

Network Topology Uncovers Function, Disease, and Phylogeny

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Imperial College London

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Overview

1. Introduction and Background:

- PPI and other networks
- Network analysis and modeling challenges

2. New Network Analysis and Modeling:

(A) Analysis:

- Structure vs. biological function and disease
- Network alignment

(B) Modeling – Geometric Graphs:

- Graphlet Degree Distributions
- Network embedding
- Trained Geometric Model

- Application: De-noising PPI networks

(C) Why PPI networks might be geometric?

3. Conclusions

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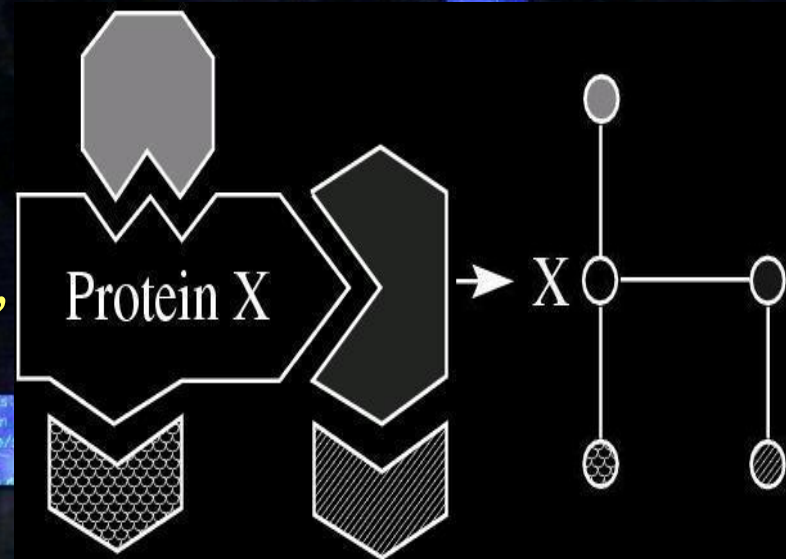
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- Large Networks model many real-world phenomena
 - technological: www, internet, electric circuits,...
 - social: friendship, collaboration, disease spread,...
 - biological:
 - protein structure,
 - transcriptional regulation,
 - metabolic,
 - protein-protein interaction (PPI),
 - ...

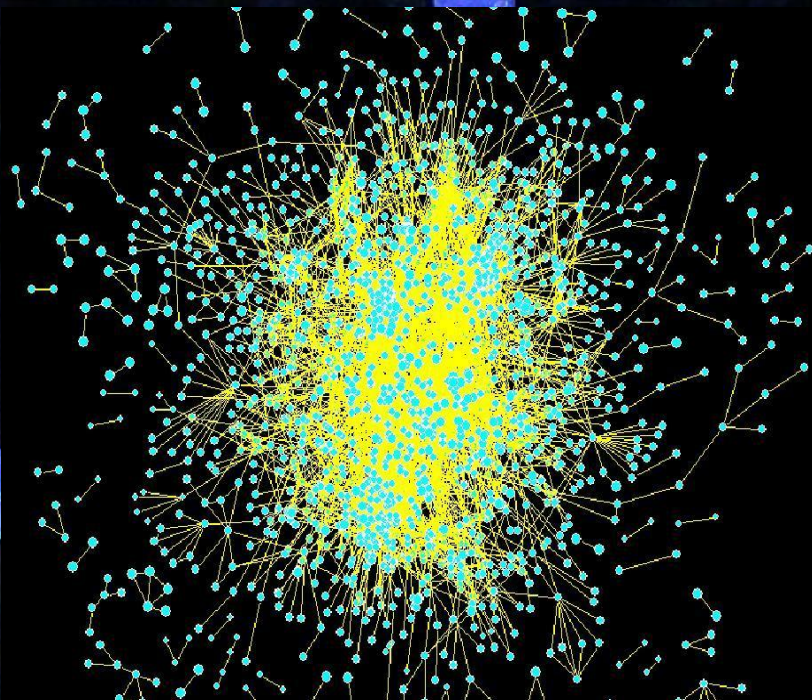
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- Large-scale networks in bioinformatics:
 - Technological advances in experimental biology
 - data
 - Important computational problems
 - Algorithmic and modeling advances contribute:
 - biological understanding (function, disease, pathogens,...)
 - therapeutics
- ➔ Booming research area

1. Introduction and Background

Problems:

1. Noise → revise models as data sets evolve
2. “Hardness” of graph theoretic problems

E.g. NP-completeness of subgraph isomorphism

- Cannot exactly compare/align networks
 - heuristics (approximate solutions)
- Exact comparison inappropriate in biology
 - due to biological variation

1. Introduction and Background

Properties of Large Networks (heuristic comparisons)

- *Global*

- Degree distribution
- Diameter
- Clustering coefficient/spectrum

- *Local:*

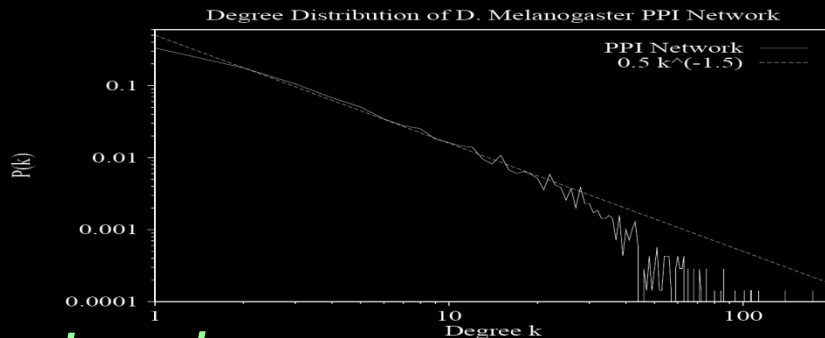
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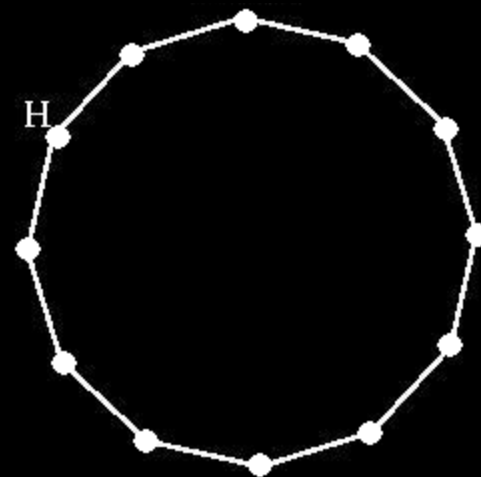
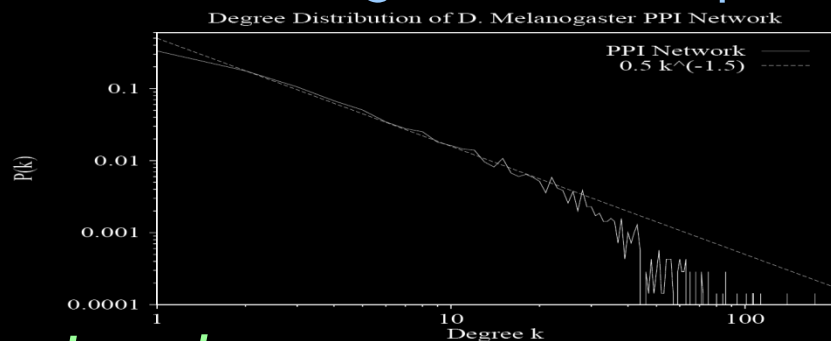
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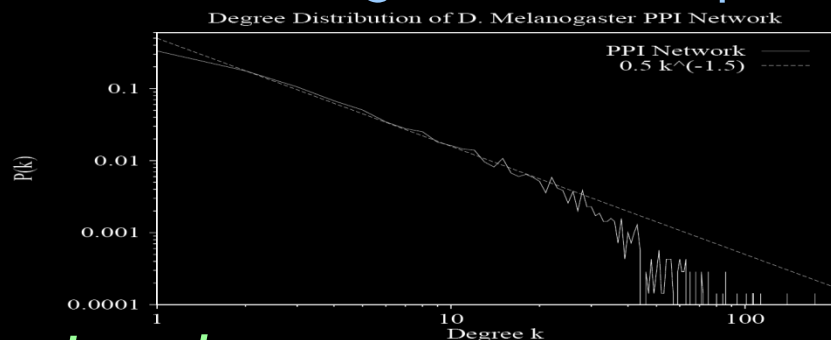
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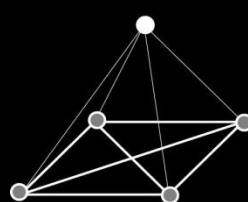
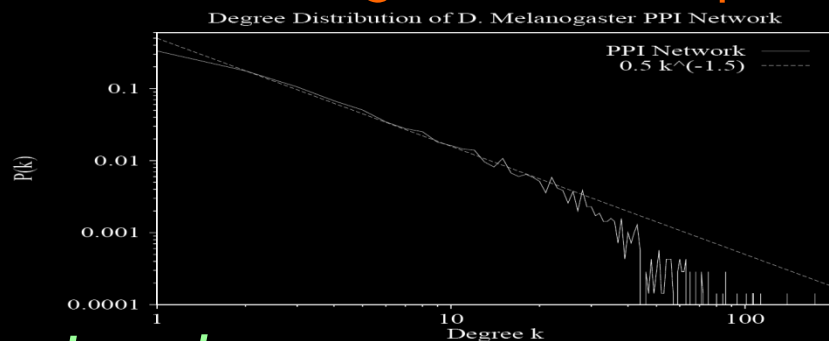
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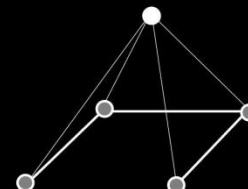
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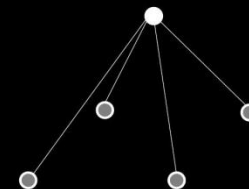
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$$C=6/6=1$$



$$C=3/6=1/2$$



$$C=0/6=0$$

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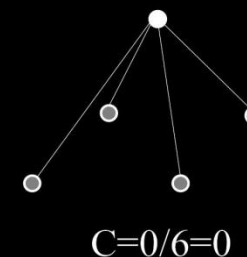
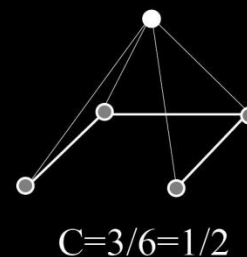
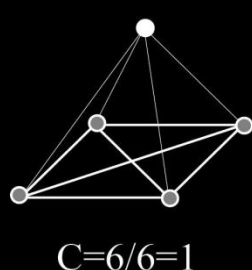
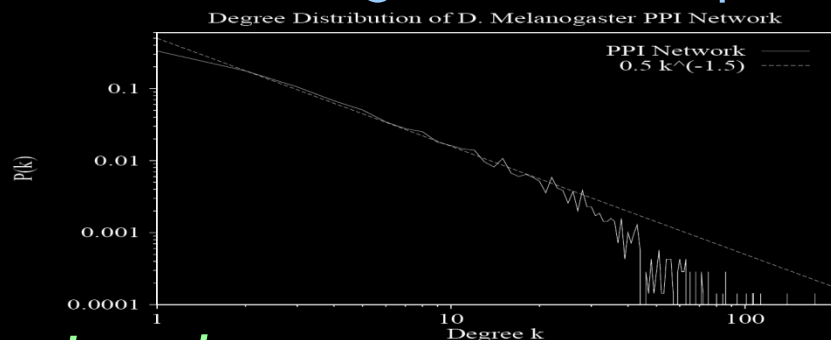
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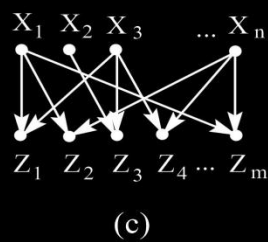
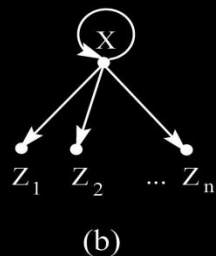
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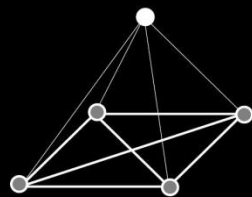
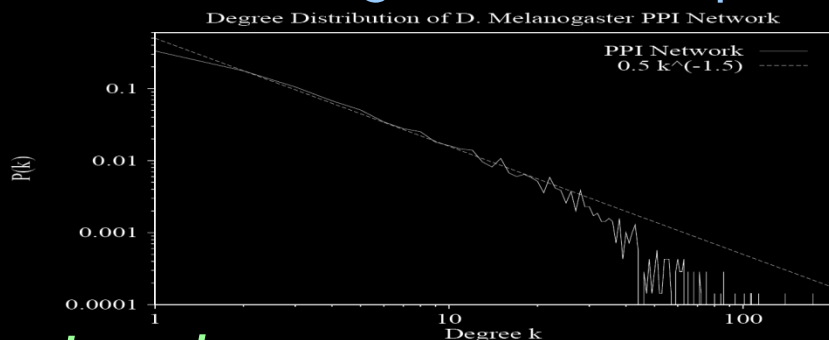


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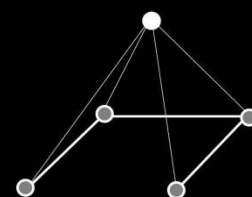
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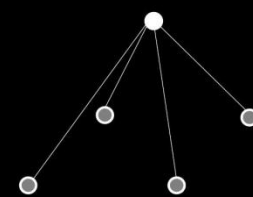
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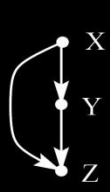
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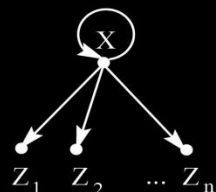
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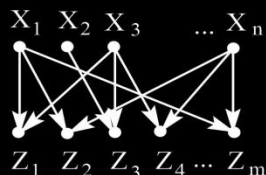
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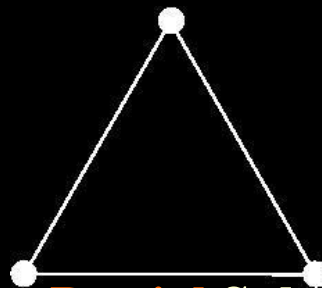
(a)



(b)



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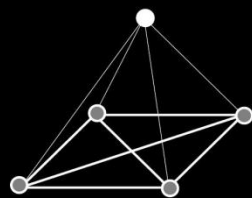
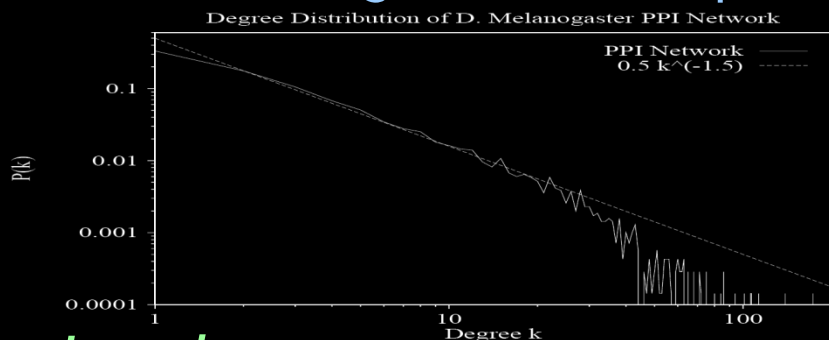
Induced vs. Partial Subgraphs

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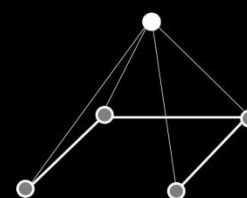
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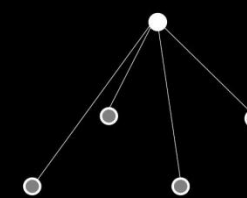
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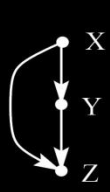
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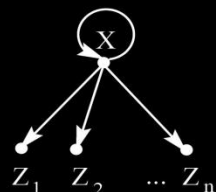
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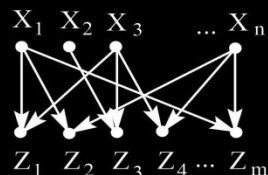
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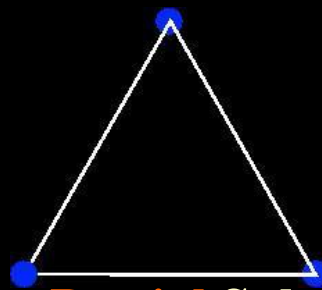
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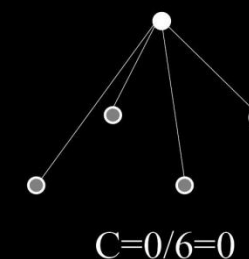
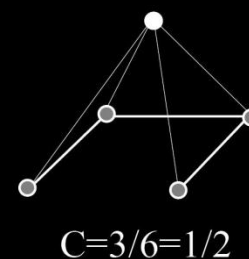
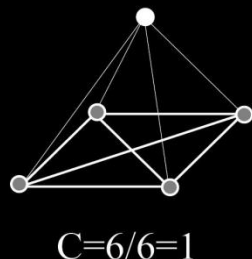
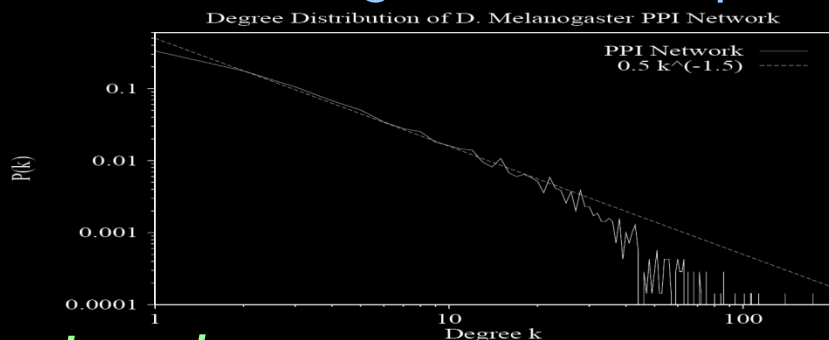
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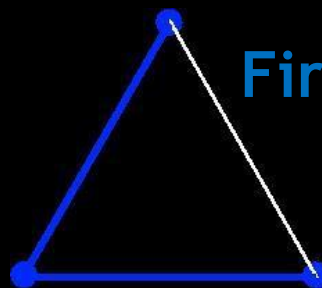
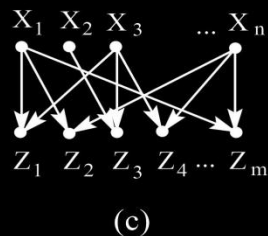
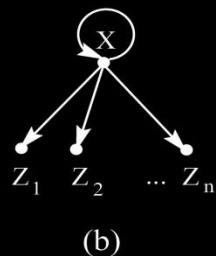
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First 3-node path

