Large Linear Classification When Data Cannot Fit In Memory

Hsiang-Fu Yu

Department of Computer Science
National Taiwan University

Joint work with C.-J. Hsieh, K.-W. Chang, and C.-J. Lin

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Outline

- Introduction
- A Block Minimization Framework for Linear SVMs
- Implementations for SVM
- Techniques to Reduce the Training Time
- Other Functionalities
- Experiments
- Conclusions

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Linear Classification

- Recently linear classification is a popular research topic
- By linear we mean that kernel is not used
- Though linear may not be as good as nonlinear
- for some problems:

  accuracy by linear is as good as nonlinear, and

  training and testing are much faster
- This talk addresses on large linear classification

Motivation

Existing approaches for large linear classification:

- Data smaller than memory:
  Efficient methods are well-developed
- Data beyond disk size:
  Usually handled in a distributed way

Can we have something in the between?

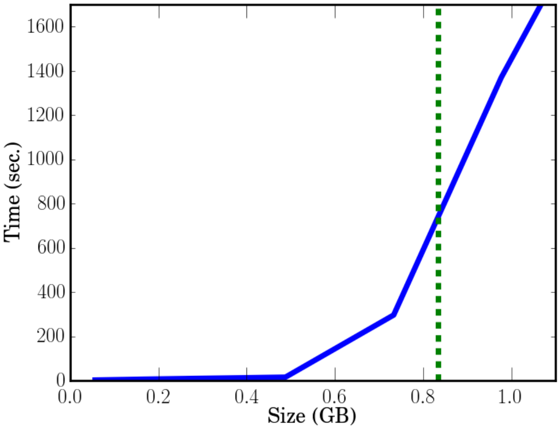
- A simple setting

  $$\text{memory} < \text{data} < \text{disk}$$

- Ferris and Munson (2003) proposed a method, but only for data with # features ≪ # instances

When Data Cannot Fit In Memory

LIBLINEAR on a machine with 1 GB memory:



Disk swap causes lengthy training time

The Goal

Goal: construct large linear classifiers for ordinary users on a single machine

Assumptions

- memory $<$ data $<$ disk
- Sub-sampling causes lower accuracy

Requirement: must be simple so that it supports

- Multi-class classification
- Parameter selection,
- Other functionalities

Modeling the Training Time

$$\text{train time} = \text{time to train in-memory data} + \text{time to access data from disk}$$

- Now need to pay attention to the second part
- Loading time may dominate the training time *even data can fit in memory*
- `> ./liblinear-1.51/train rcv1_test.binary`

  rcv1_test.binary: $>$ half millions of documents

  Loading time: $>$ 1 minute

  Computing time: $<$ 5 seconds

Conditions for a Viable Method

1. Each optimization step reads a *continuous* chunk of training data.
2. The optimization procedure converges toward the optimum.
3. The number of optimization steps should not be too large.

Linear SVM as the Linear Classifier

We consider SVM as the linear classifier

- Training data $\{(y_i, \mathbf{x}_i)\}_1^l$, $\mathbf{x}_i \in R^n$, $y_i = \pm 1$
- n: # of features, l: # of data
- Primal SVM:

$$\min_{\mathbf{w}} \ \frac{1}{2}\mathbf{w}^T\mathbf{w} + C\sum_{i=1}^{l} \max\left(0, 1 - y_i\mathbf{w}^T\mathbf{x}_i\right)$$

- Dual SVM:

$$\begin{aligned} \min_{\boldsymbol{\alpha}} \quad & \frac{1}{2}\boldsymbol{\alpha}^T Q\boldsymbol{\alpha} - \mathbf{e}^T\boldsymbol{\alpha} \\ \text{subject to} \quad & 0 \le \alpha_i \le C, \forall i, \end{aligned}$$

- $\mathbf{e} = [1, \ldots, 1]^T$, $Q_{ij} = y_i y_j \mathbf{x}_i^T\mathbf{x}_j$
- $\boldsymbol{\alpha} \in R^l$, each α_i corresponds to $\mathbf{x}_i$

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A Block Minimization Framework

