Carnegie Mellon

Mining Billion-node Graphs: Patterns, Generators and Tools

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CMU

(on sabbatical at google)

Thank you!

- José Balcázar,
- Francesco Bonchi
- Aristides (Aris) Gionis
- Michèle Sebag

- Ricard Gavaldà

Our goal:

Open source system for mining huge graphs:

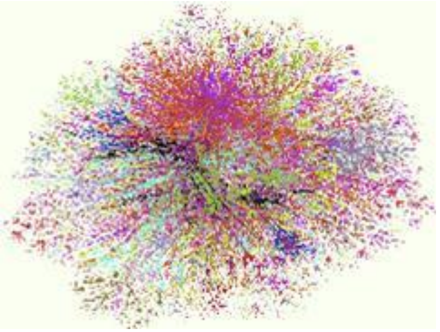
PEGASUS project (PEta GrAph mining System)

- www.cs.cmu.edu/~pegasus
- code and papers

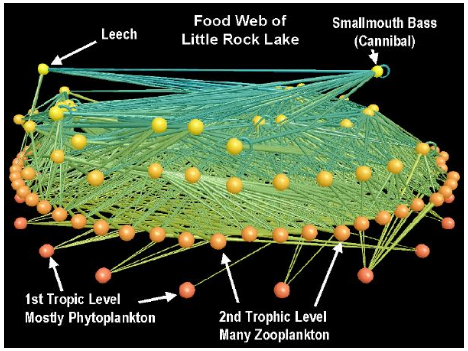
Outline

- Introduction – Motivation
- Problem#1: Patterns in graphs
- Problem#2: Tools
- Problem#3: Scalability
- Conclusions

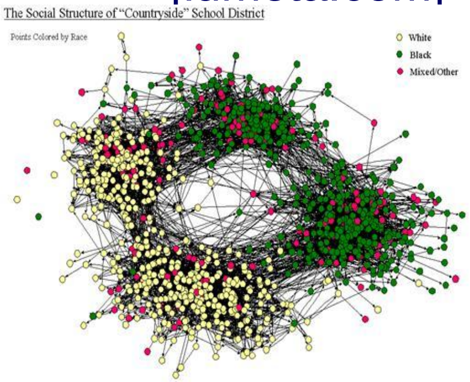
Graphs - why should we care?



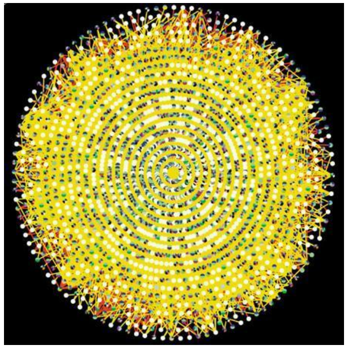
Internet Map
[lumeta.com]



Food Web
[Martinez '91]



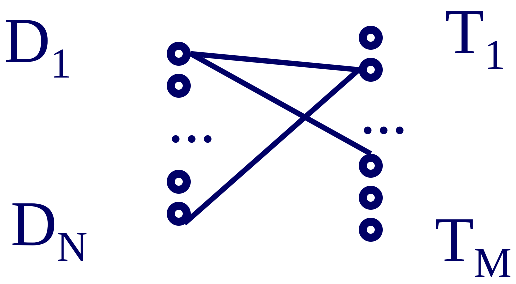
Friendship Network
[Moody '01]



Protein Interactions
[genomebiology.com]

Graphs - why should we care?

- IR: bi-partite graphs (doc-terms)



- web: hyper-text graph

- ... and more:

Graphs - why should we care?

- network of companies & board-of-directors members
- ‘viral’ marketing
- web-log (‘blog’) news propagation
- computer network security: email/IP traffic and anomaly detection
- ....

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  - Static graphs
  - Weighted graphs
  - Time evolving graphs
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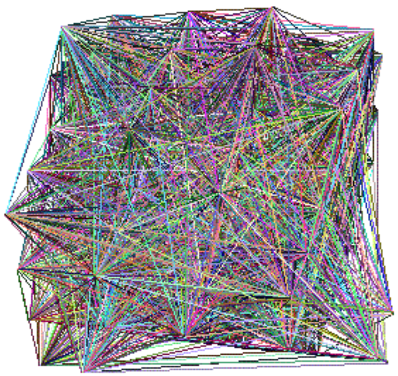
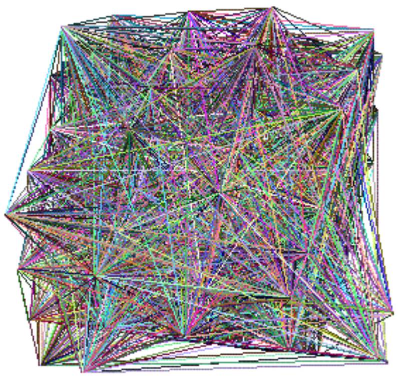
Problem #1 - network and graph mining



- What does the Internet look like?
- What does FaceBook look like?

- What is ‘normal’/‘abnormal’?
- which patterns/laws hold?

Problem #1 - network and graph mining



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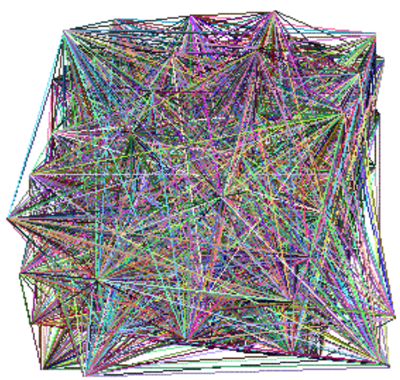

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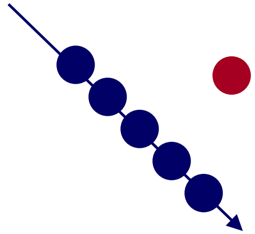


- What is ‘normal’/‘abnormal’?
- which patterns/laws hold?
  - To spot **anomalies** (rarities), we have to discover **patterns**
  - **Large** datasets reveal patterns/anomalies that may be invisible otherwise…

Graph mining

- Are real graphs random?

Laws and patterns

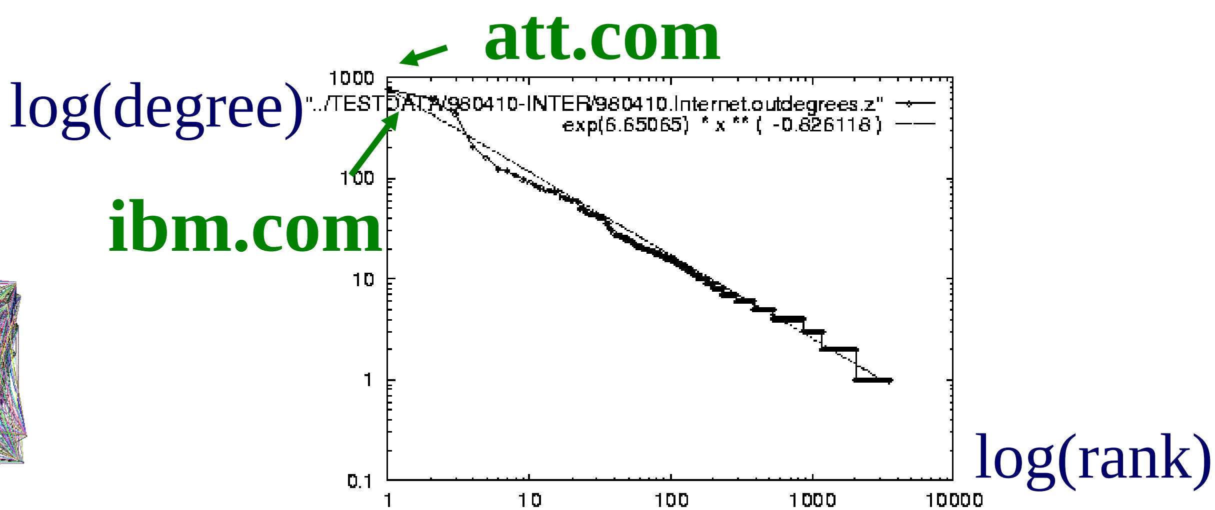
- Are real graphs random?
- A: NO!!
  - Diameter
  - in- and out- degree distributions
  - other (surprising) patterns

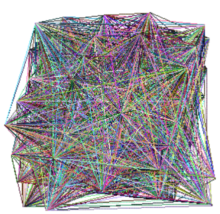
- So, let's look at the data

Solution# S.1

- Power law in the degree distribution [SIGCOMM99]