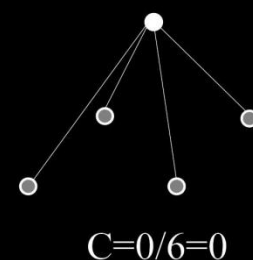
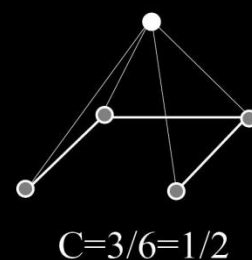
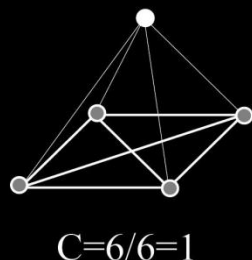
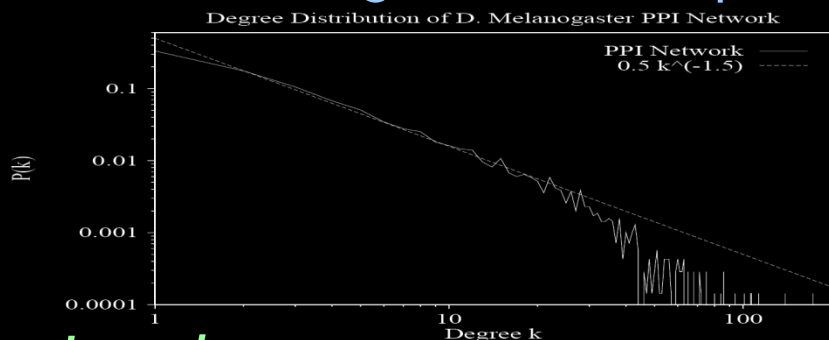
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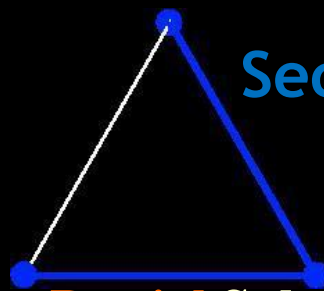
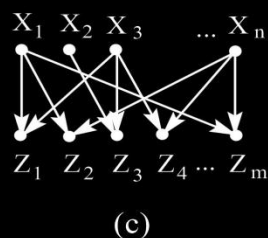
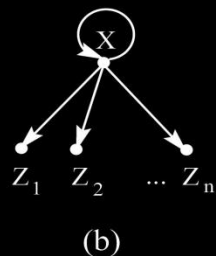
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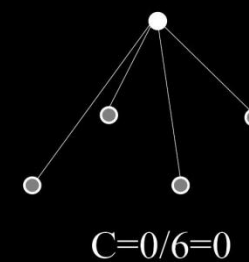
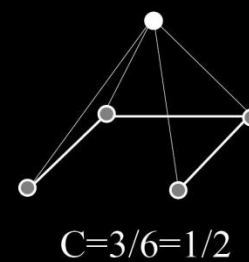
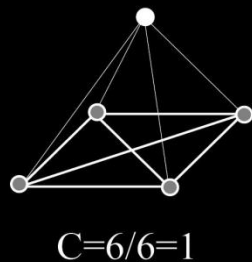
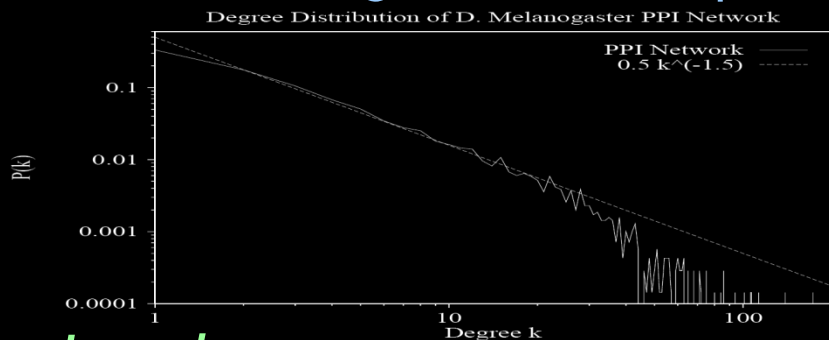
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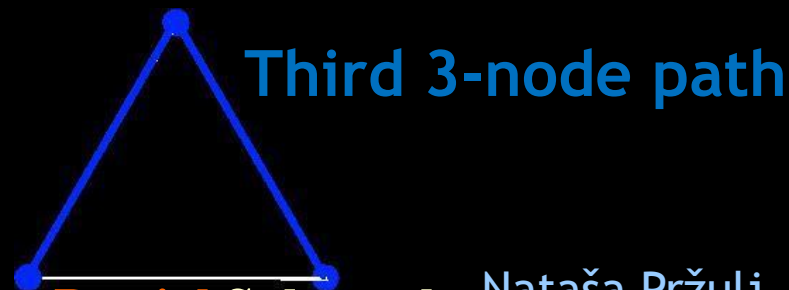
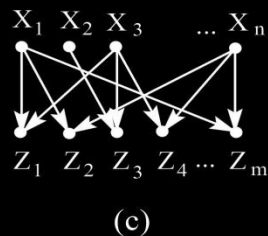
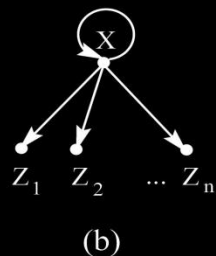
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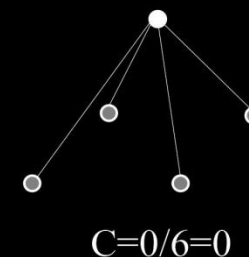
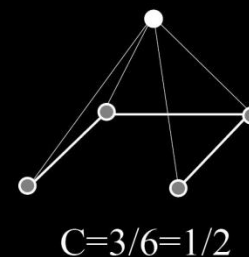
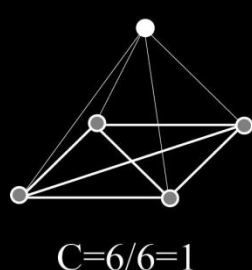
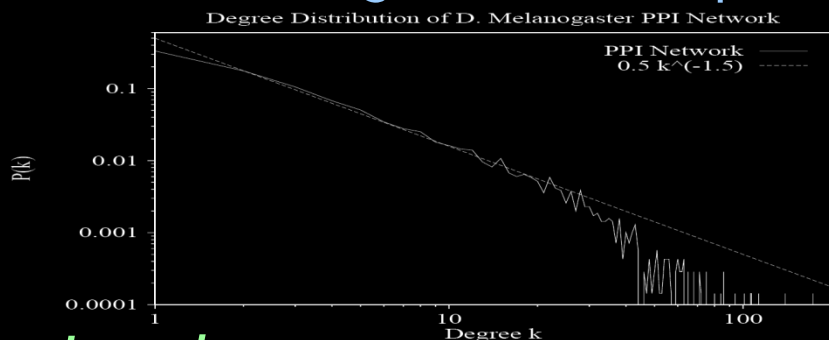
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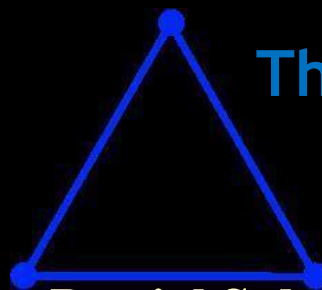
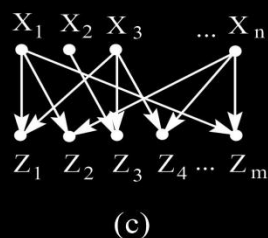
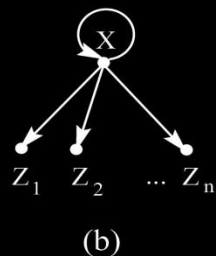
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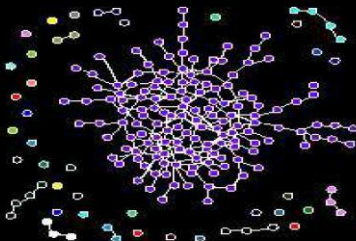
The only triangle

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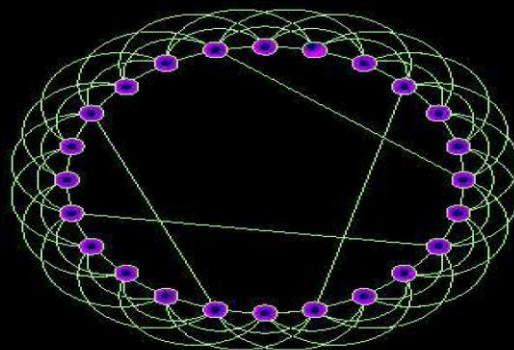
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Examples of different **model networks**:

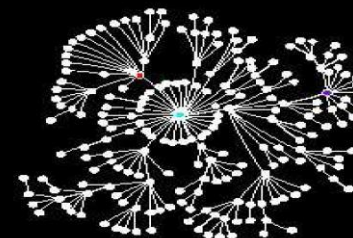
Erdős-Rényi (ER)



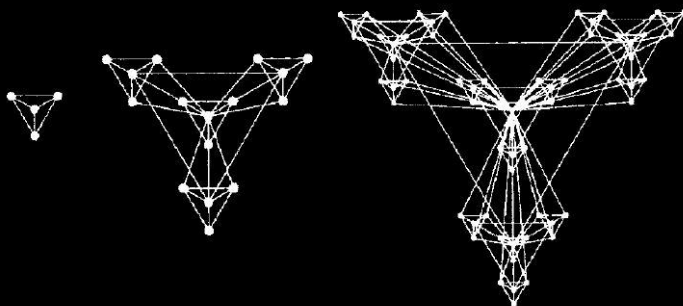
Small-World



Scale-Free (SF)



Hierarchical

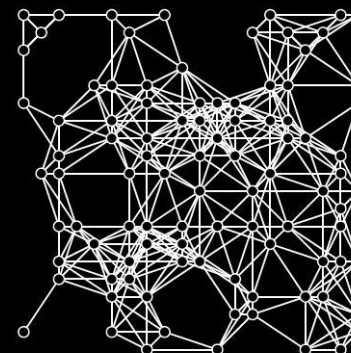


(a) $n=1$; $N=4$

(b) $n=2$; $N=16$

(c) $n=3$; $N=64$

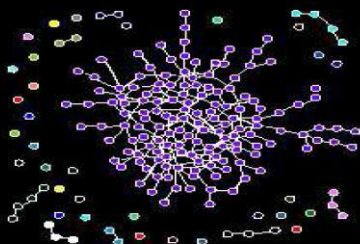
Geometric (GEO)



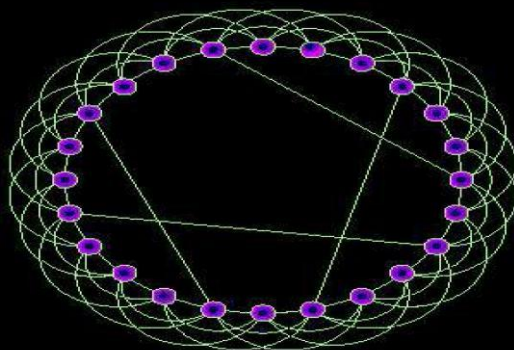
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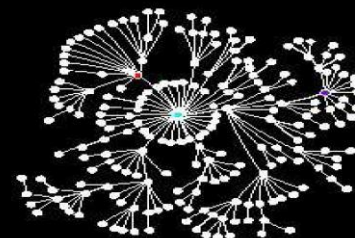
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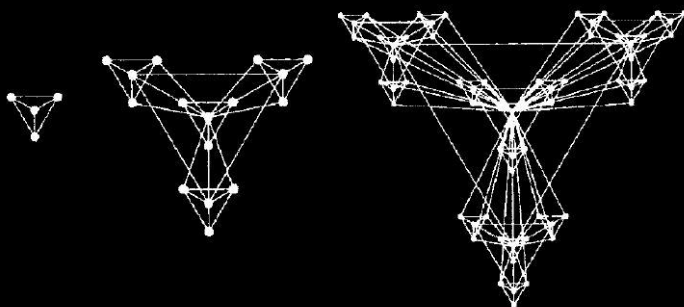
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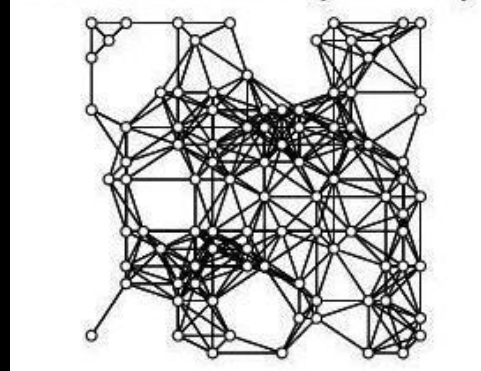


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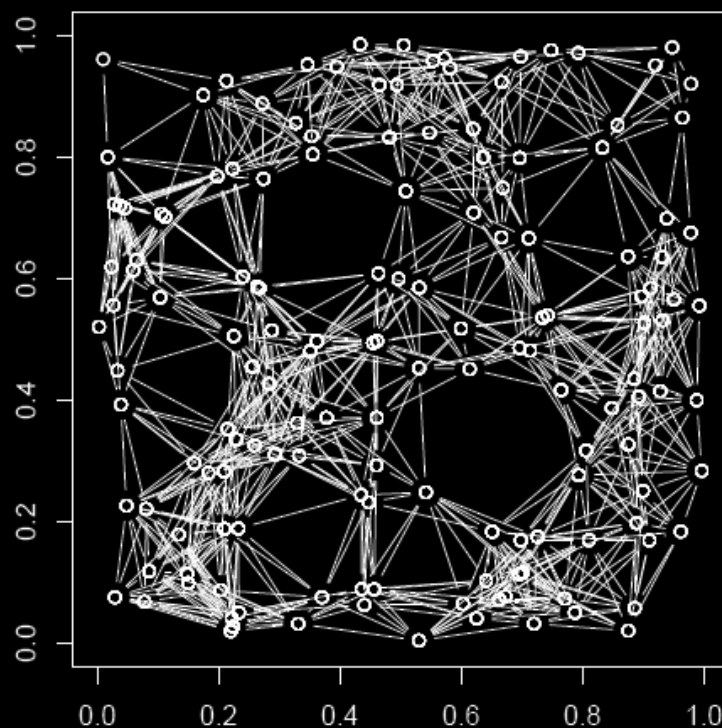
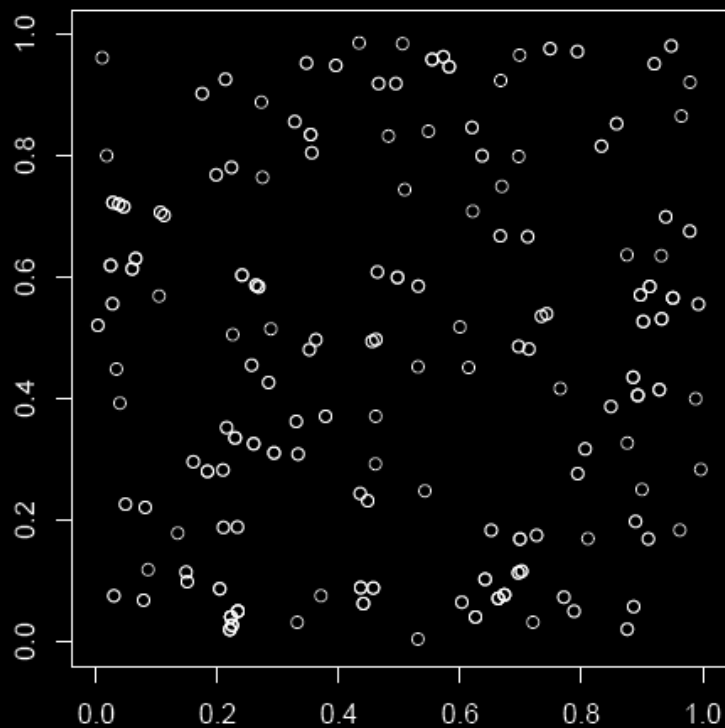
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Geometric (GEO)



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Geometric random graphs, generated using the Euclidean distance.

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1. Introduction and Background:

- PPI and other networks
- Network analysis and modeling challenges

2. New Network Analysis and Modeling:

(A) Analysis:

- Structure
- Network alignment

(B) Modeling – Geometric Graphs:

- Graphlet Degree Distributions
- Network embedding
- Trained Geometric Model

- Application: De-noising PPI networks

(C) Why PPI networks might be geometric?

3. Conclusions

All robust to noise

2. Network Analysis and Modeling

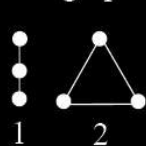
New measures of network local structure

Definition 3 Graphlets are small connected non-isomorphic subgraphs of a graph G induced on $n \geq 3$ nodes of G .

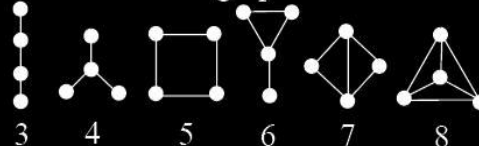
For $n = 3, 4, 5, \dots, 10$, there are 2, 6, 21, \dots , 11716571 graphlets!

All Graphlets on 3-5 nodes:

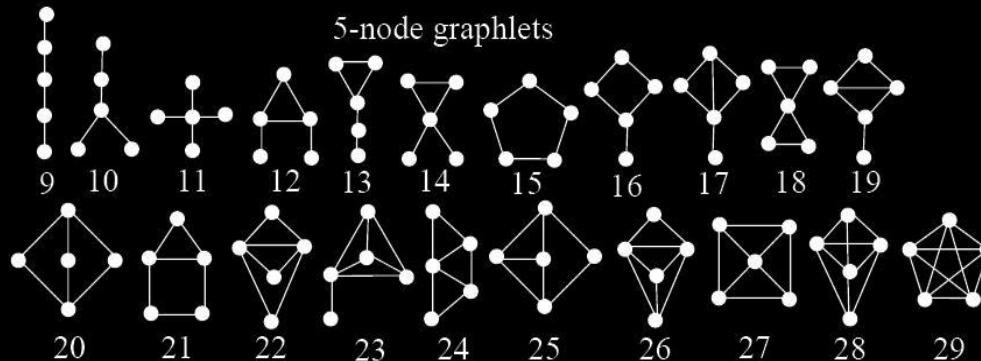
3-node graphlets



4-node graphlets



5-node graphlets



N. Przulj, D. G. Corneil, and I. Jurisica, "Modeling Interactome: Scale Free or Geometric?," *Bioinformatics*, vol. 20, num. 18, pg. 3508-3515, 2004.