Algorithm 1

```
1. Split $\{1, \ldots, l\}$ to $B_1, \ldots, B_m$ such that $B_i$ fits in memory, and store data into $m$ files accordingly.
2. Set initial $\boldsymbol{\alpha}$ or $\mathbf{w}$
3. For $k = 1, 2, \ldots$    (outer iteration)
       For $j = 1, \ldots, m$    (inner iteration)
           (a) Read $\mathbf{x}_r, \forall r \in B_j$ from disk
           (b) Conduct operations on $\{\mathbf{x}_r \mid r \in B_j\}$
           (c) Update $\boldsymbol{\alpha}$ or $\mathbf{w}$
```

Here we do not specify operations on each block

Block Minimization

A classical optimization method

- Block of variables
- Widely used in nonlinear SVM
- Here need a connection between a block of data and a block of variables

In the situation, data $>$ memory

- to avoid random access on the disk
- cannot use holistic methods to select block variables
- $B_1, \ldots, B_m$: fixed partition of $\{1, \ldots, l\}$

Number of Blocks and Block Size

How to decide m ($\#$ of blocks)

- Assume all blocks have similar size $|B|$
- $\#$ blocks: $m = \frac{l}{|B|}$

Block size

- Cannot be too large: each B_j must fit in memory
- Cannot be too small: should be as large as possible

Total time for an outer iteration:

$$(T_m(|B|) + T_d(|B|)) \times \frac{l}{|B|} \qquad m = \frac{l}{|B|}$$

- $T_m(|B|)$: time cost of one inner iteration in memory
- $T_d(|B|)$: time cost of reading B from disk
- Both $T_m(|B|)$ and $T_d(|B|)$ are functions of $|B|$

Block Size Should Be Large

Total time for an outer iteration:

$$(T_m(|B|) + T_d(|B|)) \times \frac{l}{|B|}$$

Past, $T_m(|B|)$ only: $T_m(|B|)$ more than linear to $|B|$

- Total time $= T_m(|B|) \times \frac{l}{|B|}$
- previous SVM works: smaller $|B|$ is better

Now, $T_d(|B|)$ added: $T_d(|B|)$: initial cost $+ O(|B|)$

- Total reading time $=$ initial cost $\times \frac{l}{|B|} + O(1)$
- Larger $|B|$ is better (but can't exceed memory)

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Sub-problem for Dual SVM

Let $f(\boldsymbol{\alpha})$ be the dual function:

$$f(\boldsymbol{\alpha}) \equiv \frac{1}{2}\boldsymbol{\alpha}^T Q \boldsymbol{\alpha} - \mathbf{e}^T \boldsymbol{\alpha}$$

Each block of variables corresponds to a block of data

$$\begin{aligned} \min_{\mathbf{d}_{B_j}} \quad & f(\boldsymbol{\alpha} + \mathbf{d}) \\ \text{s.t.} \quad & \mathbf{d}_{\bar{B}_j} = \mathbf{0} \text{ and } 0 \leq \alpha_i + d_i \leq C, \ \forall i \in B_j \end{aligned} \qquad (1)$$

- $\bar{B}_j = \{1, \ldots, l\} \backslash B_j$; only $\boldsymbol{\alpha}_{B_j}$ is changed
- (1) involves all data; handled by some techniques (details omitted)

An Implementation for Dual SVM

Algorithm 2 A special case of Algorithm 1

```
1. Split $\{1, \ldots, l\}$ to $B_1, \ldots, B_m$ and store data to $m$ files
2. Set initial $\alpha$ and $\mathbf{w}$
3. For $k = 1, 2, \ldots$    (outer iteration)
     For $j = 1, \ldots, m$    (inner iteration)
       (a) Read $\mathbf{x}_r, \forall r \in B_j$ from disk
       (b) Approximately solve the
           sub-problem to obtain $\mathbf{d}^*_{B_j}$.
       (c) Update $\boldsymbol{\alpha}_{B_j} \leftarrow \boldsymbol{\alpha}_{B_j} + \mathbf{d}^*_{B_j}$ and $\mathbf{w}$
```

Solving Sub-problem By LIBLINEAR

Any bound-constrained method can be used

- We consider LIBLINEAR: a coordinate descent method

Two-level block minimization

- Used in some algorithms (e.g., Memisevic, 2006; Pérez-Cruz et al., 2004; Rüping, 2000)

  But here inner ⇒ memory, outer ⇒ disk
- An approximate solution for the sub-problem in practice

  Sub-problem stopping criterion and convergence are issues

Sub-problem Stopping Condition and Overall Convergence

Two approaches

1. A fixed number of passes to all variables in B_j

   Need to decide the number of passes

2. Gradient-based stopping condition

   Easy to know how accurate the sub-problem's solution is; we use the one in LIBLINEAR

