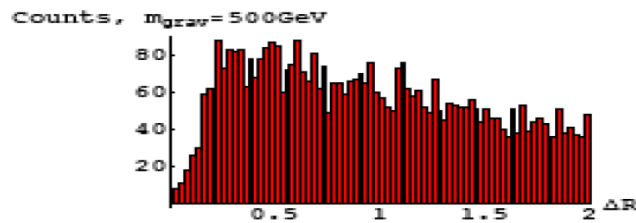
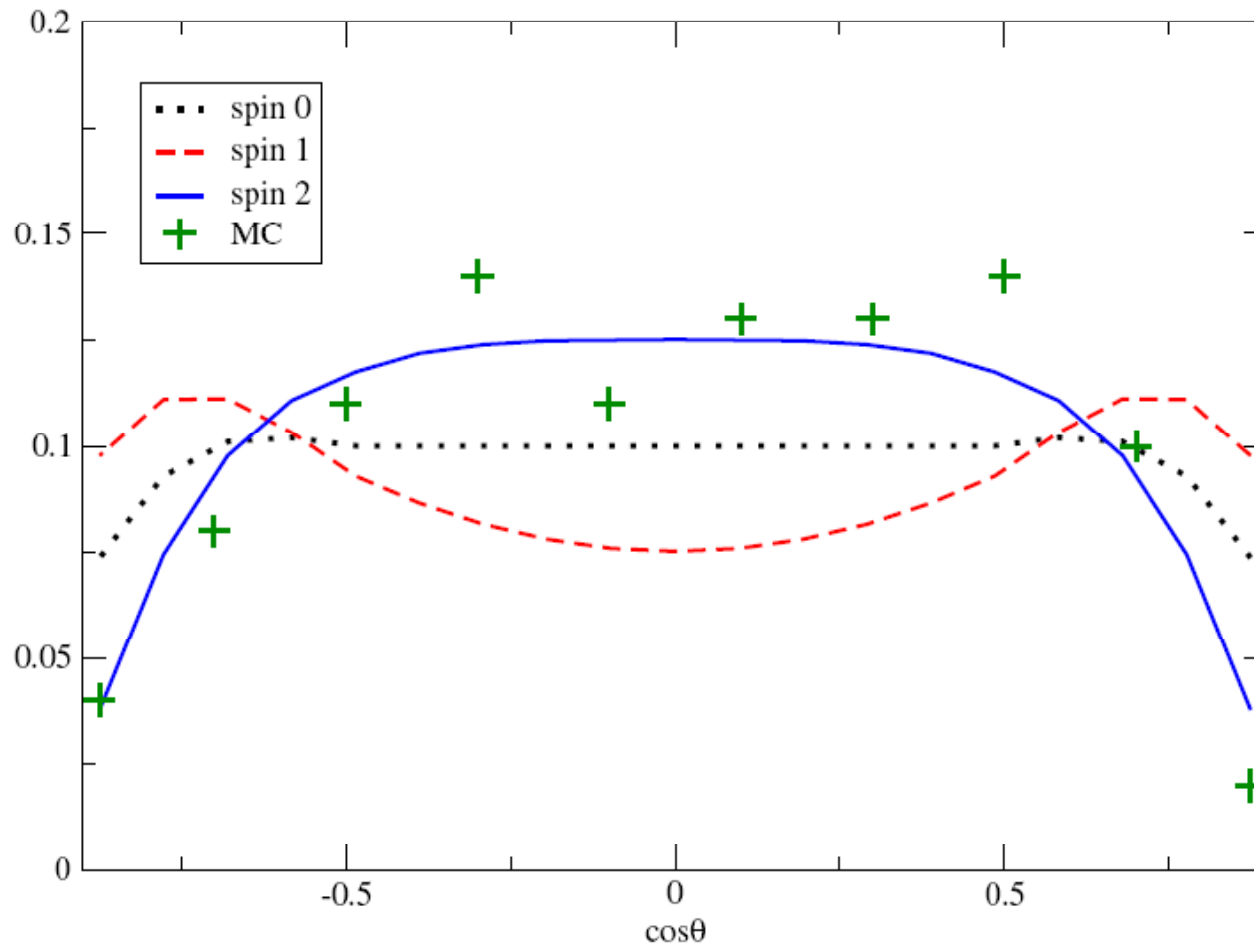


Figure 3: The $s/\sqrt{b} = 5$ reach as a function of graviton mass and the parameter ($M_4 L$). From top to bottom is shown for $\nu = 1.0, 0.5, 0.0, -0.1$.

Determining top jets: delta R: Angle between decay products



Angular dependence: spin determination



Graviton: some reach Other Bulk Modes?