internet domains





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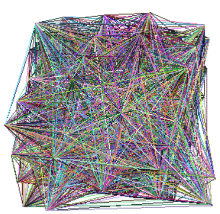
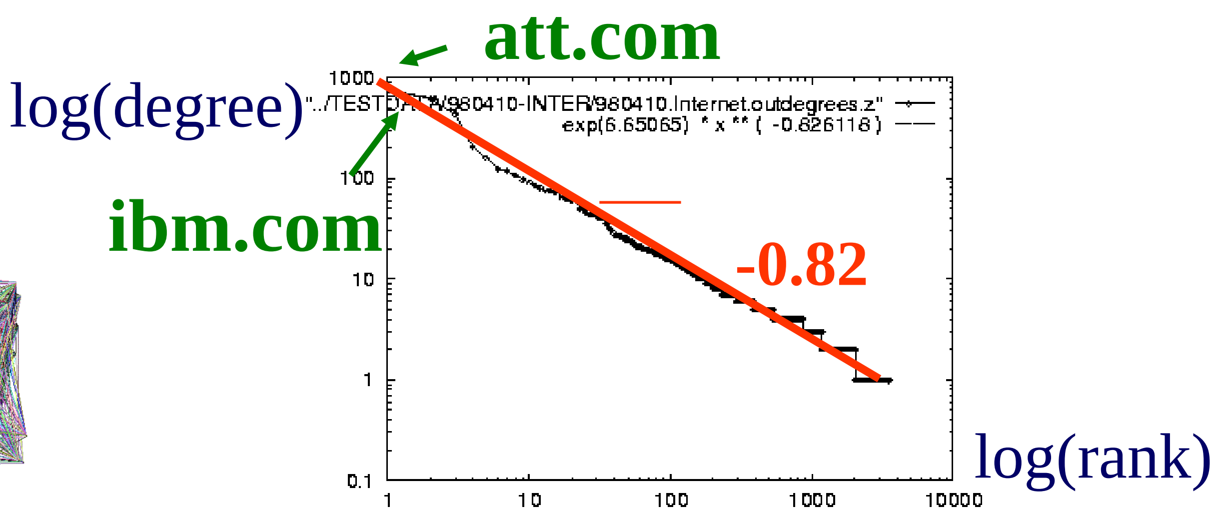
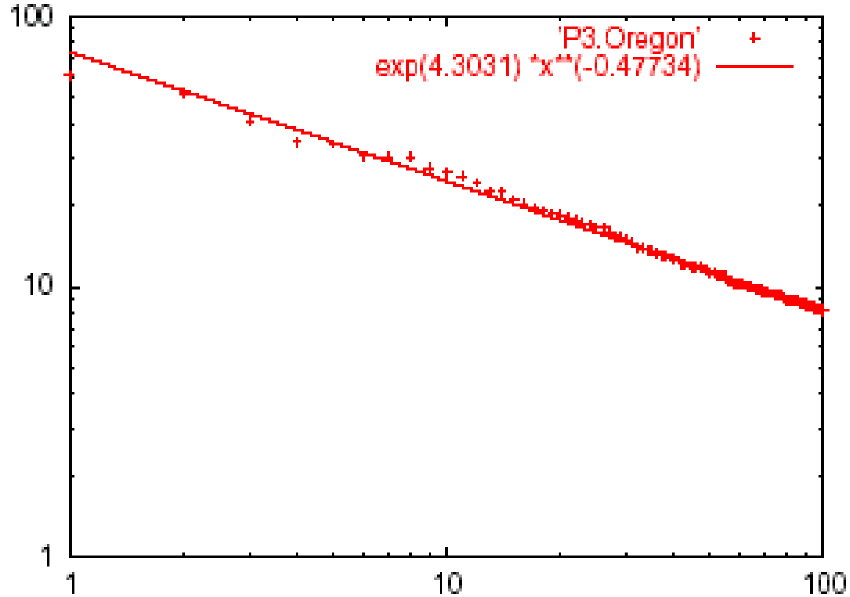




Solution# S.2: Eigen Exponent *E*

Eigenvalue



Exponent = slope

E = -0.48

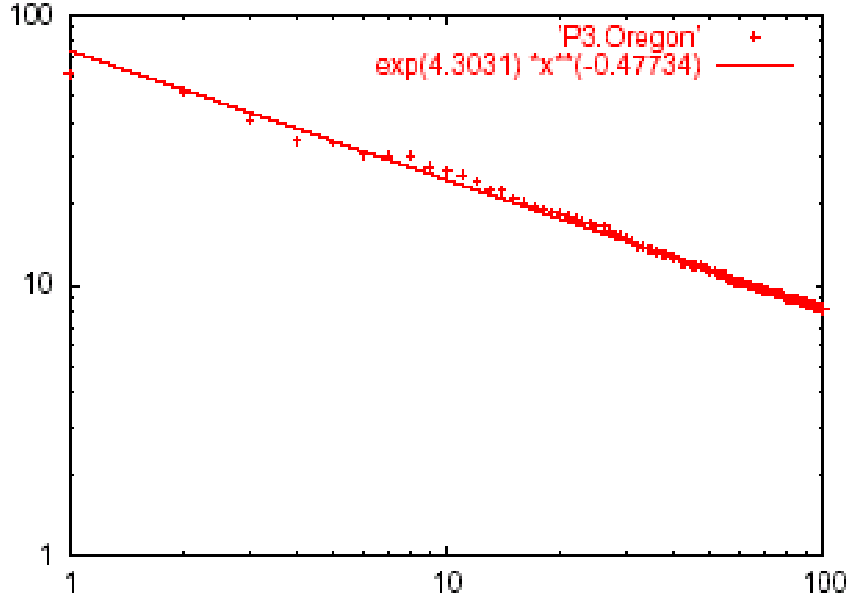
May 2001

Rank of decreasing eigenvalue

- A2: power law in the eigenvalues of the adjacency matrix

Solution# S.2: Eigen Exponent *E*

Eigenvalue



Rank of decreasing eigenvalue

Exponent = slope

E = -0.48

May 2001

- [Mihail, Papadimitriou '02]: slope is ½ of rank exponent

But:

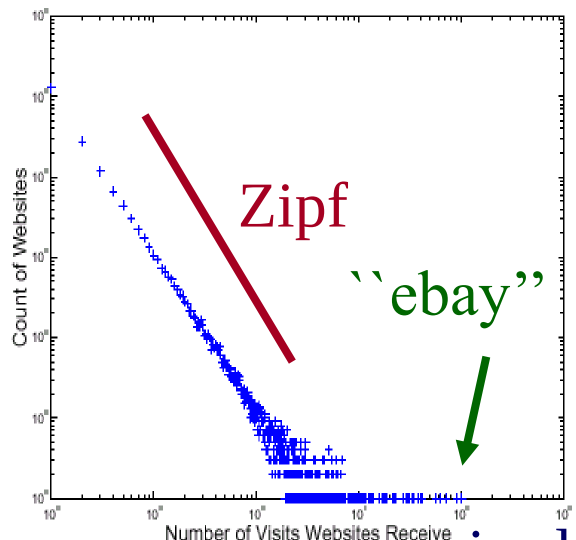
How about graphs from other domains?

More power laws:

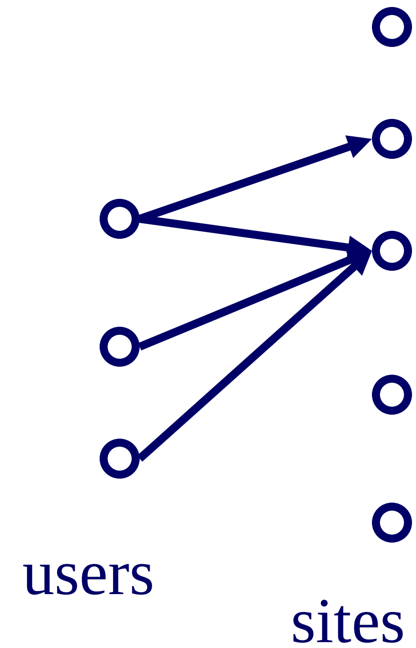
- web hit counts [w/ A. Montgomery]

Web Site Traffic

Count
(log scale)



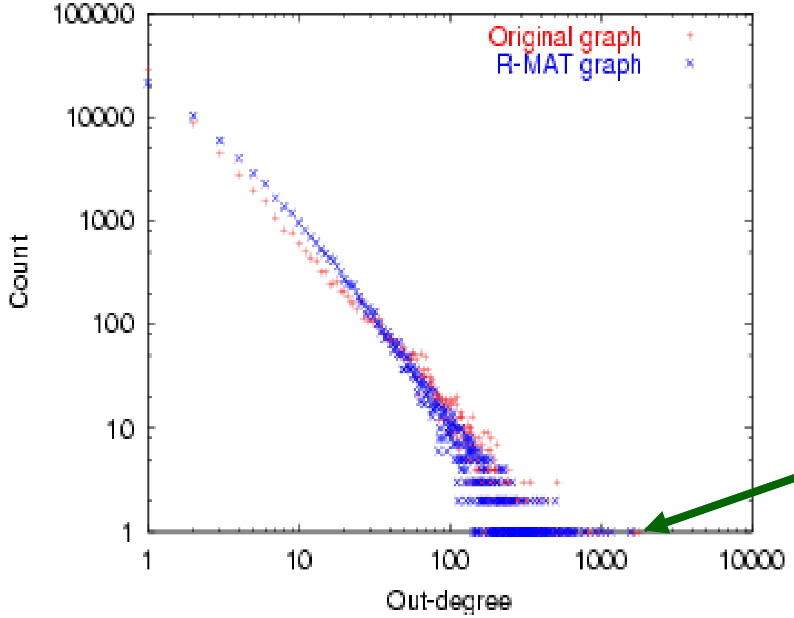
in-degree (log scale)



epinions.com

- who-trusts-whom [Richardson + Domingos, KDD 2001]

count



trusts-2000-people user

(out) degree

And numerous more

- # of sexual contacts
- Income [Pareto] –'80-20 distribution'
- Duration of downloads [Bestavros+]
- Duration of UNIX jobs ('mice and elephants')
- Size of files of a user
- …
- 'Black swans'

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    - triangles
    - cliques
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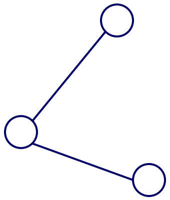
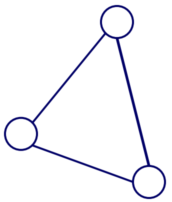
Solution# S.3: Triangle 'Laws'



- Real social networks have a lot of triangles

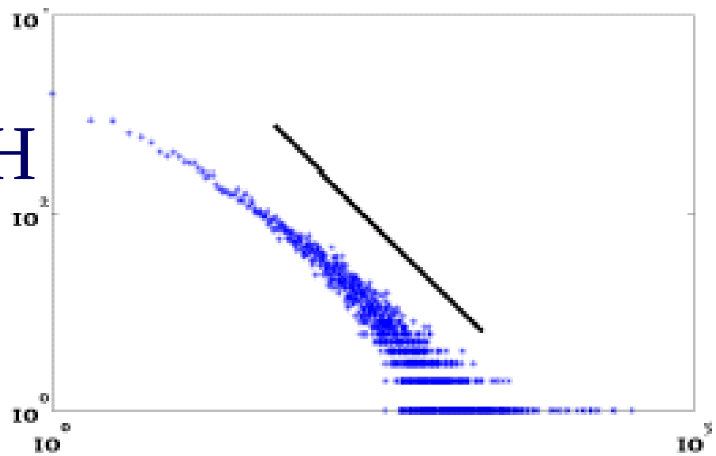
Solution# S.3: Triangle ‘Laws’

