Multi-criteria Anomaly Detection using Pareto Depth Analysis

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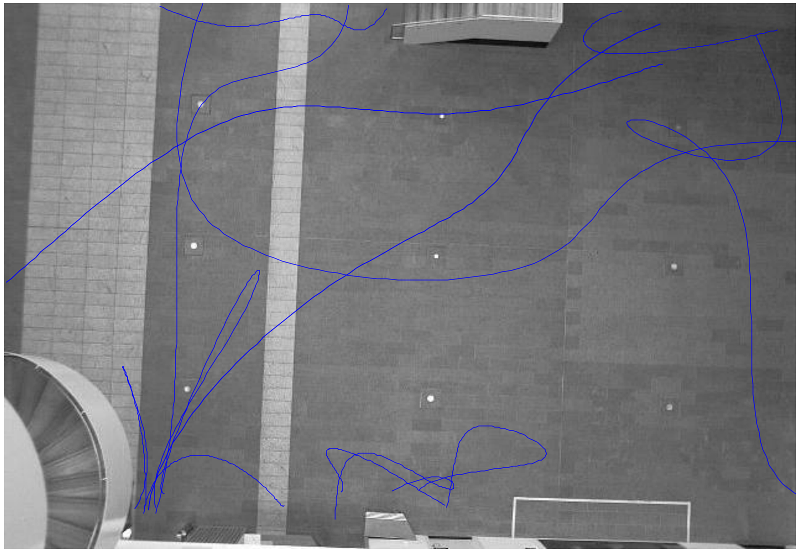
Motivation: Detect anomalous pedestrian trajectories.

Question: Which one of these groups of trajectories are anomalous?



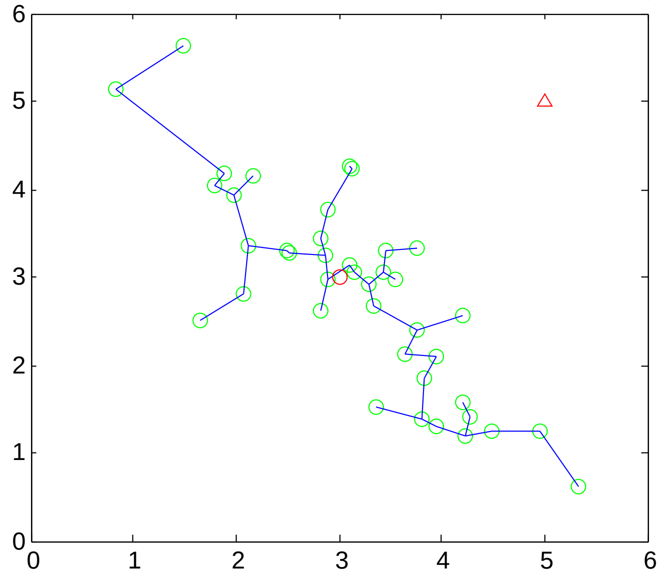
Anomalous trajectories

Nominal trajectories

Curve features: curve length, shape, walking speed.

Graph-based anomaly detection

MST graph using similarities $D(i,j)$

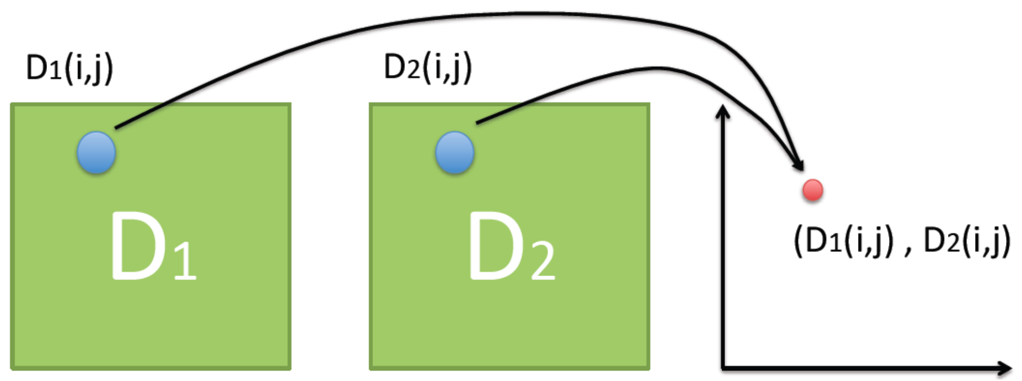


Trajectory X is anomalous if:

- X is most distant from its k-NN points **(Byers&Raftery 1998)**
- X is most distant from avg of its k-NNs **(Angiulli&Pizutti 2002)**
- X lies outside of capture range of k-MST **(Hero 2006)**

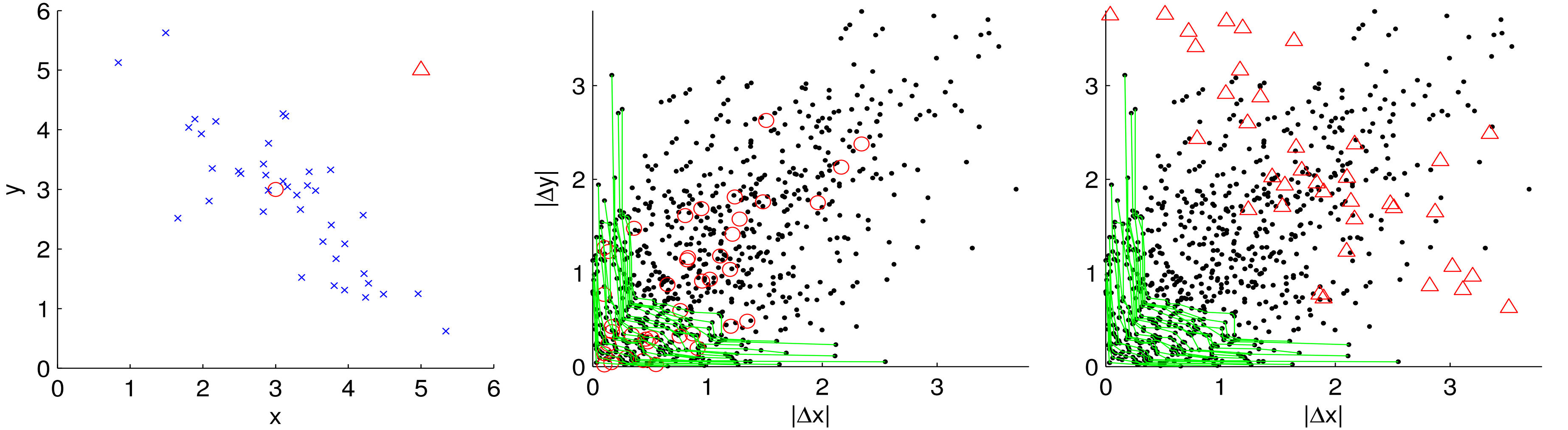
How to do anomaly detection with multiple features/criteria?

1. Scalarization:
   $$D_\lambda(i,j) = \lambda D_1(i,j) + (1-\lambda) D_2(i,j)$$
2. Pareto depth analysis:
   $(D_1(i,j), D_2(i,j))$ → one *dyad*



When Pareto fronts are convex, 1 is special case of 2. (n.b. Thm 2 in paper.)