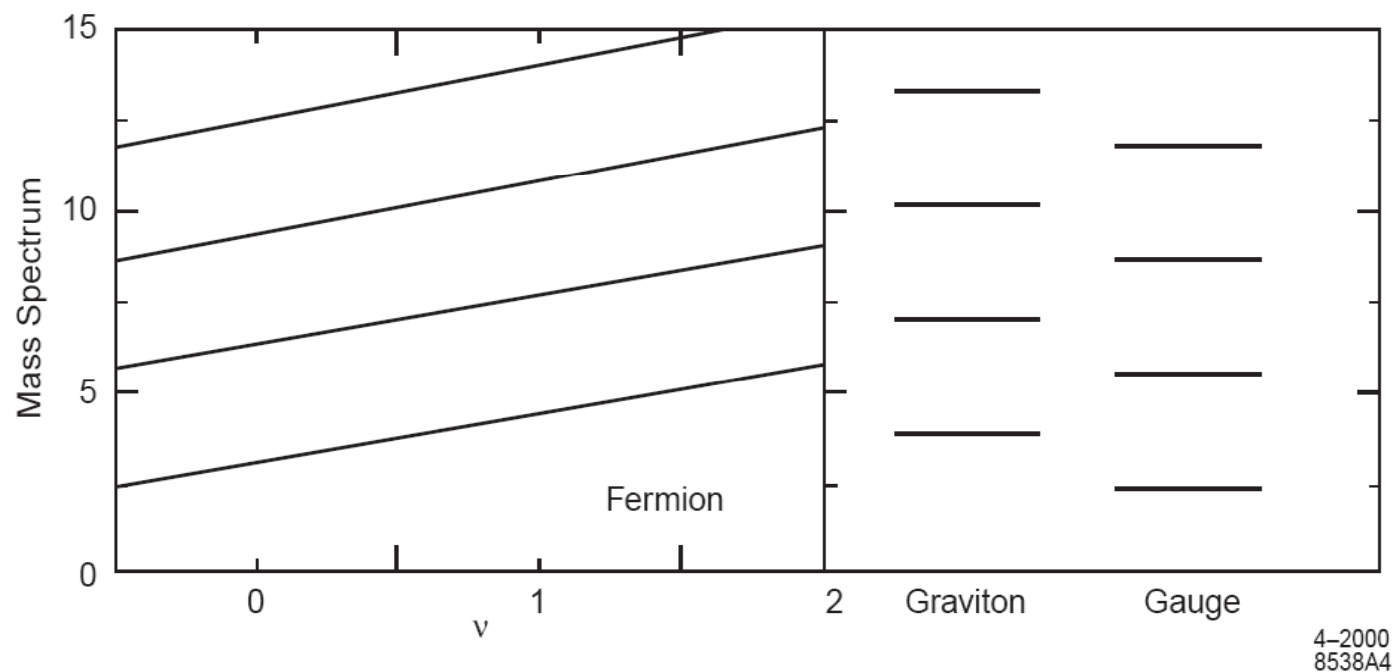
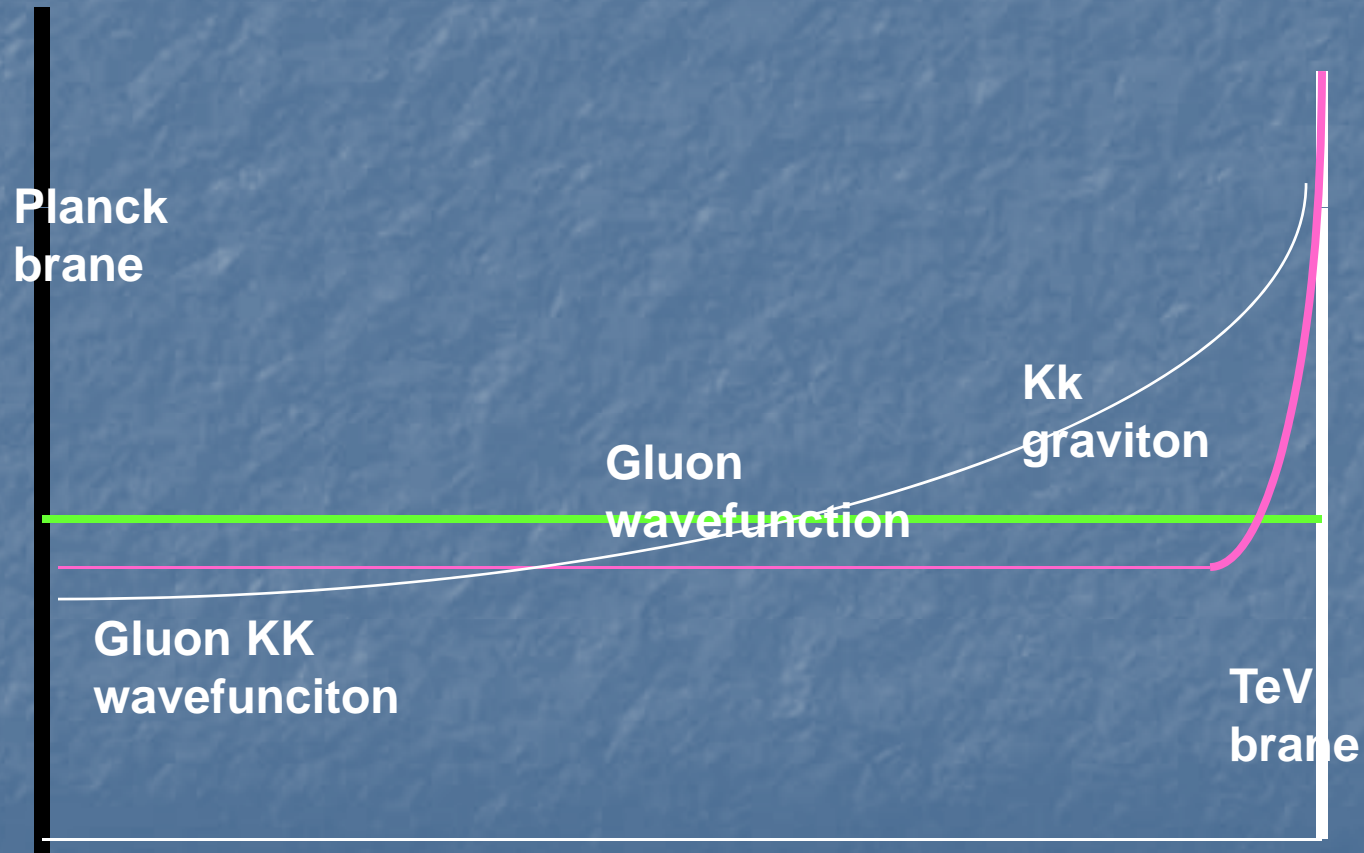


Figure 1: Relative mass spectra in units of $ke^{-kr_c\pi}$ of the KK excitations of the fermion fields as a function of their bulk mass parameter ν , as well as for the graviton and the gauge boson fields as described in the text.

Gluon, Gluon KK WFs



Gluon KK Mode!

- Gluon KK mode coupling to light quarks is much bigger than graviton
 - Gluon KK mode wave function peaked at TeV brane
 - But relatively flat in bulk;
 - Only square root volume
- Also expect gluon KK mode lighter by factor 1.5
- Finally no $1/M$
- Much larger reach for gluon KK mode