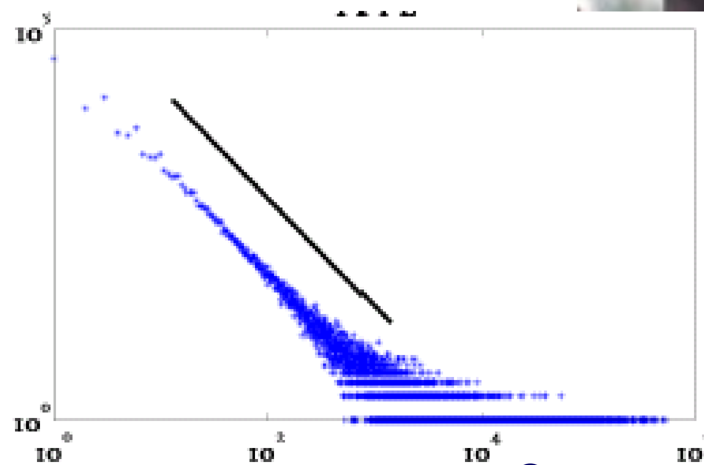


- Real social networks have a lot of triangles
  - Friends of friends are friends
- Any patterns?

Triangle Law: #S.3
[Tsourakakis ICDM 2008]

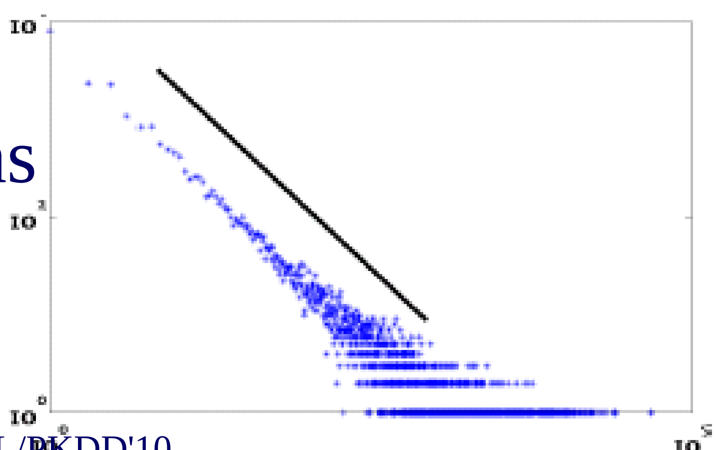
HEP-TH

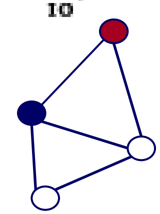




ASN

Epinions





X-axis: # of participating triangles

Y: count (~ pdf)

Triangle Law: #S.3
[Tsourakakis ICDM 2008]

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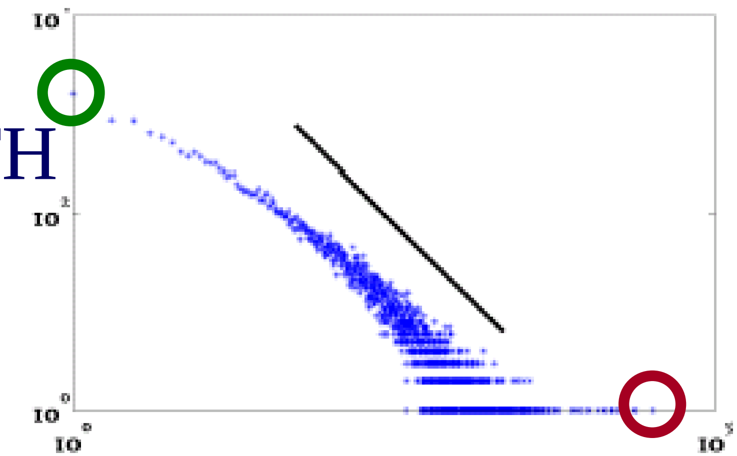

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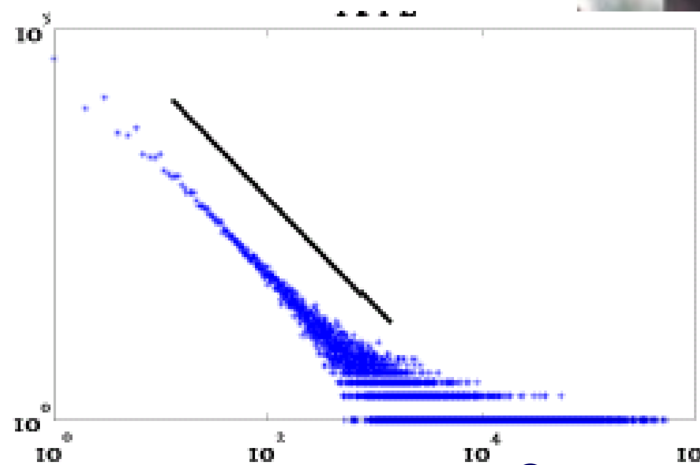

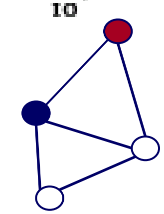

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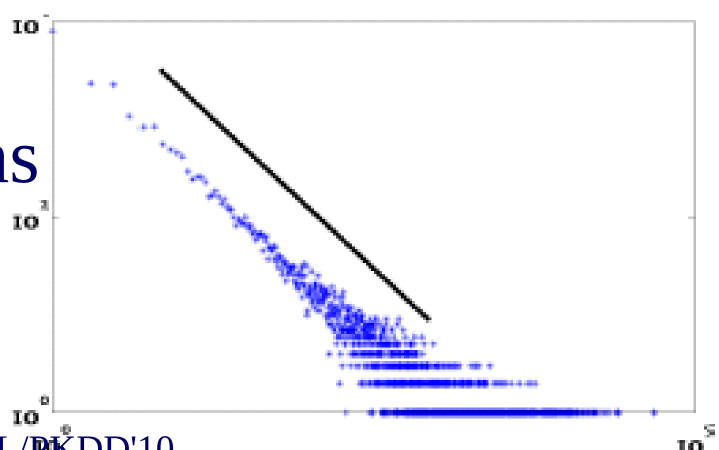

X-axis: # of participating triangles

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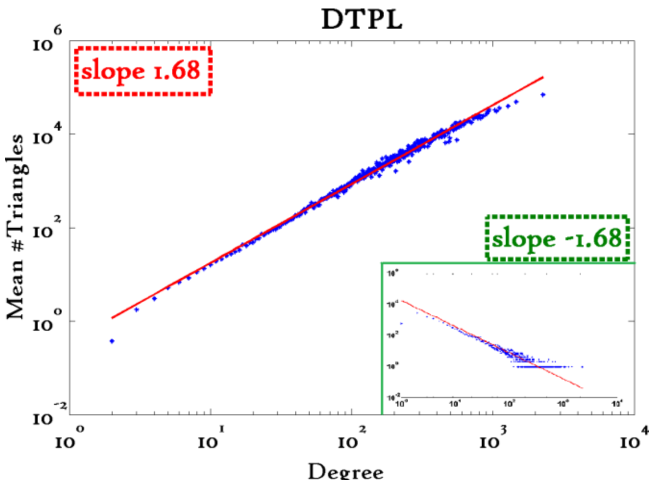
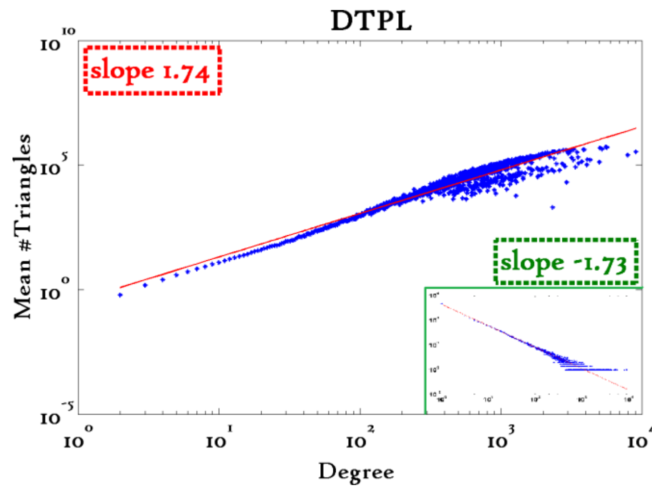
Triangle Law: #S.4
[Tsourakakis ICDM 2008]

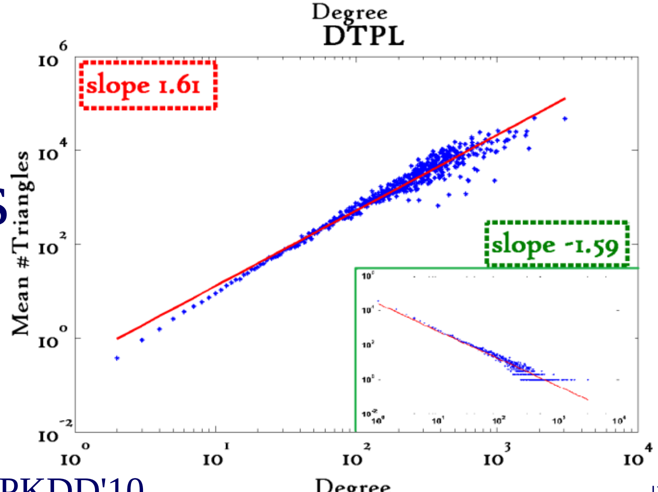
Reuters





SN

Epinions



X-axis: degree

Y-axis: mean # triangles

n friends -> $\sim n^{1.6}$ triangles

details

Triangle Law: Computations
[Tsourakakis ICDM 2008]

But: triangles are expensive to compute

(3-way join; several approx. algos)

Q: Can we do that quickly?

details

Triangle Law: Computations
[Tsourakakis ICDM 2008]

But: triangles are expensive to compute

(3-way join; several approx. algos)

Q: Can we do that quickly?

