UNIFYING DEPENDENT CLUSTERING AND DISPARATE CLUSTERING FOR NON-HOMOGENEOUS DATA

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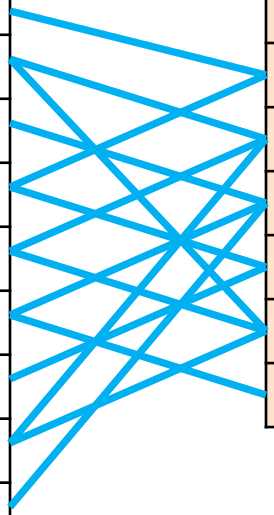
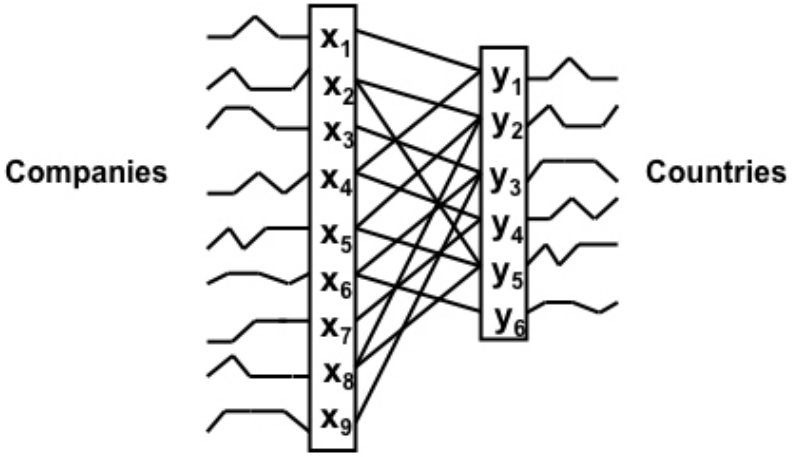
Problem Setting

Companies	Avg. salary of Employees	Stock values	Profit margins
x_1	1.0 K	25.11	11%
x_2	1.1 K	21.32	20%
x_3	1.2 K	28.81	12%
x_4	1.2 K	31.85	22%
x_5	1.1 K	85.32	5%
x_6	1.2 K	10.71	32%
x_7	0.9 K	11.61	18%
x_8	1.1 K	35.81	12%
x_9	1.2 K	20.81	4%



Countries	GDP	GNP	Inflation
y_1	$11832 B	$12970 B	-0.4%
y_2	$8219 B	$8153 B	2.0%
y_3	$6732 B	$7812 B	-0.3%
y_4	$1761 B	$2852 B	1.8%
y_5	$5022 B	$4391 B	0.0%
y_6	$7224 B	$8312 B	1.1%

Objective



Fortunes of individual companies are inter-twined with the fortunes of the countries.

Performances of companies may not necessarily be tied to the economies of the countries.

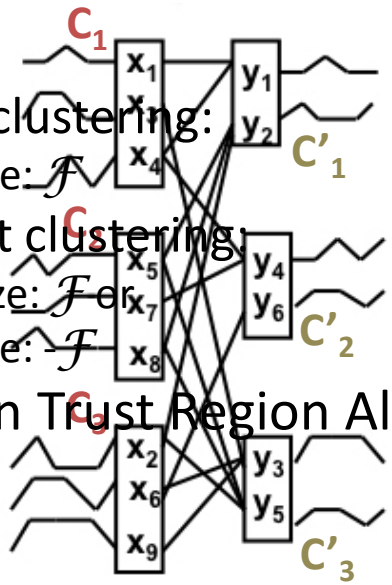
Companies \ Countries	C'_1	C'_2	C'_3
C_1	4	0	0
C_2	0	6	0
C_3	0	0	4