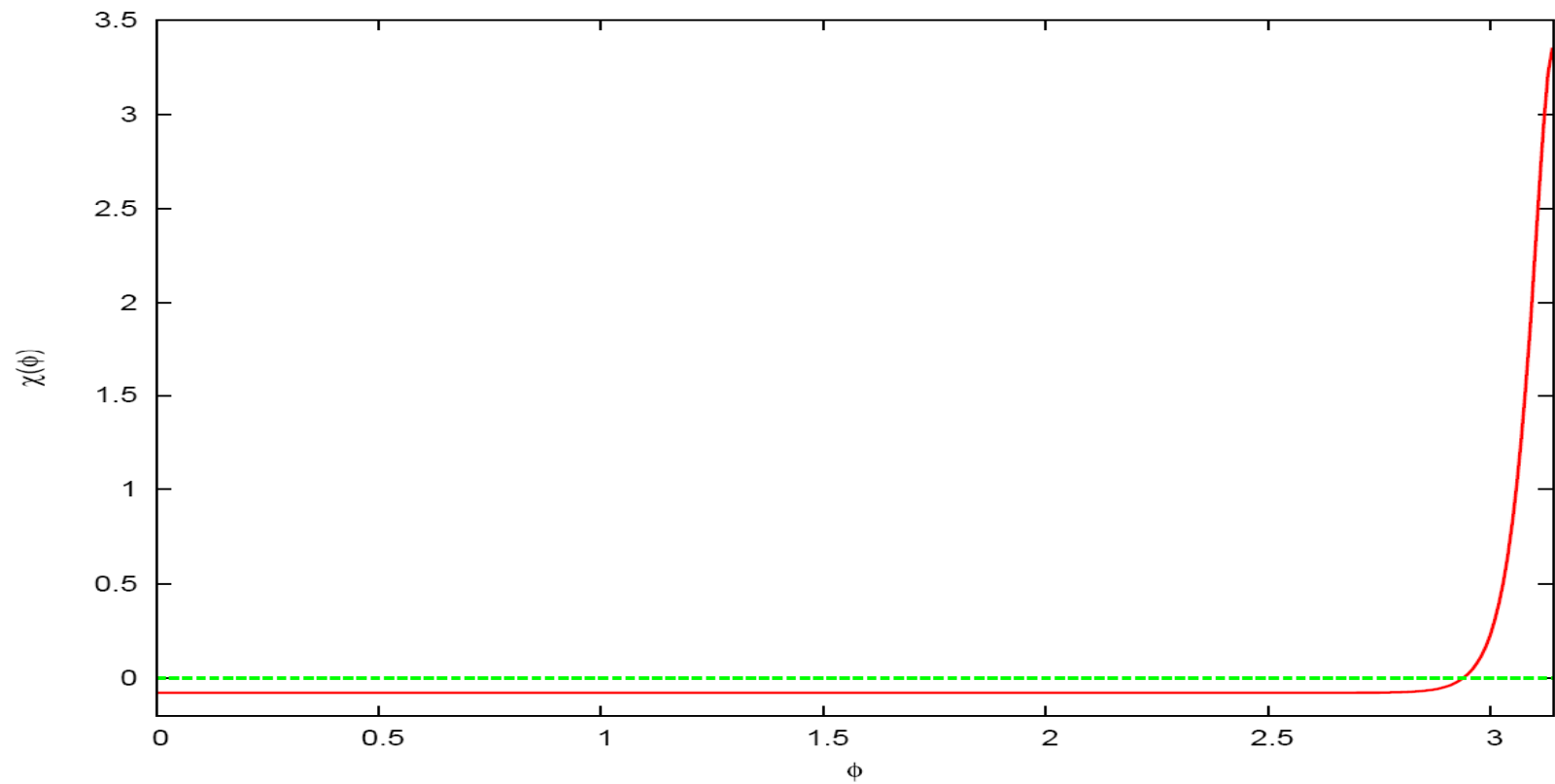
$$\frac{g_f \bar{f} A^{(n)}}{g_{SM}} \approx \frac{1}{\sqrt{\pi k r_c}} \frac{1}{\sqrt{n - 0.2}} + \mathcal{O}[(\pi k r_c)^{-3/2}].$$

Understand from dual point of view in terms of gluon KK mixing-vector meson dominance

Note only gluon production from quarks.

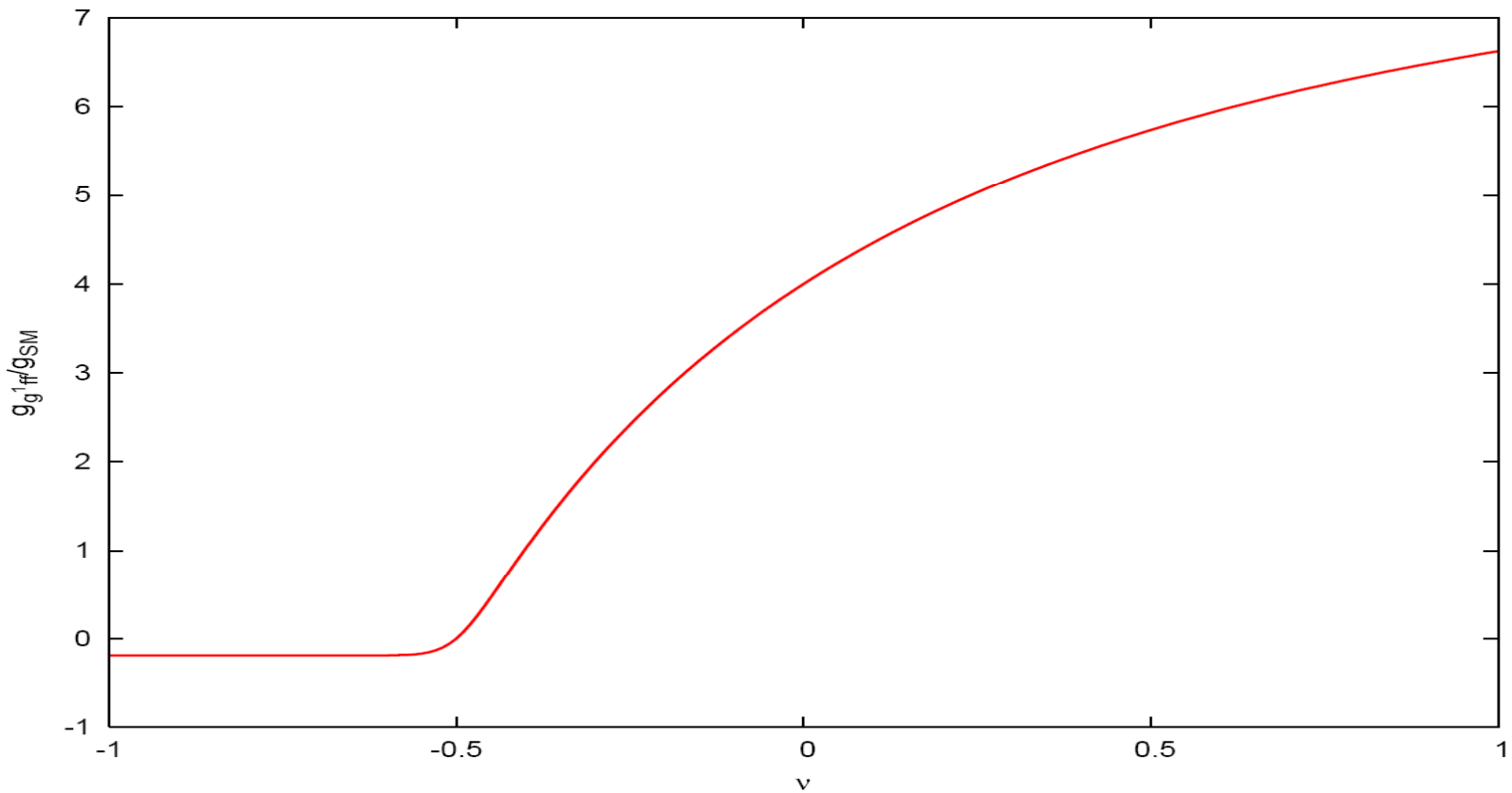
At tree level, gluon coupling vanishes.