A: Yes!

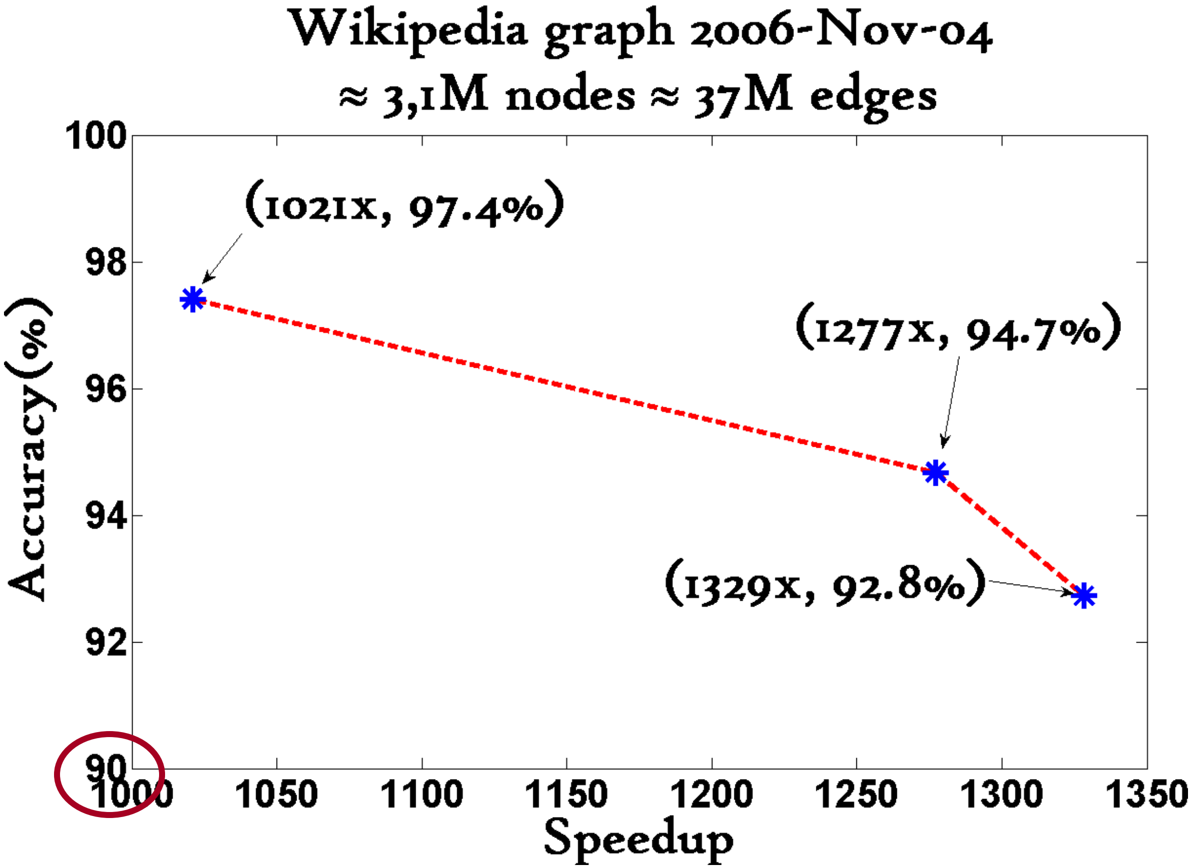
#triangles = 1/6 Sum (λ_i^3)

(and, because of skewness (S2) ,

we only need the top few eigenvalues!

details

Triangle Law: Computations [Tsourakakis ICDM 2008]



1000x+ speed-up, >90% accuracy

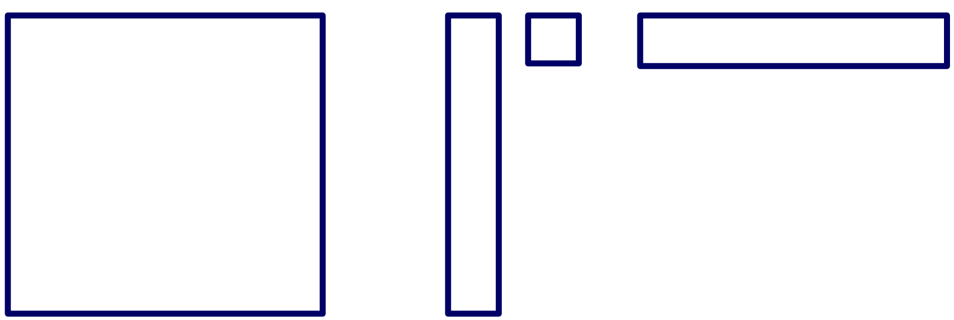
EigenSpokes

B. Aditya Prakash, Mukund Seshadri, Ashwin Sridharan, Sridhar Machiraju and Christos Faloutsos: *EigenSpokes: Surprising Patterns and Scalable Community Chipping in Large Graphs*, PAKDD 2010, Hyderabad, India, 21-24 June 2010.

EigenSpokes

- Eigenvectors of adjacency matrix
  - equivalent to singular vectors (symmetric, undirected graph)

$$A = U \Sigma U^T$$



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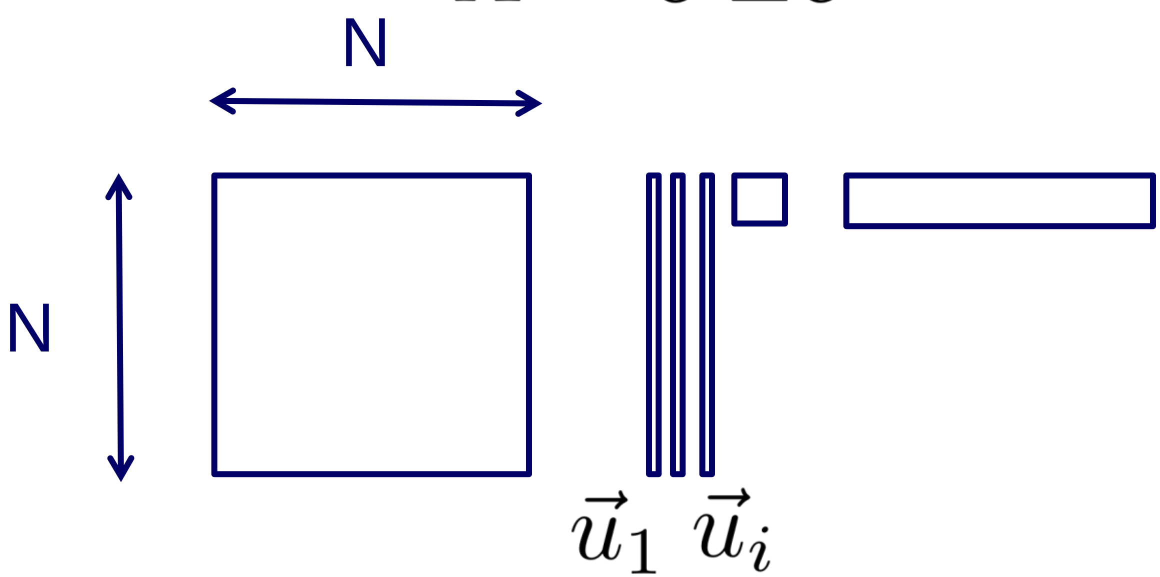

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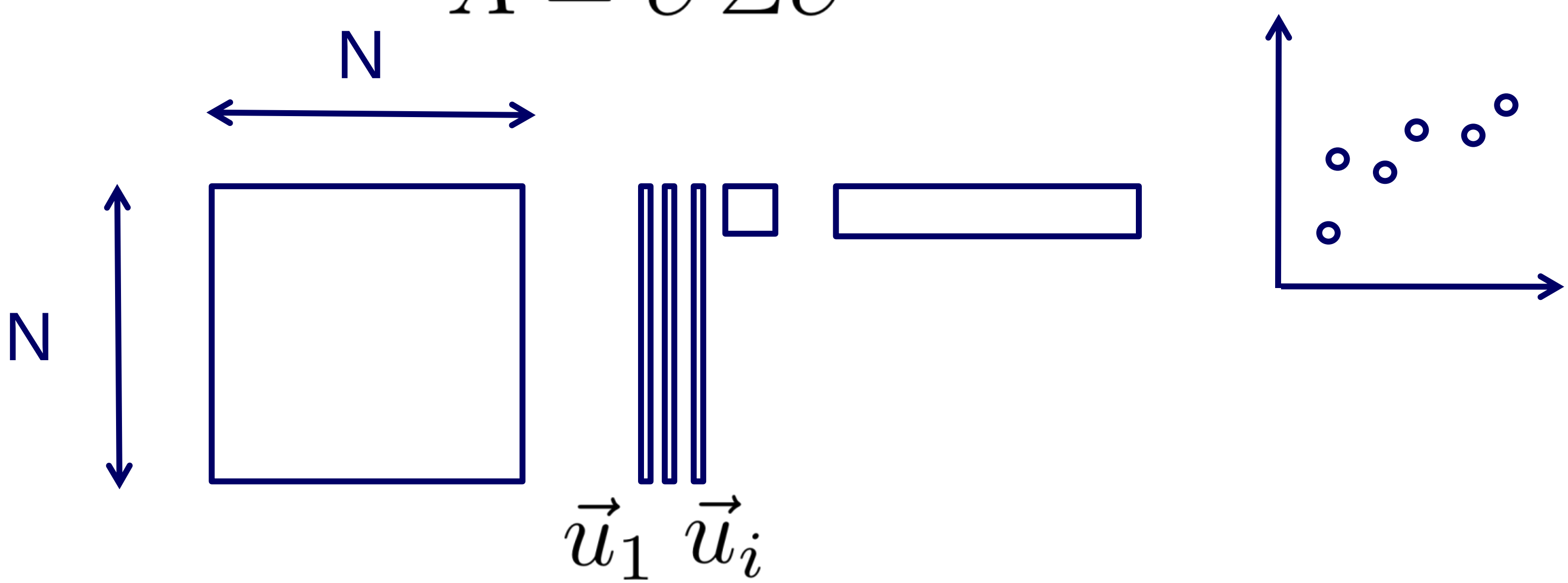

