Kernel Topic Models

Thore Graepel

Microsoft Research Cambridge

Joint work with **Philipp Hennig**, David Stern, and Ralf Herbrich

Overview

- Introduction and Motivation
- The Kernel Topic Model
- The Laplace Bridge: From Hilbert Space to Probabilities and back
- Experimental Results

Talk based on:
Kernel Topic Models. Philipp Hennig, David Stern, Ralf Herbrich, Thore Graepel ; JMLR W&CP 22: 511-519, 2012

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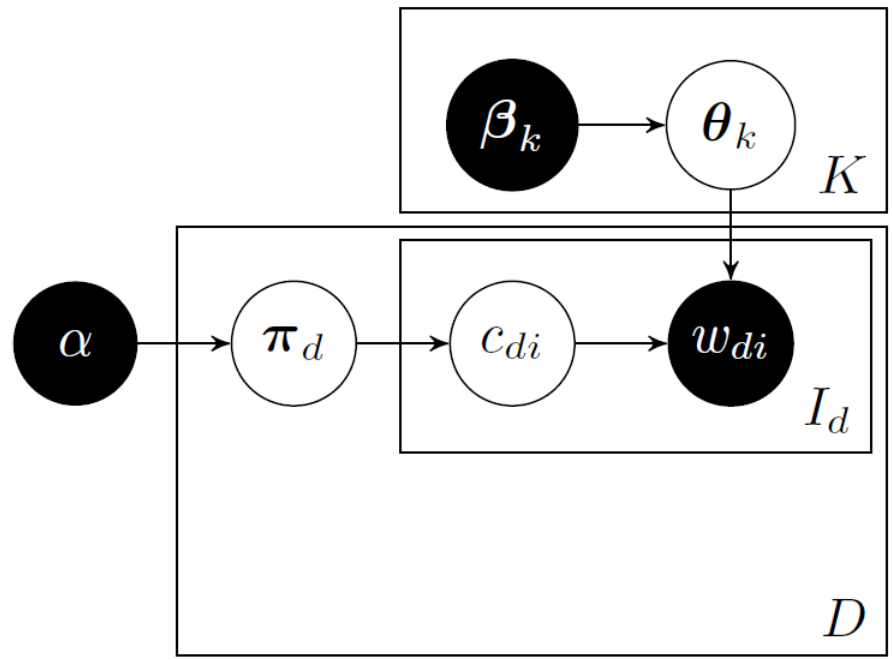
Topic Models

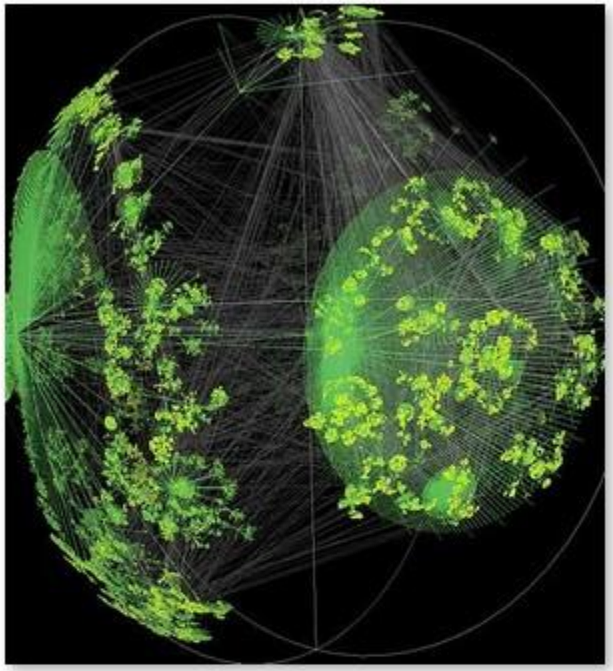
- Model documents as bags-of-words
- For each document d: Draw probability vector $\boldsymbol{\pi}_d$ over topics from Dirichlet ($\boldsymbol{\alpha}$)
- For each topic k: Draw probability vector $\boldsymbol{\theta}_k$ over words from Dirichlet ($\boldsymbol{\beta}_k$)
- For each document d and word slot i, draw topic c_{di} from topic distribution $\boldsymbol{\pi}_d$, then word w_{di} from word distribution $\boldsymbol{\theta}_{c_{di}}$



Topic Models and Metadata

- Topic Models are useful to organize text corpora in a meaningful but unsupervised way
- The classical Latent Dirichlet Allocation disregards context information and meta data such as
  - Author
  - Time and Place of creation
  - Part of book, journal, proceedings, series etc.
  - People to which document is addressed
  - Web links and citations

Documents in Context



You forwarded this message on 09/07/2012 04:43.
Extra line breaks in this message were removed.

From: lesong <lsong@cc.gatech.edu> Sent: Thu 05/07/2012 21:12
To: Thore Graepel
Cc: arthur gretton; Alex Smola
Subject: Speaking at (Proposed) NIPS 2012 workshop on kernel methods

Message | nips2012workshop.txt (12 KB)

Dear Thore,

Arthur Gretton, Alex Smola and I are planning to submit a workshop proposal on Kernel Methods and Graphical Models at this year's NIPS conference.
We were wondering if you would like to be an invited speaker to share your contributions in the field (kernel topic models). We think your perspective would help add a

D.M. Blei and J.D. Lafferty. Dynamic topic models. In *Proceedings of the 23rd International Conference on Machine Learning*, pages 113–120, 2006.

Related Work

- Latent Dirichlet Allocation [Blei, Ng, & Jordan, 2003]
- Dynamic Topic Models [Blei and Lafferty, 2006]
- A correlated topic model of Science [Blei and Lafferty 2007]
- Topic models conditioned on arbitrary features with Dirichlet-multinomial regression. [Mimno and McCallum, 2008]
- Relational Topic Models for document networks [Chang & Blei, 2009]