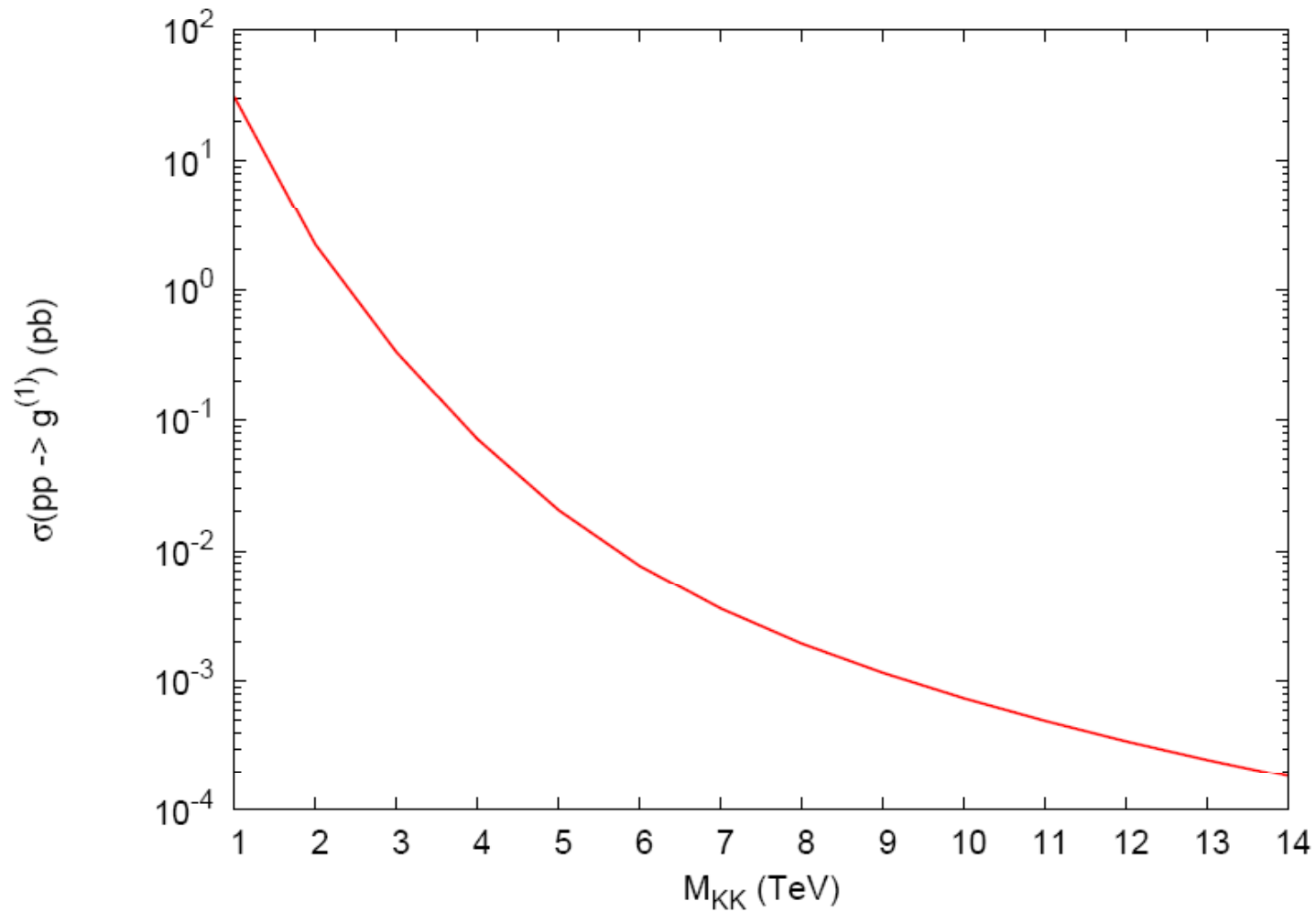
$$\chi_A^{(n)} = \frac{e^\sigma}{N_n^A} \left[J_1(z_n^A) + \alpha_n^A Y_1(z_n^A) \right] ,$$



Gluon wave function

KK Gluon fermion interaction (relative to g_s)

