A Formal Semantics for Weighted Ontology Mappings

Manuel Atencia Alexander Borgida Jérôme Euzenat
Chiara Ghidini Luciano Serafini

November 13, 2012

Crisp vs. Weighted Mappings

Crisp Mapping

$1 : car \sqsubseteq 2 : vehicle$
$1 : person \sqsupseteq 2 : student$
$1 : dog \equiv 2 : cane$
$1 : nutella \perp 2 : healthyFood$

Weighted mappings

$1 : pickUp \sqsupseteq 2 : automobile$ [0.9]
$1 : technician \sqsupseteq 2 : engineer$ [0.7]
$1 : spagetti \equiv 2 : pasta$ [0.8]
$1 : turtle \perp 2 : wildAnimal$ [0.9]

Motivations and objectives

Motivation

- Ontology matchers mostly return mappings with weights
- mapping refinement, mapping inconsistency check, mapping minimality, mapping compositions, ... are all useful operations, which are base on an logical inference on mappings
- no shared view on how weights should be interpreted $\longrightarrow$ no well founded inference on weighted mappings

Motivations and objectives

Motivation

- Ontology matchers mostly return mappings with weights
- mapping refinement, mapping inconsistency check, mapping minimality, mapping compositions, . . . are all useful operations, which are base on an logical inference on mappings
- no shared view on how weights should be interpreted $\longrightarrow$ no well founded inference on weighted mappings

Objectives

- Define a formal semantics for weighted ontology mappings
- Characterize a notion of logical consequence between weighted mappings
- Design a set of sound inferences on weighted mappings

Classificational semantics

Classificational interpretation of mappings

If O_1 and O_2 are two ontologies used to classify a common set of objects X, then mappings between O_1 and O_2 are interpreted to encode how elements of X classified in the concepts of O_1 are re-classified in the concepts of O_2, and weights are interpreted to measure how precise and complete re-classifications are.

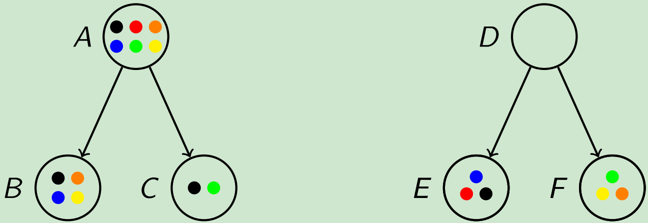
Example

Classificational semantics

Classificational interpretation of mappings

If O_1 and O_2 are two ontologies used to classify a common set of objects X, then mappings between O_1 and O_2 are interpreted to encode how elements of X classified in the concepts of O_1 are re-classified in the concepts of O_2, and weights are interpreted to measure how precise and complete re-classifications are.

Example



Classificational semantics

Classificational interpretation of mappings

If O_1 and O_2 are two ontologies used to classify a common set of objects X, then mappings between O_1 and O_2 are interpreted to encode how elements of X classified in the concepts of O_1 are re-classified in the concepts of O_2, and weights are interpreted to measure how precise and complete re-classifications are.

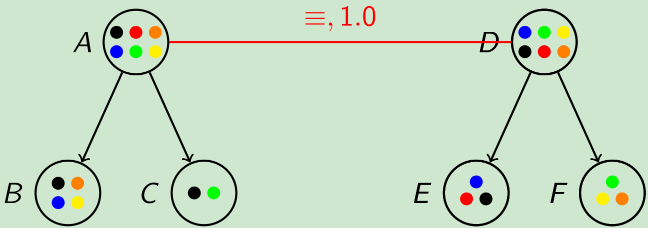
Example

Classificational semantics

Classificational interpretation of mappings

If O_1 and O_2 are two ontologies used to classify a common set of objects X, then mappings between O_1 and O_2 are interpreted to encode how elements of X classified in the concepts of O_1 are re-classified in the concepts of O_2, and weights are interpreted to measure how precise and complete re-classifications are.

