# On the Quantum Complexity of Classical Words

#### Markus Müller

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Markus Müller On the Quantum Complexity of Classical Words

## Outline



### Motivation

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#### **Kolmogorov Complexity**

- Classical Kolmogorov Complexity
- Qubit Strings
- Quantum Kolmogorov Complexity

### 3 Main Theorem

- Statement of the Main Theorem
- Outline of Proof, Part 1
- Outline of Proof, Part 2

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# Outline



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### **Motivation**

# Are quantum computers more powerful than classical computers?

- Quantum computers can solve some problems faster than classical computers (→ Shor's factoring algorithm).
   Answer for *Computational Complexity*. Yes.
- What about description length (compression)? Can classical words be compressed further by allowing quantum descriptions? Answer for Kolmogorov Complexity, 222

For fixed classical words like x = 00100010, compare its classical and its quantum minimal description lengths.

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# **Classical Kolmogorov Complexity**

Finite binary words:  $\{0,1\}^* := \{\varepsilon, 0, 1, 00, 01, 10, 11, 000, ...\}$ A computer is a partial recursive function  $U : \{0,1\}^* \rightarrow \{0,1\}^*$ 

Definition of Kolmogorov Complexity

Let U be a (fixed, but arbitrary) universal computer. Then,

 $C(x) := \min\{\ell(p) \mid U(p) = x\} \qquad (x \in \{0, 1\}^*).$ 

Example



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# **Qubit Strings**

#### Quantum information theory: study superpositions like

$$|\psi
angle:=rac{1}{\sqrt{2}}\left(|10
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#### **Definition (Qubit Strings)**

A qubit string  $\sigma$  is a state vector or density operator on  $\mathcal{H}_{\{0,1\}^*}$ , the Hilbert space with  $\{0,1\}^*$  as orthonormal basis.

Thus,  $|\psi
angle$  is a qubit string, and so is  $\sigma:=rac{2}{3}|\psi
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Distance measure: hace norm ||ρ − σ||<sub>0</sub> = ½T(ρ − σ)
 Length: ℓ(σ) := max{ℓ(s) | (s|σ|s) > 0}.

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#### Properties

- Distance measure: trace norm  $\|\rho \sigma\|_{Tr} := \frac{1}{2}Tr|\rho \sigma$
- Length:  $\ell(q) := \max\{\ell(s) \mid \langle s|q|s \rangle$

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#### **Properties**

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For example  $\ell(|\psi\rangle) = 4$ , and  $\||\psi\rangle\langle\psi| - \sigma\|_{\mathrm{Tr}_{4}} = \frac{1}{34}$ 

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# **Quantum Kolmogorov Complexity**

Similarly as classical computers, quantum computers are partial maps U: input qubit string  $\sigma \mapsto$  output qubit string  $U(\sigma)$ .

#### Definition (pprox Berthiaume et al. 2001)

Let *U* be a universal quantum computer and  $\delta > 0$ . Then, for every qubit string  $\rho$ , define

$$\mathsf{Q}\mathsf{C}^{\delta}(
ho) := \min\{\ell(\sigma) \mid \|
ho - \mathsf{U}(\sigma)\|_{\mathrm{Tr}} \leq \delta\}.$$

Moreover, we set

$$\mathsf{QC}(\rho) := \min \left\{ \ell(\sigma) \mid \| \rho - U(\sigma, k) \|_{\mathrm{Tr}} \leq \frac{1}{k} \text{ for every } k \in \mathbb{N} 
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As classically,  $\mathsf{QC}(
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Statement of the Main Theorem Outline of Proof, Part 1 Outline of Proof, Part 2

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#### Main Theorem

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Statement of the Main Theorem Outline of Proof, Part 1 Outline of Proof, Part 2

### **Statement of the Theorem**

Result: Concerning minimal description lengths, quantum computers are not more powerful than classical computers:

Theorem (Quantum Complexity of Classical Words)

 $C(\mathbf{x}) = QC(|\mathbf{x}\rangle) + O(1).$ 

If  $0 < \delta < \frac{1}{6}$ , then

$$\mathsf{QC}^{\delta}(|x
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If  $0 < \delta < \frac{1}{6}$ , then

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Equation (1) follows from (2) by an appropriate limit  $\delta \rightarrow 0$ . It remains to show Equation (2).

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### **Outline of Proof, Part 1**

#### Theorem (Quantum Complexity of Classical Words)

 $\mathsf{QC}^{\delta}(|\mathbf{x}\rangle) \leq \mathsf{C}(\mathbf{x}) + \mathrm{const.} \leq \frac{\mathsf{QC}^{\delta}(|\mathbf{x}\rangle)}{1-4\delta} + \mathrm{const'}.$ 

#### Proof of $QC^{\delta}(|x\rangle) \leq C(x) + \text{const.}$ :

- Bennett: Every classical computation can be done reversibly...
- ... and can thus be simulated by a universal quantum computer.
- Thus, quantum computers are at least as powerful in compression as classical computers.

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Theorem (Quantum Complexity of Classical Words)

 $\mathsf{QC}^{\delta}(|x\rangle) \leq \mathsf{C}(x) + \mathrm{const.} \leq \frac{\mathsf{QC}^{\delta}(|x\rangle)}{1-4\delta} + \mathrm{const'}.$ 

Proof of  $QC^{\delta}(|x\rangle) \leq C(x) + \text{const.}$ :

- Bennett: Every classical computation can be done reversibly...
- ... and can thus be simulated by a universal quantum computer.
- Thus, quantum computers are at least as powerful in compression as classical computers.

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Statement of the Main Theorem Outline of Proof, Part 1 Outline of Proof, Part 2

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## **Outline of Proof, Part 2**

#### Theorem (Quantum Complexity of Classical Words)

$$C(x) \leq \frac{\mathsf{Q}C^{\delta}(|x\rangle)}{1-4\delta} + \mathrm{const.}$$

Outline of Proof:

- Classical words are mutually orthogonal qubit strings, i.e.  $\langle s|t \rangle = 0$  if  $s, t \in \{0, 1\}^*$  with  $s \neq t$ .
- (Almost) orthogonal outputs must have (almost) orthogonal inputs. There are only few short orthogonal qubit strings.
- They can all be discovered by short classical computer programs that simulate the quantum computer with brute force.

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### Conclusions

 Classical and quantum Kolmogorov complexities agree up to an additive constant on the classical words, e.g.

 $C(x) = QC(|x\rangle) + O(1)$  for every  $x \in \{0, 1\}^*$ .

- Concerning description length alone, quantum and classical computers are equally powerful.
- As C is a special case of QC, both complexities can thus be treated in a single mathematical framework.

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