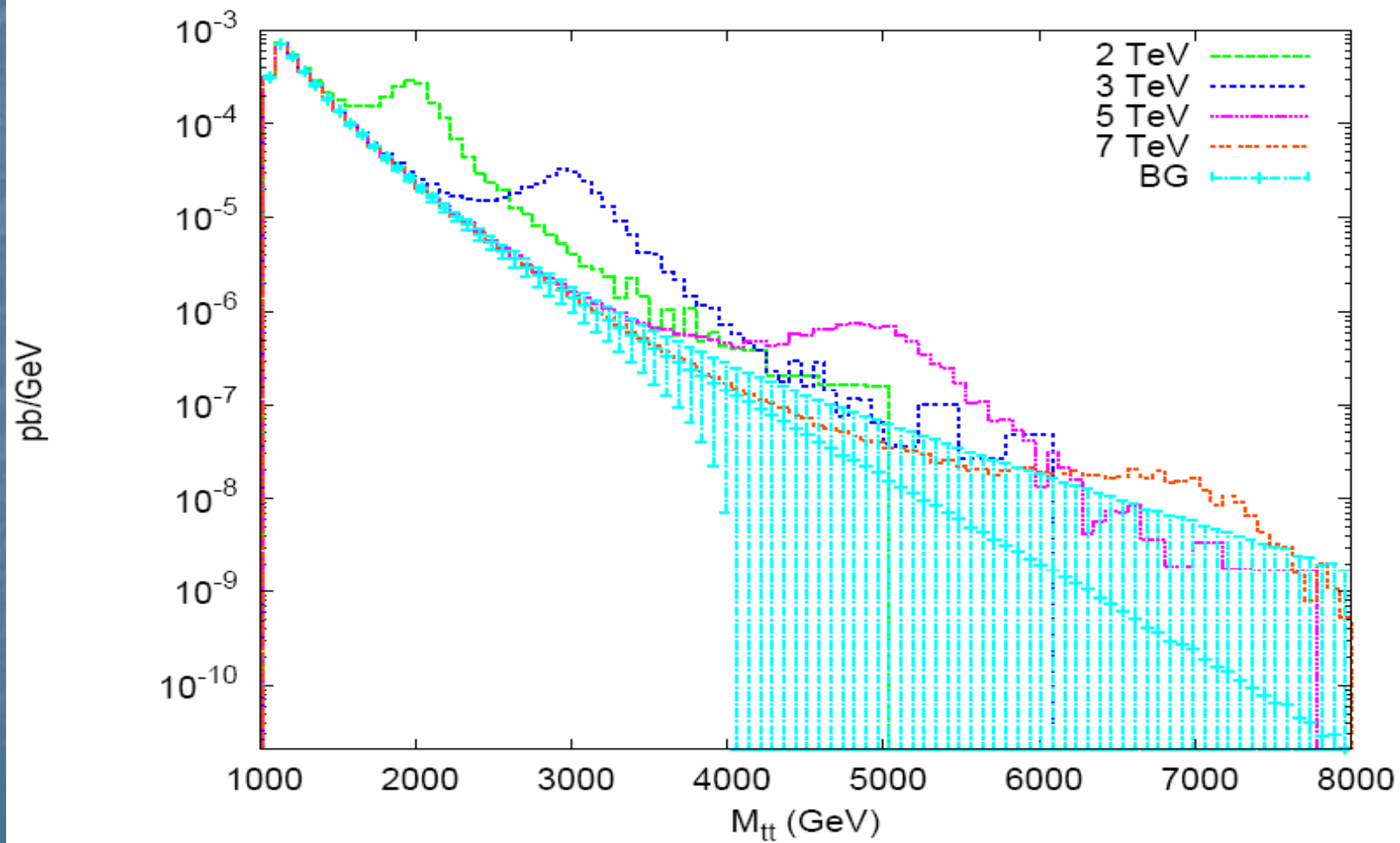




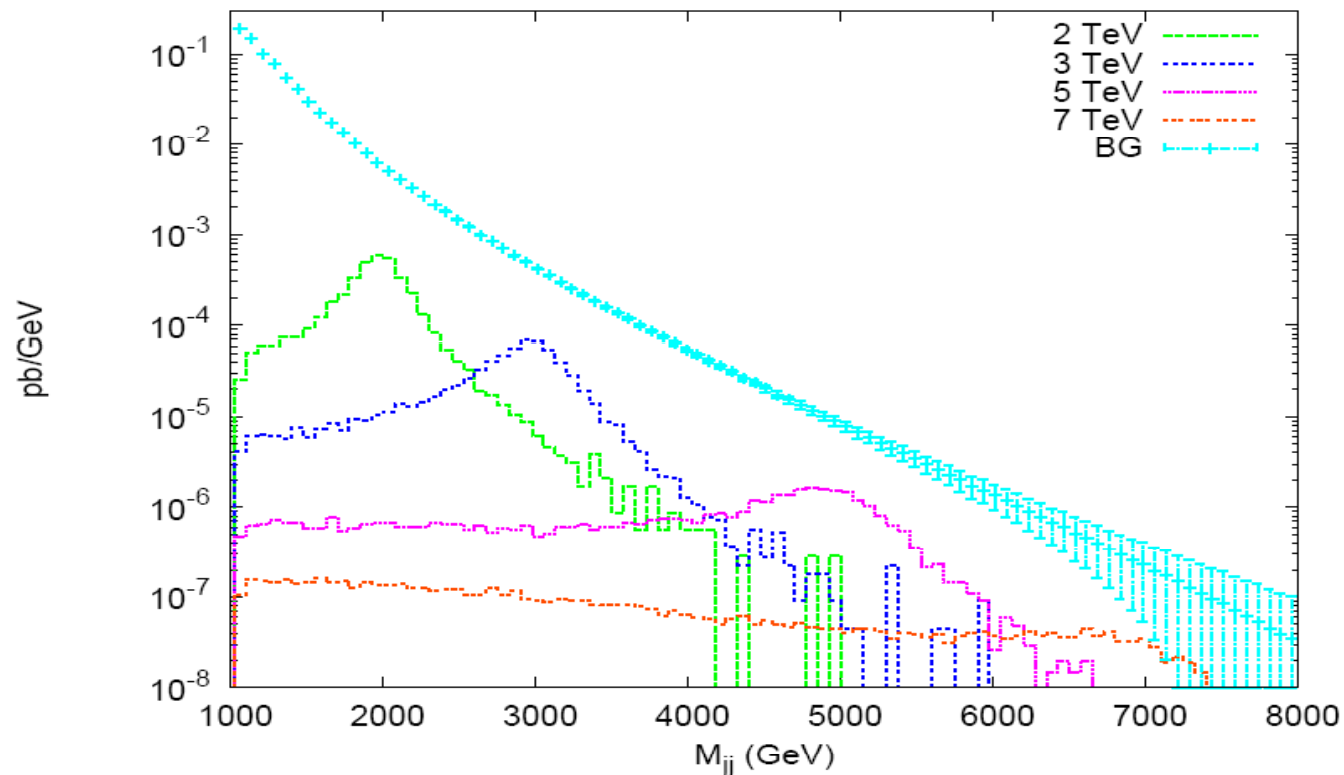
Total cross-section for production of the first KK gluon, as a function of KK mass