Companies \ Countries	C'_1	C'_2	C'_3
C_1	2	1	1
C_2	2	1	2
C_3	1	1	3

Objective Function

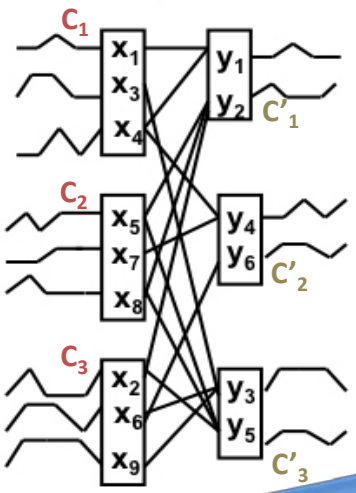
$v_i^{x_s}$ $v_j^{y_s}$

- Optimize $\mathcal{F}$
  - Disparate clustering:
    - minimize: $\mathcal{F}$
  - Dependent clustering:
    - maximize: $\mathcal{F}$ or
    - minimize: $-\mathcal{F}$
- Quasi Newton Trust Region Algorithm



	C'1	C'2	C'3
C1	2	1	1
C2	2	1	2
C3	1	1	3

Formulations



	C'1	C'2	C'3
C1	2	1	1
C2	2	1	2
C3	1	1	3

w

	C'1	C'2	C'3
C1	0.33	0.33	0.33
C2	0.33	0.33	0.33
C3	0.33	0.33	0.33

U

α

	C'1	C'2	C'3
C1	0.5	0.25	0.25
C2	0.4	0.2	0.4
C3	0.2	0.2	0.6

Row distributions

β

	C'1	C'2	C'3
C1	0.4	0.33	0.17
C2	0.4	0.33	0.33
C3	0.2	0.33	0.5

Col. distributions

Membership Probability

$$v_i^{(\mathbf{x}_s)} = \frac{\exp(-\frac{\rho}{D}||\mathbf{x}_s - \mathbf{m}_{i,\mathcal{X}}||^2)}{\sum_{i'=1}^{k_x}\exp(-\frac{\rho}{D}||\mathbf{x}_s - \mathbf{m}_{i',\mathcal{X}}||^2)}$$

Contingency Table Counts

$$w_{ij} = \sum_{s=1}^{n_x}\sum_{t=1}^{n_y} B(s,t)v_i^{(\mathbf{x}_s)}v_j^{(\mathbf{y}_t)}$$

Probability Distributions (row and column)

$$p(\alpha_i = j) = p(C_{(y)} = j|C_{(x)} = i) = \frac{w_{ij}}{w_{i.}}$$

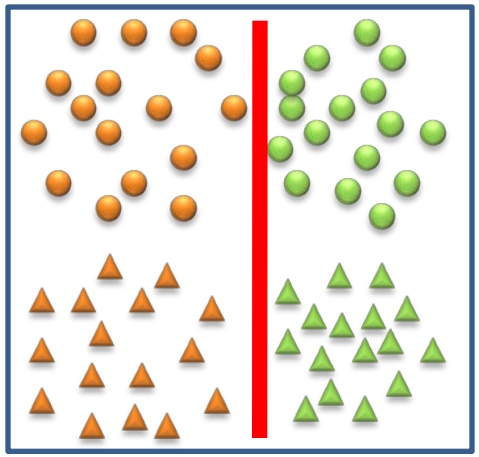
$$p(\beta_j = i) = p(C_{(x)} = i|C_{(y)} = j) = \frac{w_{ij}}{w_{.j}}$$