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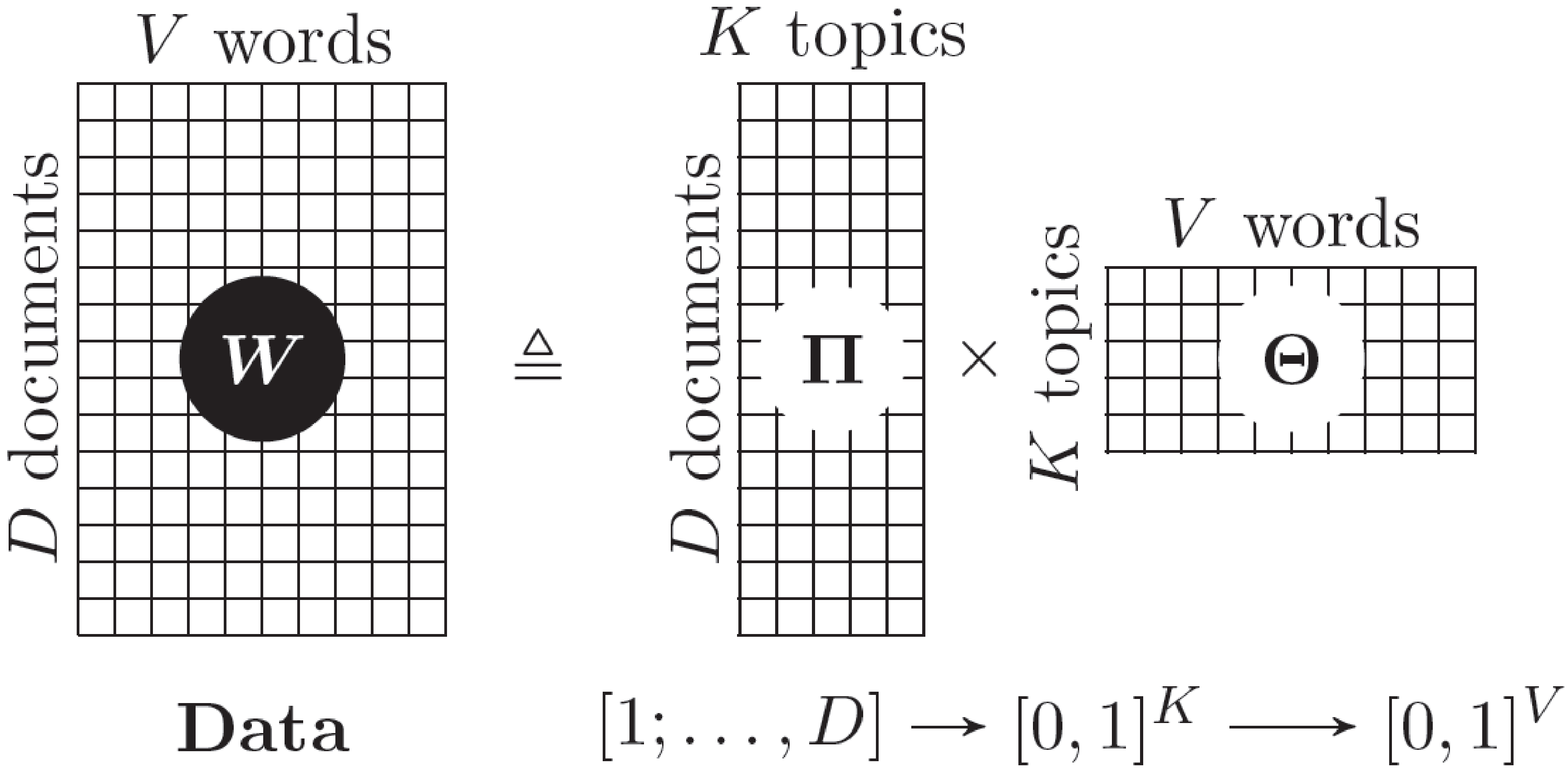
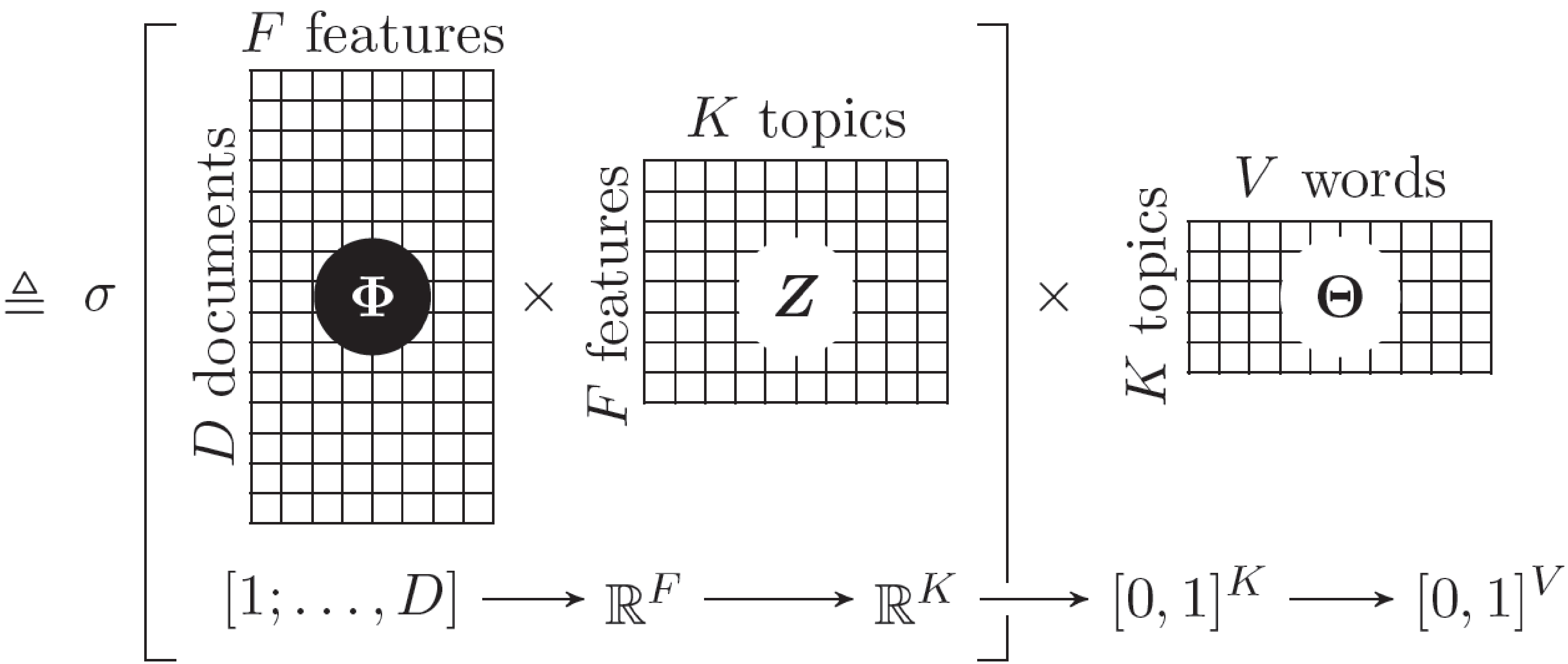
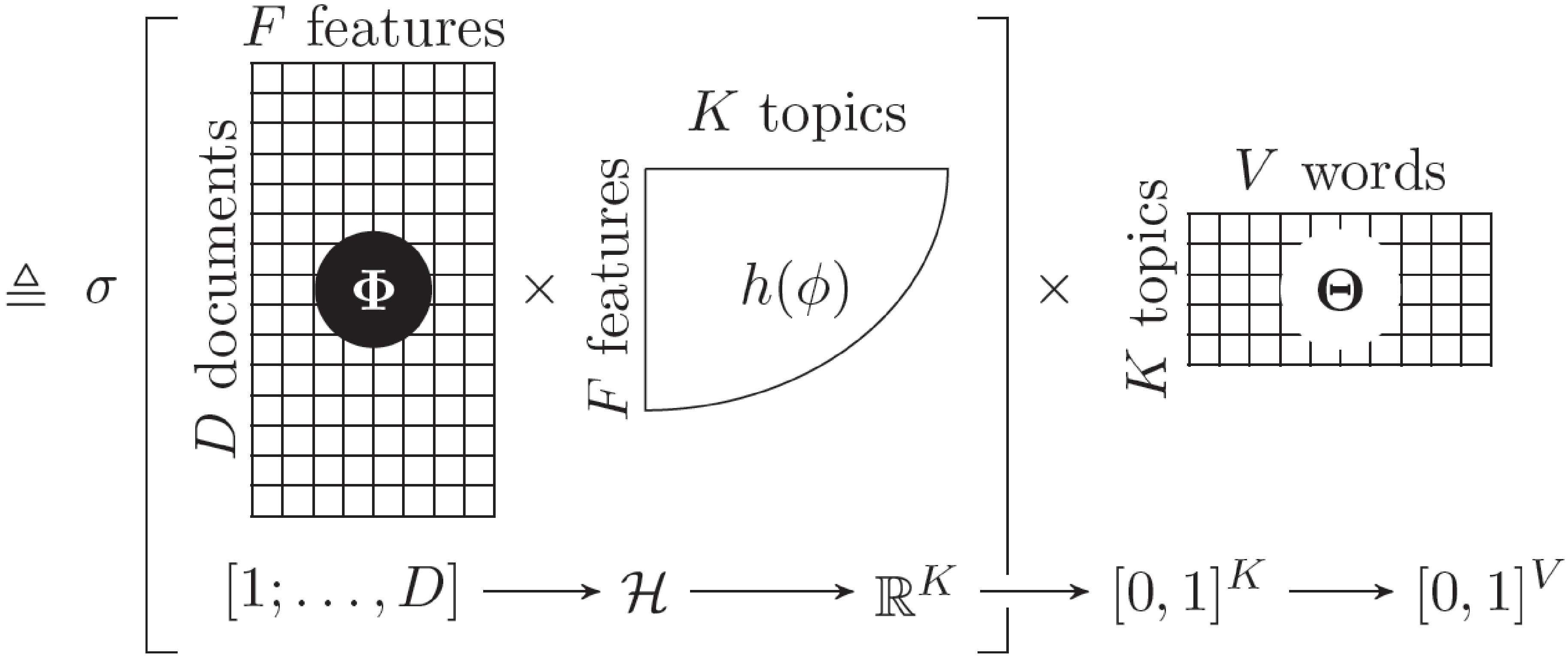
Topic Model as Matrix Factorization