Convergence holds for both conditions (details omitted)

Block Minimization for Primal SVM

Let f^P be the primal function

$$f^P(\mathbf{w}) = \frac{1}{2}\mathbf{w}^T\mathbf{w} + C\sum_{i=1}^{l} \max\left(0, 1 - y_i\mathbf{w}^T\mathbf{x}_i\right)$$

A block of primal variable **w**

- corresponds to a subset of features
- no connection to a block of data

Stochastic gradient descent (SGD) approach

- For each update only a block of data is needed
- We use Pegasos (Shalev-Shwartz et al., 2007)

Pegasos for Each Block

Algorithm 3 A special cases of Algorithm 1

```
1. Split $\{1, \ldots, l\}$ to $B_1, \ldots, B_m$ and store data into $m$
   files accordingly.
2. $t = 0$ and initial $\mathbf{w} = \mathbf{0}$
3. For $k = 1, 2, \ldots$
      For $j = 1, \ldots, m$
         (a) Find a partition of $B_j$: $B_j^1, \ldots, B_j^{\bar{r}}$
         (b) For $r = 1 \ldots, \bar{r}$
            (b.1) Apply Pegasos update on $B_j^r$
            (b.2) $t \leftarrow t + 1$
```

- $\bar{r} = 1$: only one update on the whole block
- $\bar{r} = |B|$: $|B|$ updates, one for each data instance

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Techniques To Reduce the Training Time

Data compression for disk reading time $T_d(|B|)$

- Except initial time, $T_d(|B|) \propto$ data size $|B|$
- Data compression effectively reduces the disk reading time (Details not shown)

Initial Split of Data

- If original data ordered by labels

  a whole block with same label

  $\Rightarrow$ slow convergence
- A random split is needed

Techniques To Reduce the Training Time

Two tasks in the beginning:

- random split
- data compression

Data $>$ memory:

- avoid re-reading data from disk
- A carefully design ensures

  Random split+data compression by going data only **once**

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Other Functionalities

Due to the simplicity and block design, we can support

- Cross validation
- Multi-class classification
- Incremental/Decremental setting

Details omitted here.

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Data and Environment

Data set	l (data)	n (features)	Mem
yahoo-korea	460,554	3,052,939	2.5GB
webspam	350,000	16,609,143	20.8GB
epsilson	500,000	2,000	16.0GB

- 64-bit machine with 1 GB RAM

  Data 20 times larger

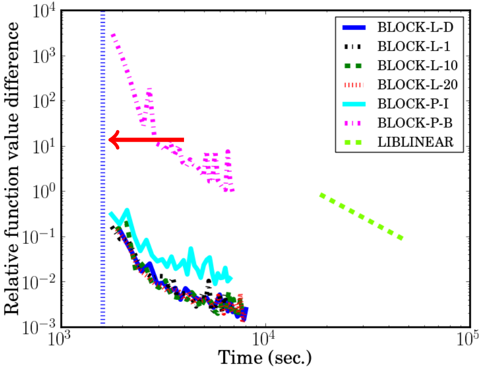
Compared Methods

BLOCK-$\star$-$\star$: Block minimization methods.

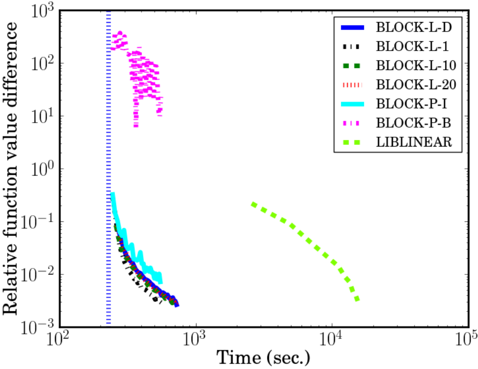
1. BLOCK-L-N: Solving dual. LIBLINEAR goes through each block N rounds; $N = 1, 10, 20$.
2. BLOCK-L-D: Solving dual. LIBLINEAR default stopping condition for each block.
3. BLOCK-P-B: Solving primal. Pegasos on each whole block (one update).
4. BLOCK-P-I: Solving primal. Pegasos on each data instance of the block ($|B|$ updates).
5. LIBLINEAR: The standard LIBLINEAR without any modification.