w/Ben Lillie,
Liantao Wang

Dominates over top jet background

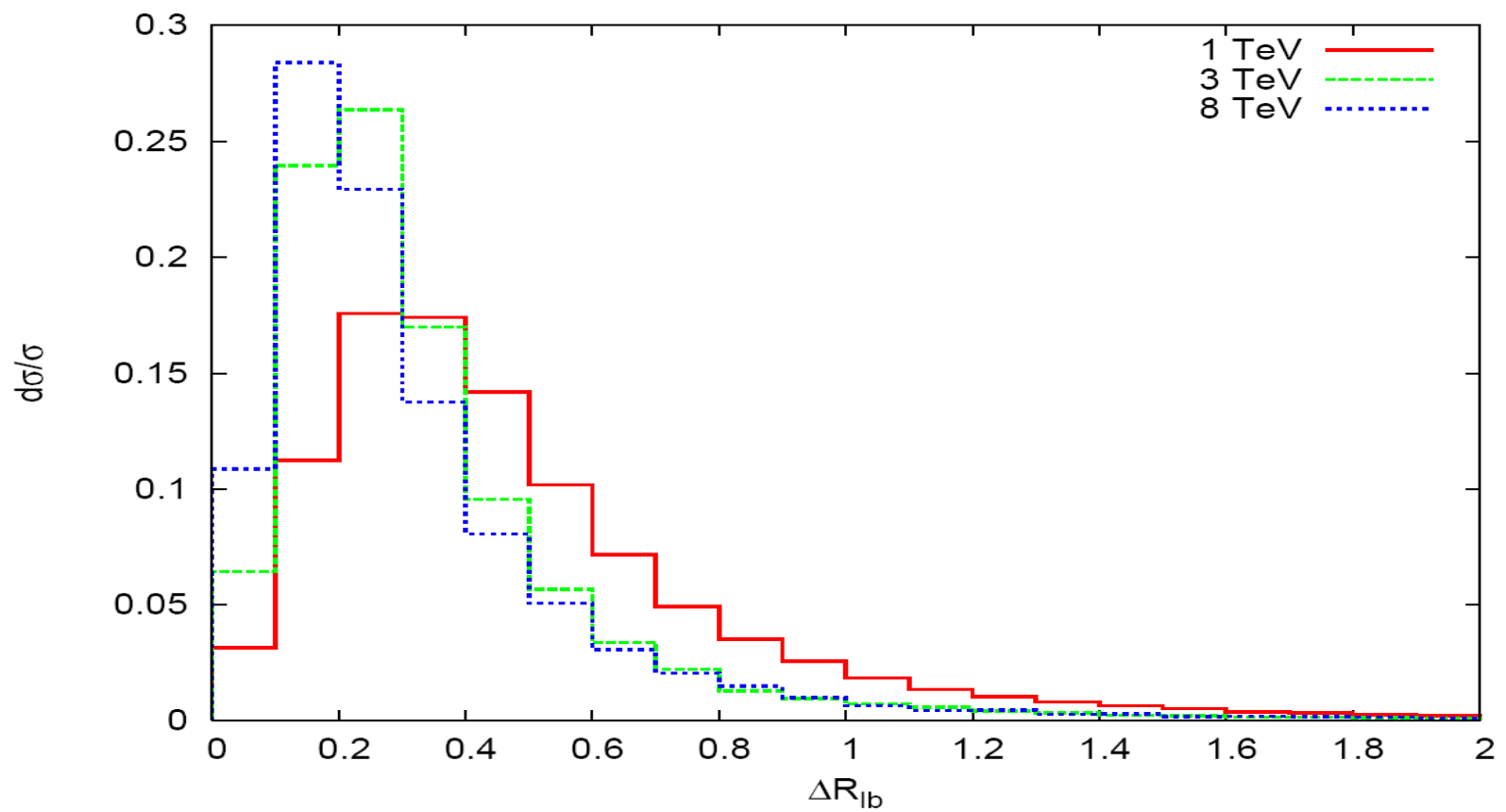


However, signal doesn't dominate over jet background



7: Invariant mass distribution of the decay products for several masses of the KK gluon. Assumes all $t\bar{t}$ events are fully collimated. “BG” is QCD dijet production. All jets are

Top determination: dR?