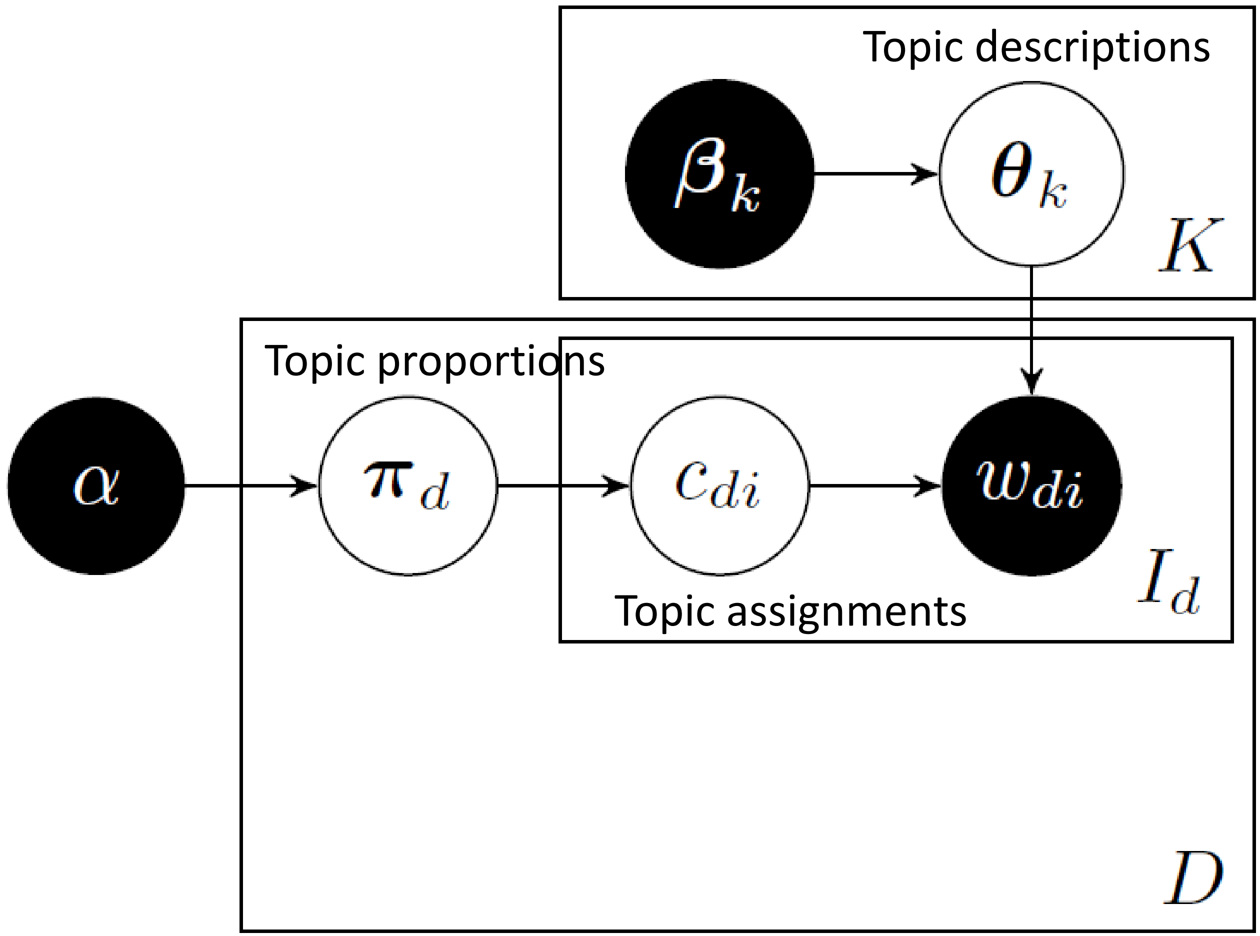
Conditional Topic Model



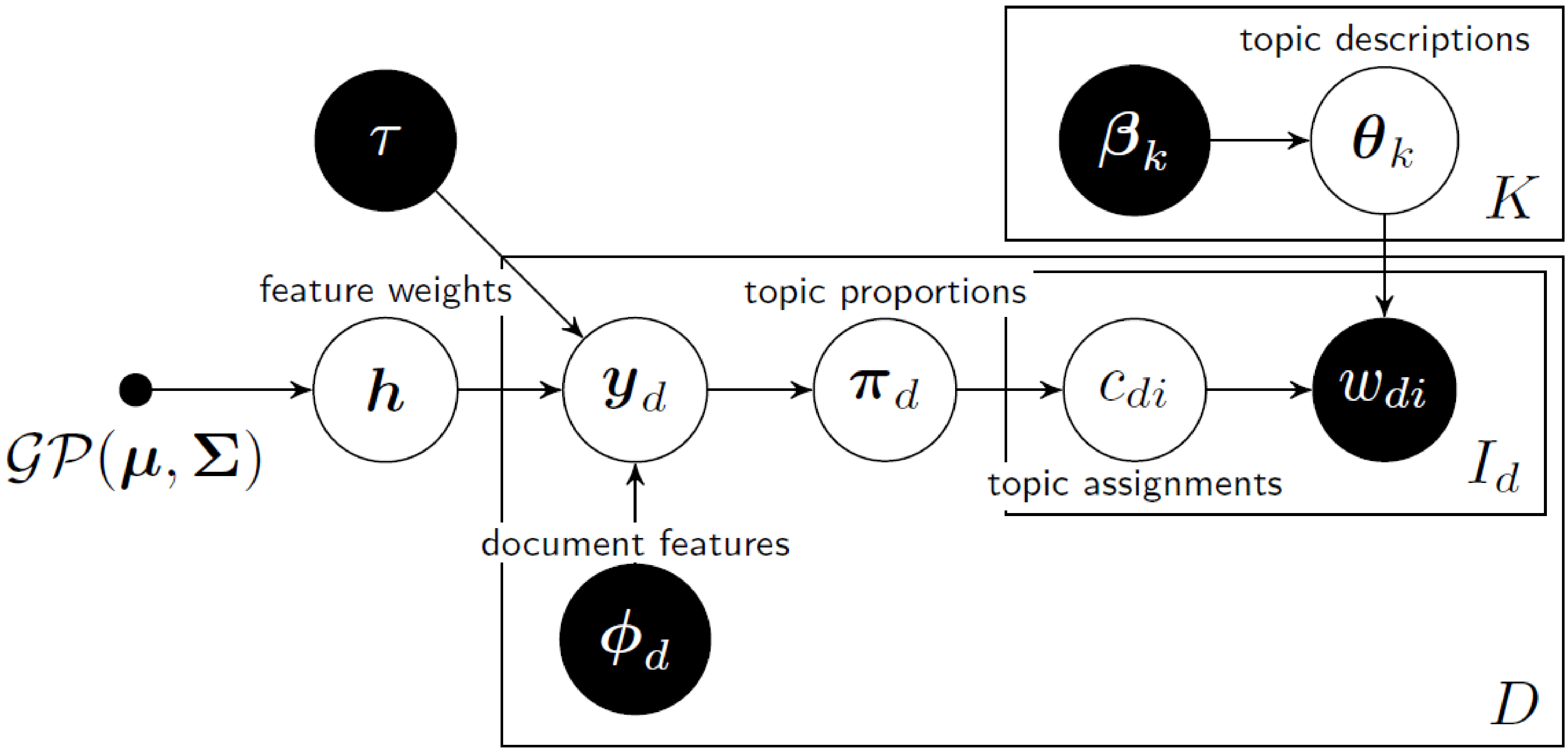
Kernel Topic Model



Latent Dirichlet Analysis



Kernel Topic Model



Kernels/Covariance Functions on Documents

- Kernel over documents based on meta-data
  - Author
  - Time