Objective Function

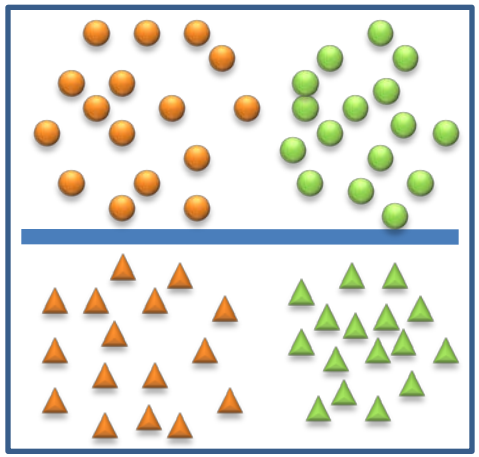
$$\mathcal{F} = \frac{1}{k_x}\sum_{i=1}^{k_x} D_{KL}\left(\alpha_i||U(\frac{1}{k_y})\right) + \frac{1}{k_y}\sum_{j=1}^{k_y} D_{KL}\left(\beta_j||U(\frac{1}{k_x})\right)$$

Single Dataset Scenarios

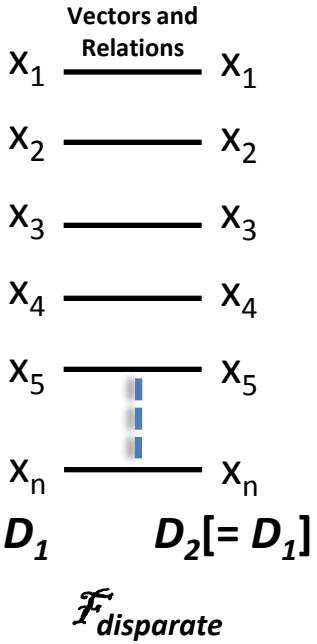
- **ALTERNATIVE CLUSTERING**



D_1



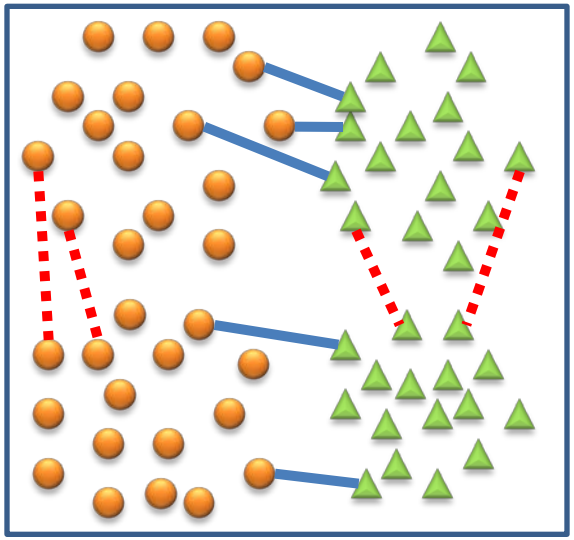
D_2



$\mathcal{F}_{disparate}$

Single Dataset Scenarios

- **CONSTRAINED CLUSTERING**
  - Instance-level constraints

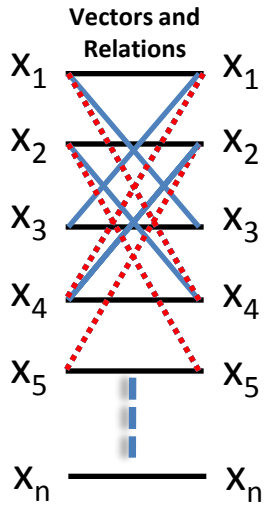


D_1

Must-Link (ML)

Must-Not-Link (MNL)

- **Clustering of D_1 is given.**
- **The desired constrained clustering is obtained in D_2.**