2. Network Analysis and Modeling

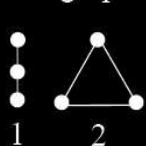
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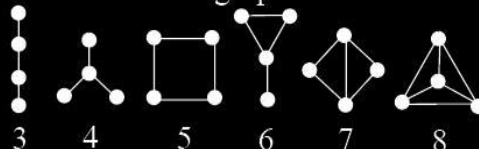
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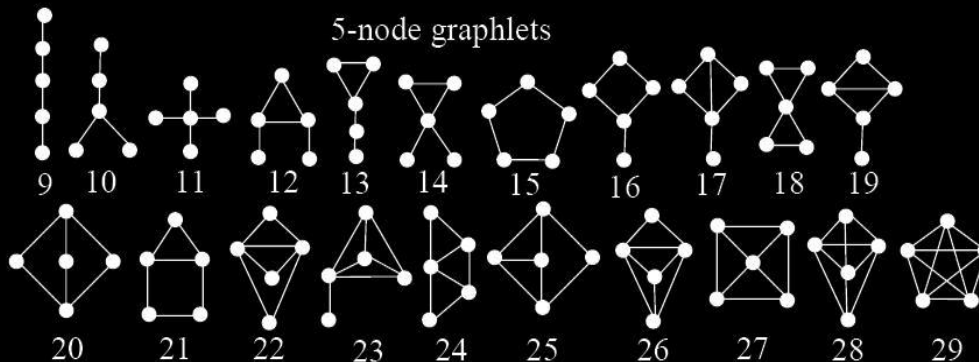
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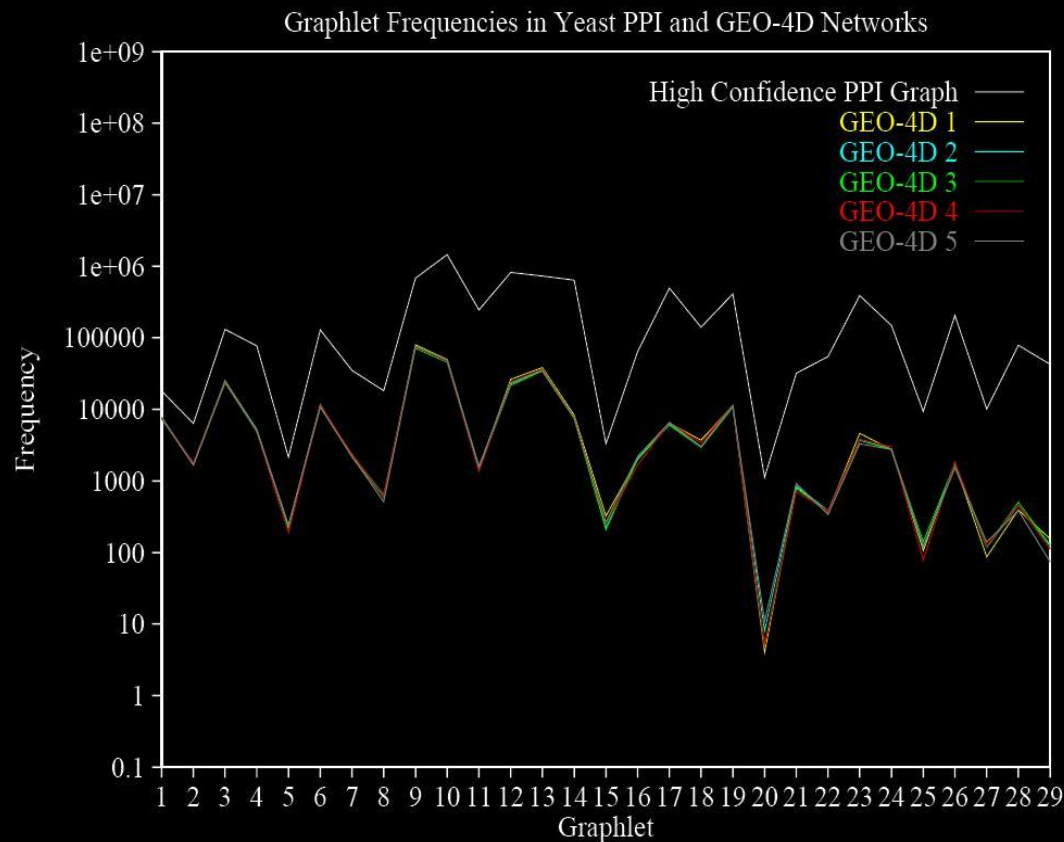
- Induced
- Of any frequency

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2. Network Analysis and Modeling

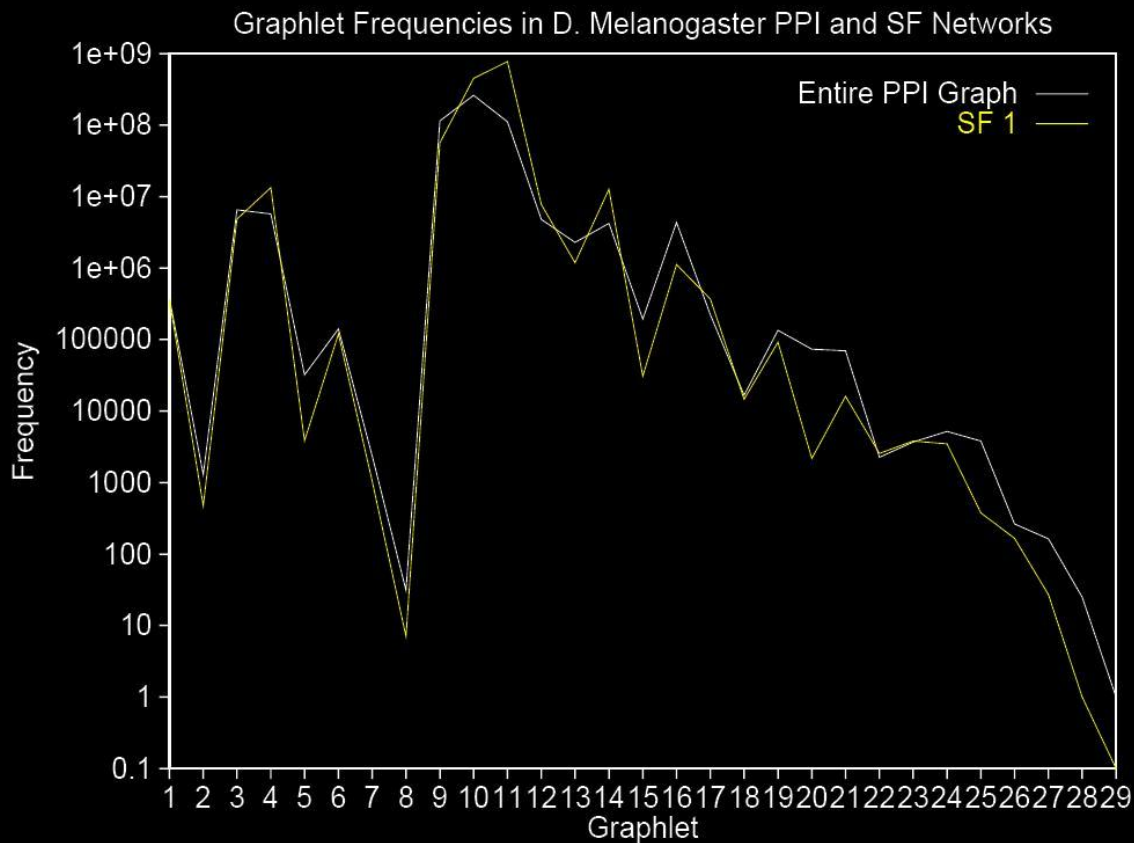
Graphlet Frequencies: *S.cerevisiae* High-Confidence PPI Network
(von Mering *et al.*, *Nature* 417)



N. Przulj, D. G. Corneil, and I. Jurisica, “Modeling Interactome: Scale Free or Geometric?,” *Bioinformatics*, vol. 20, num. 18, pg. 3508-3515, 2004.

2. Network Analysis and Modeling

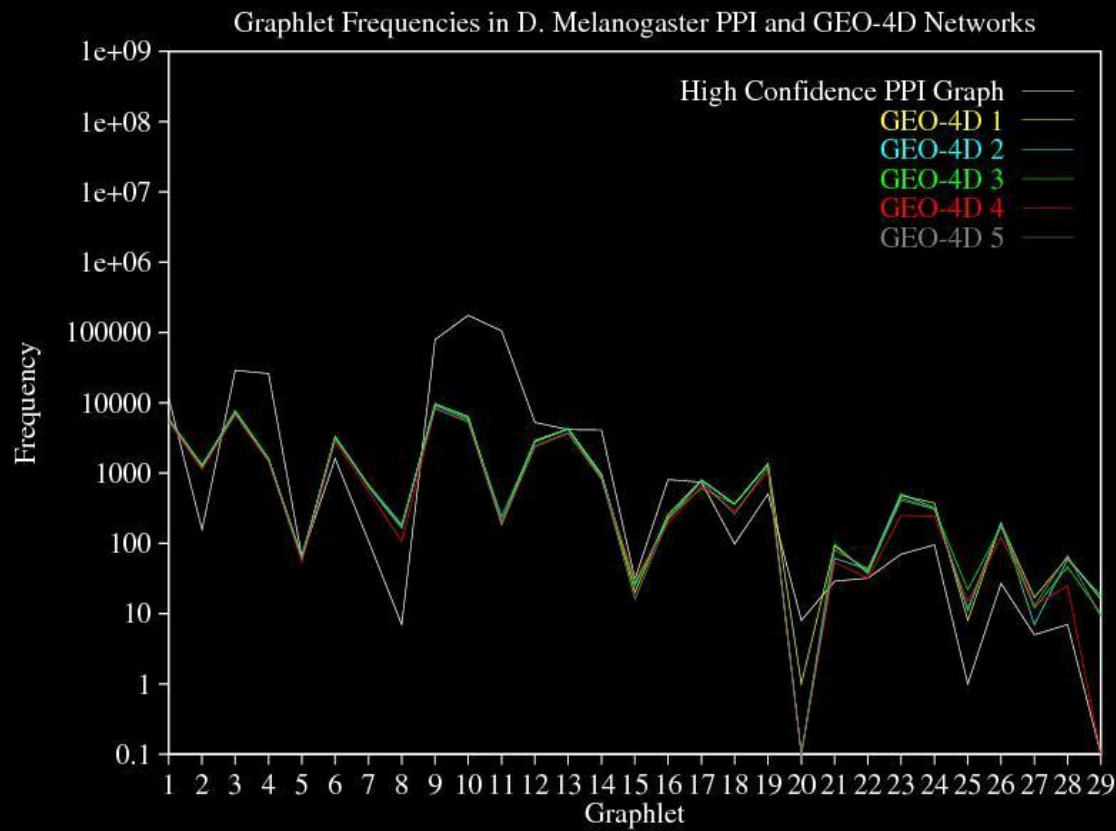
Graphlet Frequencies: *D. melanogaster* Noisy PPI Network
(Giot *et al.*, *Science* 302) (77% of edges are of low confidence)



N. Przulj, D. G. Corneil, and I. Jurisica, “Modeling Interactome: Scale Free or Geometric?,”
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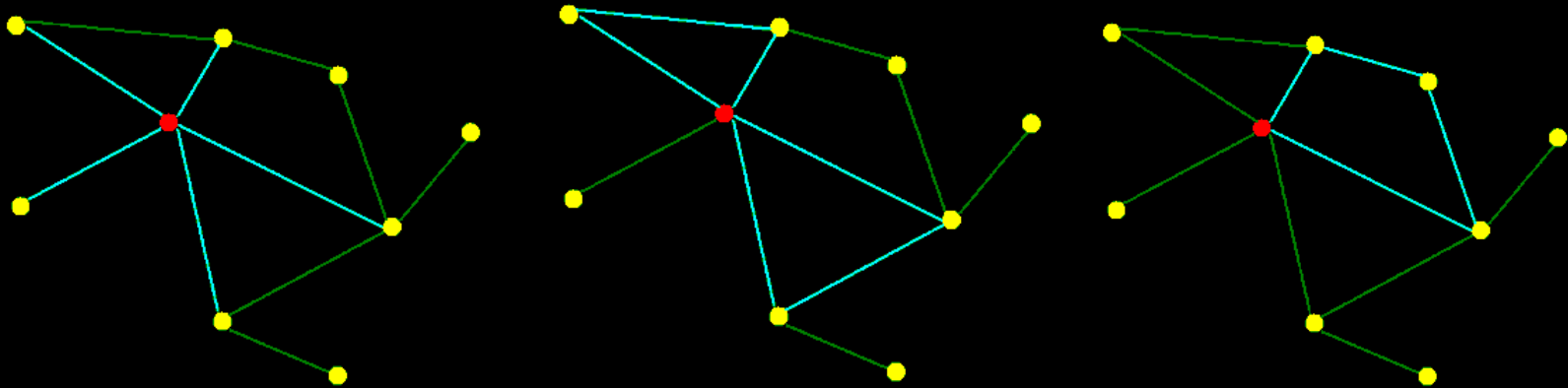
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2. Network **Analysis** and Modeling

Generalize node degree

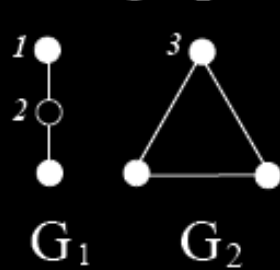


2. Network Analysis and Modeling

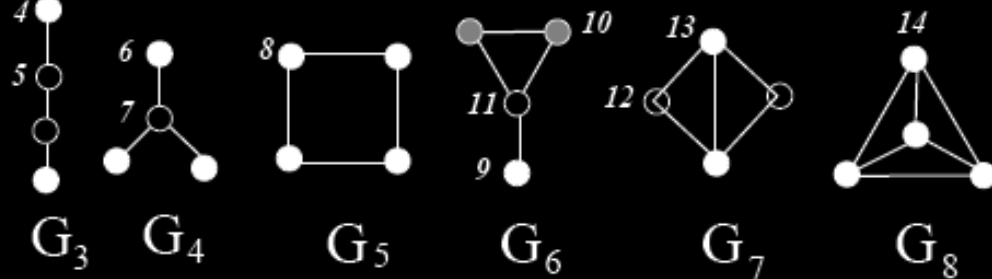
2-node graphlet



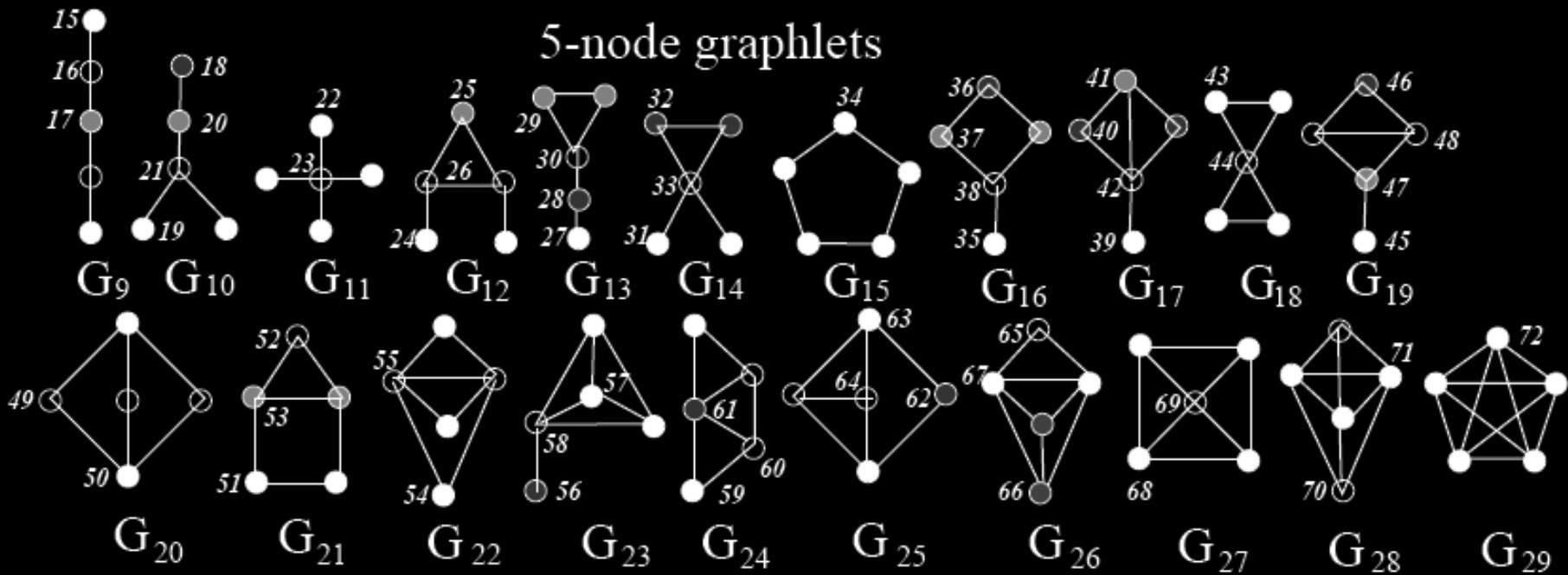
3-node graphlets



4-node graphlets



5-node graphlets



N. Przulj, "Biological Network Comparison Using Graphlet Degree Distribution," *ECCB, Bioinformatics*, vol. 23, pg. e177-e183, 2007.

2. Network **Analysis** and Modeling

Definitions:

An *isomorphism* f from graph G to graph H is a bijection: $f : V(G) \rightarrow V(H)$ such that xy is an edge of G iff $f(x)f(y)$ is an edge of H .

An *automorphism* is an isomorphism from a graph to itself.

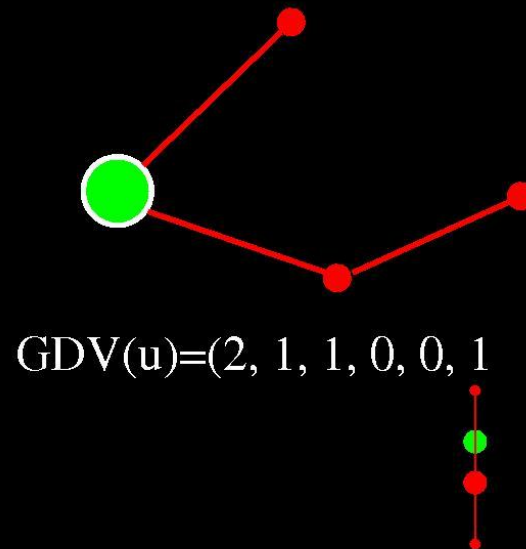
The automorphisms of a graph G form a *group*, called the *automorphism group of G* , and commonly denoted by $Aut(G)$.

For a node x of graph G , the *automorphism orbit of x* is $Orb(x) = \{y \in V(G) \mid y = f(x) \text{ for some } f \in Aut(G)\}$, where $V(G)$ is the set of nodes of graph G .

2. Network **Analysis** and Modeling

Network structure vs. biological function & disease

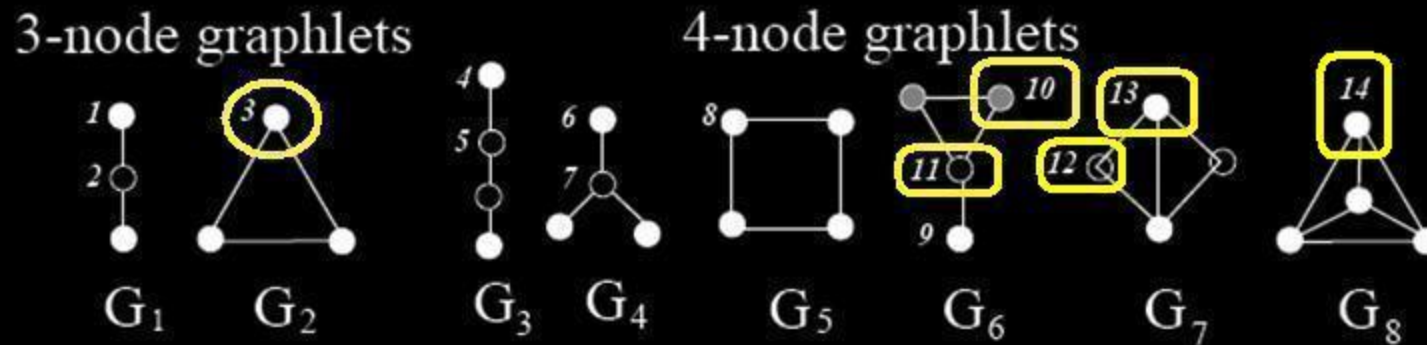
Graphlet Degree (GD) vectors, or “node signatures”



2. Network **Analysis** and Modeling

Network structure vs. biological function & disease

Similarity measure between nodes' Graphlet Degree vectors



2. Network Analysis and Modeling

Signature Similarity Measure

- o_i is number of orbits that affect orbit $i \in \{0, \dots, 72\}$
- $w_i = 1 - \frac{\log(o_i)}{\log(73)}$
- Distance between the i^{th} orbits of nodes u and v is

$$D_i(u, v) = w_i \times \frac{|\log(u_i+1) - \log(v_i+1)|}{\log(\max\{u_i, v_i\} + 2)}$$

