Dynamic linear combination of two-class classifiers

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Introduction

Linear combination of the outputs (**scores**) of an ensemble of classifiers is widely used to attain higher performance with respect to pick out the "best" single classifier (that is not a trivial task).

Different methods to estimate the weights of a combination had been proposed, but generally to each classifier is assigned a unique weight that does not depend on the pattern to be classified (improving the average performance).

In this work we investigate some techniques aimed at *dynamically* estimate the coefficients of the combination on a pattern per pattern basis.

For each pattern, and for each classifier, we propose the computation of an index called ***Score Decidability Index***, based on the Wilcoxon-Mann-Whitney statistic. The proposed index measures, for each classifier, and for each pattern, the confidence in classifying the pattern either as *positive* or a *negative* pattern. Thus, it can be used to estimate the parameters for a *dynamic combination*.

Score Decidability Index

Let now us consider a set of patterns whose class is known for a generic classifier C_k, and let:

$$S_k^+ = \left\{ s_i^+ = f_k(x_i) \mid x_i \in \omega_+ \right\},\ \forall i \qquad S_k^- = \left\{ s_i^- = f_k(x_i) \mid x_i \in \omega_- \right\},\ \forall i \qquad AUC_k = \frac{\sum_{i=0}^{n_+} \sum_{j=0}^{n_-} I(s_{ik}^+, s_{jk}^-)}{n_+ \cdot n_-}$$

Let us define:

$$r_-(s) = \frac{\sum_{i=0}^{n_+} I(s_i^+, s)}{n_+} \simeq P(S^+ > s) \qquad r_+(s) = \frac{\sum_{j=0}^{n_-} I(s, s_j^-)}{n_-} \simeq P(s > S^-)$$

$$\frac{\sum_{i=0}^{n_+} \sum_{j=0}^{n_-} I(s_{ik}^+, s_{jk}^-)}{n_+ \cdot n_-} \rightarrow \begin{cases} \dfrac{\sum_{j=0}^{n_-} r_-(s_{jk}^-)}{n_-} \\ \dfrac{\sum_{i=0}^{n_+} r_+(s_{ik}^+)}{n_+} \end{cases}$$

The **Score Decidability Index** (SDI) is defined as:

$$\Delta(s_{ik}) = r_+(s_{ik}) - r_-(s_{ik})$$

The Score Decidability Index can also be seen as a different representation of the original score assigned by each classifier to a given pattern, as it represents the likelihood with which the original score is drawn from either the **positive** or **negative** distributions of scores.

Dynamic Combination rules based on the Score Decidability Index

Dynamic Linear Combination: in this case the weights depend both on the classifier, and the pattern to be classified. Generally it is an hard-task of computing dynamic weights, our solution is the following:

$$s_i^* = \sum_{k=1}^{N} \alpha_{ik} \cdot s_{ik} \qquad \alpha_{ik} = \frac{\Delta(s_{ik}) + 1}{2}$$

Score Decidability Index as a normalized score:

$$s_i^* = \frac{1}{N} \sum_{k=1}^{N} \frac{\Delta_{ik} + 1}{2}$$

Dynamic Score Combination: this framework [1] basically combines only two values among all the scores produced by the ensemble of classifiers, namely the smallest and the biggest values, while the behavior of the ensemble is used to compute the values of the weights. Two different formulations has been proposed:

$$s_i^* = \beta_{1i} \cdot \max_k(s_{ik}) + \beta_{2i} \cdot \min_k(s_{ik})$$

$$s_i^* = \beta_i \cdot \max_k(s_{ik}) + (1 - \beta_i) \cdot \min_k(s_{ik})$$

Dynamic Score Combination by *Δ* voting [2]:

$$\beta_{1i} = \frac{1}{N} \sum_{k=1}^{N} I(\Delta(s_{ik}), \alpha) \qquad \beta_{2i} = \frac{1}{N} \sum_{k=1}^{N} I(-\Delta(s_{ik}), \alpha)$$

Dynamic Score Combination by *Δ* mean [2]:

$$\Delta^*(\mathbf{s}_i) = \frac{\frac{1}{N} \sum_{k=1}^{N} \Delta(s_{ik})}{\sigma_{\Delta(s_{ik})}} \qquad \beta_i = \frac{1}{1 + e^{-\gamma \cdot \Delta^*(\mathbf{s}_i)}}$$