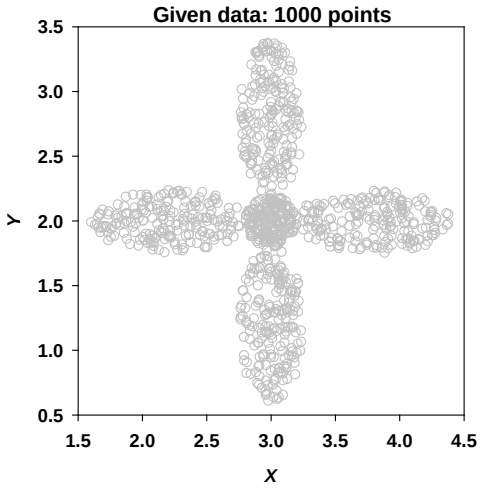


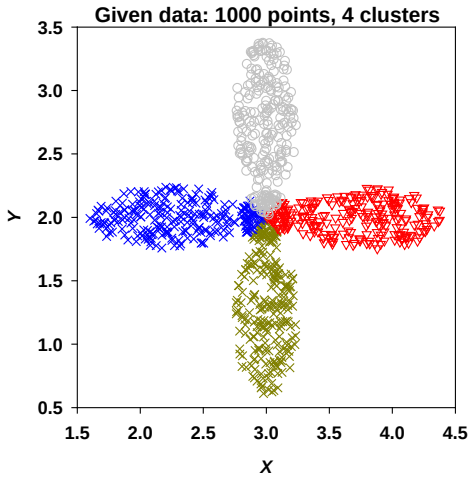
D_1 $D_2[= D_1]$

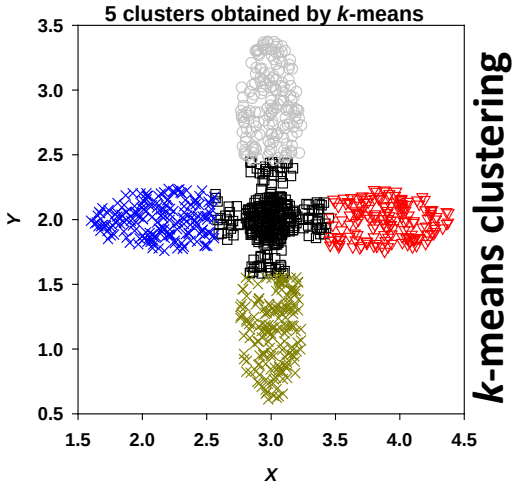
$$\mathcal{F}=\alpha\,\mathcal{F}_{dep}+(1-\alpha)\,\mathcal{F}_{disparate}$$