- The total distance between nodes u and v is

$$D(u, v) = \frac{\sum_{i=0}^{72} D_i}{\sum_{i=0}^{72} w_i}$$

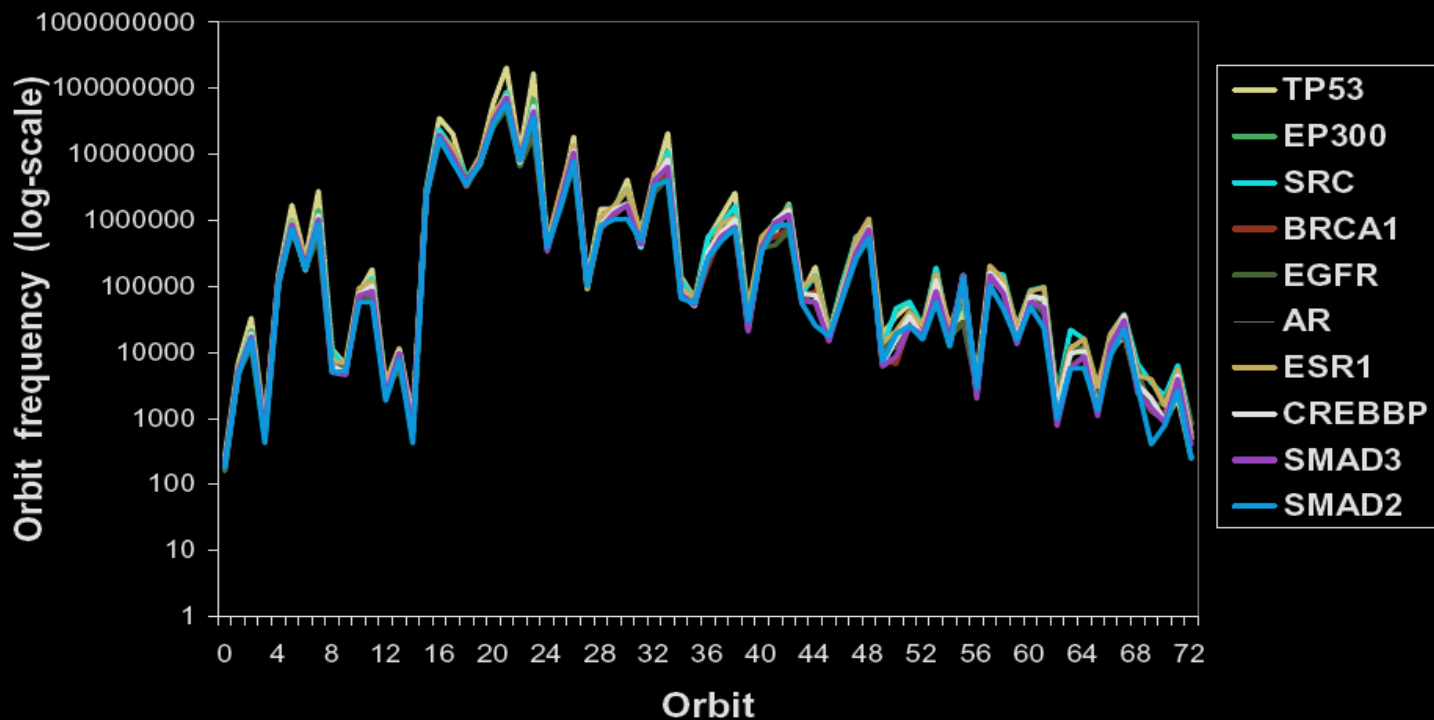
- The *signature similarity* between nodes u and v is

$$S(u, v) = 1 - D(u, v)$$

2. Network Analysis and Modeling

Network structure vs. biological function & disease

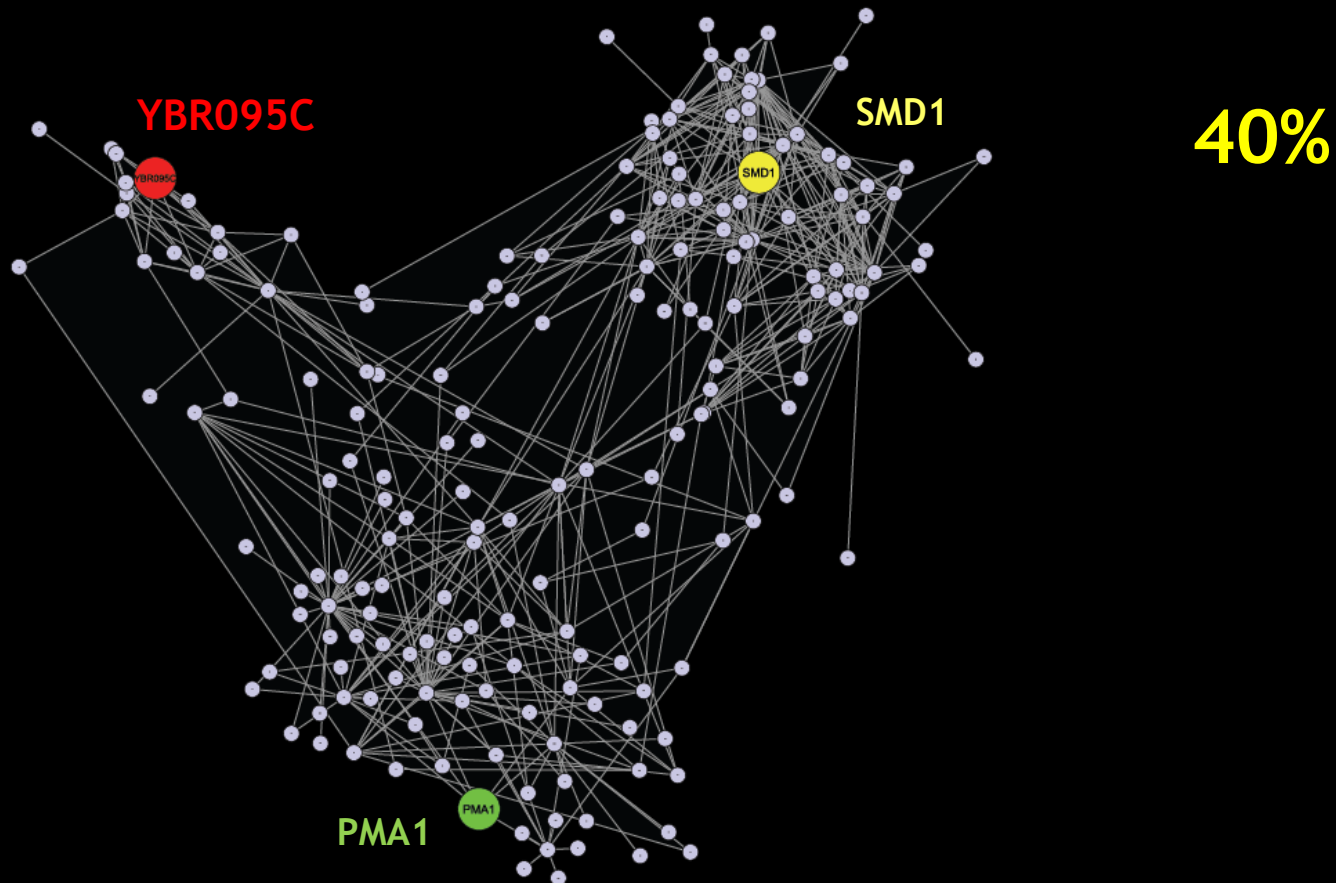
Signatures of proteins belonging to the TP53 cluster



96%

2. Network **Analysis** and Modeling

Network structure vs. biological function & disease

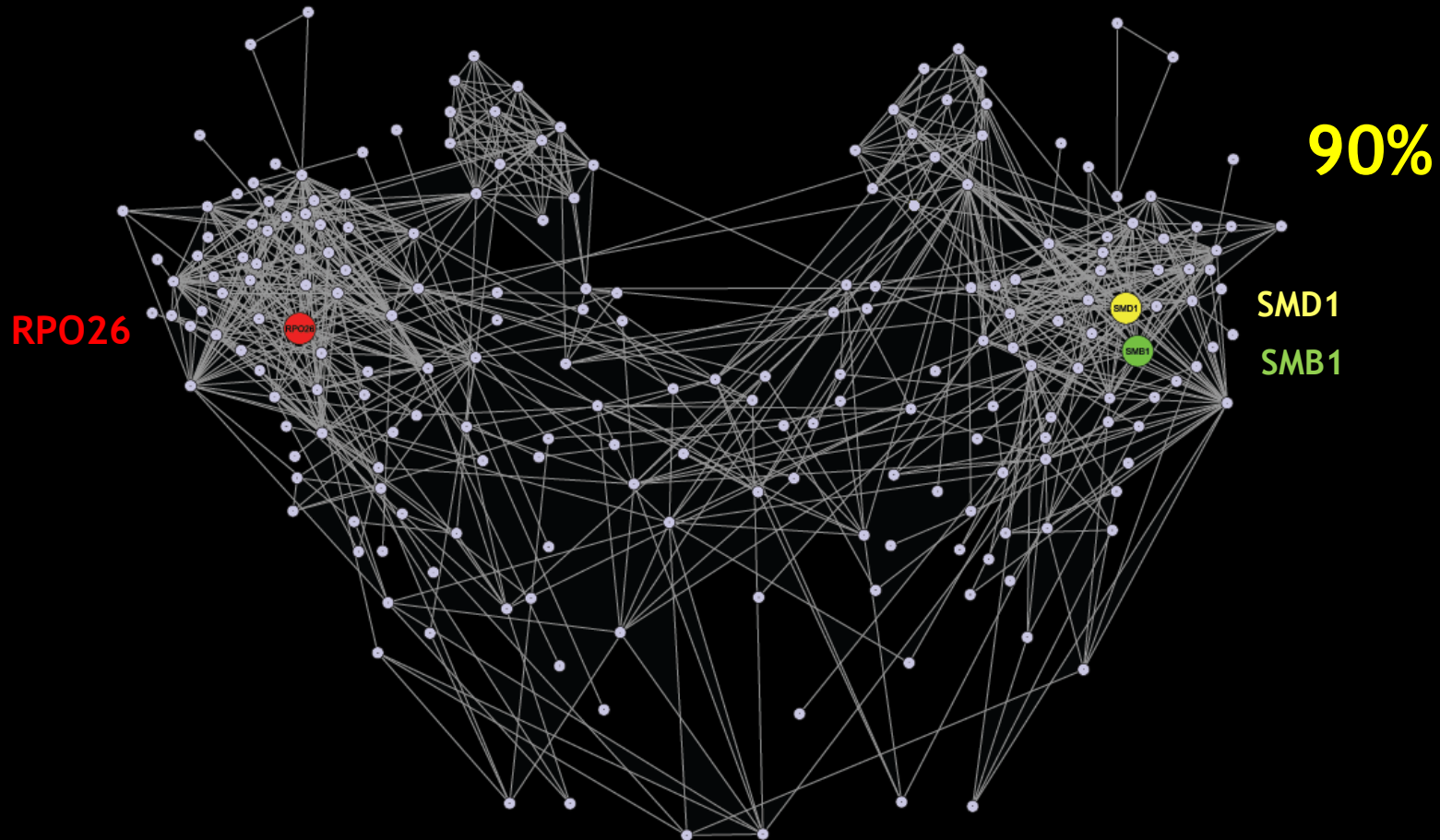


T. Milenkovic and N. Pržulj, “Uncovering Biological Network Function via Graphlet Degree Signatures”, *Cancer Informatics*, vol. 4, pg. 257-273, 2008.

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Network structure vs. biological function & disease

- For PPI networks of yeast and human:
 - Find GD signatures for all nodes in a PPI network
 - Cluster based on signature similarities
- Obtained clusters are significantly enriched in:
 - Biological function
 - Protein complexes
 - Sub-cellular localization
 - Tissue expression
 - Disease

⇒ **Predict protein function and involvement in disease**

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⇒ Find new members of melanoma pathways

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could not have been identified by existing approaches

H. Ho, T. Milenkovic, V. Memisevic, J. Aruri, N. Przulj, and A. K. Ganesan, "Protein Interaction Network Uncovers Melanogenesis Regulatory Network Components within Functional Genomics Datasets," *BMC Systems Biology*, 4:84, June 15, 2010.

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⇒ **New members of the yeast proteasome PPI network**

C. Guerrero, T. Milenkovic, N. Przulj, P. Keiser, L. Huang, "Characterization of the proteasome interaction network using a QTAX-based tag-team strategy and protein interaction network analysis," *PNAS*, 105 (36), pg. 13333-13338 2008.

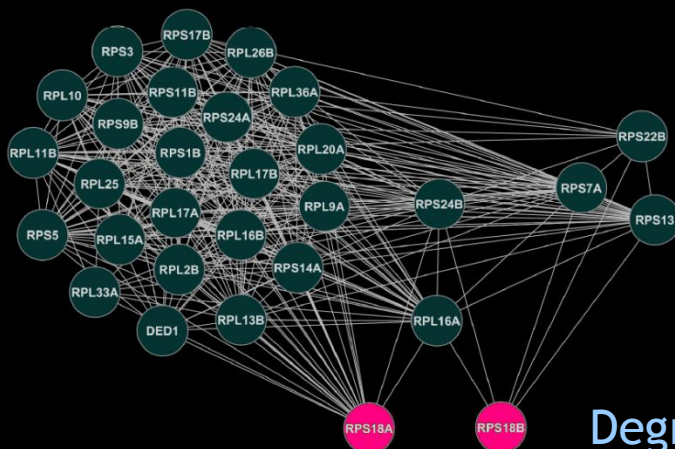
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2. Network Analysis and Modeling

Network structure vs. biological function & disease

Topology and Sequence: complementary sources of homology info.

- 59 of the yeast ribosomal proteins - retained two genomic copies
- Are duplicated proteins functionally redundant?
- No: have different genetic requirements for their assembly and localization, so are functionally distinct
- Also note: avg sequence identity of struct. similar prots ~8-10%
- E.g., two pairs with identical sequence:



100% sequence identity
50% signature similarity

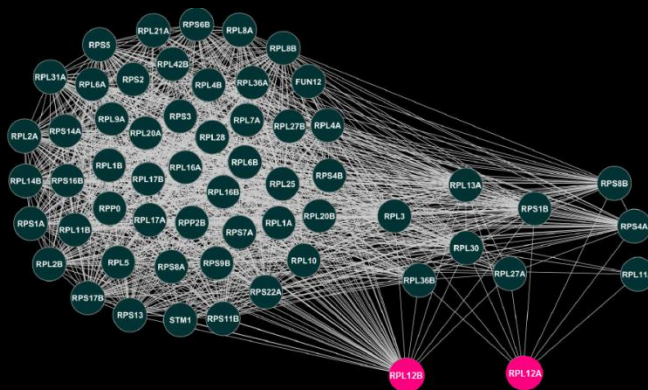
Degrees 25 and 5

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65% signature similarity

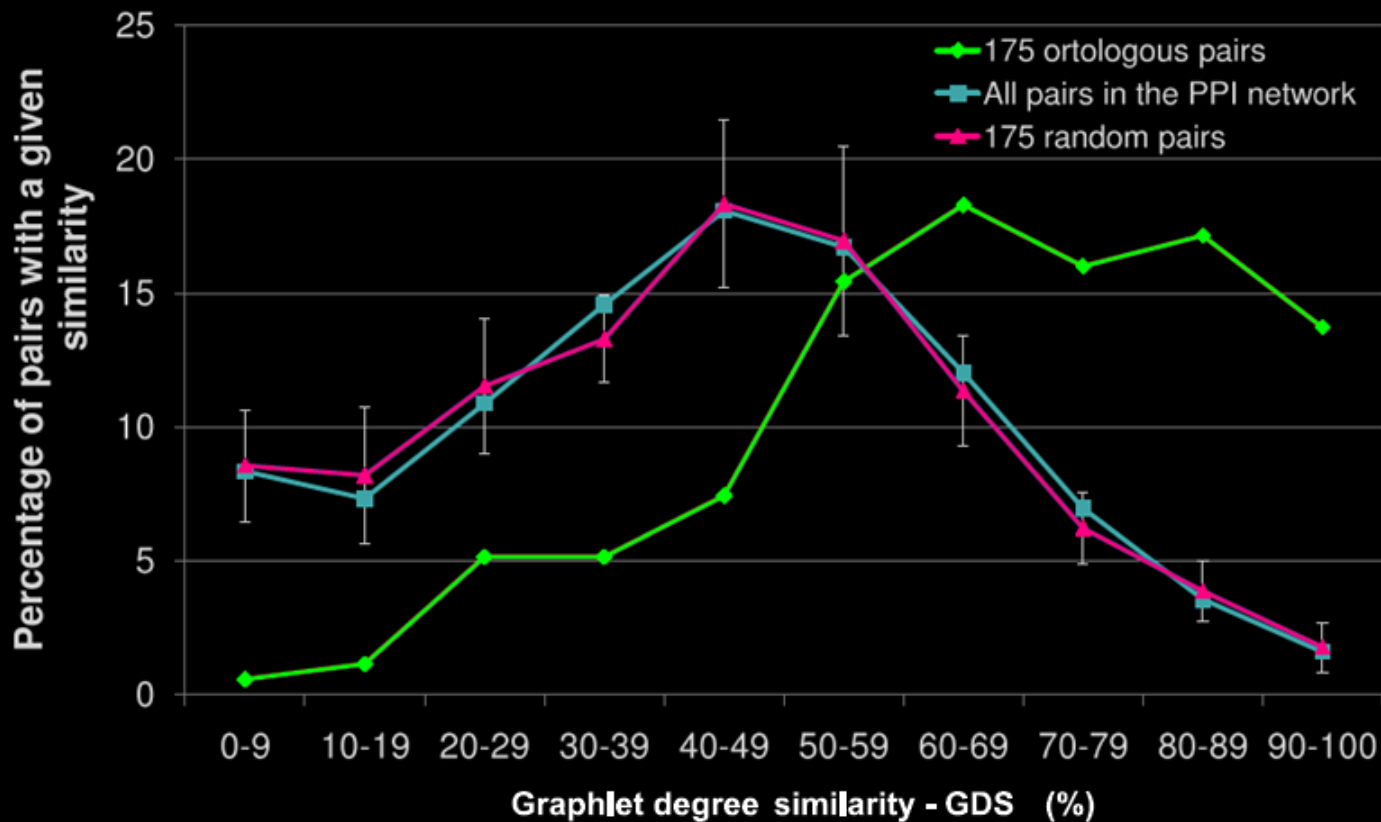
Degrees 54 and 9

2. Network Analysis and Modeling

Network Topology

- Orthologous proteins (yeast) have high GD vector similarities

Distributions of GD similarities

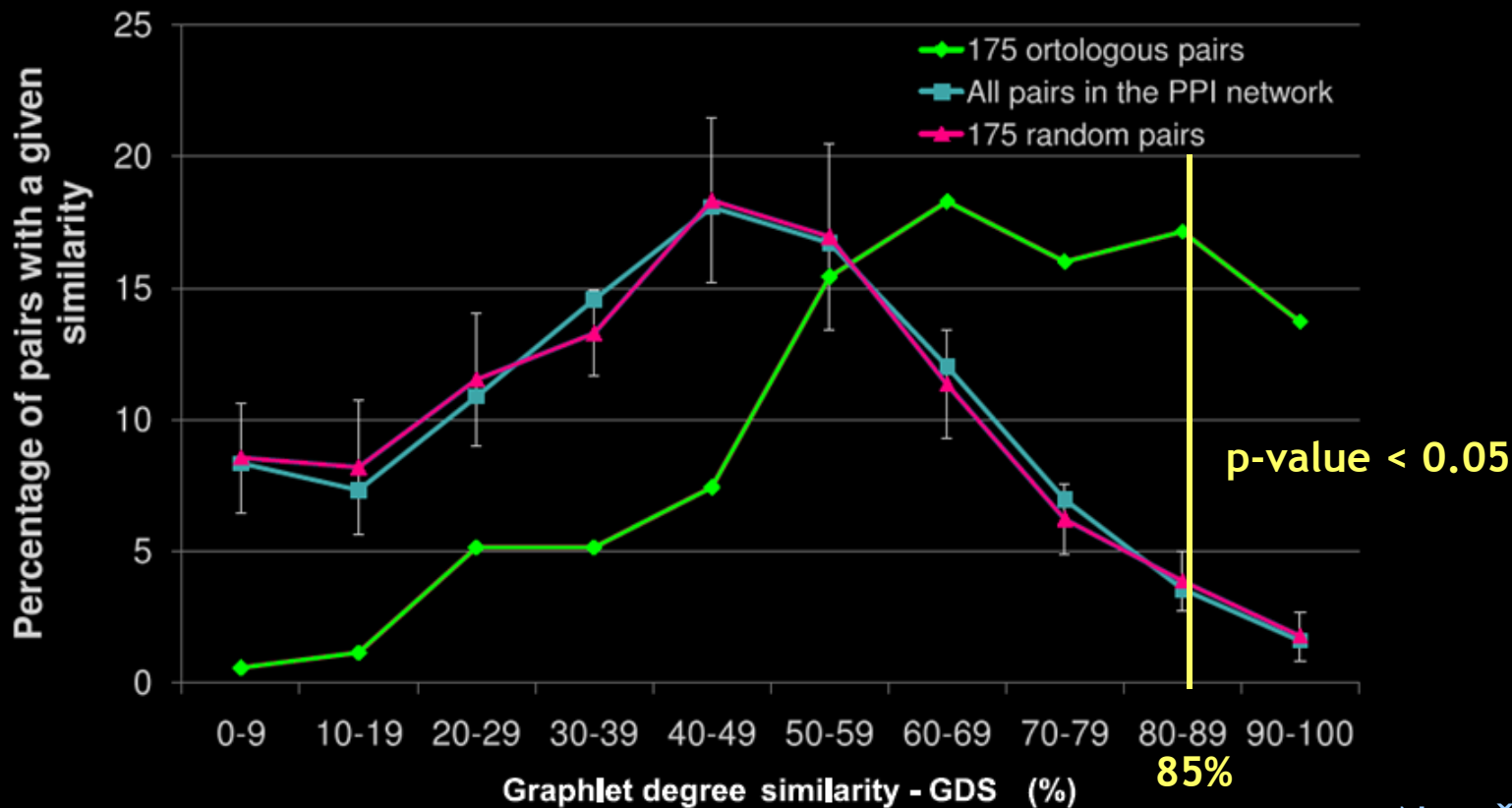


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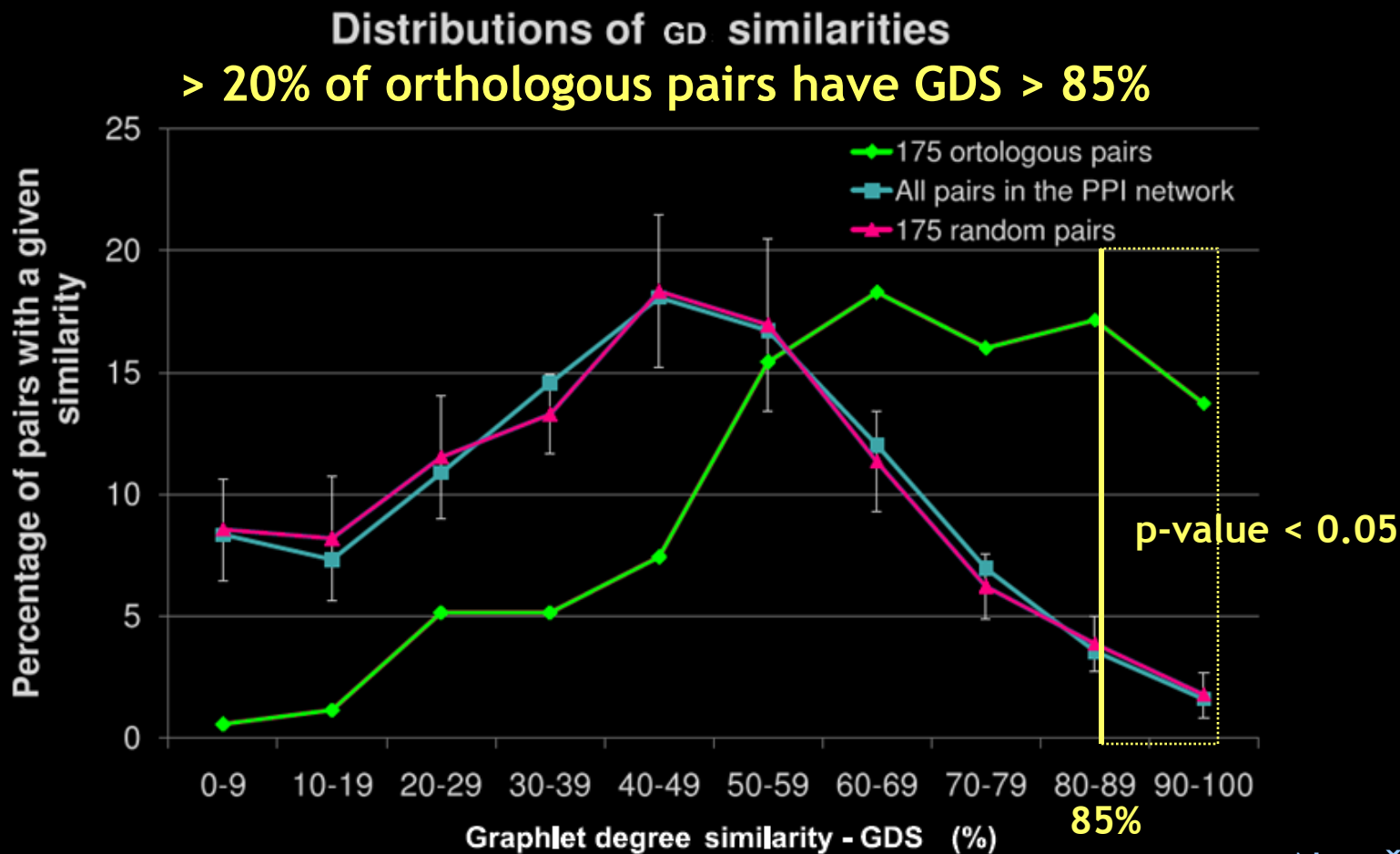
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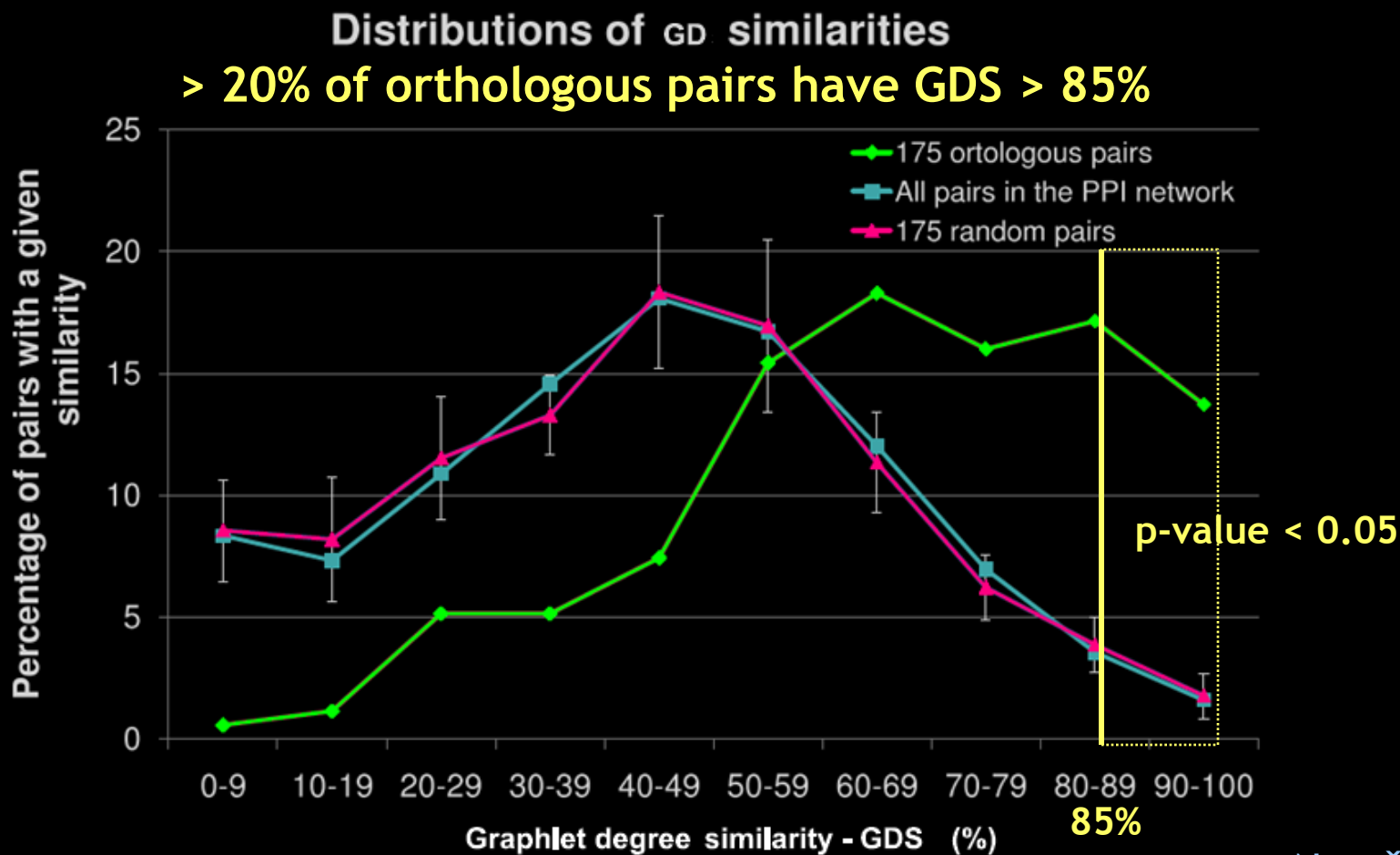
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2. Network **Robust to noise**

Network Topology

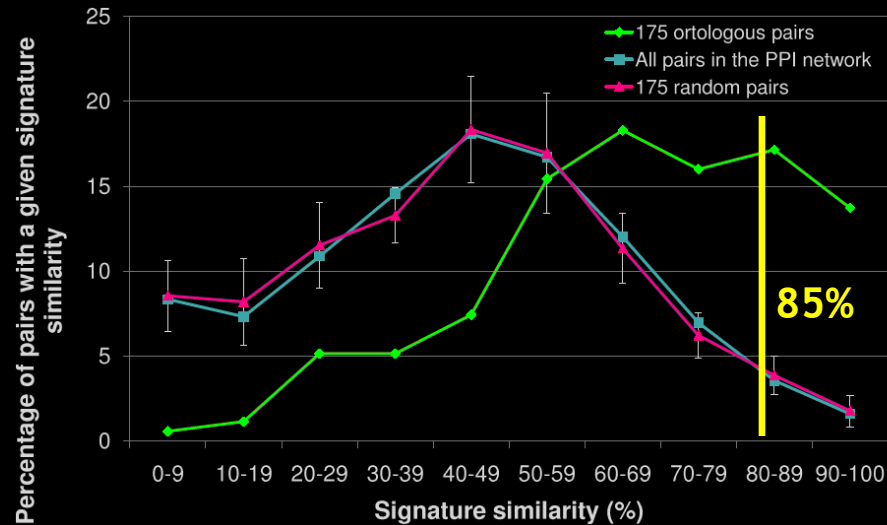
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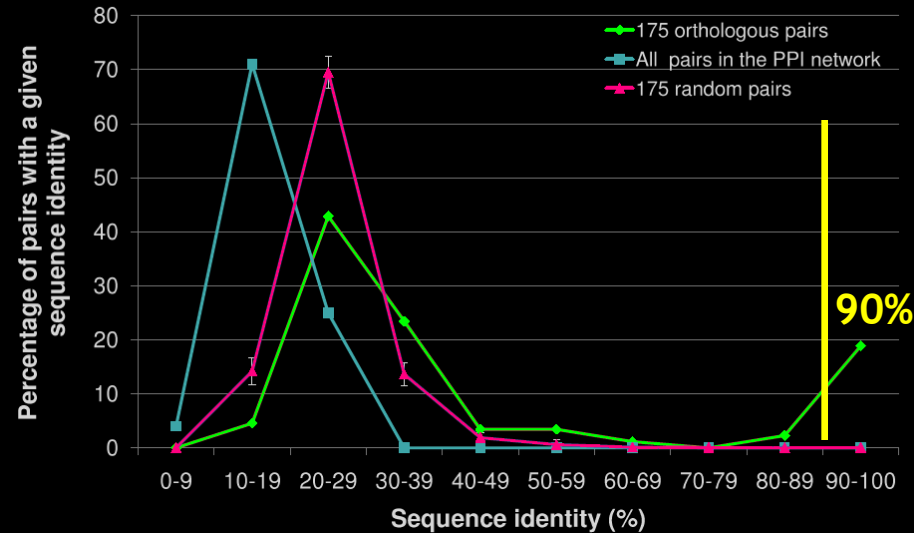
Comparison of topology and sequence w.r.t. orthology:

Distributions of signature similarities



~20% of orthologous pairs have signature similarities above 85% (35 pairs)

Distributions of sequence identities

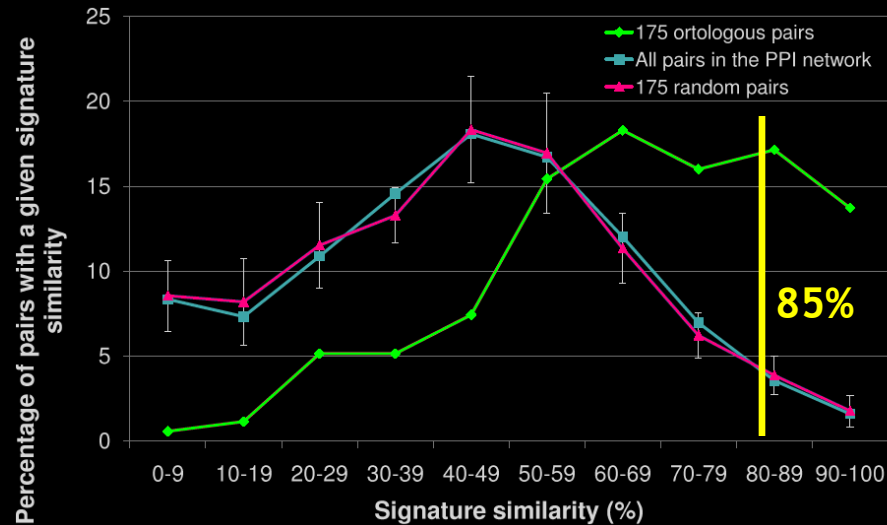


~20% orth. pairs have seq. identity > 90%

2. Network Analysis and Modeling

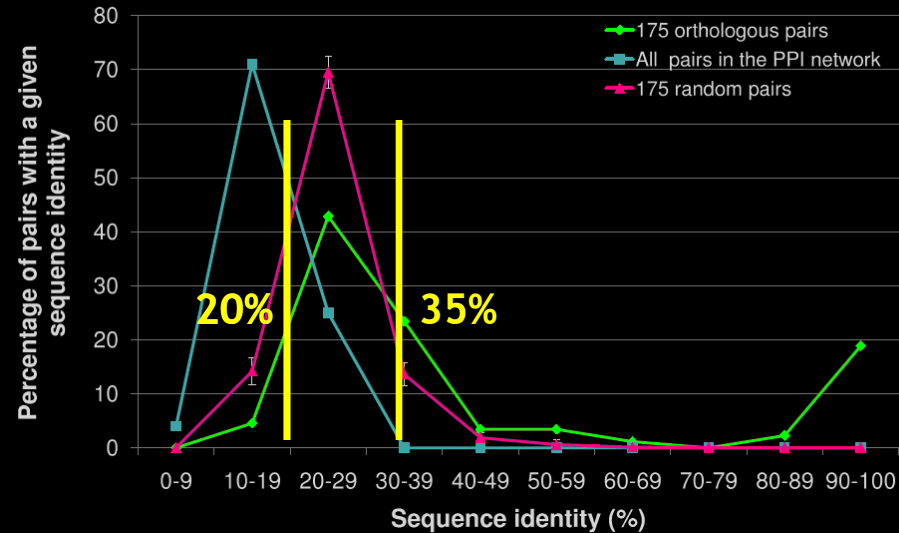
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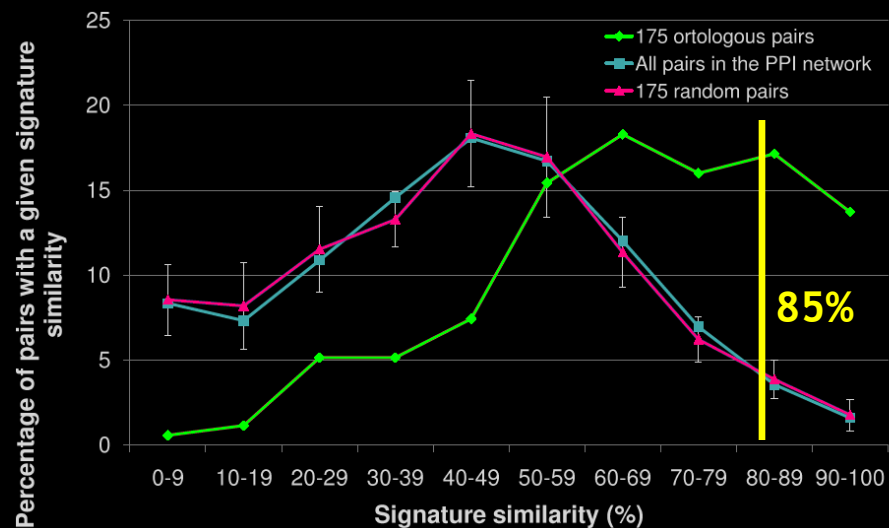


“Twilight zone” for homology
 ~70% orth. pairs have seq. identity < 35%
 ⇒ No dependence on the absolute similarity COG & KEGG, but triangles in the graph of best matches

2. Network Analysis and Modeling

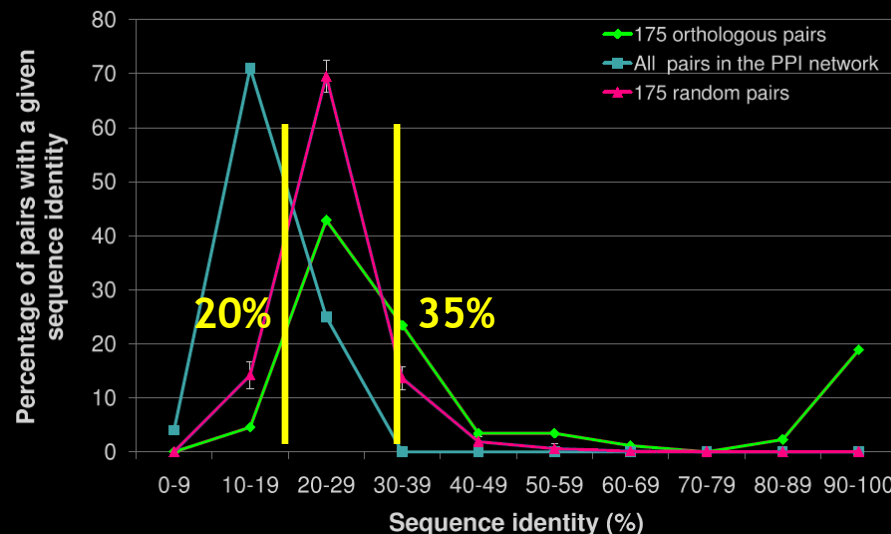
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Distributions of sequence identities



~30% of orthologous pairs have sequence identities above 35% (53 pairs)

Overlap: 22 pairs (~60% of the smaller set)

⇒ Sequence and network topology
somewhat complementary slices of homology information

V. Memisevic, T. Milenkovic and N. Przulj, "Complementarity of network and sequence information in homologous proteins", *J. Integrative Bioinformatics*, 2010.

