Probabilistic Numerics

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Lake Tahoe
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PASCAL2
Pattern Analysis, Statistical Modelling and
Computational Learning

Numerical Analysis is Inference . . .

being uncertain about deterministic problems

numerical problems: use evaluations of f to estimate

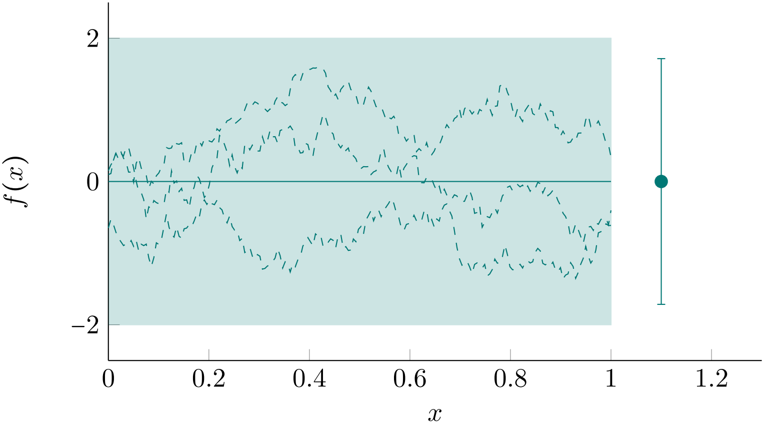
- $Z = \int f(x)\,\mathrm{d}x$ — quadrature
- samples from $p(x) = f(x)/Z$ — Monte Carlo
- $\operatorname{argmin}_x f(x)$ — optimization
- optimal trajectory under dynamics $f(x, t, u)$ — planning / control

f is deterministic for the user of the algorithm, but uncertain for the designer of the algorithm.

- B. Ajne, T. Dalenius.
  "Några tillämpningar av statistika ideer på numerisk integration"
  Nordisk Math. Tidskrift, 1960
- P. Diaconis. "Bayesian Numerical Analysis"
  Statistical Decision Theory and Related Topics, 1988
- D. Calvetti & Erkki Somersalo. "Introduction to Bayesian Scientific Computing"
  Springer, 2007

Example: ℓ_2 optimal quadrature

over a Gaussian process prior [c.f. Minka, 2000]



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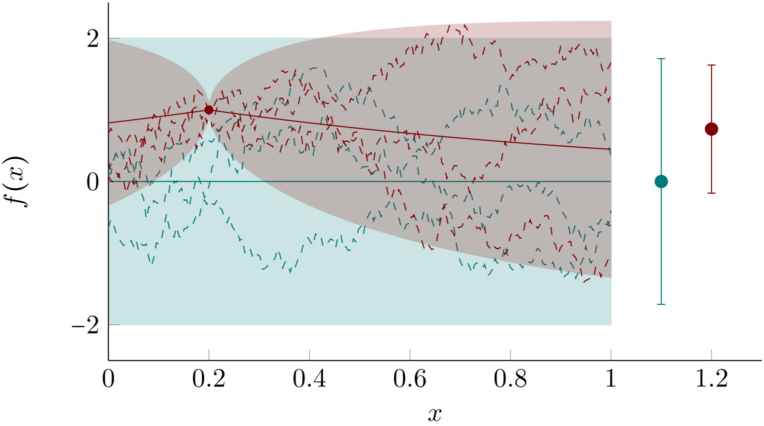

- say what functions you expect to need to integrate
- study how uncertainty drops from evaluation at x