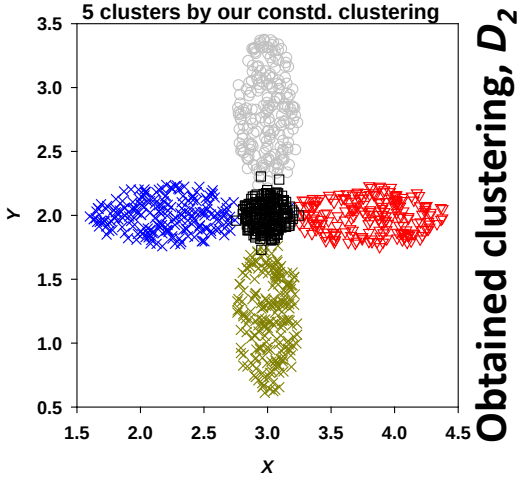
Single Dataset Scenarios

- **CONSTRAINED CLUSTERING**
  - Cluster-level constraints

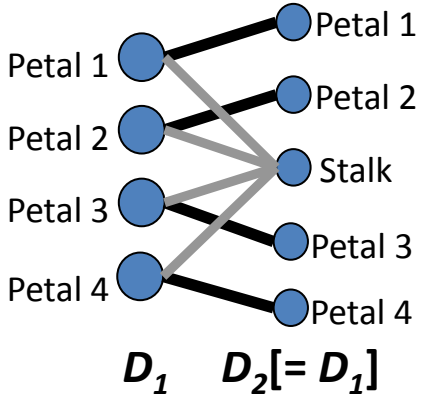






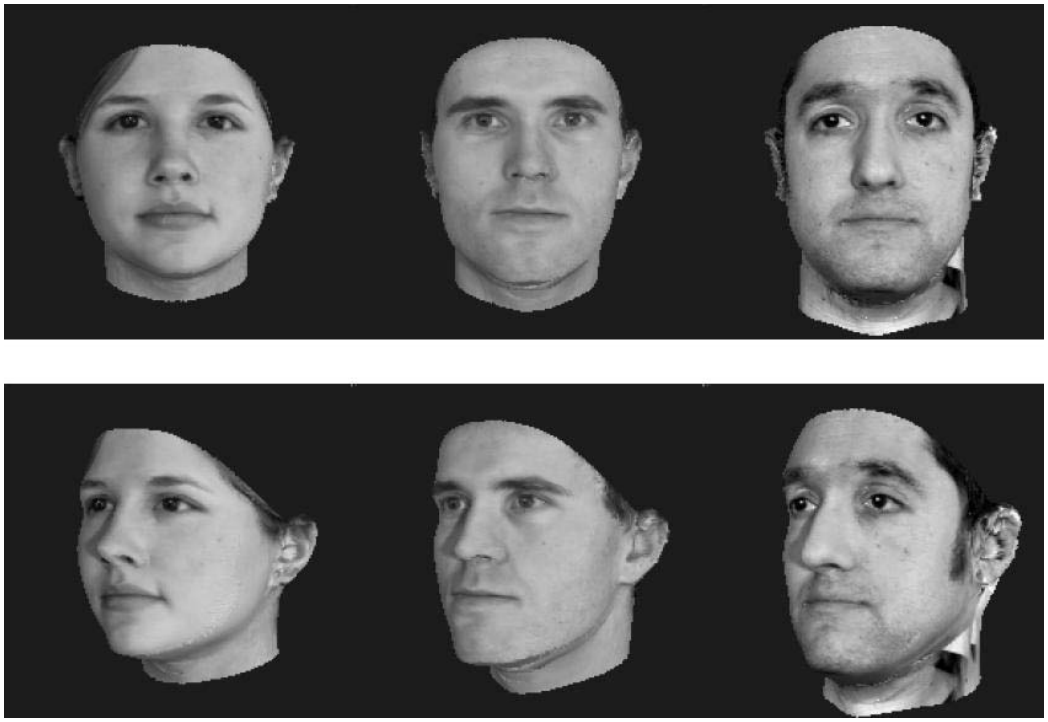


- **Clustering of D_1 is given.**
- **The desired constrained clustering is obtained in D_2.**