Example

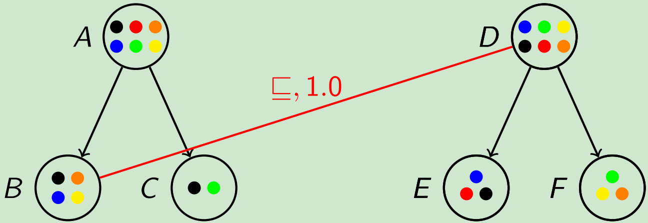


Classificational semantics

Classificational interpretation of mappings

If O_1 and O_2 are two ontologies used to classify a common set of objects X, then mappings between O_1 and O_2 are interpreted to encode how elements of X classified in the concepts of O_1 are re-classified in the concepts of O_2, and weights are interpreted to measure how precise and complete re-classifications are.

Example

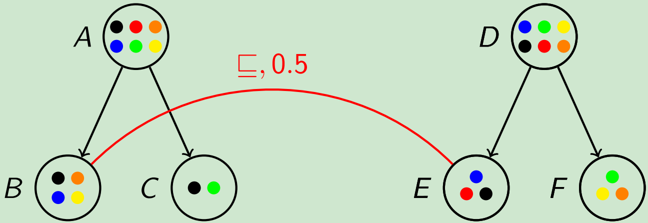


Classificational semantics

Classificational interpretation of mappings

If O_1 and O_2 are two ontologies used to classify a common set of objects X, then mappings between O_1 and O_2 are interpreted to encode how elements of X classified in the concepts of O_1 are re-classified in the concepts of O_2, and weights are interpreted to measure how precise and complete re-classifications are.

Example

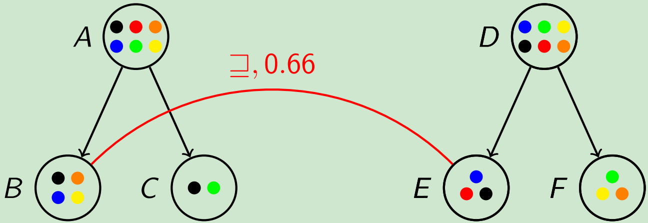


Classificational semantics

Classificational interpretation of mappings

If O_1 and O_2 are two ontologies used to classify a common set of objects X, then mappings between O_1 and O_2 are interpreted to encode how elements of X classified in the concepts of O_1 are re-classified in the concepts of O_2, and weights are interpreted to measure how precise and complete re-classifications are.

Example



Paper contribution

1. novel **formal semantics** for interpreting the confidence value associated with a mapping based on the standard quality measure of reclassification such as **precision, recall,** and **F-measures**
2. We prove that this semantics is a **conservative extension** of the semantics of crisp mappings (as given in DDL)
3. we define a notion of **logical entailment between weighted mappings**
4. we designe a set of sound **inference rules** on weighted mappings (completeness is an open problem)

Weighted Mappings

Definition (Weighted mapping)

Let $\{O_i\}_{i \in I}$ be a family of ontologies. A *weighted mapping* from O_i to O_j is an expression of the form

$$\boxed{i : C \ r_{[a,b]} \ j : D}$$

where

- C and D are concepts of O_i and O_j, respectively
- $r \in \{\sqsubseteq, \equiv, \sqsupseteq, \perp\}$
- $a, b \in [0, 1]$.

Basic properties of weighted mappings

From sup/subset mappings to equivalence mapping:
$\sqsubseteq, \sqsupseteq \quad \Longrightarrow \quad \equiv$

The following rules allows to infer equivalence mappings starting from $\sqsubseteq$ and $\sqsupseteq$ mappings:

$$\left.\begin{array}{l} i:A \sqsubseteq_{[a,b]} j:G, \\ i:A \sqsupseteq_{[c,d]} j:G \end{array}\right\} \quad \Longrightarrow \quad i:A \equiv_{[v,w]} j:G$$

Basic properties of weighted mappings

From sup/subset mappings to equivalence mapping: $\sqsubseteq, \sqsupseteq \implies \equiv$