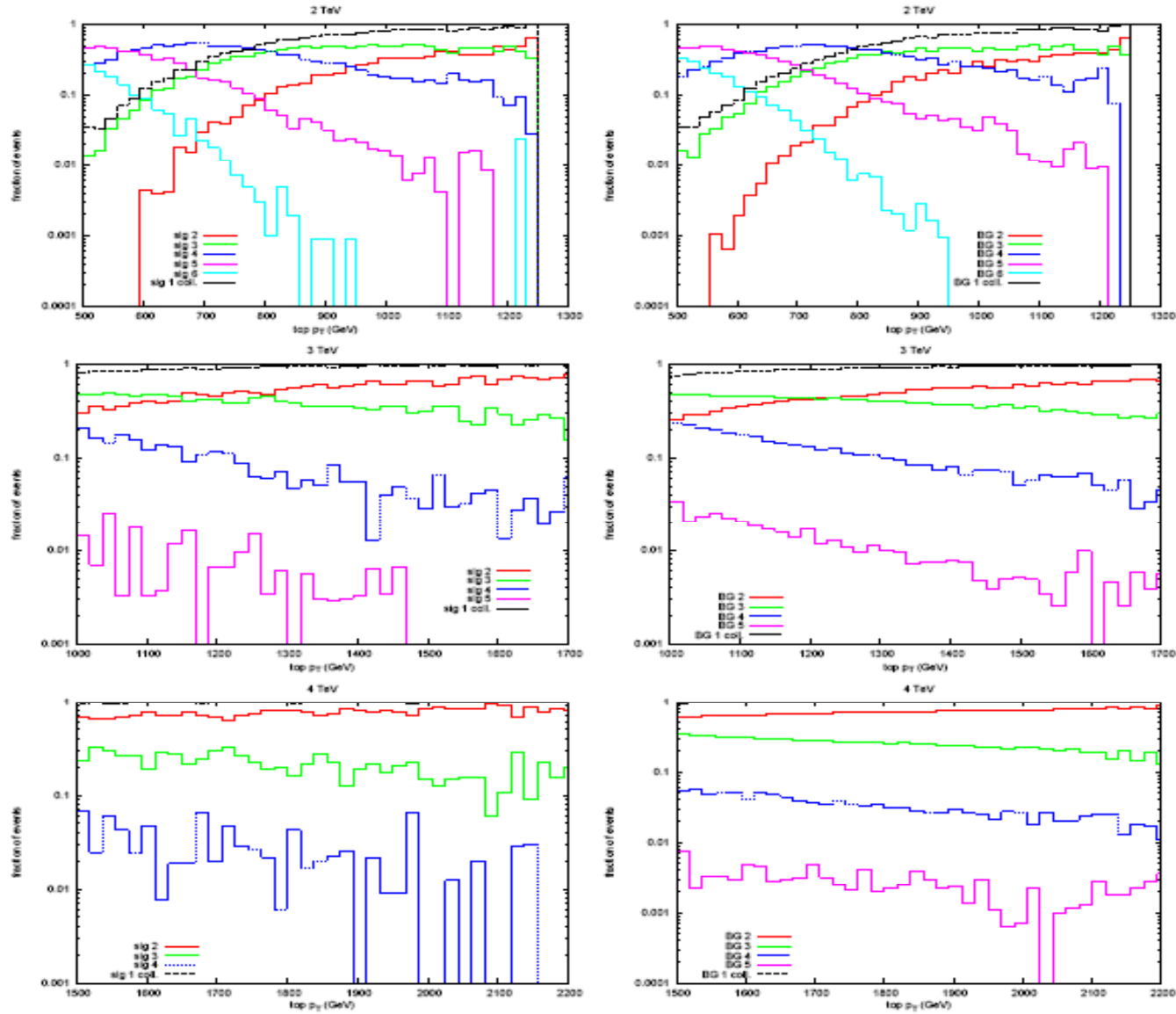


Figure 6: Left: Fraction of events for certain numbers of distinct objects for events from decay of a KK gluon, with mass (top to bottom) 2, 3, and 4 TeV as a function of p_T for events in the window $m_{KK} - 500 \text{ GeV} < m_{t\bar{t}} < m_{KK} + 500 \text{ GeV}$. Right: SM $t\bar{t}$ production using the same cuts as the corresponding plot on the right. The line labeled “1 coll.” is the fraction of events where at least one of the tops has all three decay products within the same cone. A cone size of 0.4 has been used.

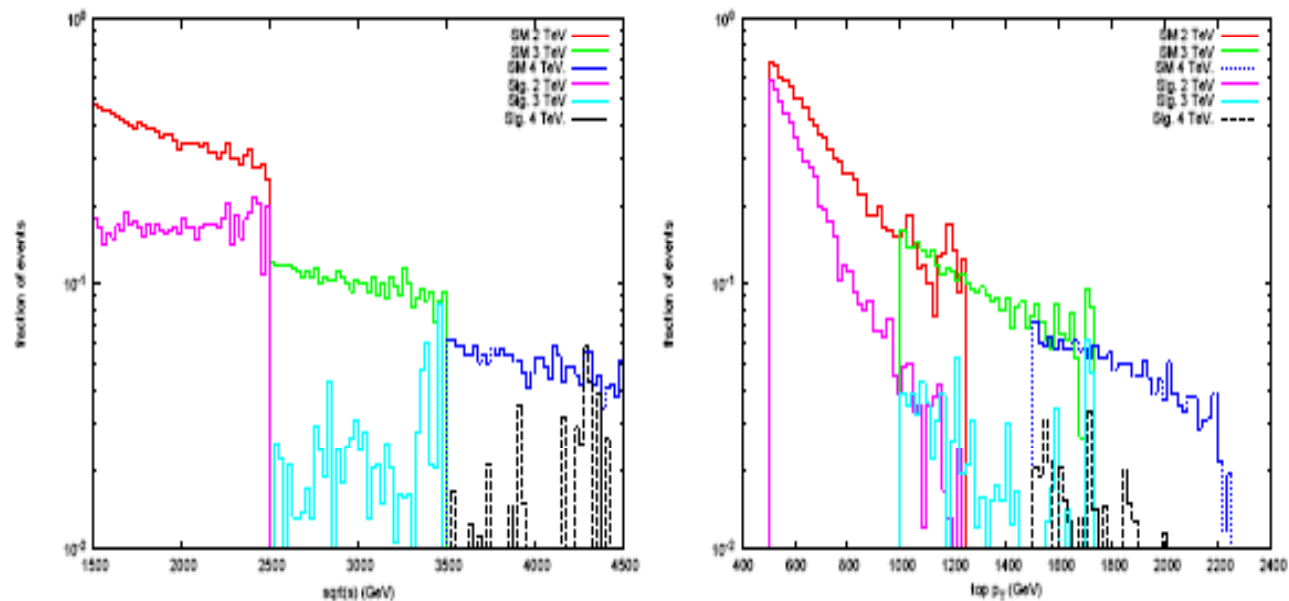


Figure 7: Left: Fraction of events where at least 1 top decays leptonically with an isolated lepton. The cuts imposed are: top $p_T > (500, 1,000, 1,500)$ GeV for KK masses (2,3,4) TeV, the invariant mass in a window 1 TeV wide around the resonance. The lines labeled "SM" are for SM $t\bar{t}$ production with the same cuts.

Clearly...

- Efficient top jet identification required, especially for heavier KK gluons
- Could be:
 - Top jet mass measurement
 - Detailed substructure of jets: eg hard lepton
- Beyond our study
- But can determine accuracy with which determination is required

Conclude require at most 10% fake
rate

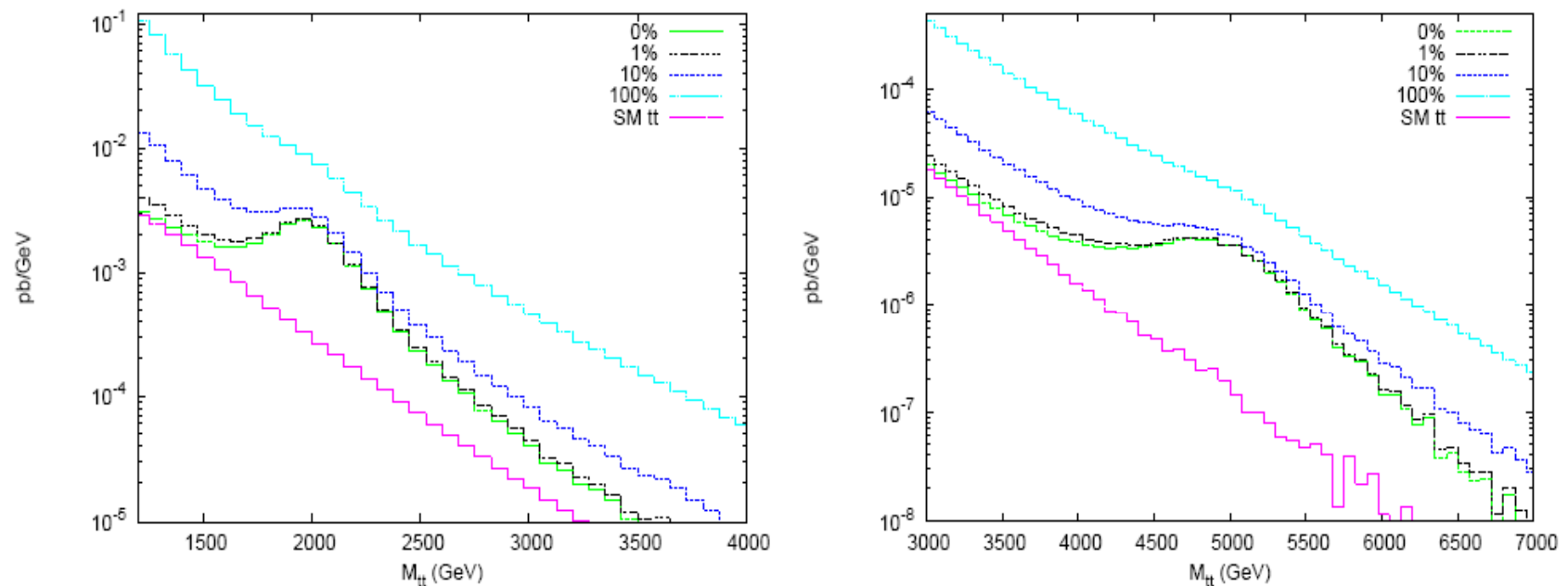


Figure 9: $t\bar{t}$ invariant mass distribution for signal plus some background assuming various values of the t -tagging fake rate for a resonance at 2 (left) and 5 (right) TeV.