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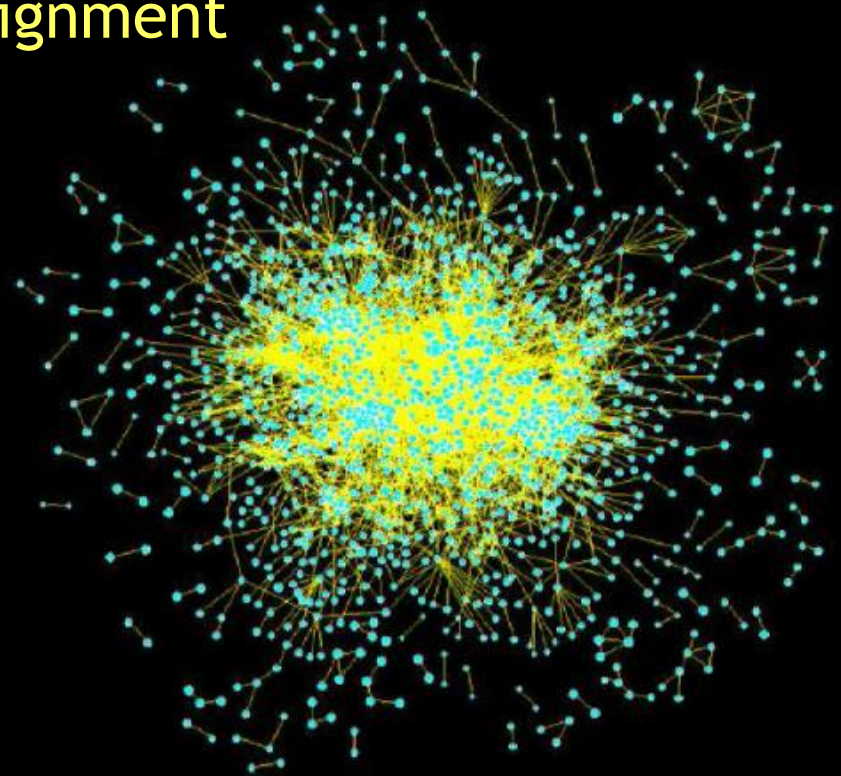
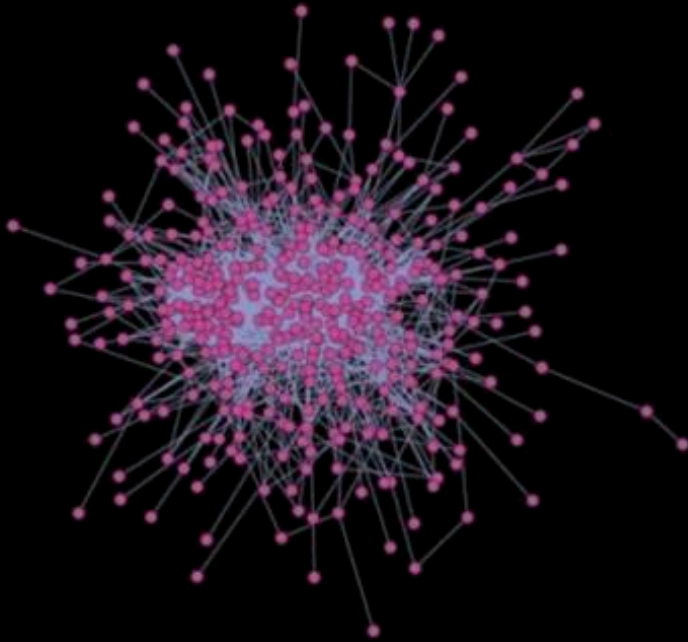
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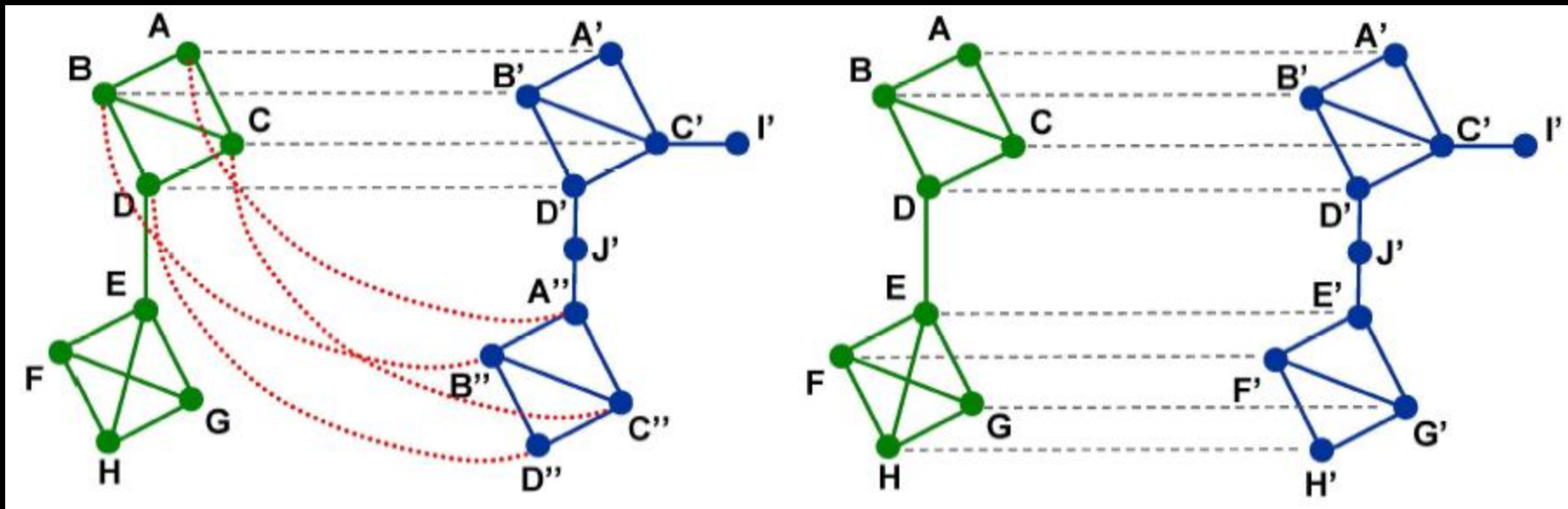
Network Alignment



- Exact network comparisons are computationally intractable
 - Subgraph Isomorphism Problem is NP-C (Cook, 1971)
 - Rely on approximate or heuristic approaches

2. Network Analysis and Modeling

Network Alignment

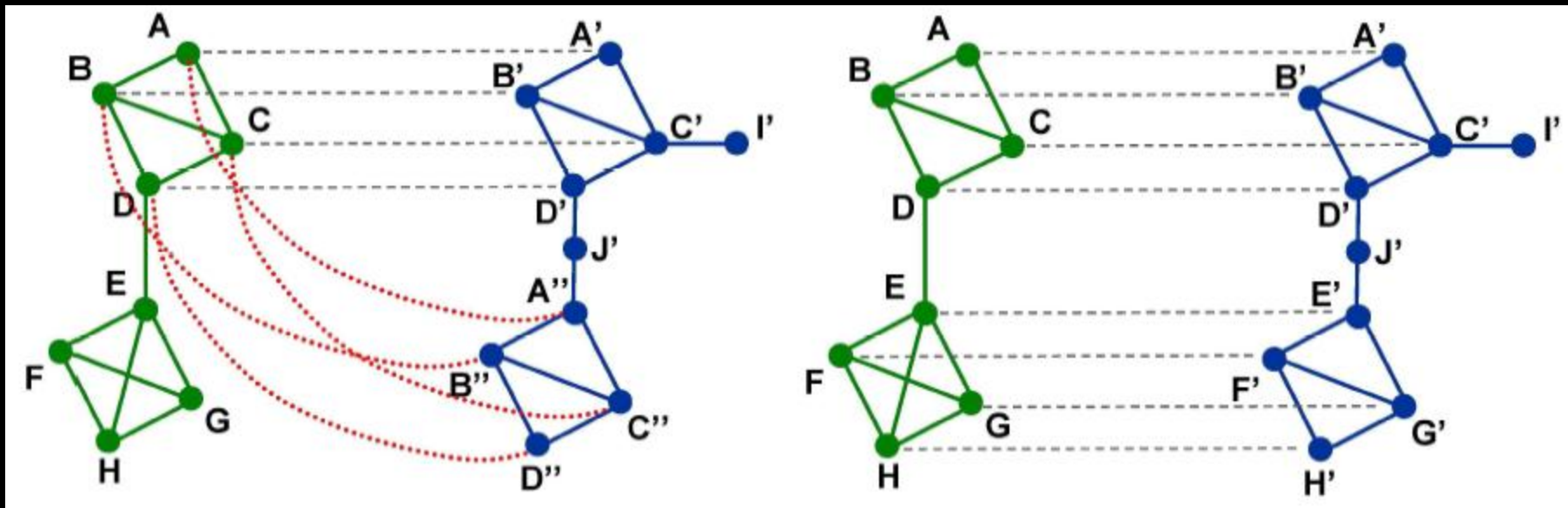


Local network alignment

Global network alignment

2. Network **Analysis** and Modeling

Network Alignment



Local network alignment

Global network alignment

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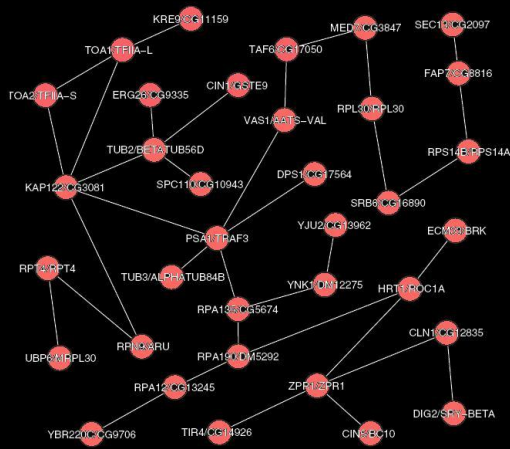
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- Find **GD node signatures** across different networks
- Align “signature-similar” nodes – “**seed nodes**” in each network
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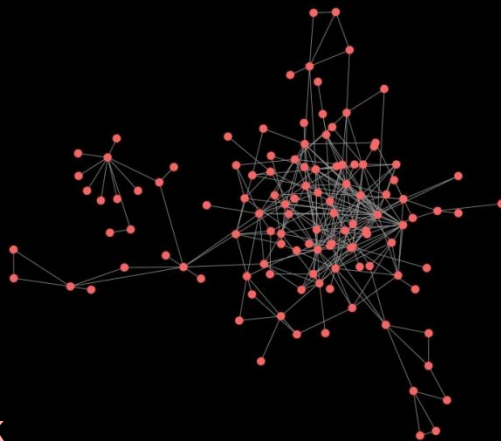


Isorank: Singh, Xu, Berger, “Pairwise Global Alignment of Protein Interaction Networks by Matching Neighborhood Topology,” *RECOMB 2007*, LNBI 4453, pp. 1631, 2007.

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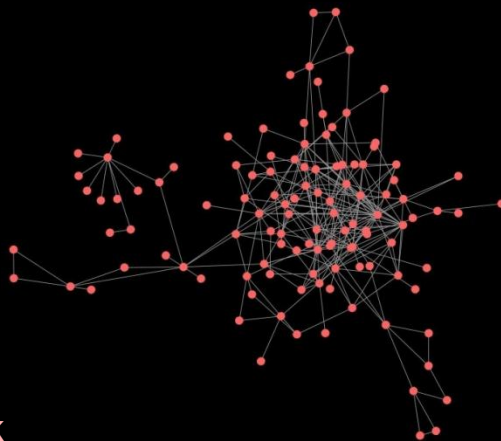
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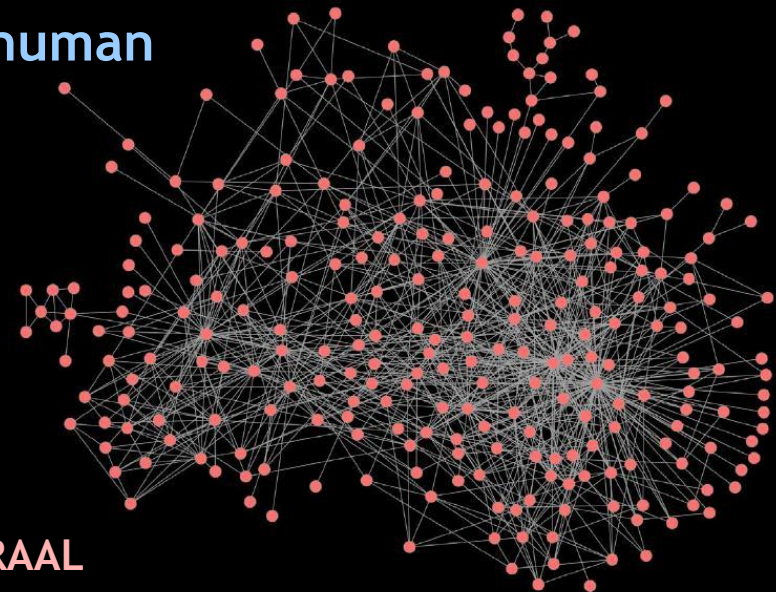
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Isorank

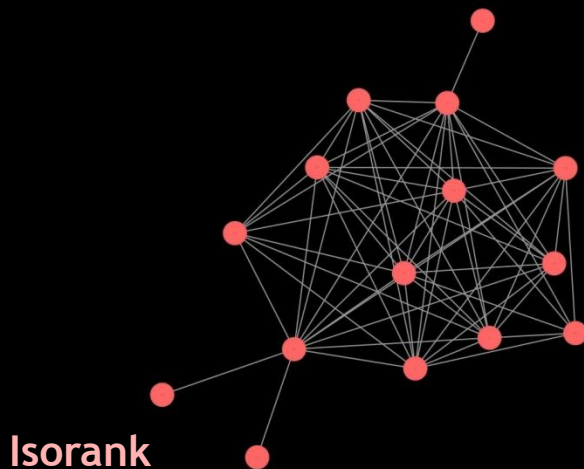


GRAAL
(GRAPh ALigner)

2. Network **Analysis** and Modeling

Network Alignment

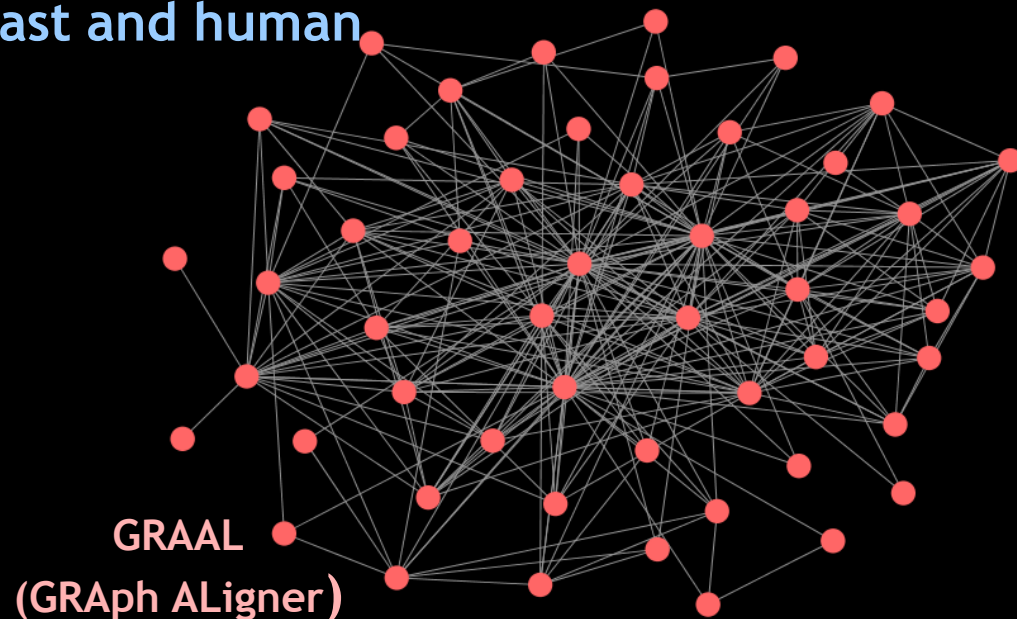
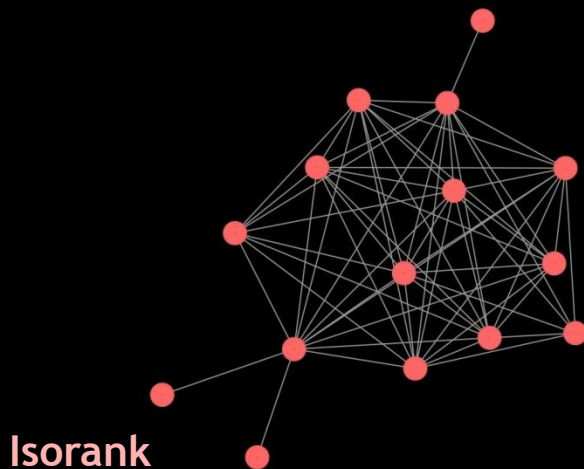
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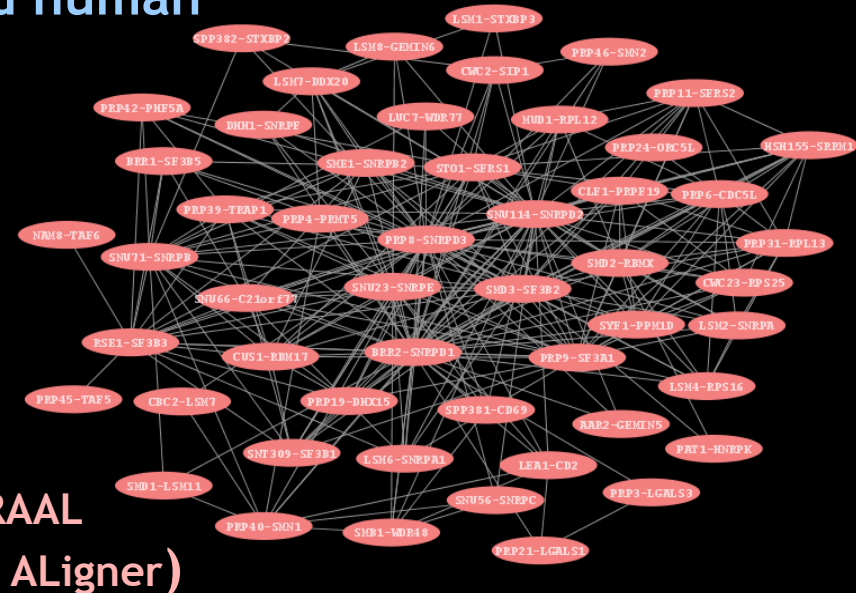
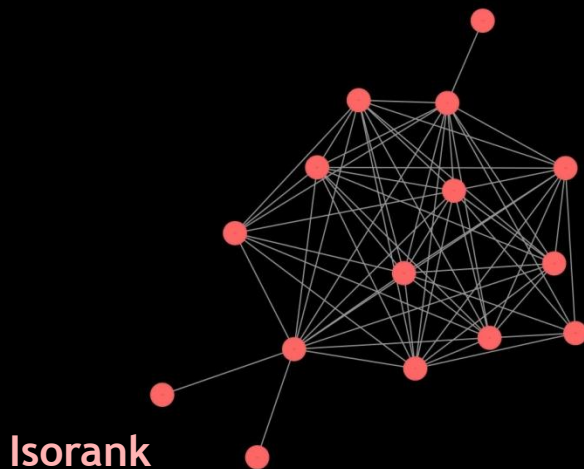
O. Kuchaiev, T. Milenkovic, V. Memisevic, W. Hayes and N. Przulj, “Topological Network Alignment Uncovers Biological Function and Phylogeny”, *J. Roy Soc. Interface*, 2010.