The following rules allows to infer equivalence mappings starting from $\sqsubseteq$ and $\sqsupseteq$ mappings:

$$\left.\begin{array}{l} i:A \sqsubseteq_{[a,b]} j:G, \\ i:A \sqsupseteq_{[c,d]} j:G \end{array}\right\} \implies i:A \equiv_{[v,w]} j:G$$

$$v = \begin{cases} \frac{2ac}{a+c} & \text{if } a \neq 0 \text{ or } c \neq 0 \\ 0 & \text{if } a = c = 0 \end{cases} \quad w = \begin{cases} \frac{2bd}{b+d} & \text{if } b \neq 0 \text{ or } d \neq 0 \\ 0 & \text{if } b = d = 0 \end{cases}$$

Basic properties of weighted mappings

From sup/subset mappings to equivalence mapping: $\sqsubseteq, \sqsupseteq \implies \equiv$

The following rules allows to infer equivalence mappings starting from $\sqsubseteq$ and $\sqsupseteq$ mappings:

$$\left.\begin{array}{l} i:A \sqsubseteq_{[a,b]} j:G, \\ i:A \sqsupseteq_{[c,d]} j:G \end{array}\right\} \implies i:A \equiv_{[v,w]} j:G$$

$$v = \begin{cases} \frac{2ac}{a+c} & \text{if } a \neq 0 \text{ or } c \neq 0 \\ 0 & \text{if } a = c = 0 \end{cases} \quad w = \begin{cases} \frac{2bd}{b+d} & \text{if } b \neq 0 \text{ or } d \neq 0 \\ 0 & \text{if } b = d = 0 \end{cases}$$

Example

$$\frac{\begin{array}{l} 1 : \textit{teacher} \ \sqsubseteq_{[0.7,1]} \ 2 : \textit{professor} \\ 1 : \textit{teacher} \ \sqsupseteq_{[0.6,0.8]} \ 2 : \textit{professor} \end{array}}{}$$

Basic properties of weighted mappings

From sup/subset mappings to equivalence mapping:
$\sqsubseteq, \sqsupseteq \quad \Longrightarrow \quad \equiv$

The following rules allows to infer equivalence mappings starting from $\sqsubseteq$ and $\sqsupseteq$ mappings:

$$\left.\begin{array}{l} i : A \sqsubseteq_{[a,b]} j : G, \\ i : A \sqsupseteq_{[c,d]} j : G \end{array}\right\} \quad \Longrightarrow \quad i : A \equiv_{[v,w]} j : G$$

$$v = \begin{cases} \frac{2ac}{a+c} & \text{if } a \neq 0 \text{ or } c \neq 0 \\ 0 & \text{if } a = c = 0 \end{cases} \quad w = \begin{cases} \frac{2bd}{b+d} & \text{if } b \neq 0 \text{ or } d \neq 0 \\ 0 & \text{if } b = d = 0 \end{cases}$$

Example

$$\frac{\begin{array}{l} 1 : \textit{teacher} \;\; \sqsubseteq_{[0.7,1]} \;\; 2 : \textit{professor} \\ 1 : \textit{teacher} \;\; \sqsupseteq_{[0.6,0.8]} \;\; 2 : \textit{professor} \end{array}}{1 : \textit{teacher} \;\; \equiv_{[0.65,0.88]} \;\; 2 : \textit{professor}}$$

A complete inference system on weighted mappings

Interesting question

The properties we have shown before correspond to a set of rules that allow to infer new mappings from existing mappings. Is these set of rules an complete inference system for logical consequence on weighted mappings?

A complete inference system on weighted mappings

Interesting question

The properties we have shown before correspond to a set of rules that allow to infer new mappings from existing mappings. Is these set of rules an complete inference system for logical consequence on weighted mappings?

Answer

We don't know. This is an open problem.

Conclusions

We have defined and investigated a (re-)classificational semantics for weighted mappings which extends the DDL semantics for crisp mappings.

In details this semantics:

- Reflects a family of approaches used in ontology matching techniques;
- Preserves the classical DDL semantics in the sense that if crisp DDL mappings are encoded as weighted mappings with $[1, 1]$ weights, the consequences correspond.
- Provides a clear definition of weighted mapping entailment.

Future work:

- apply it to mapping (or ontology) debugging by ranking given and inferred mappings according to their weight intervals and help to detect those near mappings which would not appear as crisp mappings but are mappings of high weight.
- investigate foundational aspects and properties of reclassificational mappings.