Spin Determination

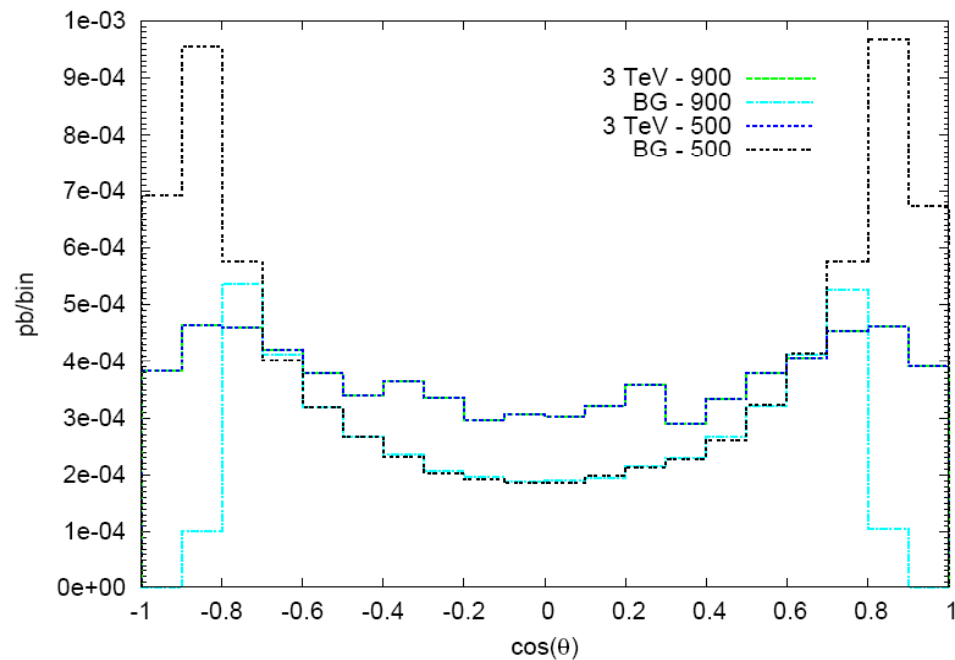


Figure 10: Distribution of $\cos \theta$, the decay angle in the hard-scattering CM frame, from a 3 TeV KK-gluon resonance. We show signal and background for two different choices of p_T cuts.

Forward region significance enhanced with p_T cut

For light masses, can determine
model structure: t_R

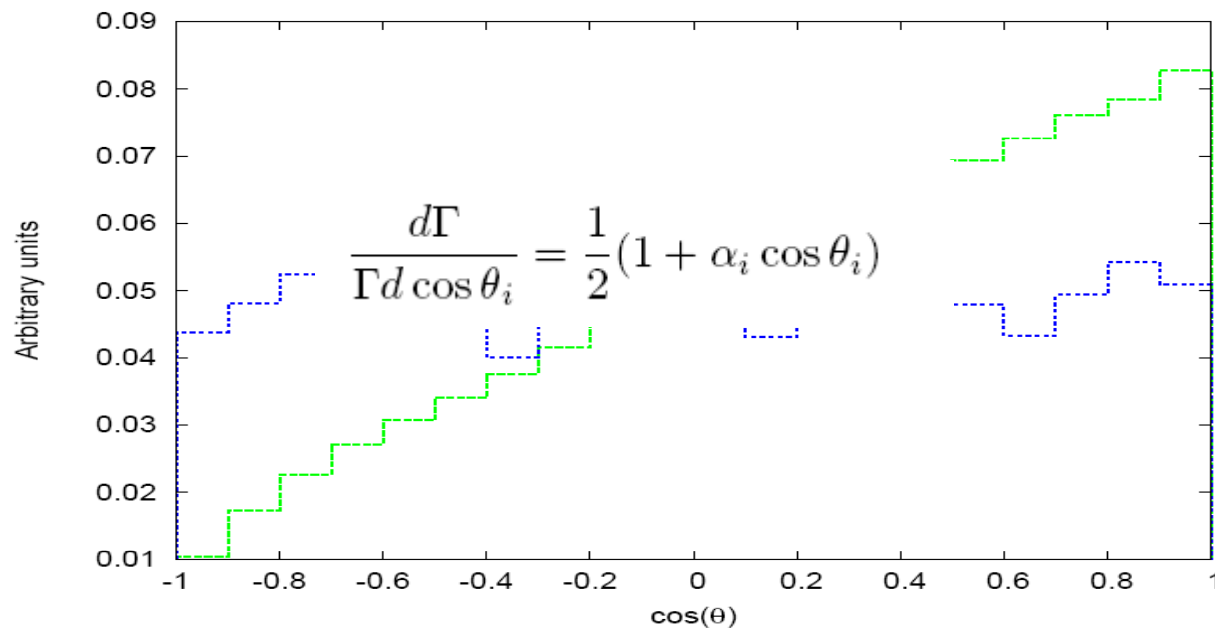


Figure 11: Normalized distribution $\cos(\phi)$, defined in the text, for a 3 TeV resonance (blue, dotted), and for QCD top production (green, dashed).

Angle between lepton and top momentum axis

Summary

- Weak scale physics should be testable at LHC
- Including RS1, whatever the implementation
- Best signature: spin-2 resonance and mass gap
- In bulk, gluon KK mode will be important
- Decays into tops critical
- Challenge is to maximize energy reach
- But hope for pinning this down

To be done

- Think about jet characteristics
 - Detailed substructure
- Push b tagging to limit
- Explore hard leptons

Conclude

- If RS1 solves the hierarchy problem, we should be able to tell
- Clean KK graviton signal if SM on brane
- Far-reaching KK gluon signal if SM in bulk
- Supplemented by graviton KK signal if sufficiently low scales
- But challenges: top quark ID foremost!