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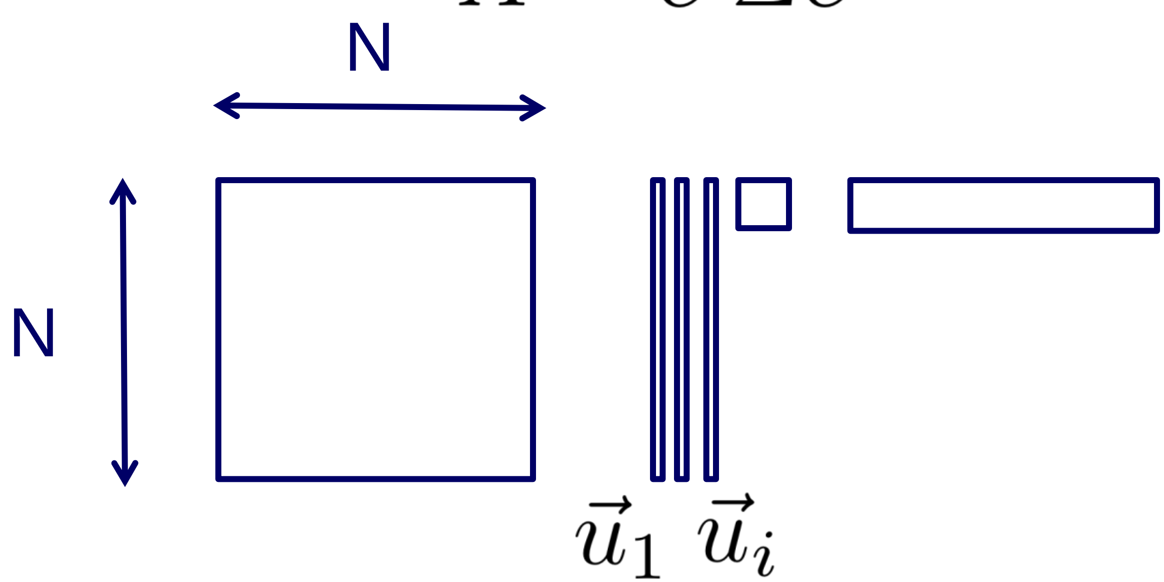
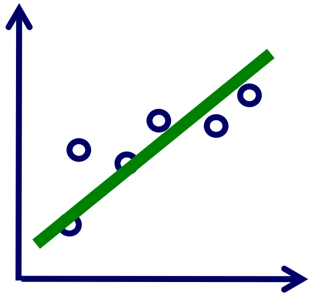
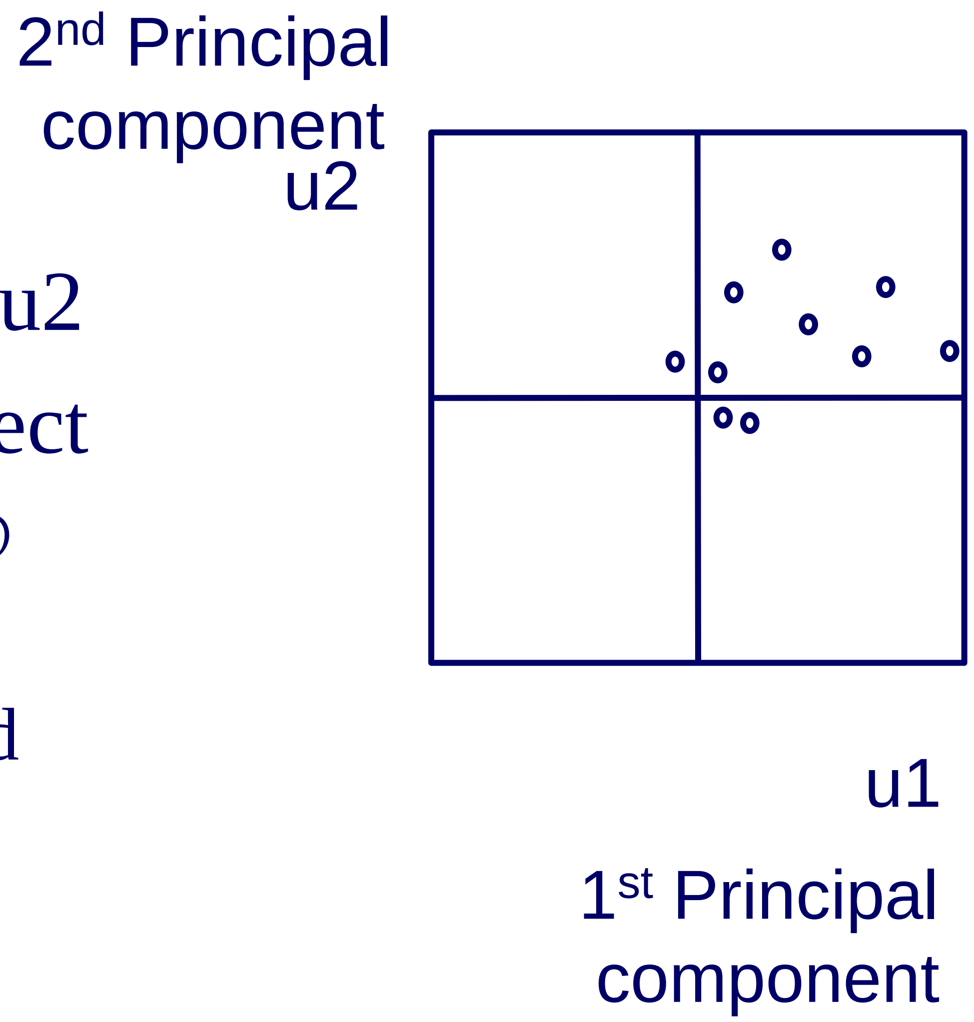




EigenSpokes

- EE plot:
- Scatter plot of scores of u1 vs u2
- One would expect
  - Many points @ origin
  - A few scattered ~randomly



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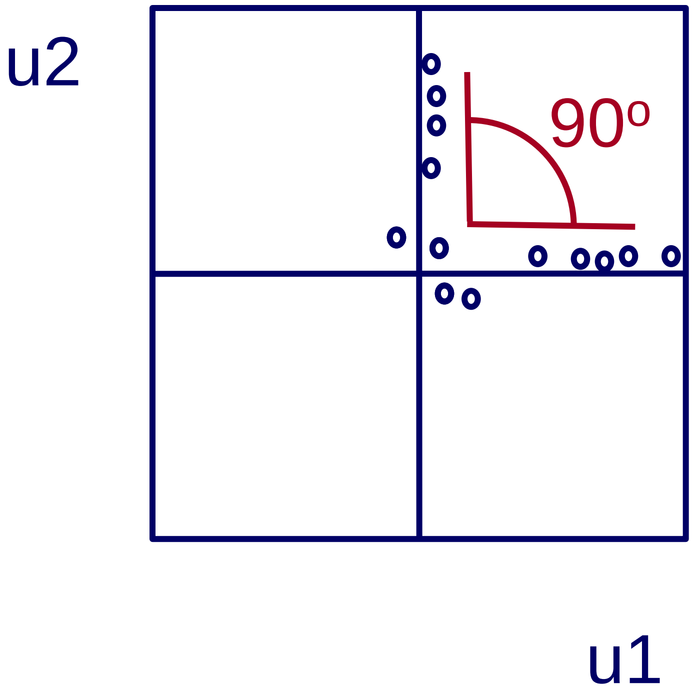
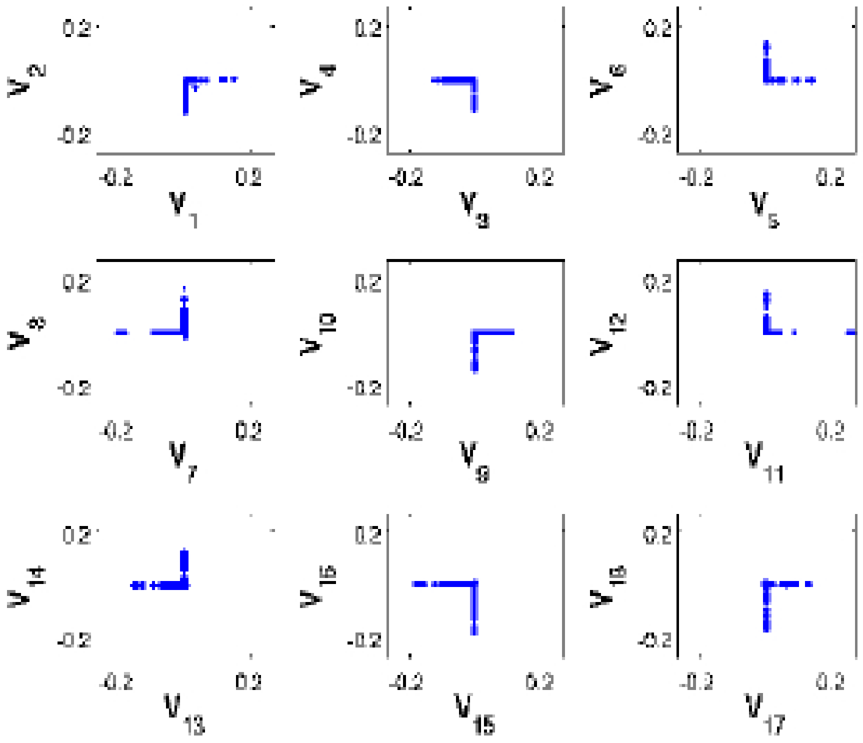
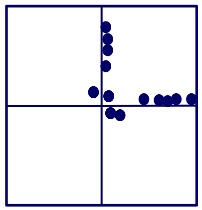