  - Location
  - Publication
- Kernel over documents based on their link structure
  - Web Link structure
  - Citation structure
  - Social network of authors

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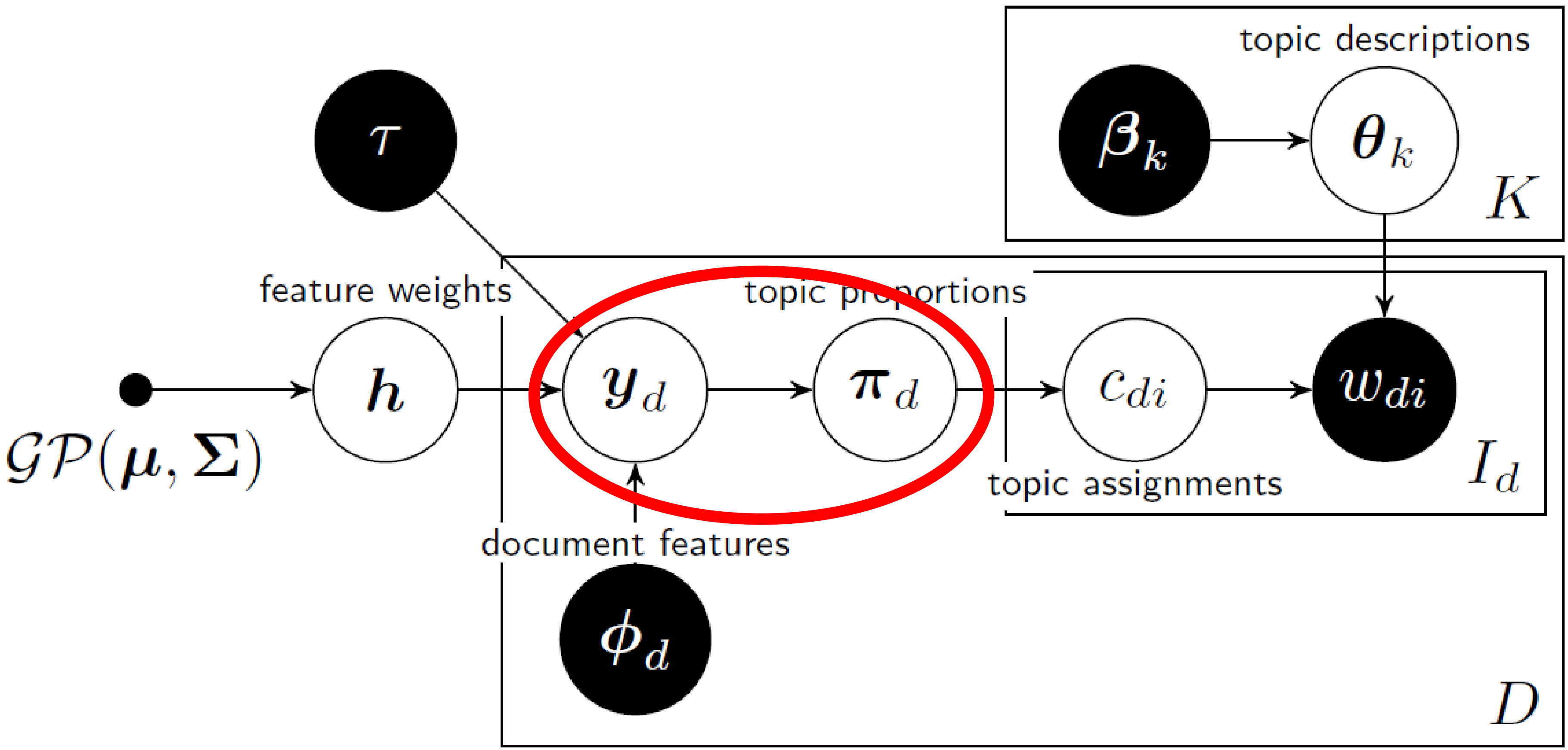
Kernel Topic Model: Laplace Bridge