Experimental Results

- **ALTERNATIVE CLUSTERING**
- Portrait Dataset

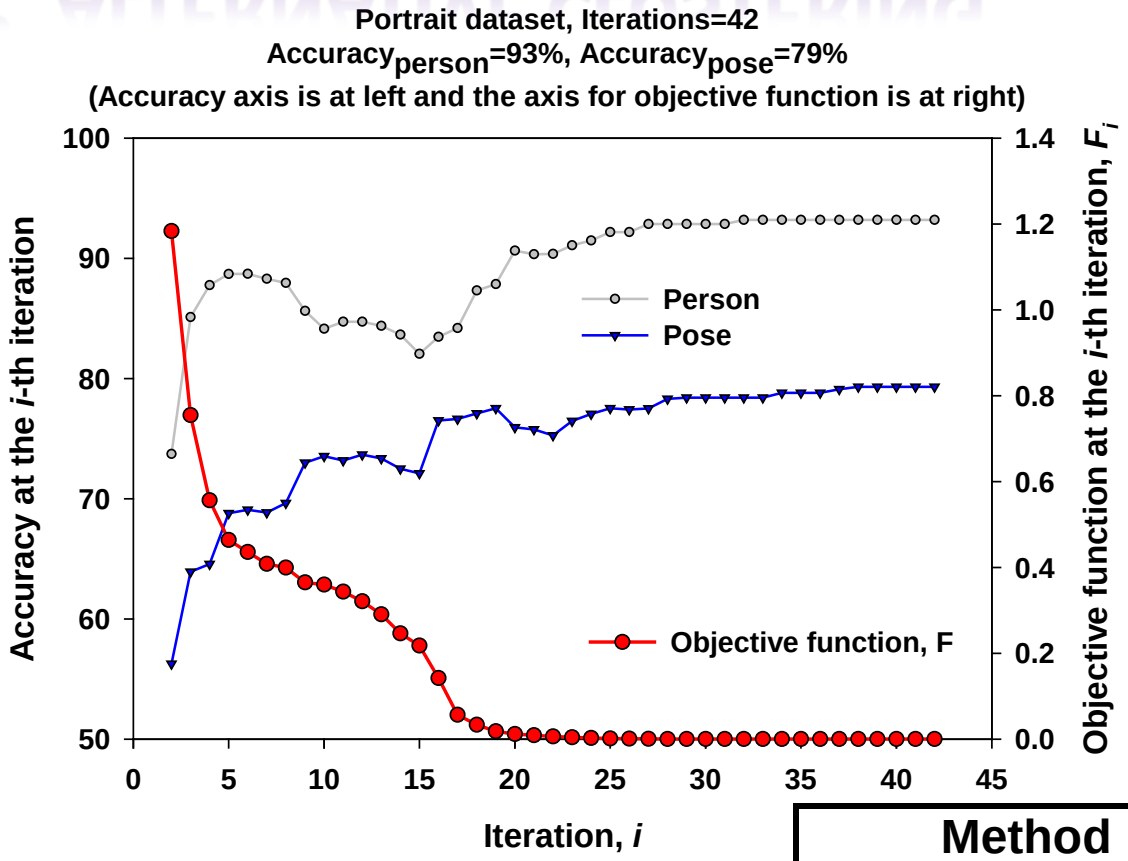


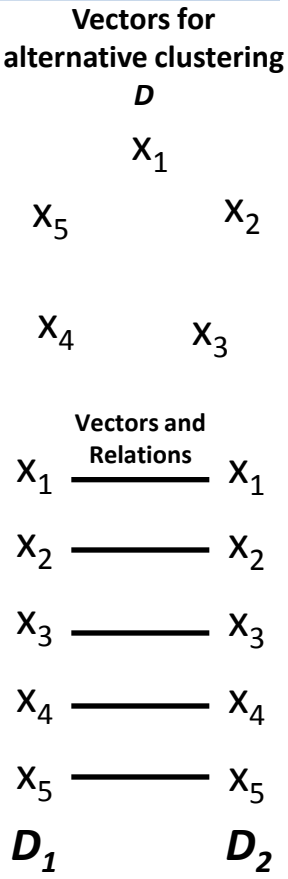
Prateek Jain et al. 2008

- **3** people each in **3** poses and **36** illuminations (i.e., **324** images.)
- **300** features