EigenSpokes - pervasiveness

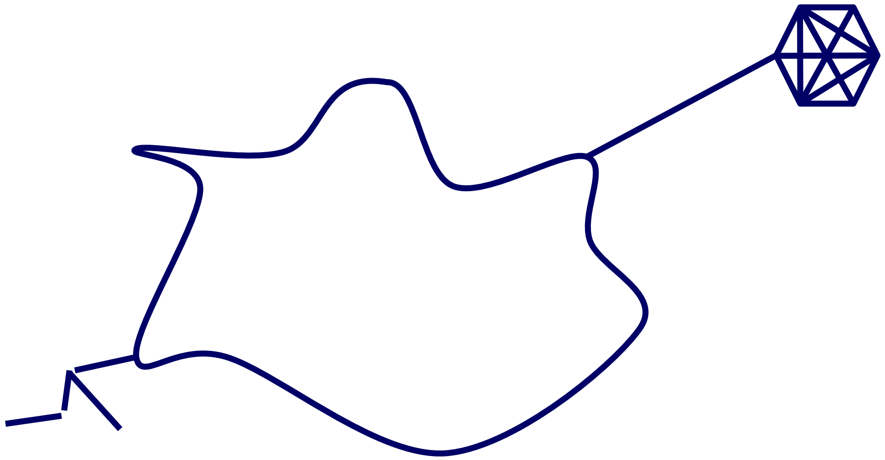
- Present in mobile social graph
  - across time and space
- Patent citation graph



EigenSpokes - explanation



Near-cliques, or near-bipartite-cores, loosely connected



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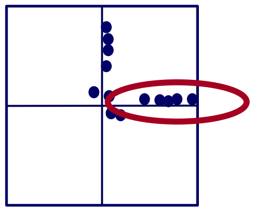

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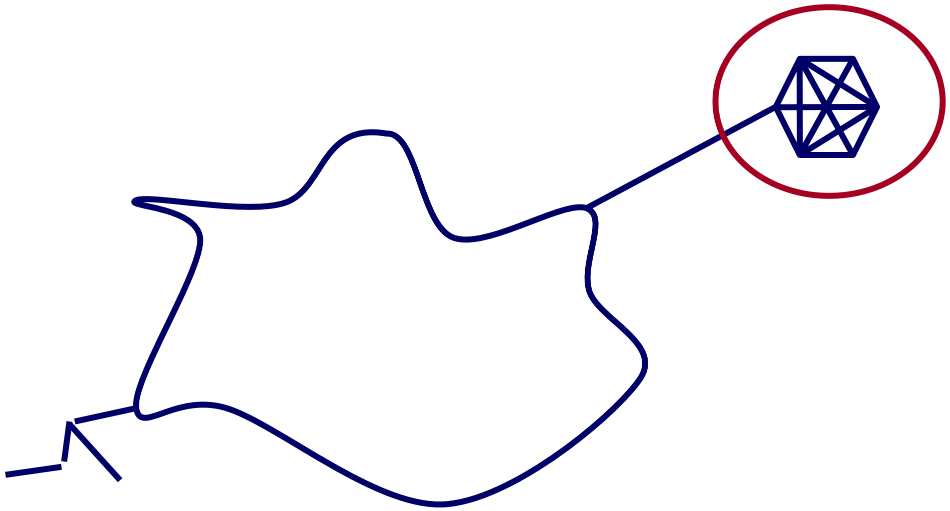

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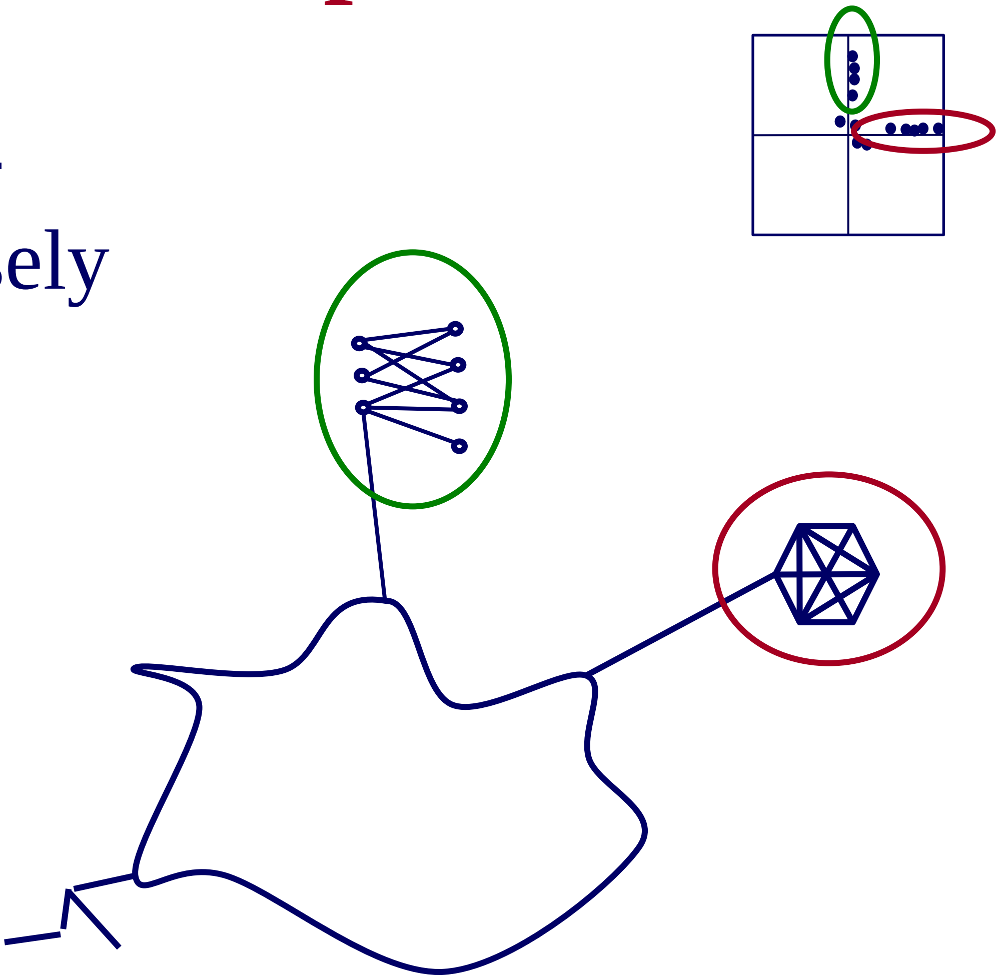

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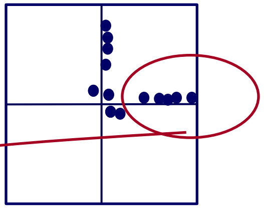
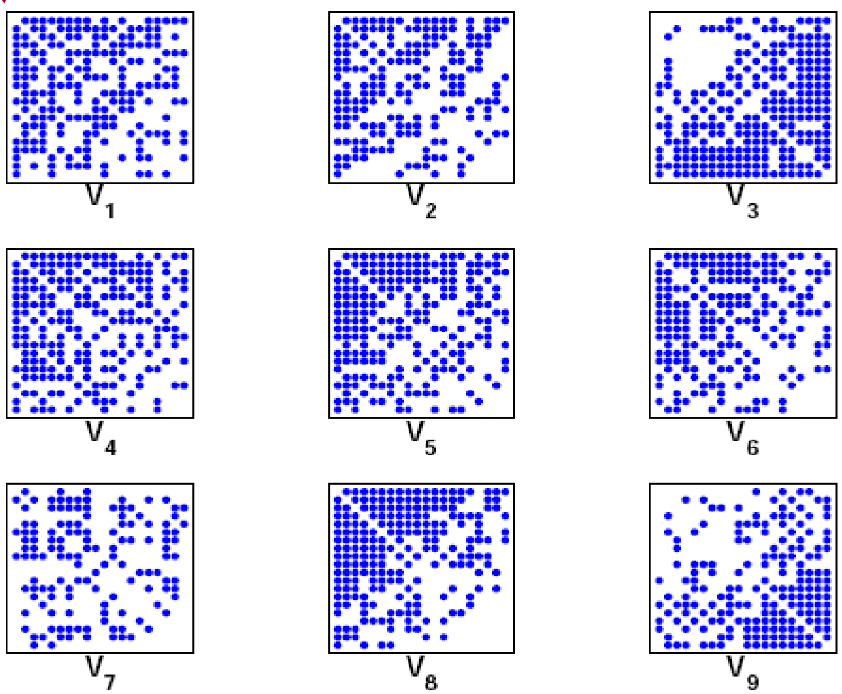


Near-cliques, or near-bipartite-cores, loosely connected

spy plot of top 20 nodes



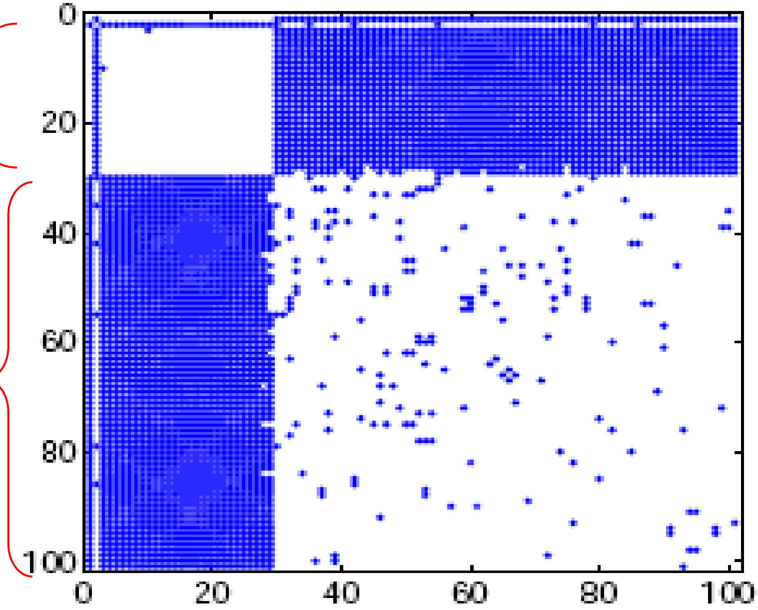
So what?