[1] Tronci, R., Giacinto, G., Roli, F.:*Dynamic Score Combination: A Supervised and Unsupervised Score Combination*. MLDM 2009, LNAI 5632:163-177

[2] Lobrano, Tronci, Giacinto, Roli :*A score decidability index for dynamic score combination*. ICPR 2010

Experimental results

Dataset used and set up:

- Biometric Authentication Fusion Benchmark Database (BA-Fusion), the dataset contains similarity scores from **8** classifiers, and the scores have been normalized by the Tanh rule.
- Experiments have been carried out by creating ensembles where the number of classifier in the ensemble ranges from 2 to 8. In this way, we create ensembles that contain **all possible subsets of classifiers** from the original pool of 8 classifiers.
- In order to get unbiased results, a **4-fold cross-validation** technique has been used.

Comparisons with:

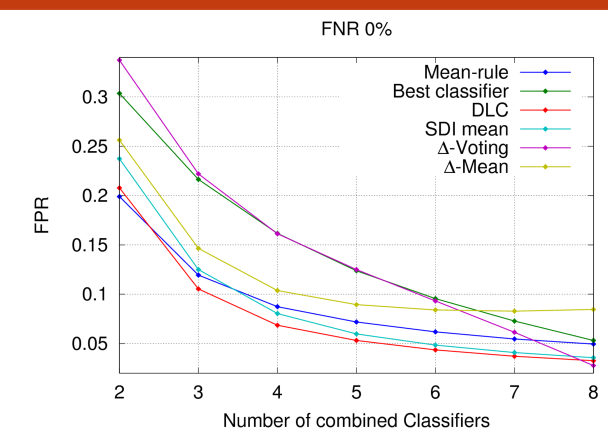
- the "best" classifier, as it is the basic target for a combination.
- the *Mean rule*, as this is a simple and effective way of combining multiple scores.

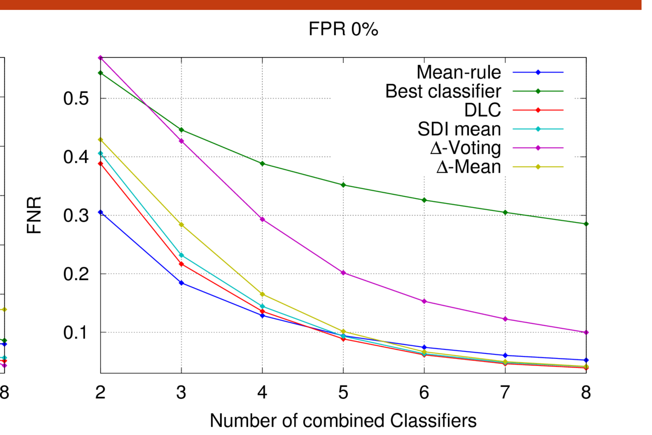
Figures: Performance for all the possible ensemble of classifiers in terms of average value.

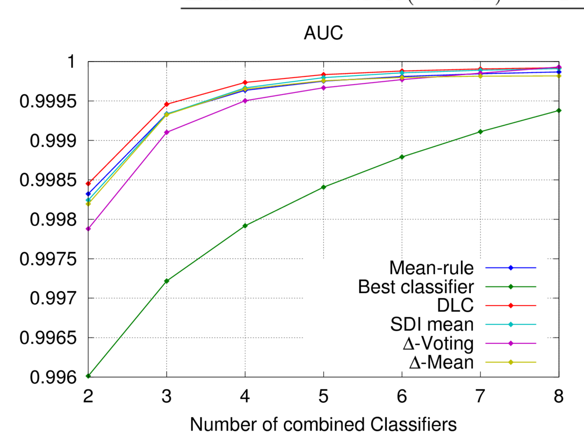
Tables: Performance for all the ensembles of 5 classifiers. **o** indicates that the difference in performance from the Mean-rule is not statistically significant.