Experimental Results

- **ALTERNATIVE CLUSTERING**

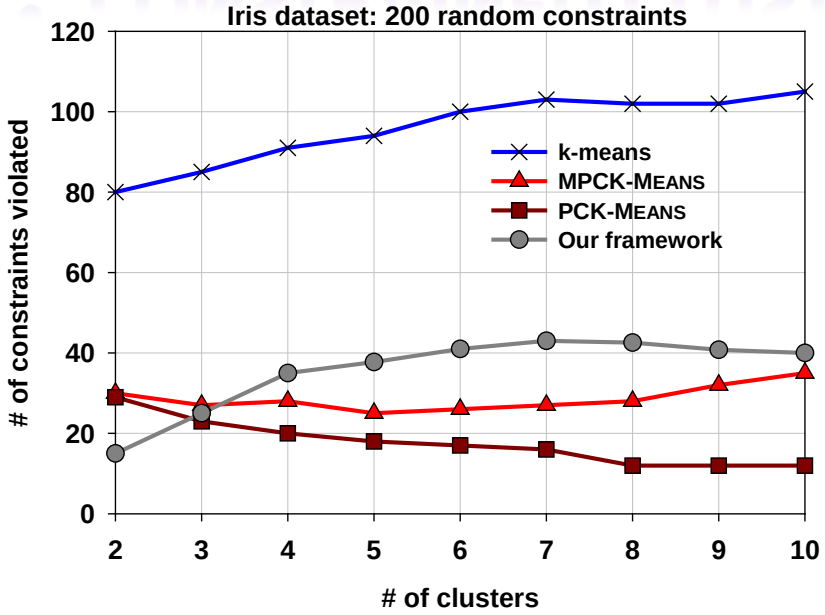


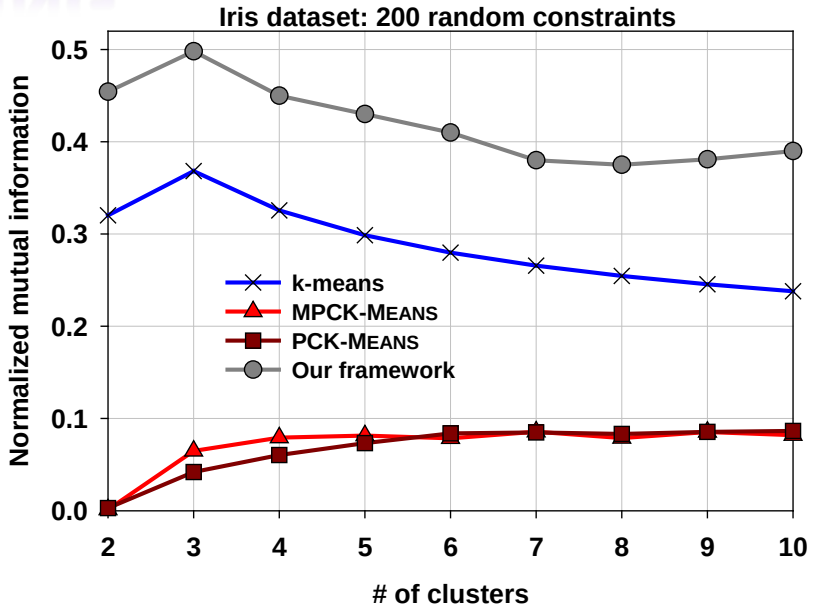


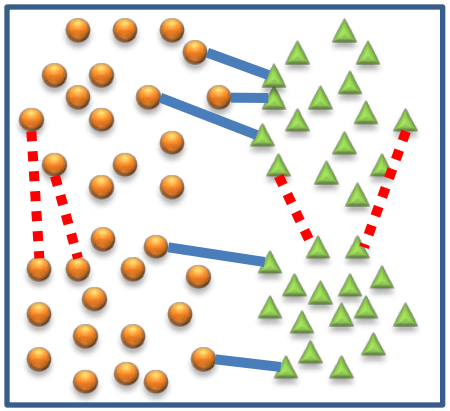
Method	Person	Pose
k-means	0.65	0.55
Conv-EM	0.69	0.72
Dec-kmeans	0.84	0.78
Our framework	0.93	0.79