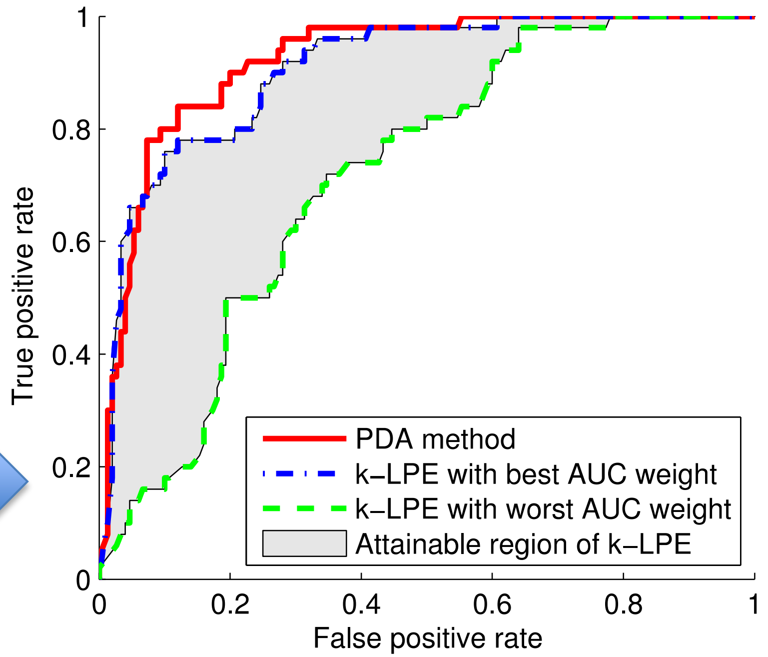
Pareto depth analysis

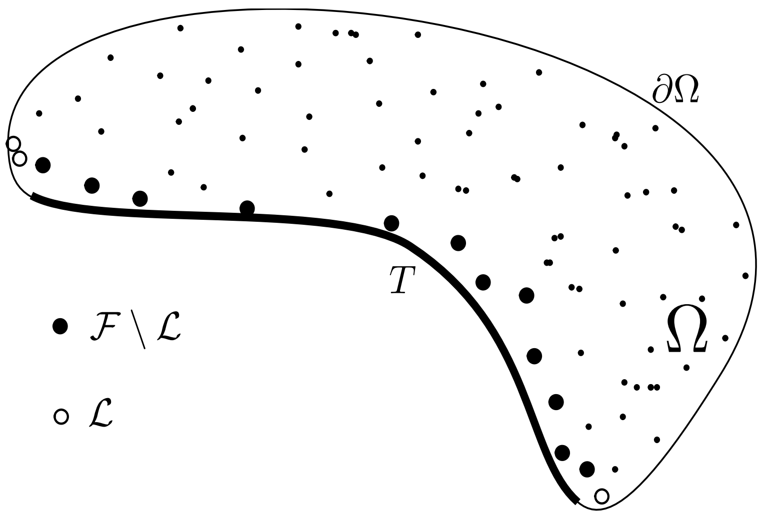


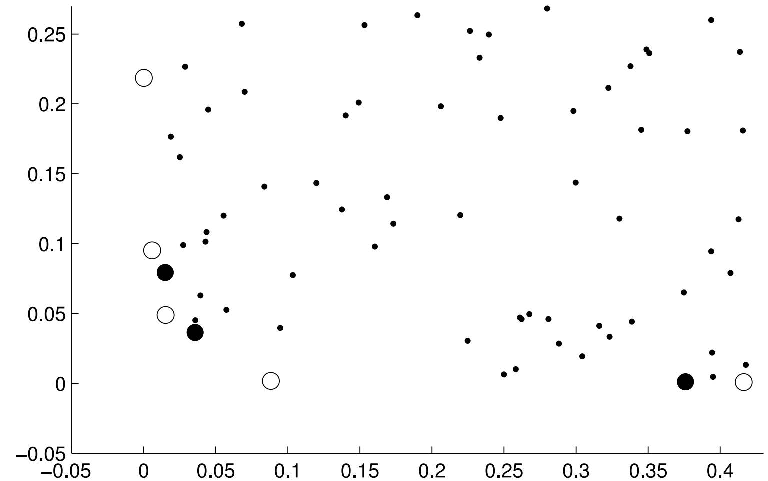
PDA Algorithm:

- Embed N choose 2 dyads onto plane
- Build Pareto fronts of non-dominated dyads.
- Compute anomaly scores = depth of front.

PDA outperforms scalarization







Inadequacy of scalarization. Non-convexities in the Pareto front induced by: **Left** the geometry of the domain (Thm. 1); **Right** randomness in the samples (Thm. 2).

Theorem 1. *Let* $f \in C^1(\overline{\Omega})$ *with* $\inf_\Omega f > 0$. *Let* $T \subset \partial\Omega$ *be open and connected such that*

$$\inf_{z \in T} \min(\nu_1(z), \ldots, \nu_d(z)) \geq \delta > 0, \quad \textit{and} \quad \{y \in \overline{\Omega} : y \preceq x\} = \{x\}, \;\; \text{for} \;\; x \in T.$$

Then for $h > 0$ *sufficiently small, we have*

$$E|\mathcal{F}_{T_h}| = \gamma n^{\frac{d-1}{d}} + \delta^{-d-1} O\left(n^{\frac{d-2}{d}}\right) \;\; \text{as} \;\; n \to \infty,$$

where $\gamma = d^{-1}(d!)^{\frac{1}{d}} \Gamma(d^{-1}) \int_T f(z)^{\frac{d-1}{d}} (\nu_1(z) \cdots \nu_d(z))^{\frac{1}{d}} dz.$