Function Value Reduction

webspam



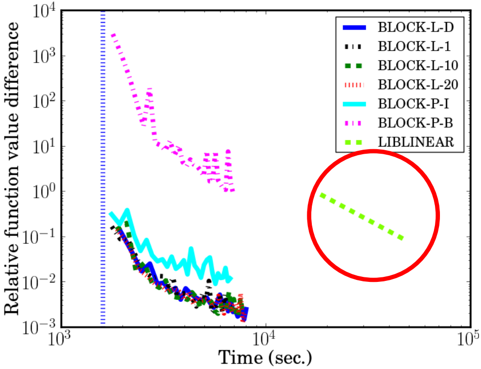
yahoo-korea



Time for initial block split

Function Value Reduction

webspam



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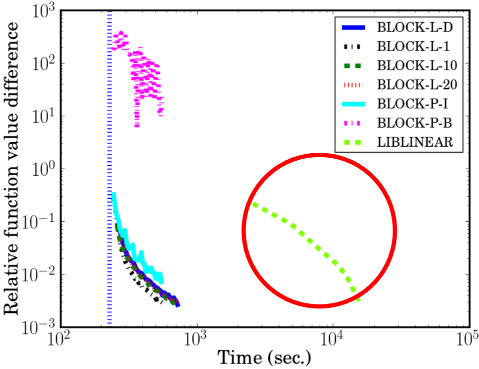
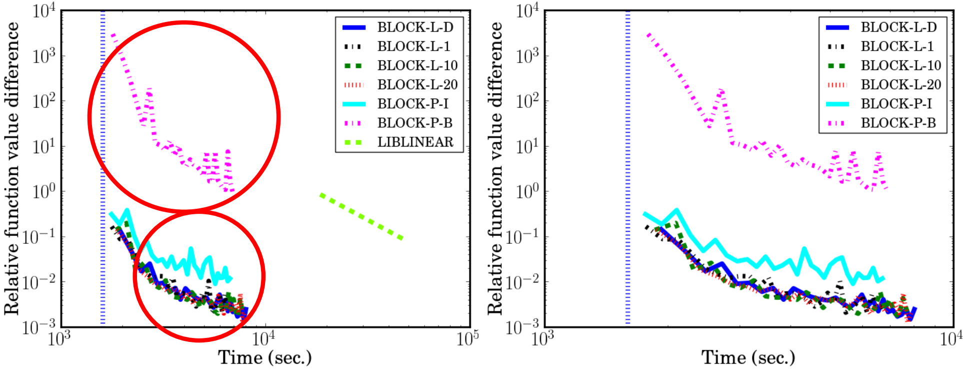


Proposed methods are faster than LIBLINEAR

Function Value Reduction



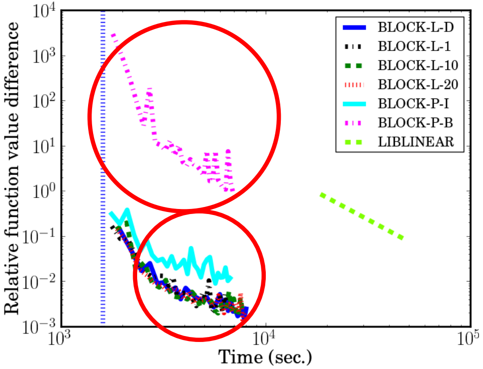
webspam

Magnified view

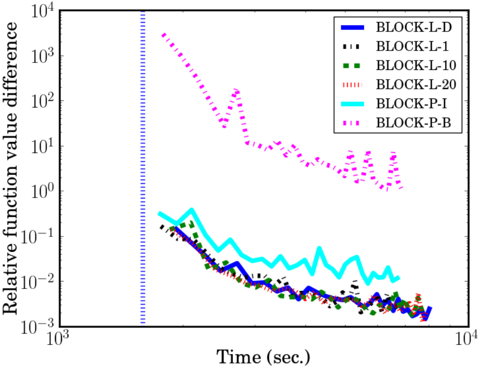
BLOCK-P-⋆ are worse than BLOCK-L-⋆

Function Value Reduction

webspam



Magnified view



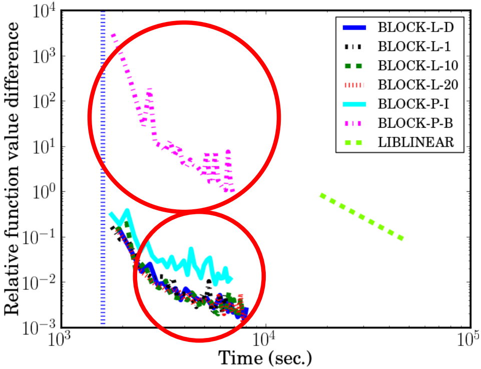
BLOCK-P-$\star$ are worse than BLOCK-L-$\star$