Inference: The Laplace Bridge

- Laplace approximation:
  - Find mode of posterior distribution
  - Fit Gaussian distribution based on mode/curvature
  - Evaluate approximate posterior integrals etc.
- Laplace approximation is basis dependent and MacKay, 1998, suggests using a softmax basis
- Here we use the Laplace approximation in the softmax basis to transform probability messages into unconstrained (Gaussian) messages and back

Laplace approximation: softmax basis

- Dirichlet distribution over probability vectors:

$$D_{\boldsymbol{\pi}}(\boldsymbol{\pi};\boldsymbol{\alpha}) \propto \prod_{i=1}^{K} \pi_i^{\alpha_i - 1}\,\delta\left(\sum_i \pi_i - 1\right)$$

- Transform to new softmax basis

$$\pi_i(\boldsymbol{y}) = \sigma_i(\boldsymbol{y}) = \frac{\exp(y_i)}{\sum_j \exp(y_j)}$$

- New parameterized form:

$$D_{\boldsymbol{y}}(\boldsymbol{y}|\boldsymbol{\alpha}) \propto \prod_{i=1}^{K} \big(\pi_i(\boldsymbol{y})\big)^{\alpha_i} g(\mathbf{1}^T\boldsymbol{y})$$

- Choose $g(x) = \exp\left(-\frac{\epsilon}{2}x^2\right)$, later $\epsilon \to 0$

From Dirichlet to Gaussian (and back)

- Laplace approximation of Dirichlet in form $N(\boldsymbol{y}; \boldsymbol{\mu}, \boldsymbol{\Sigma})$
- Mean/Mode is given by

$$\mu_k = \log \alpha_k - \frac{1}{K} \sum_{\ell=1}^{K} \log \alpha_\ell$$

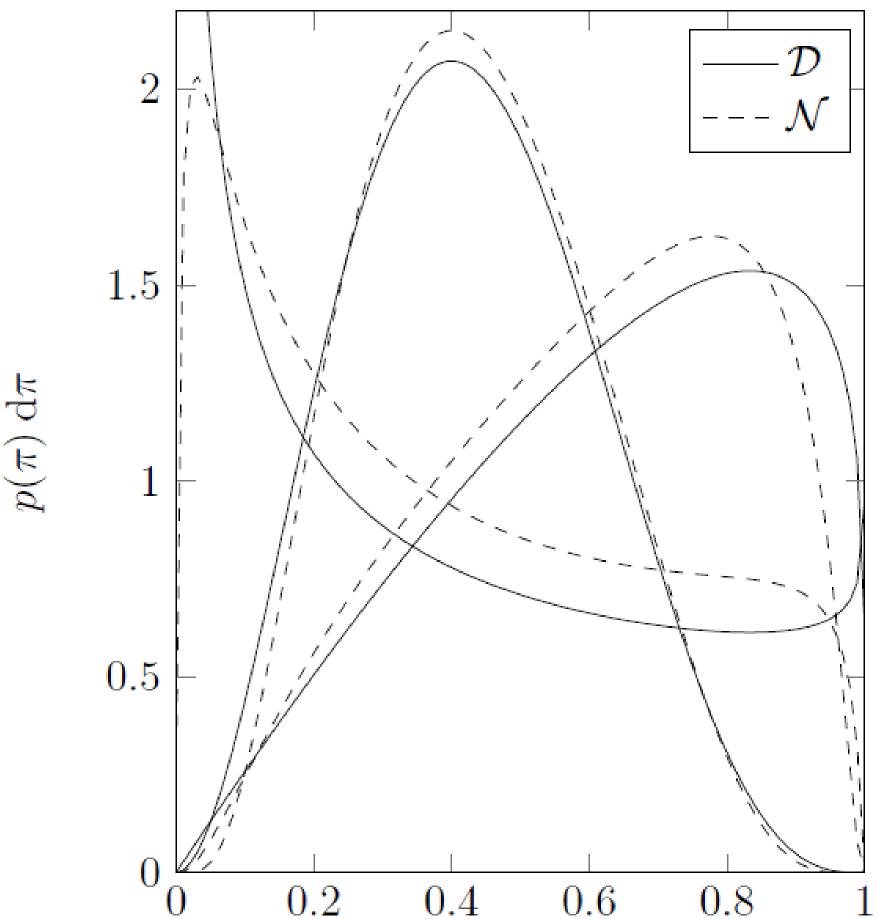
- Approximately diagonal covariance given by

$$\Sigma_{kk} = \frac{1}{\alpha_k}\left(1 - \frac{2}{K}\right) + \frac{1}{K^2} \sum_{\ell=1}^{K} \frac{1}{\alpha_\ell}$$

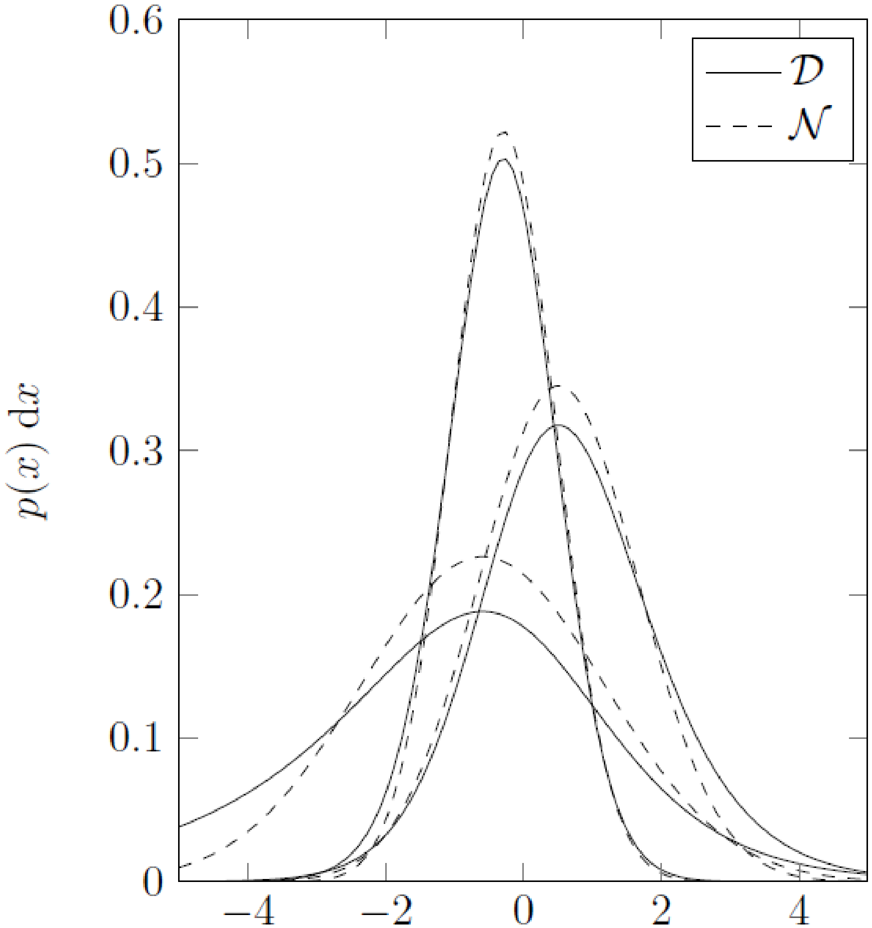
- And the inverse mapping:

$$\alpha_k = \frac{1}{\Sigma_{kk}}\left(1 - \frac{2}{K} + \frac{\exp(-\mu_k)}{K^2} \sum_{\ell=1}^{K} \exp(-\mu_\ell)\right)$$