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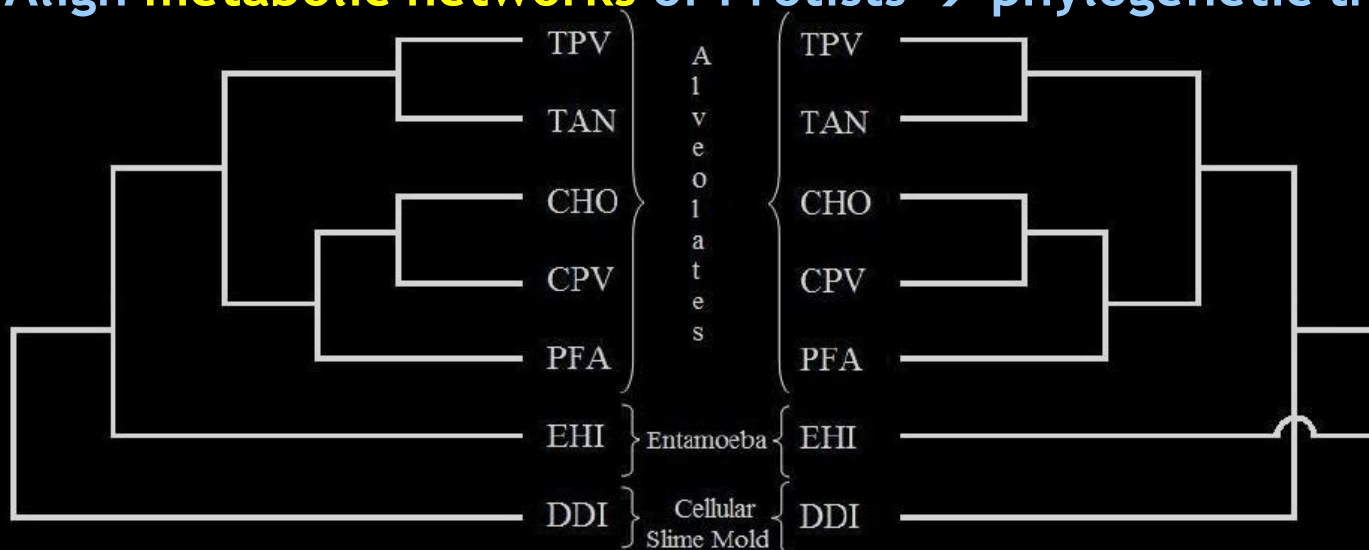
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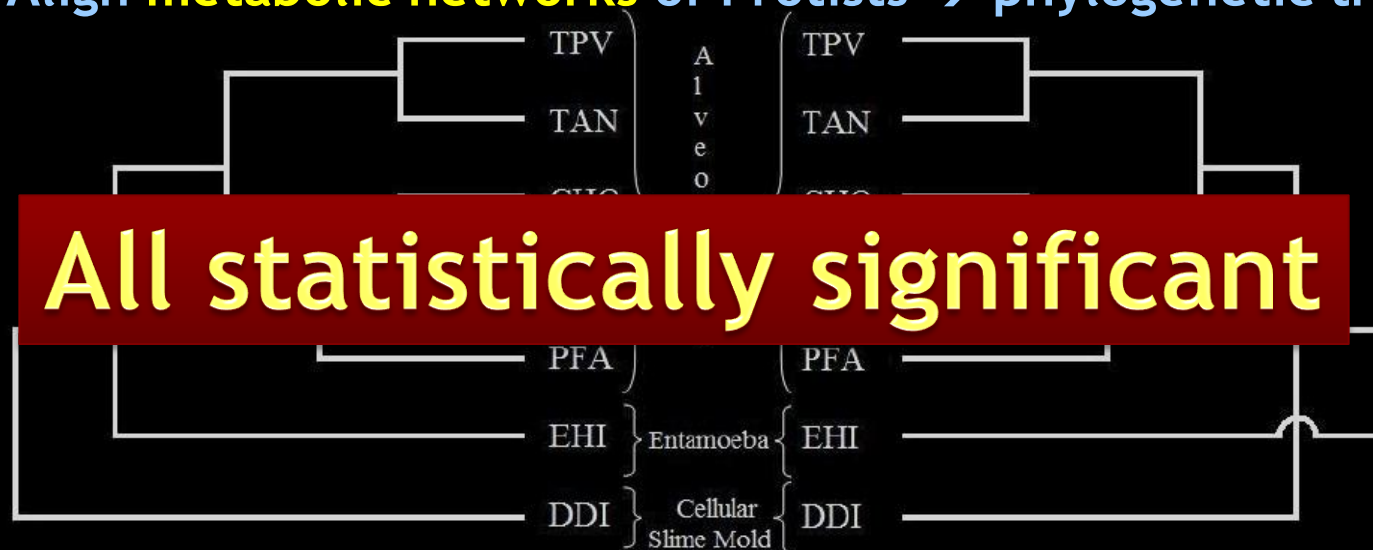
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 - Align **metabolic networks** of Protists → phylogenetic tree



2. Network **Analysis** and Modeling

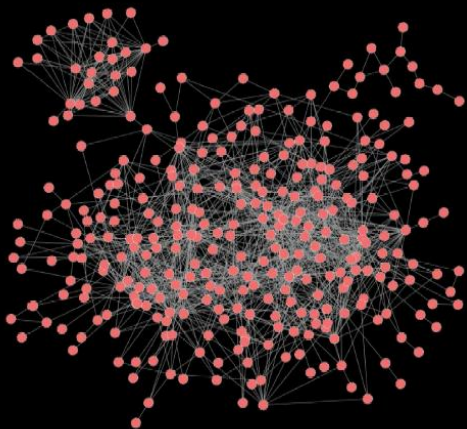
Network Alignment

- Find **GD node signatures** across different networks
- Align “signature-similar” nodes
- BUT, not in a seed-and-extend greedy way
- Use the *Hungarian Algorithm* for minimum weight bipartite matching
 - Hence termed **H-GRAAL**
 - Find **an optimal alignment** with respect to the cost function (GDV)
 - **“Core (stable) alignment”** - present in all optimal alignments

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Network Alignment

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Yeast - Human alignment:

- 72% of possible pairs are in core alignment
- Both GRAAL & H-GRAAL align: 67% human prots
- H-GRAAL core align. in GRAAL: 63% human prots

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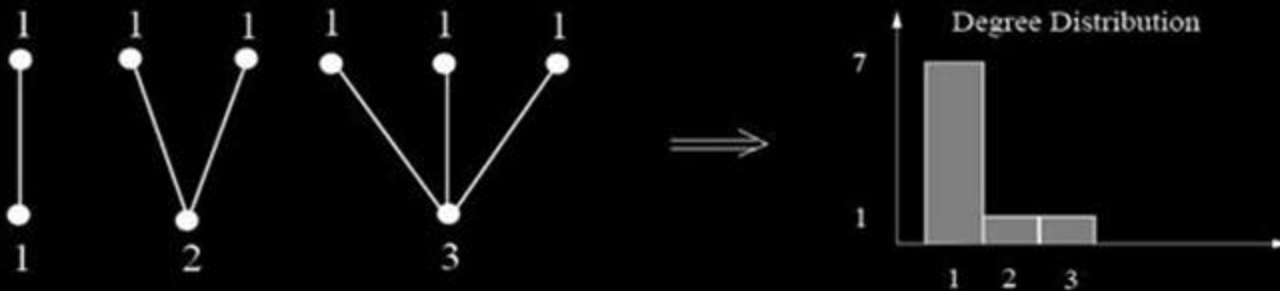
Generalize Degree Distribution

The **degree distribution** measures:

- the number of nodes “touching” k edges for each value of k .

Example:

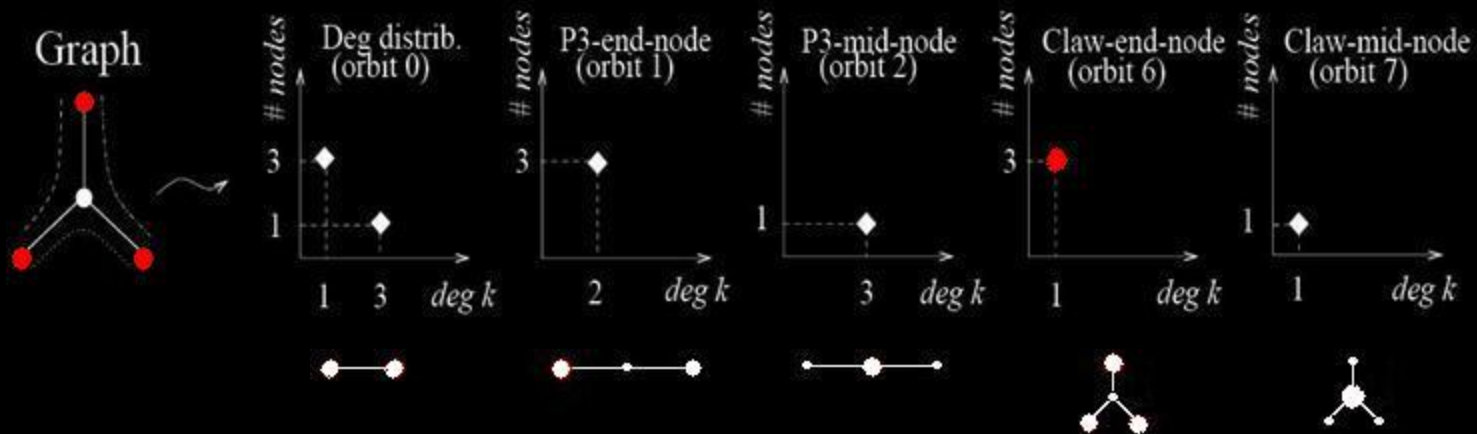
7 nodes touch 1 edge, 1 touches 2 edges, 1 touches 3 edges.



2. Network Analysis and Modeling

For each of these 73 automorphism orbits, we count:

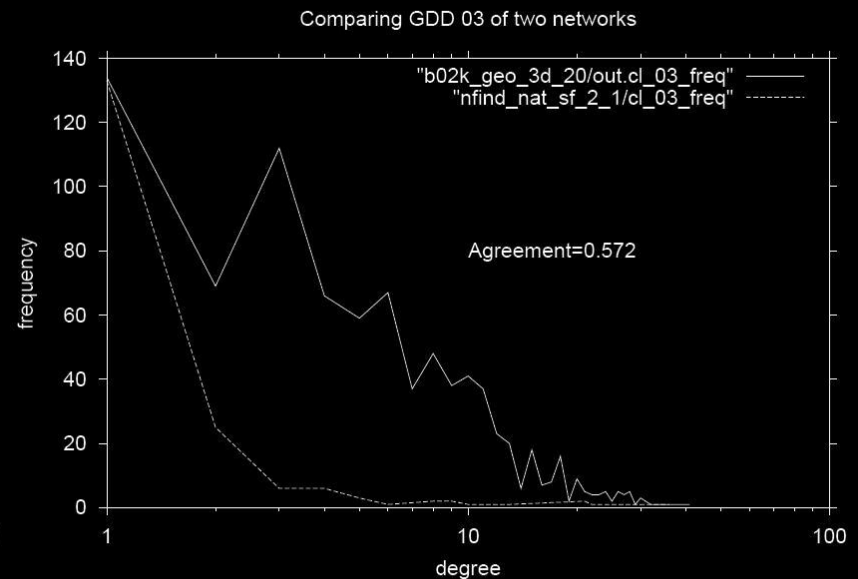
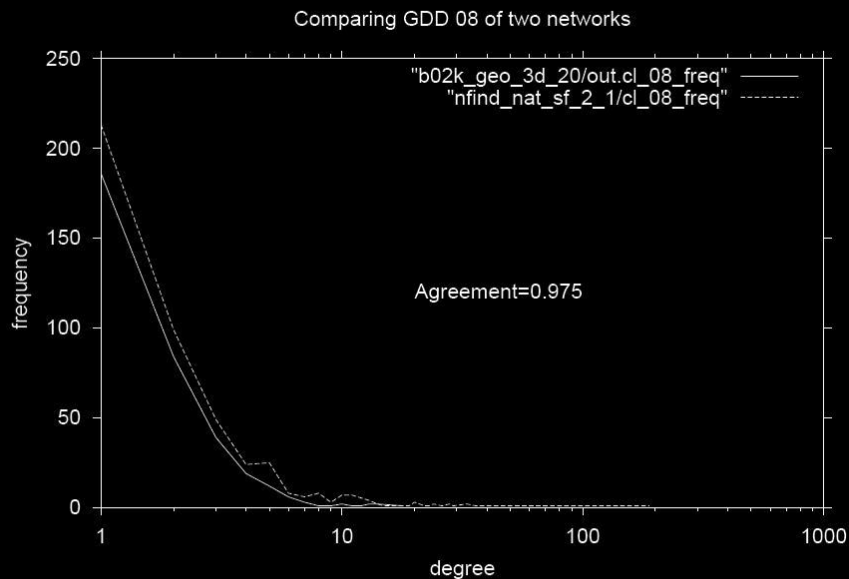
- the number of nodes touching a particular graphlet at a particular orbit.



⇒ The *spectrum* of 73 “graphlet degree distributions (GDDs)” for G_0, \dots, G_{29} measuring local structural properties of a network.

2. Network Analysis and Modeling

Illustration of comparing i^{th} GDDs of two networks:



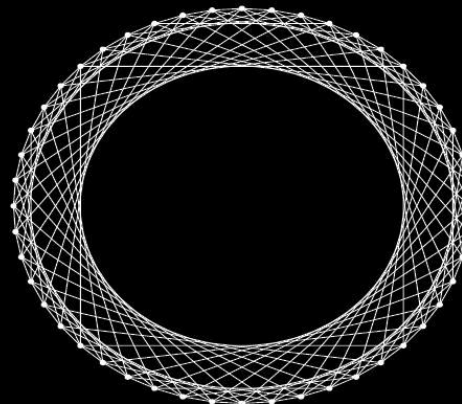
There are 73 such GDD agreements, one for each graphlet orbit.

The *total agreement* between two networks is the average of these 73 GDD agreements.

2. Network Analysis and Modeling

Range of Network “Agreement” Measure:

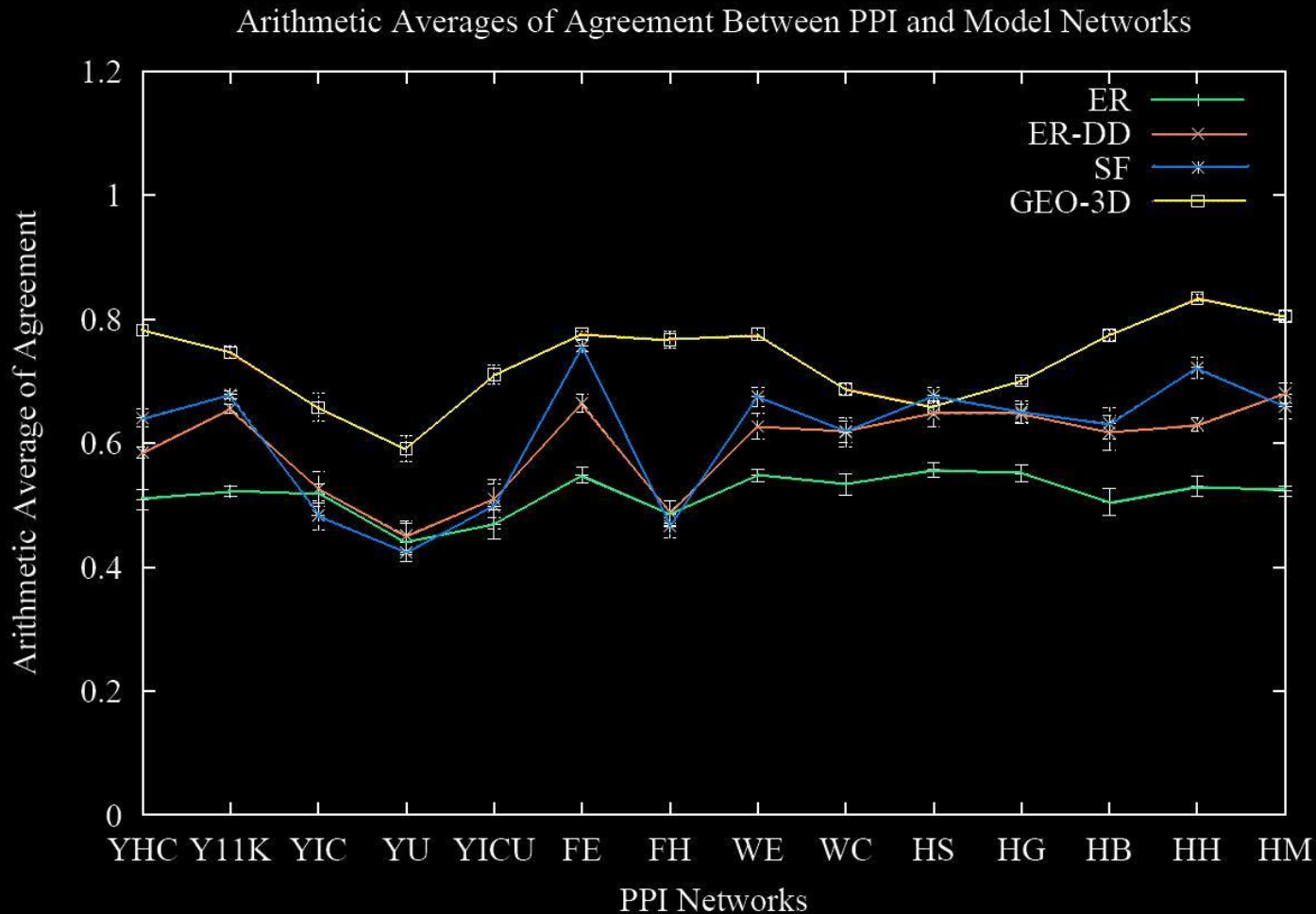
- Agreement between the same model networks: 0.84 ± 0.07
- \Rightarrow Agreements between data and model networks of 0.75 are exceptionally high
- Agreements can be very close to zero:
e.g. between a $\pm 6, 12$ -chord *circulant* and the data < 0.08



2. Network Analysis and Modeling

- Yeast *S. cerevisiae*:
 - von Mering (*Nat* 417, 2002): “YHC”, “Y11K” (TAP, HMS-PCI)
 - Ito *et al.* (*PNAS* 97, 2000) “core”: “YIC” (Y2H)
 - Uetz *et al.* (*Nat* 403, 2000): “YU” (Y2H)
 - Union of “YIC” and “YU”: “YICU” (to increase coverage) (Y2H)
- Fruit fly *D. melanogaster*:
 - Giot *et al.* (*Sci* 302 2003): “FE” and “FH” (Y2H)
- Worm *C. elegans*:
 - Li *et al.* (*Sci* 303, 2004): “WE” and “WC” (Y2H)
- Human:
 - Stelzl (*Cell* 122, 2005): “HS”; Rual (*Nat* 437, 2005): “HG” (Y2H)
 - BIND, HPRD, and MINT: “HB”, “HH”, “HM” (curated)

2. Network Analysis and Modeling



N. Pržulj, "Biological Network Comparison Using Graphlet Degree Distribution,"
Bioinformatics, vol. 23, pg. e177-e183, 2007.

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2. Network Analysis and Modeling

- Different models best-fitting w.r.t. different network properties
- Integrate a variety of local and global network properties into the “network fingerprint” vector:
 1. The average degree
 2. The average clustering coefficient
 3. The average diameter
 4. Frequencies of appearance of all 31 1-5-node graphlets
- Apply a series of machine learning classifiers to network fingerprints:
 1. Backpropagation method (BP)
 2. Probabilistic neural networks (PNN)
 3. Decision tree (DT)
 4. Multinomial naive Bayes classifier (MNB)
 5. Support vector machine (SVM)
- Conclusion: the structure of PPI networks is that of a noisy GEO.

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2. Network Analysis and Modeling

Network Embedding Algorithm

- Use Multi Dimensional Scaling (MDS)
- Paths play a role of distances

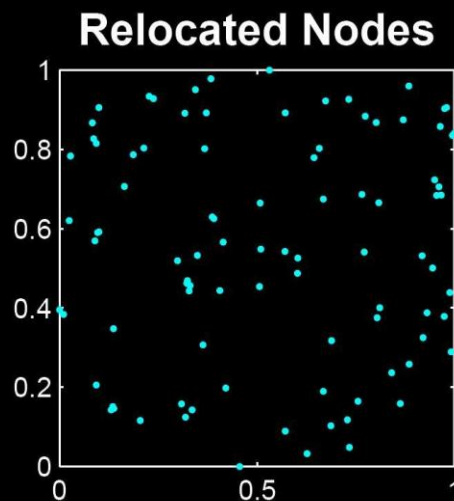
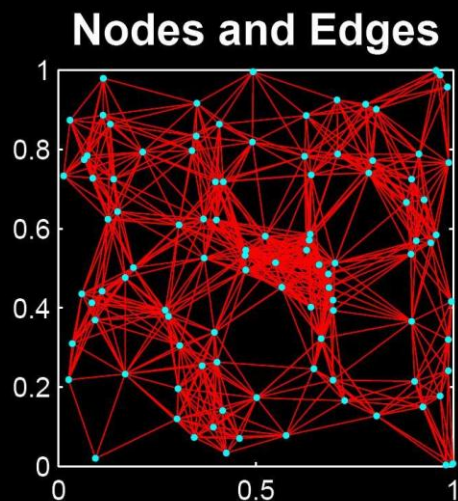
D. J. Higham, M. Rasajski, N. Przulj, “Fitting a Geometric Graph to a Protein-Protein Interaction Network”, *Bioinformatics*, 24(8), 1093-1099, 2008.