- Extract nodes with high *scores*
- high connectivity
- Good “communities”

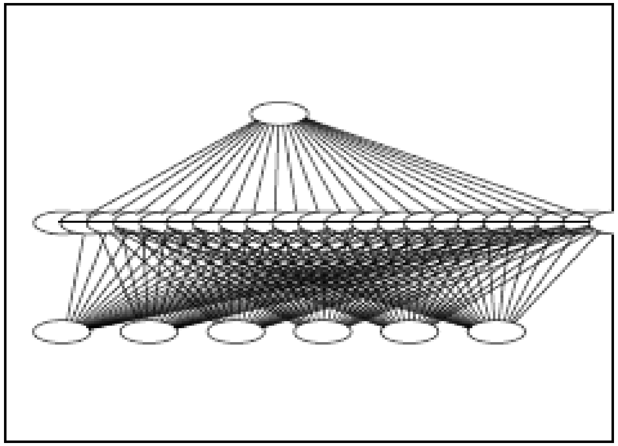
Bipartite Communities!

patents from
same inventor(s)

cut-and-paste
bibliography!



magnified bipartite community



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    - cliques
  - Weighted graphs
  - Time evolving graphs
- Problem#2: Tools

Observations on weighted graphs?

- A: yes - even more 'laws'!

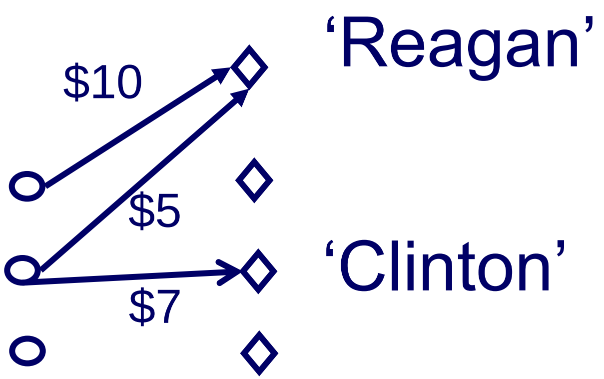
M. McGlohon, L. Akoglu, and C. Faloutsos
Weighted Graphs and Disconnected Components: Patterns and a Generator.
SIG-KDD 2008

Observation W.1: Fortification

Q: How do the weights of nodes relate to degree?

Observation W.1: Fortification

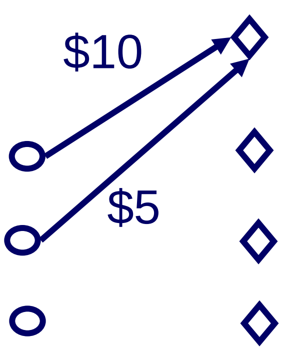
More donors, more $?



Observation W.1: fortification: Snapshot Power Law

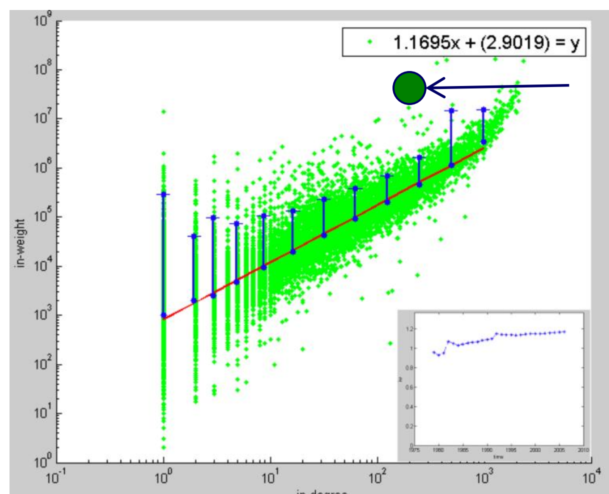
- Weight: super-linear on in-degree
- exponent ‘iw’: 1.01 < iw < 1.26

More donors, even more $



In-weights ($)

Orgs-Candidates



e.g. John Kerry, $10M received, from 1K donors

Edges (# donors)

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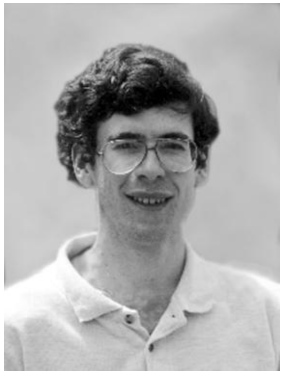
Problem: Time evolution

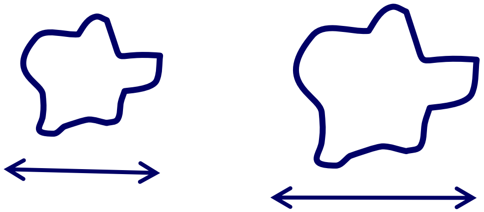
- with Jure Leskovec (CMU -> Stanford)

- and Jon Kleinberg (Cornell – sabb. @ CMU)



T.1 Evolution of the Diameter

- Prior work on Power Law graphs hints at **slowly growing diameter**:
  - diameter ~ O(log N)
  - diameter ~ O(log log N)
- What is happening in real data?



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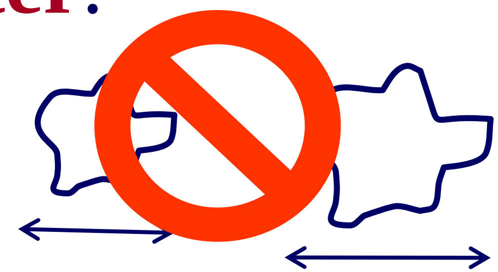
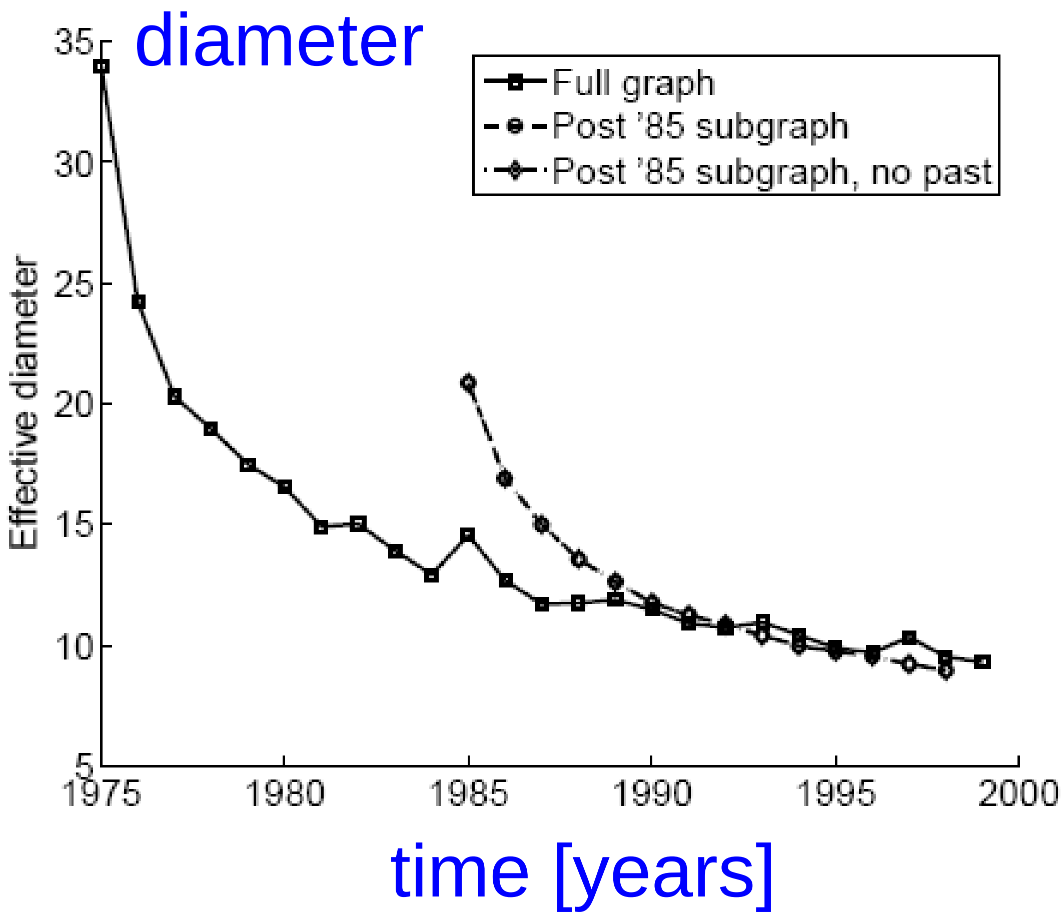
  - diameter ~ O(log N)
  - diameter ~ O(log log N)