Beta distributions approximated in Softmax basis

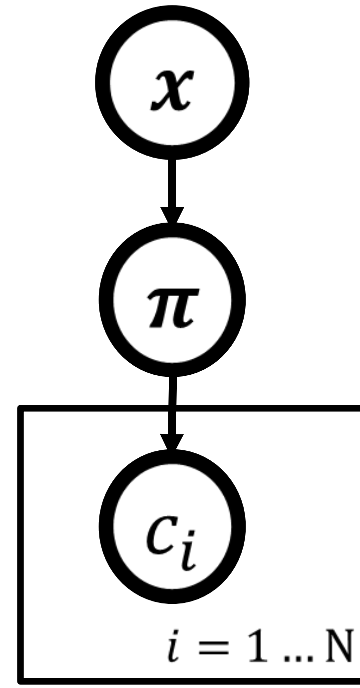


Simplex Basis



Softmax Basis

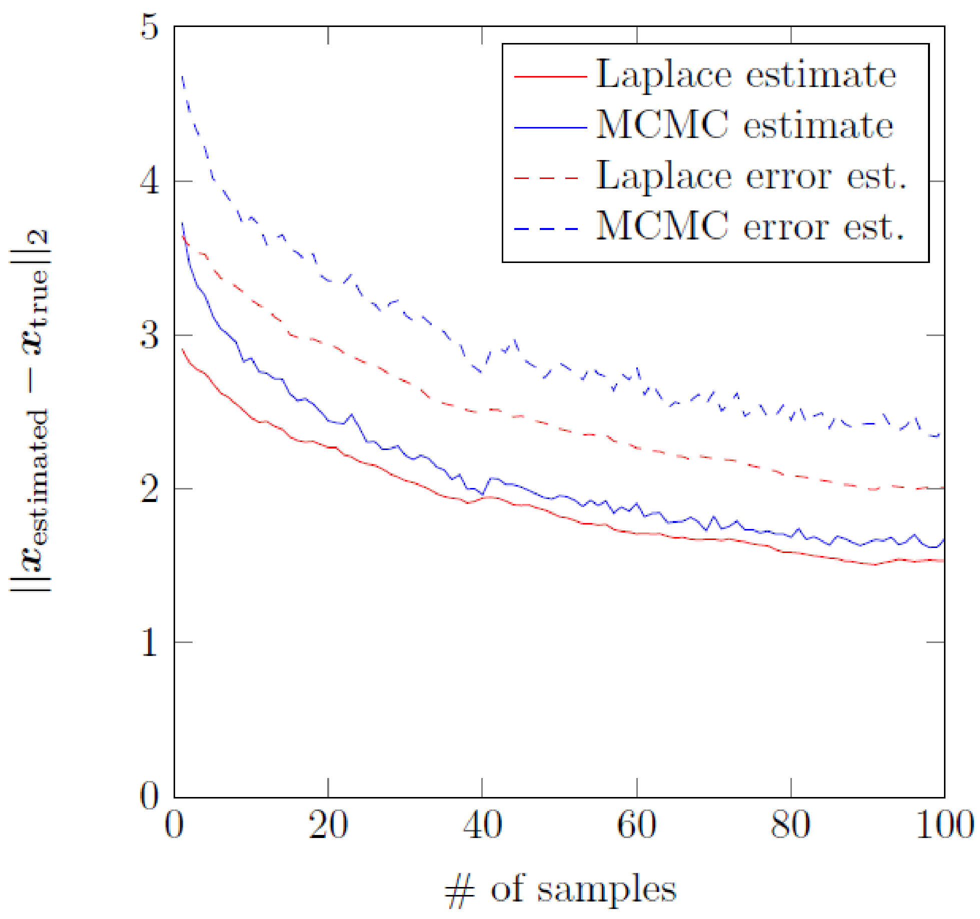
Gaussian "Dirichlet" in Action



- Generation
  - Sample $\boldsymbol{x} \in \mathbb{R}^K \sim N(\boldsymbol{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma}) N(0; \mathbf{1}^{\mathrm{T}}\boldsymbol{x}, \epsilon^2)$
  - Map $\boldsymbol{\pi} \coloneqq \boldsymbol{\sigma}(\boldsymbol{x})$
  - Sample data c from $p(c = k|\boldsymbol{\pi}) = \boldsymbol{\pi}$
- Inference
  - Use Laplace bridge to obtain Dirichlet belief on $\boldsymbol{\pi}$ from Gaussian prior
  - Update Dirichlet belief on data c (conjugacy, easy!)
  - Reverse Laplace bridge to obtain Gaussian belief on $\boldsymbol{x}$.

Inference: Laplace Bridge vs MCMC

- Multivariate Gaussian $\boldsymbol{x}$ in $K = 10$ dimensions
- Sample from resulting Discrete distribution
- Infer point estimate of $\boldsymbol{x}$ using MCMC and Laplace
- Estimates are stable and accurate