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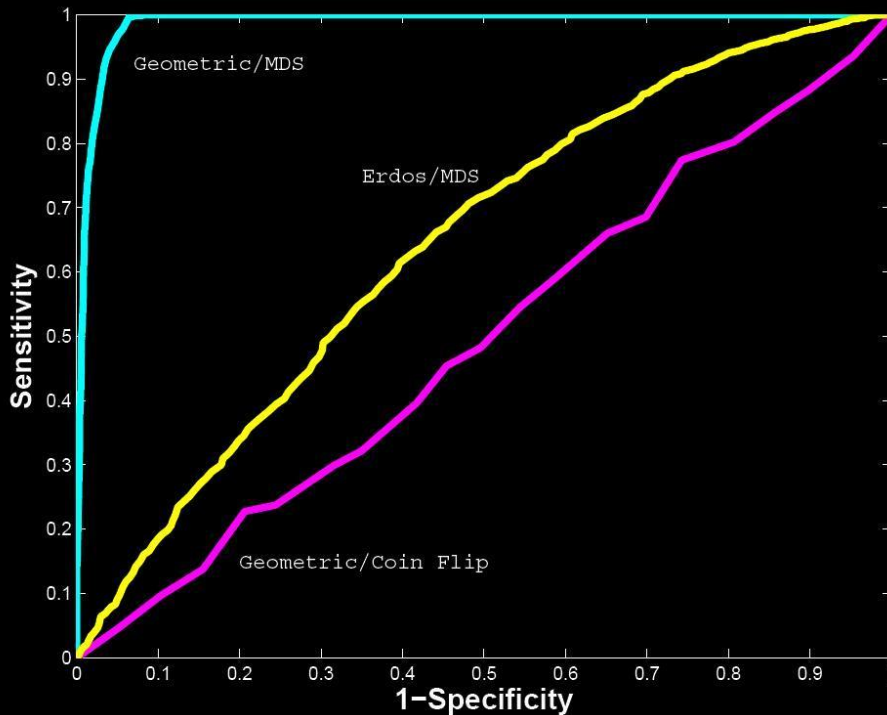
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TN=no edge in original,
no edge in embedded

...

$\text{Specificity} = \text{TN} / (\text{TN} + \text{FP})$

$\text{Sensitivity} = \text{TP} / (\text{TP} + \text{FN})$

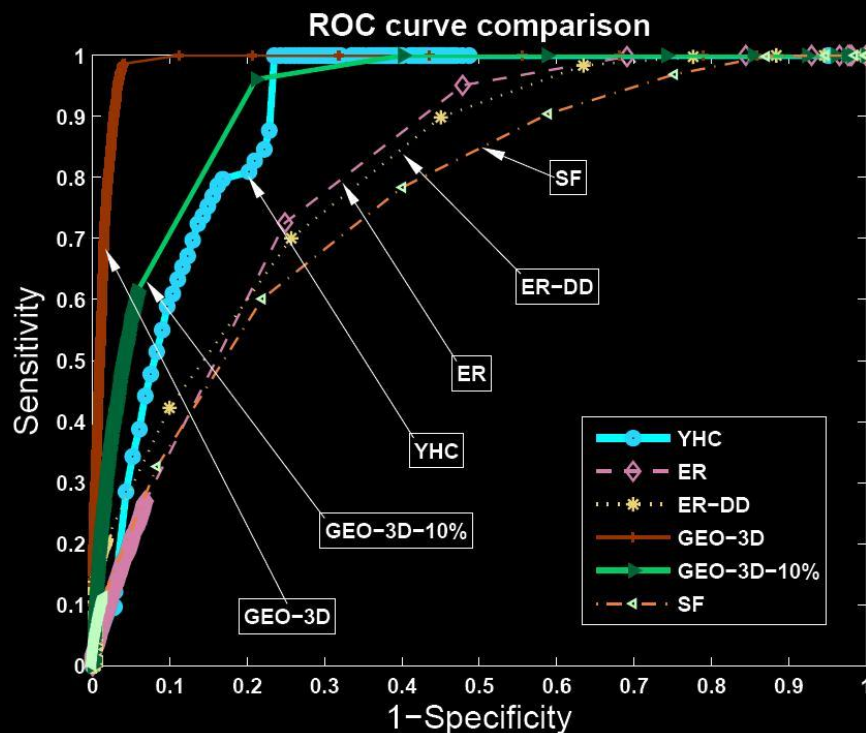
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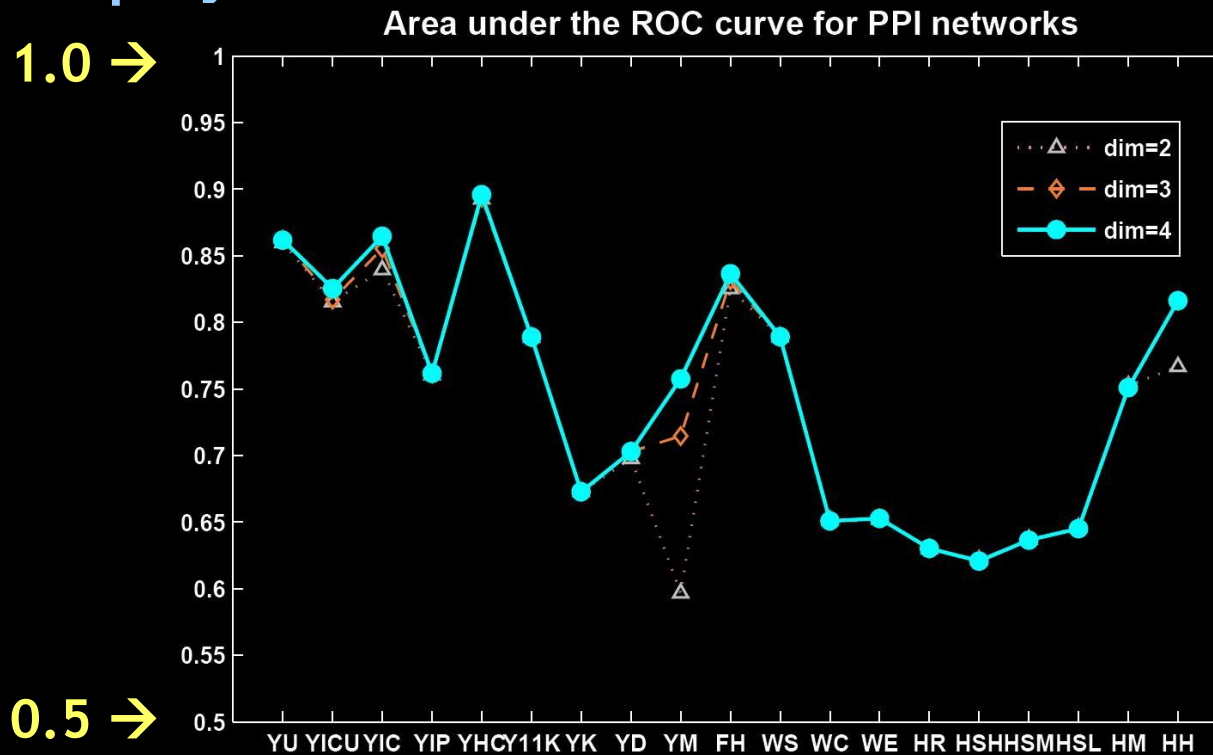


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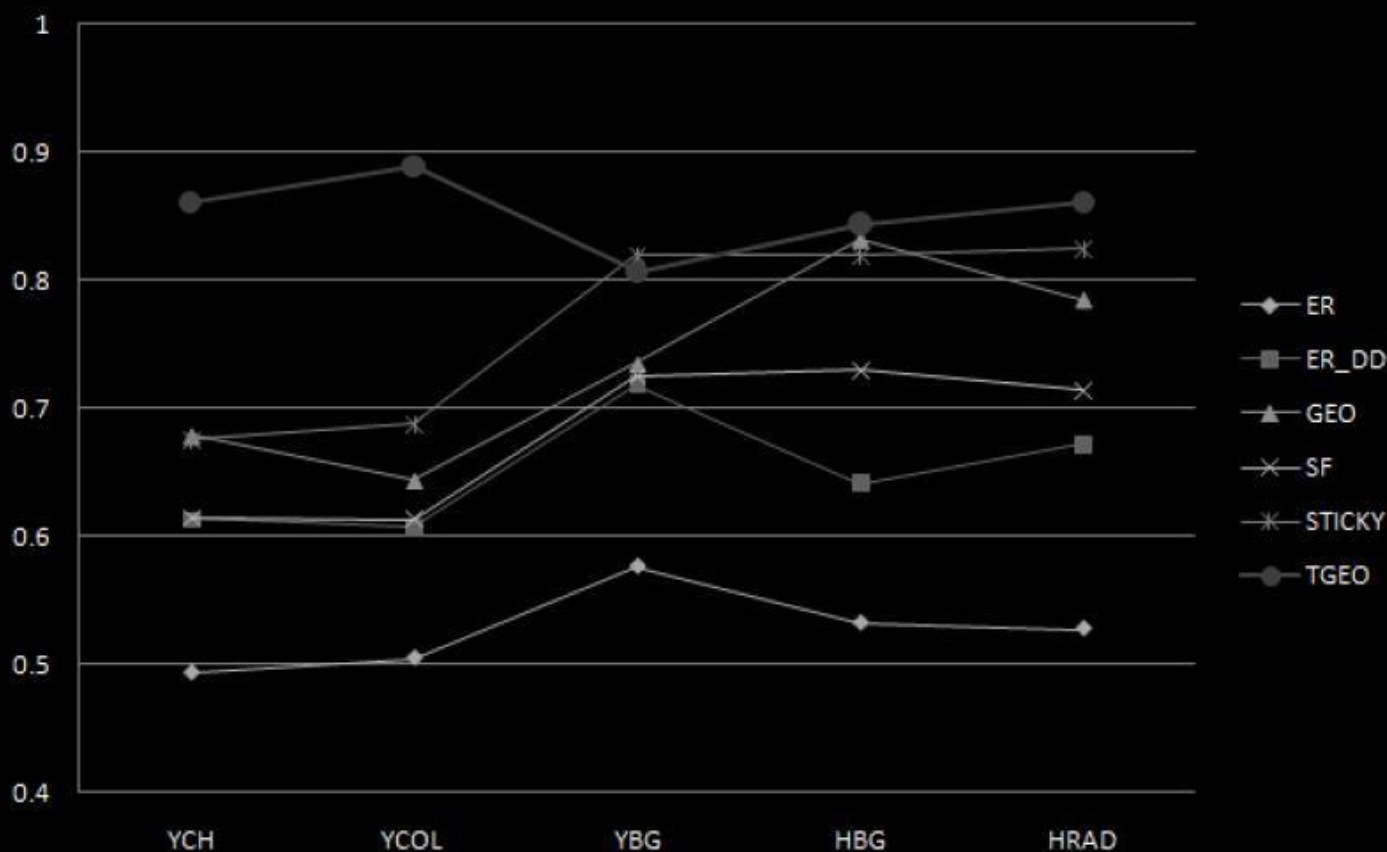
2. Network Analysis and Modeling

Trained Geometric Model

- Embed a high-confidence PPI network in space
- Learn distribution of points in this space using:
 - Mixture of Gaussians
 - Learn the required parameters using Expectation-Maximization algorithm (EM)
- Generate model networks of arbitrary size by:
 - distributing points in the space according to the trained distribution
 - connecting two nodes by an edge if they are close enough in space.

2. Network Analysis and Modeling

Trained Geometric Model



GDD-agreement between the data and model networks.

O. Kuchaiev and N. Pržulj, "Learning the Structure of Protein-Protein Interaction Networks", *Pacific Symposium on Biocomputing (PSB'09)*, Hawaii, USA, 2009.

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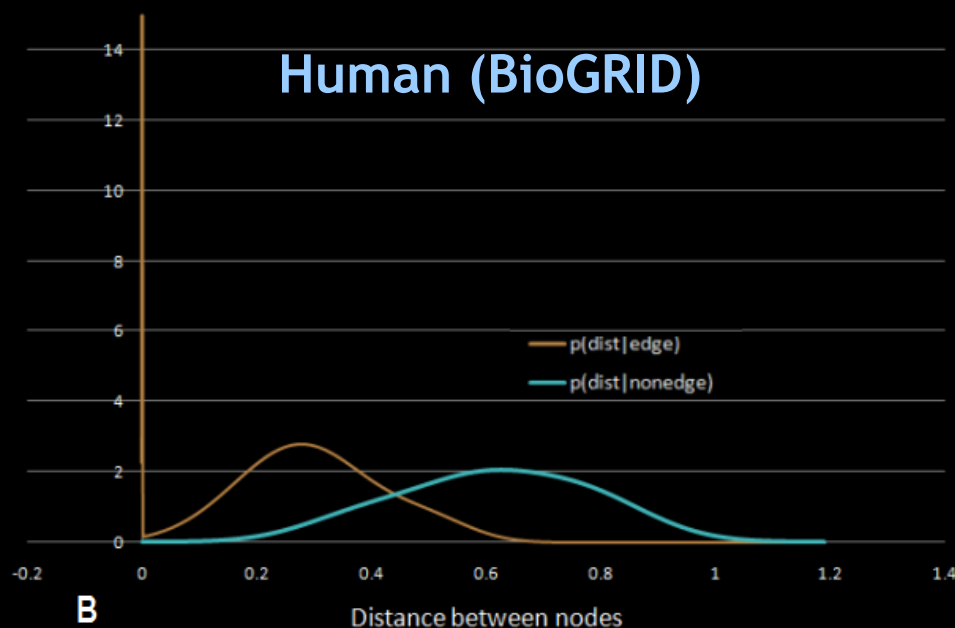
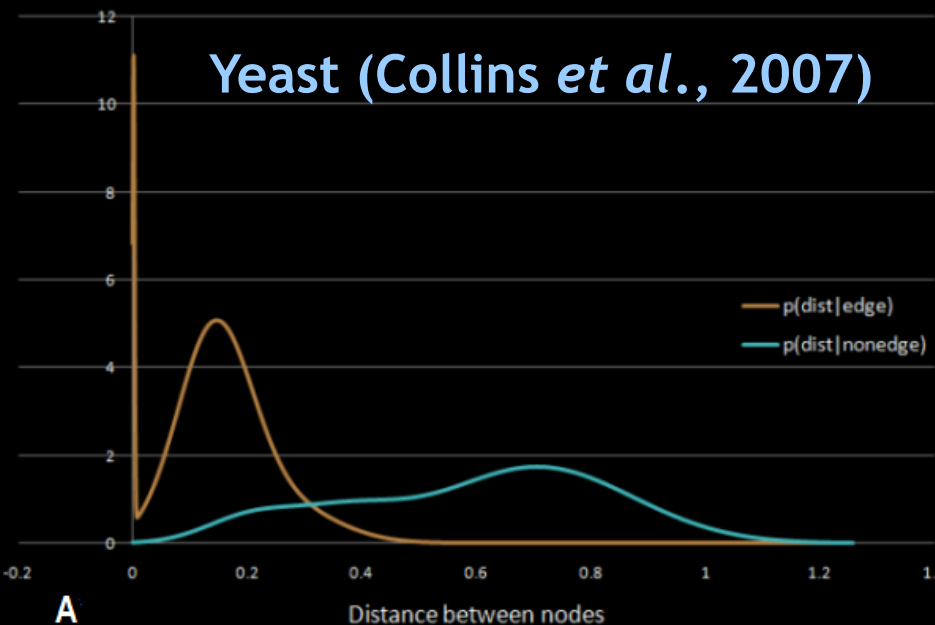
Application: De-Noising PPI Networks

- Embed a PPI network into space
- Learn from coordinates of node embedding points probability densities $p(dist | edge)$ and $p(dist | non-edge)$:
 - Mixture of Gaussians
 - Learn parameters using Expectation-Maximization (EM)

2. Network Analysis and Modeling

Application: De-Noising PPI Networks

- Embed a PPI network into space
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2. Network Analysis and Modeling

Application: De-Noising PPI Networks

- Embed a PPI network into space
- Learn from coordinates of node embedding points probability densities $p(dist | edge)$ and $p(dist | non-edge)$
- Choose a threshold δ
- For each pair of nodes at distance $\leq \delta$, compute its Confidence Score:

$$CS(i, j) = \frac{p(edge(i, j) | dist(i, j))}{p(edge(i, j) | dist(i, j)) + p(nonedge(i, j) | dist(i, j))}$$

⇒ Predict new PPIs

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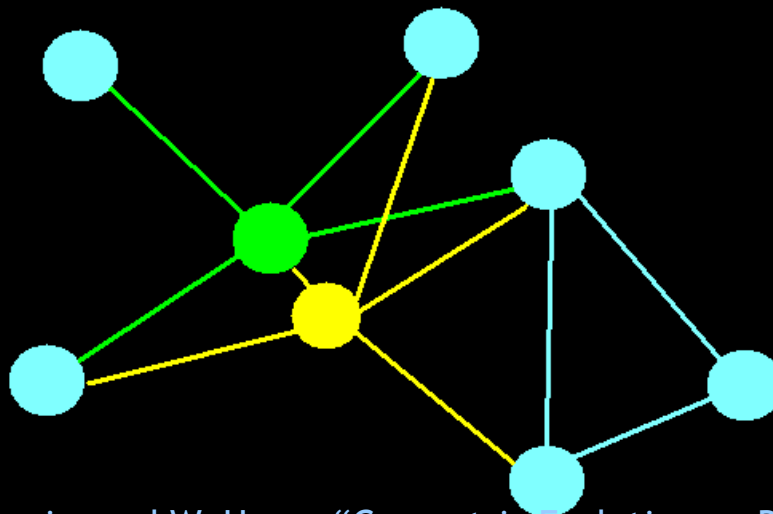
(C) Why PPI networks might be geometric?

3. Conclusions

2. Network Analysis and Modeling

Why PPI networks might be geometric?

- Intuitive “geometricity” of PPI networks:
 - Genes exist in some bio-chemical space
 - Gene duplications and mutations
 - Natural selection = “evolutionary optimization”



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Conclusions

New Network Analysis and Modeling:

(A) Analysis – Graphlet Signatures :

- Structure vs. biological function and disease
- Homology from topology
- Network alignment

(B) Modeling – Geometric Graphs:

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- Network embedding
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- Application: De-noising PPI networks

(C) Why PPI networks might be geometric?

Other Research Projects

- 1) **GraphCrunch** - network analysis software:
 - Milenković, Lai, & Pržulj, *BMC Bioinformatics*, 2008. [Highly Accessed.](#)
- 2) **Residue Interaction Graphs:**
 - New null model:
 - Milenković, Filippis, Lappe, & Pržulj, *PLoS ONE* 4(6), 2009.
- 3) **Structure of brain functional networks:**
 - Kuchaiev, Weng, Nenadić, & Przulj, *IEEE Engineering in Medicine and Biology Society (EMBC'09)*, 2009.
- 4) **Planar Cell Polarity** - ommatidial rotation in fruit-fly eye caused by net. structural perturbation

- 5) **Etc.**

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Council on Research & Computing UCI (2008–2009)

- Alumni:

1. Tijana Milenković, Ph.D.
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2. Oleksii Kuchaiev, Ph.D.
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3. Vesna Memišević, Ph.D.
US Army, Bioinformatics Res.
4. Aleksandar Stevanović, M.Sc.



Undergraduate programmer: Jason Lai

- Collaborators: D.Higham, L.Huang, M.Lappe,
W.Hayes, Z.Nenadić, P.Kaiser, A.Ganesan...

1. O. Kuchaiev, T. Milenkovic, V. Memisevic, W. Hayes and N. Przulj, "Topological Network Alignment Uncovers Biological Function and Phylogeny", *J. Roy Soc. Interface*, 2010.
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