Experimental Results

- **CONSTRAINED CLUSTERING**







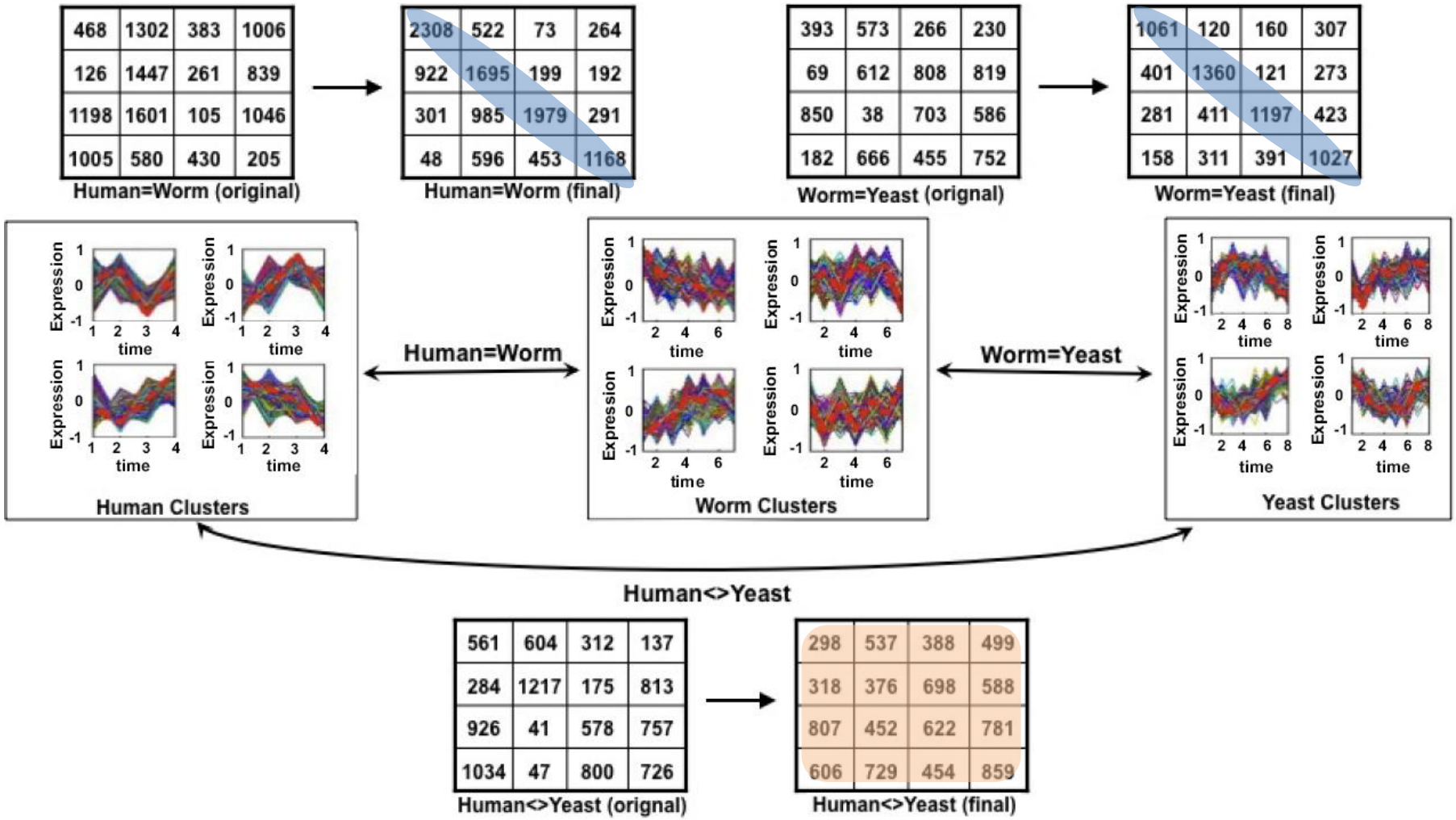
D_1

Must-Link (ML)

Must-Not-Link (MNL)

Experimental Results

- **COMPARING GENE EXPRESSION PROGRAMS**



Future Work & Conclusion

- Future directions
  - Capture more expressive relationships
    - Dependent and disparate clustering on same set of relationships
    - Different goal for different types of relationships (one-to-one, ML, MNL, etc.)
  - Clustering dependencies
- Conclusion
  - General, expressive framework for clustering non-homogenous datasets
  - The framework subsumes previously defined formulations
    - MDI (Kullback et al. ‘78), Disparate Clustering (Jain et al. ‘ 08), Clustering over Relation Graphs (Banerjee et al. ‘07), Multivariate Information Bottleneck (Friedman ‘01), etc.

Thank you