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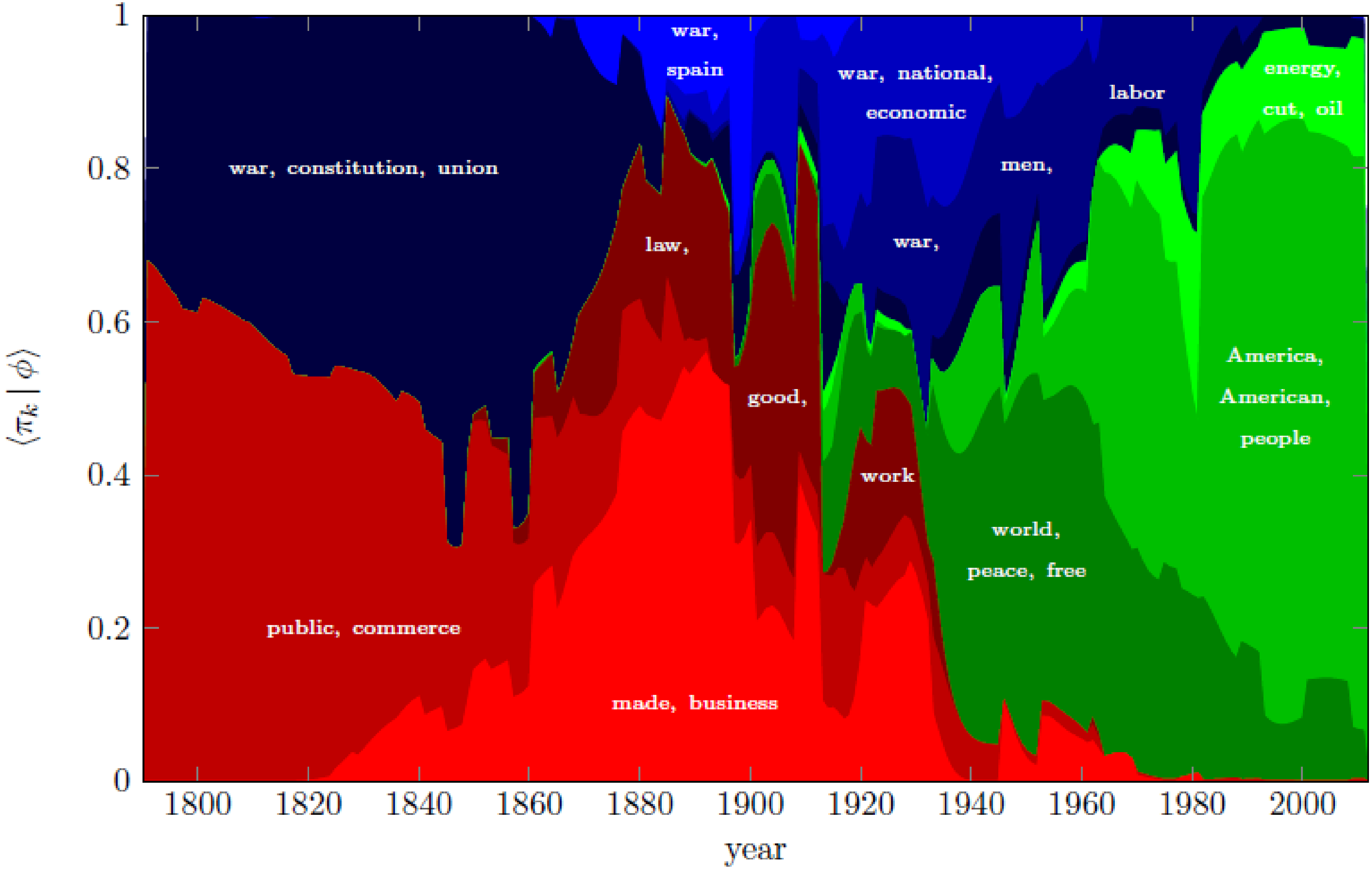
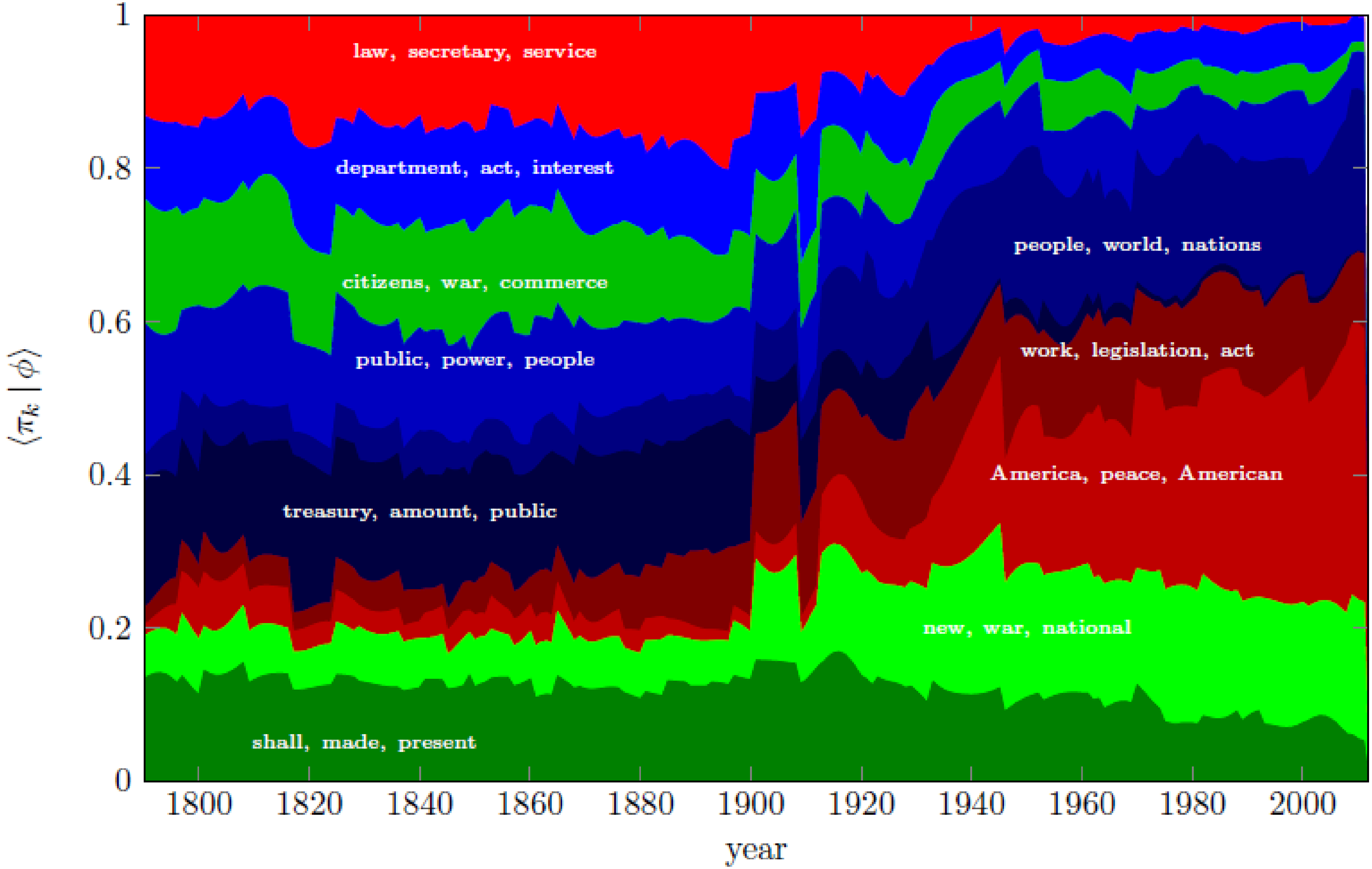
State of the Union

- Data: State of the Union corpus
  - Continuous time feature “year”, 1790-2011
  - Discrete author feature “president”, 44 values
  - Use $K = 10$ topics
- Dirichlet Multinomial Regression [Mimno et al, 2008]
  - Time features: 100 radial basis functions, width 5 years
  - Author features: 44 mutually exclusive binary features
- Kernel Topic Model
  - Rational Quadratic kernel with width 5 years
  - Additional constant term if authors are not the same