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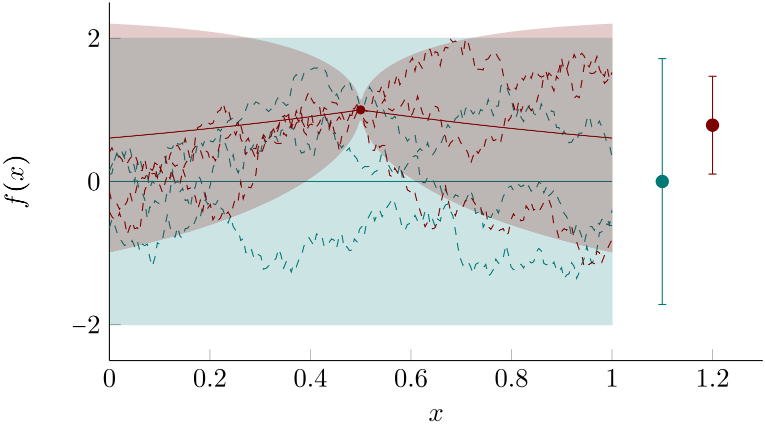

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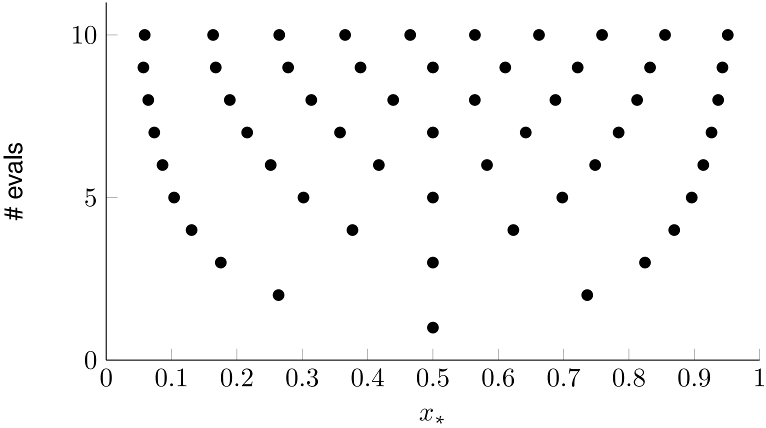

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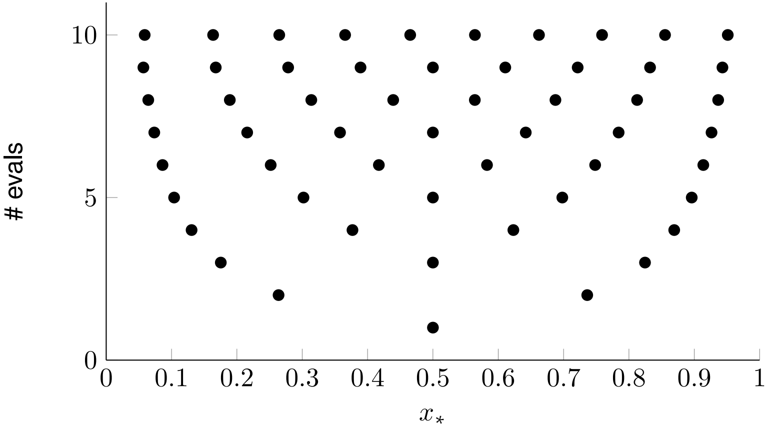

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- optimal location depends on # of evaluations (and kernel!)
- aka Bayesian Quadrature (see e.g. Osborne et al., NIPS 2012)

. . . but Numerical Analysis is not Machine Learning

some observations

Probability Theory can help Numerics . . .

- understand implicit assumptions
- improve, generalise, invent new algorithms

. . . but Numerics has unique requirements

- computational cost
- robustness

let's discuss what either field can give the other

Schedule

NIPS 2012 workshop on probabilistic numerics

07:45	Matthias Seeger	PASCAL2 invited talk
08:15	Jacek Gondzio	PASCAL2 invited talk
08:55	Coffee Break	
09:30	David Duvenaud	
10:00	Spotlights	
10:10	Posters & Discussion	

16:00	Persi Diaconis	PASCAL2 invited talk
16:45	Ben Calderhead	PASCAL2 invited talk
17:15	Coffee Break	
18:00	Ulrich Paquet (canceled) $\leftarrow$ Philipp Hennig	
18:30	Panel Discussion	
19:00	End	