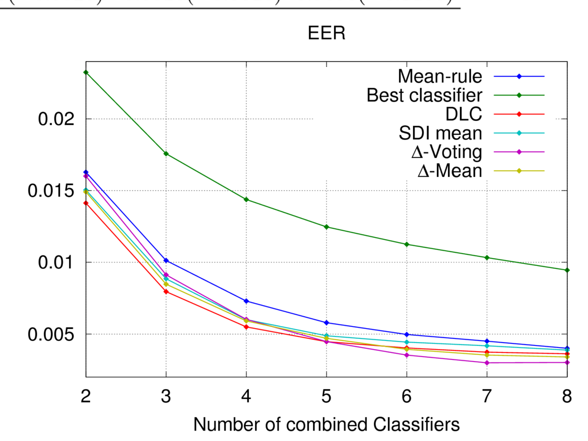
	AUC	EER
Mean-rule	*0.9998(±0.0002)*	0.0058(±0.0019)
Best classifier	0.9984(±0.0014)	0.0125(±0.0046)
DLC	*0.9998(±0.0002)*	*0.0045(±0.0017)*
SDI mean	*0.9998(±0.0002)*	0.0049(±0.0023)
Δ Voting	0.9997(±0.0005)	0.0045(±0.0021)
Δ mean	○ 0.9998(±0.0003)	0.0047(±0.0019)

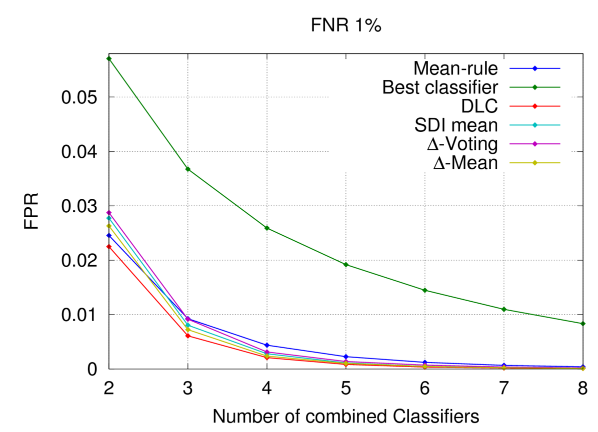
	FPR-0%	FPR-1%	FNR-1%	FNR-0%
Mean-rule	0.0941(±0.0342)	0.0040(±0.0026)	0.0023(±0.0017)	0.0719(±0.0827)
Best classifier	0.3518(±0.1148)	0.0135(±0.0092)	0.0192(±0.0181)	0.1237(±0.1120)
DLC	*0.0886(±0.0469)*	0.0028(±0.0024)	*0.0008(±0.0010)*	*0.0532(±0.0599)*
SDI mean	○ 0.0931(±0.0455)	○ 0.0038(±0.0029)	0.0011(±0.0020)	0.0598(±0.0619)
Δ Voting	0.2017(±0.1622)	*0.0026(±0.0024)*	0.0014(±0.0013)	0.1250(±0.1981)
Δ mean	○ 0.1015(±0.0723)	0.0029(±0.0025)	0.0010(±0.0013)	0.0895(±0.1150)

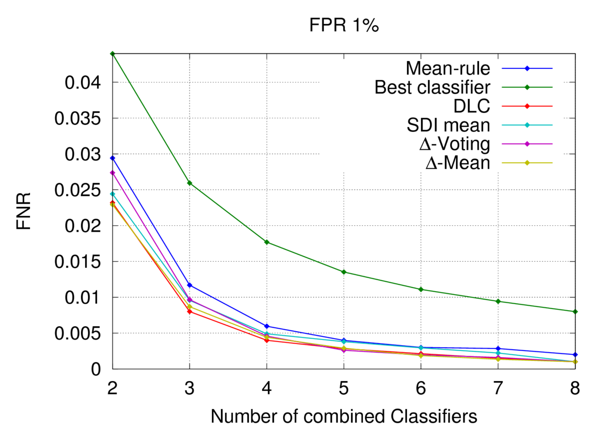












Final Remarks: the reported results allow to conclude that the proposed DLC and SDI-mean techniques based on the *Score Decidability Index* allows exploiting effectively the complementarity among different classifiers. In conclusion, it can be pointed out that the proposed Index provide an useful measure for the estimation of the parameters for combining an ensemble of two-class classifiers.