BLOCK-P-B: applies only one update on each block

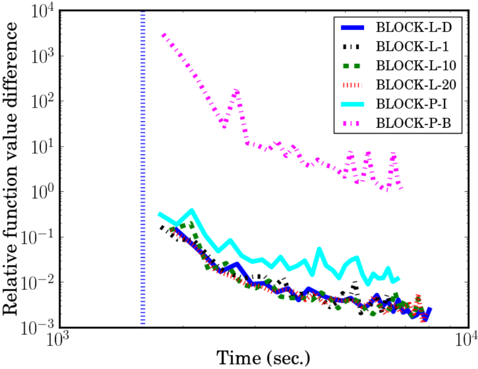
Information of a block underutilized

Function Value Reduction

webspam



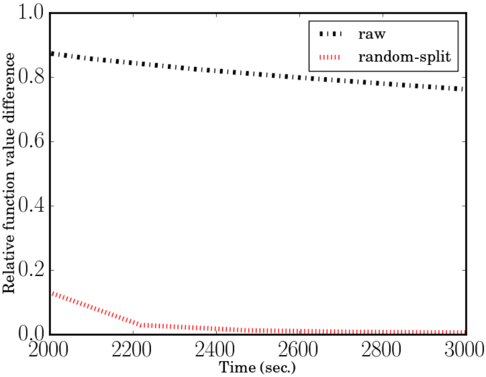
Magnified view



Due to long reading time: put more effort on each block

Other Experimental Results

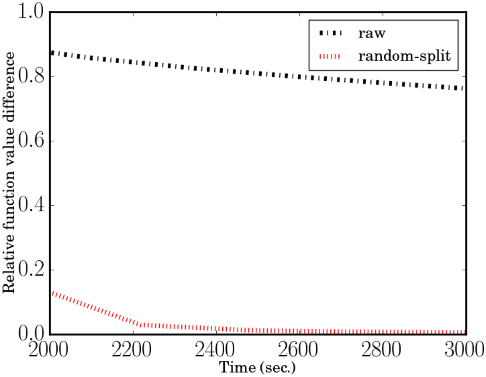
Random split vs. Raw



raw: Data are ordered according to labels
random split: Initial random split

Other Experimental Results

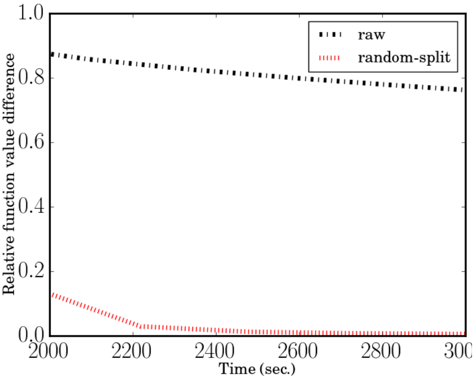
Random split vs. Raw



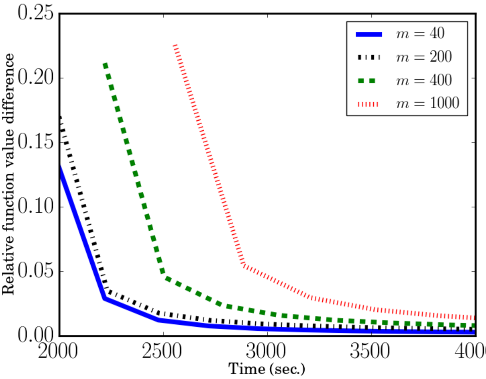
Random split is useful

Other Experimental Results

Random split vs. Raw



Different block size



m: # of blocks ⇒ smaller m; should use larger $|B|$

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Conclusions

- We have proposed methods to effectively handle data 20 times larger than memory
- Our implementation is available at:

  `http://www.csie.ntu.edu.tw/~cjlin/libsvmtools/#large_linear_classification_when_data_cannot_fit_in_memory`
- You can now train pretty large data on your laptop