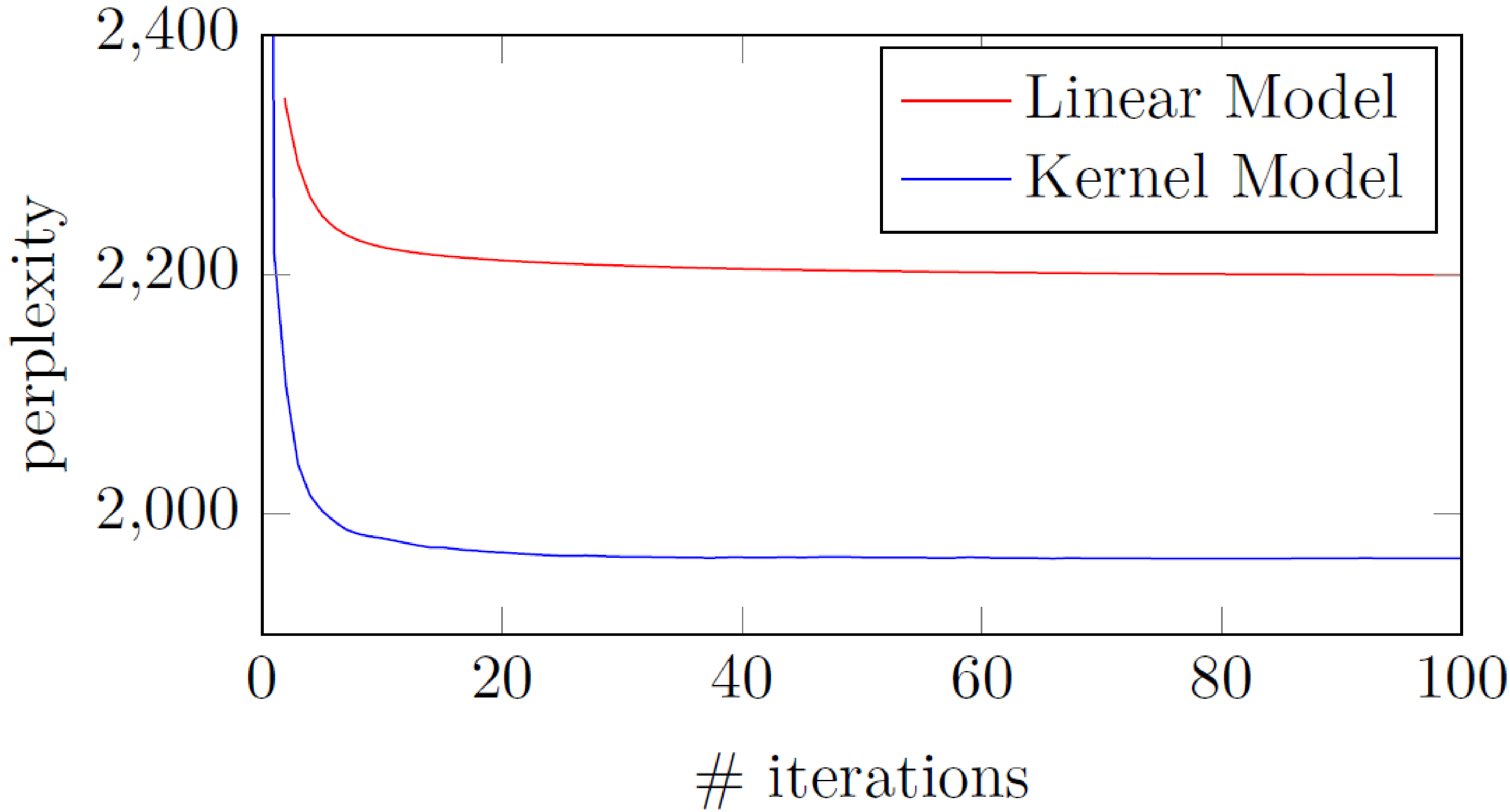
State of the Union: Kernel Topic Model



State of the Union: Linear Model



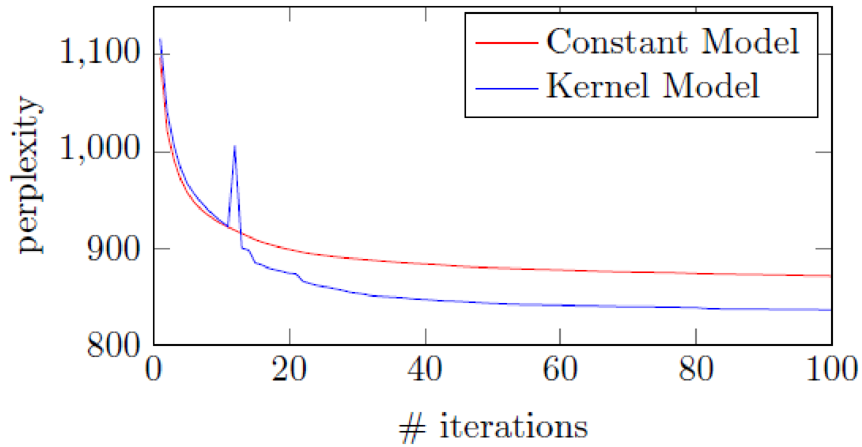
State of the Union: Perplexity

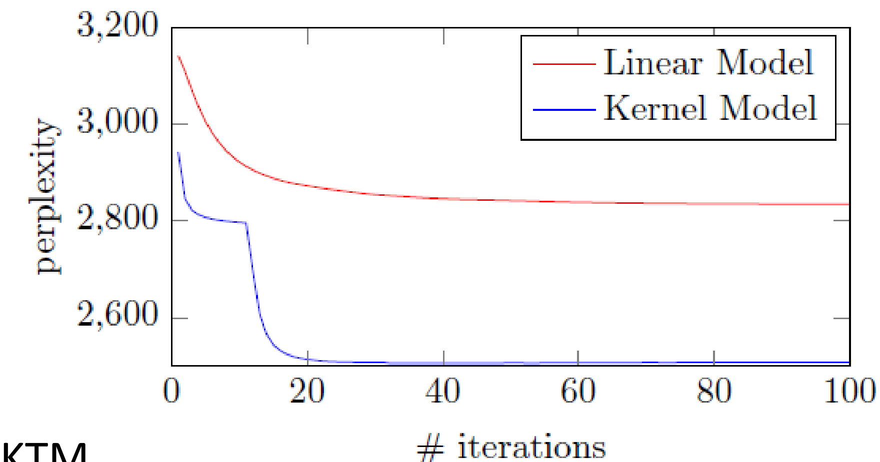


Initial perplexity equals size of vocabulary $V = 5{,}000$

More Perplexity

- Wiki dataset
  - List of probability topics
  - $D = 318$ documents
  - Squared exponential kernel on link distance
- NIPS dataset
  - Globersen et al, 2007
  - NIPS papers and their citation structure





Discontinuities in the learning curve for the KTM correspond to optimization of the hyper parameters

Conclusions

- Make use of document meta-data and link structure to improve topic models
- Use kernel/Gaussian process framework to integrate such context information into topic distribution in LDA
- Laplace bridge is a useful tool to bridge the divide between an unbounded vector space and probability vectors → other applications?