- What is happening in real data?
- Diameter **shrinks** over time

T.1 Diameter – "Patents"

- Patent citation network
- 25 years of data
- @1999
  - 2.9 M nodes
  - 16.5 M edges



T.2 Temporal Evolution of the Graphs

- N(t) … nodes at time t
- E(t) … edges at time t
- Suppose that

  N(t+1) = 2 * N(t)
- Q: what is your guess for

  E(t+1) =? 2 * E(t)

T.2 Temporal Evolution of the Graphs

- N(t) … nodes at time t
- E(t) … edges at time t
- Suppose that

  N(t+1) = 2 * N(t)

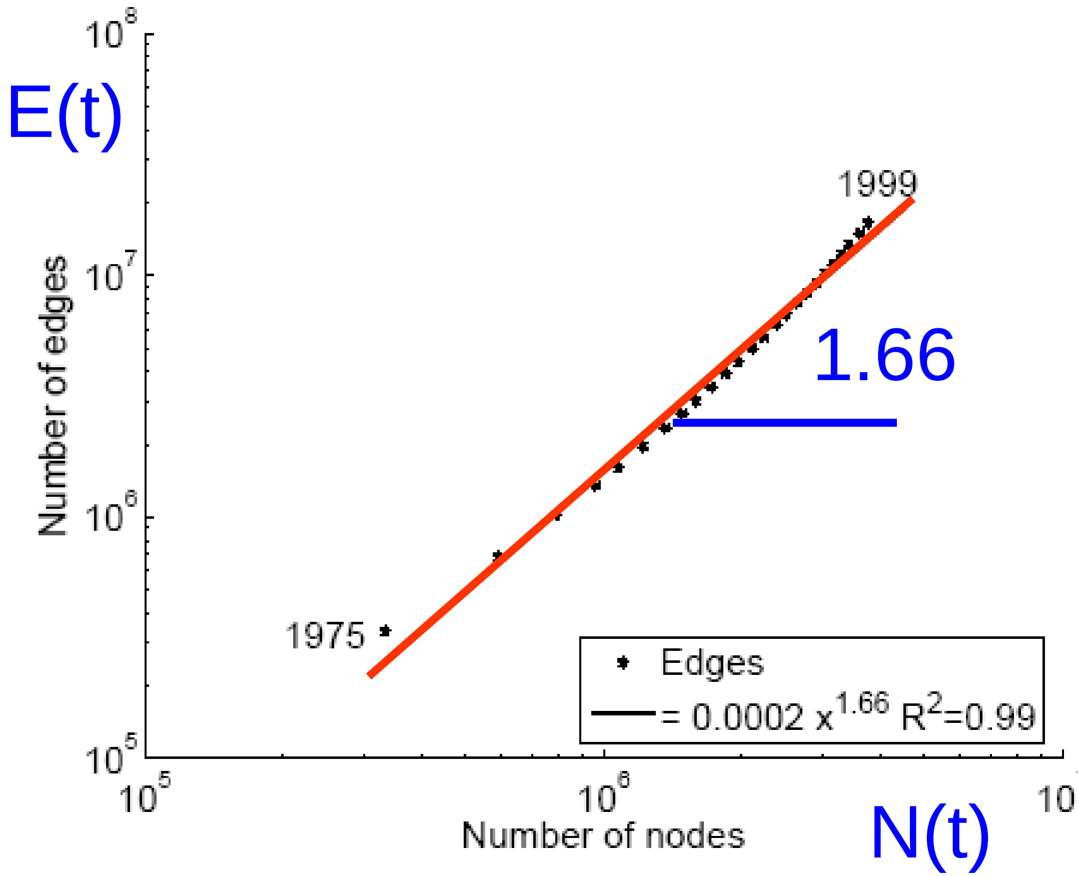
- Q: what is your guess for

  E(t+1) =?= 2 * E(t)

- A: over-doubled!
  - But obeying the ``Densification Power Law''

T.2 Densification – Patent Citations

- Citations among patents granted
- @1999
  - – 2.9 M nodes
  - – 16.5 M edges
- Each year is a datapoint



Outline

- Introduction – Motivation
- Problem#1: Patterns in graphs
  - Static graphs
  - Weighted graphs
  - Time evolving graphs
- Problem#2: Tools
- …

More on Time-evolving graphs

M. McGlohon, L. Akoglu, and C. Faloutsos
Weighted Graphs and Disconnected Components: Patterns and a Generator.
SIG-KDD 2008

Observation T.3: NLCC behavior

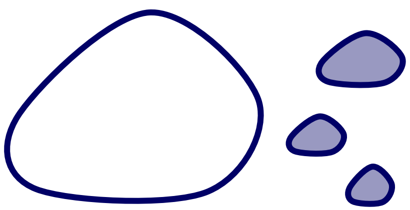
Q: How do NLCC's emerge and join with the GCC?

(``NLCC'' = non-largest conn. components)

– Do they continue to grow in size?

– or do they shrink?

– or stabilize?



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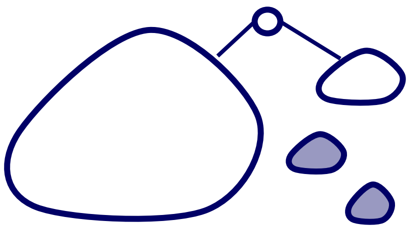
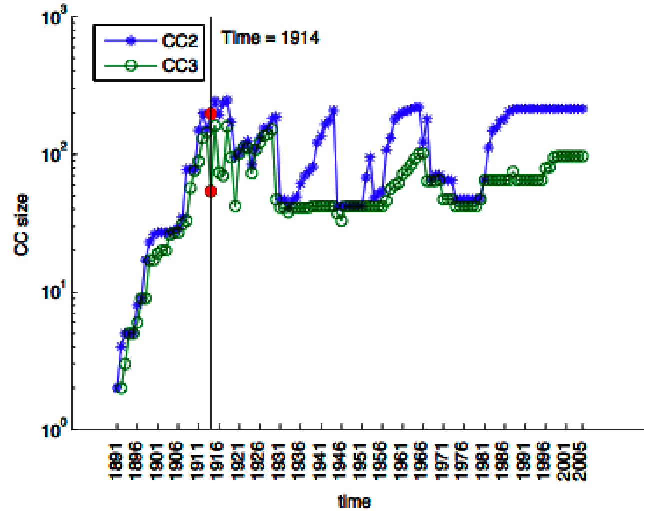


Observation T.3: NLCC behavior

- After the gelling point, the GCC takes off, but NLCC's remain ~constant (actually, oscillate).

IMDB

CC size



Time-stamp

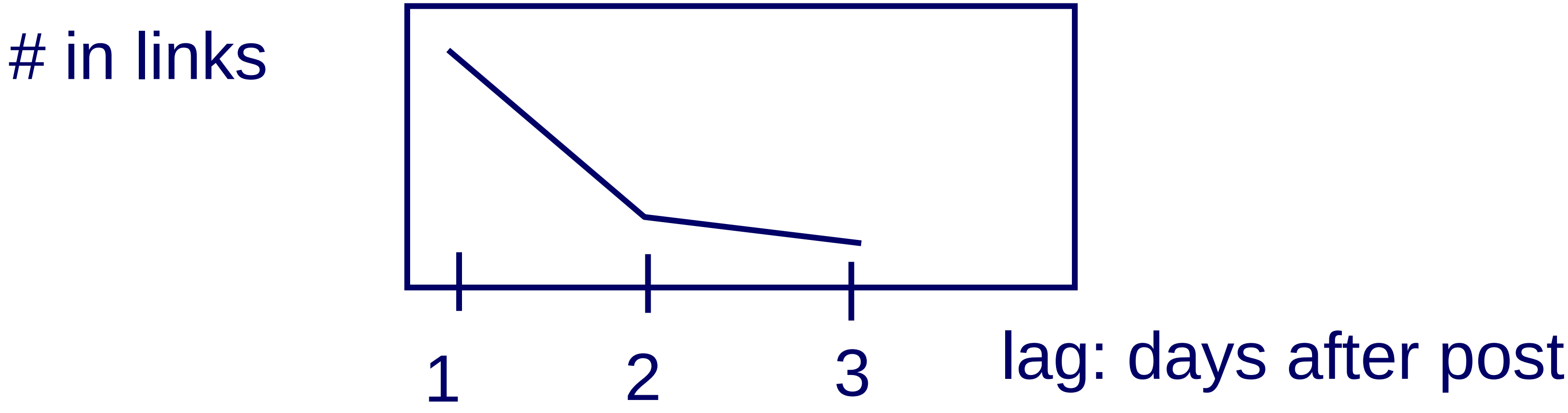
Carnegie Mellon

Timing for Blogs

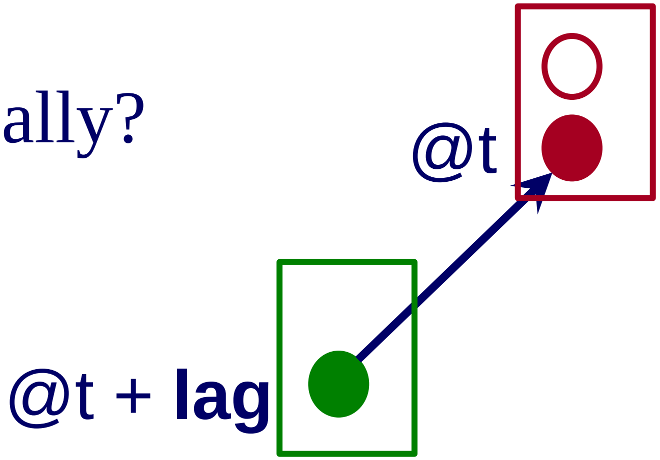
- with Mary McGlohon (CMU->google)
- Jure Leskovec (CMU->Stanford)
- Natalie Glance (now at Google)
- Mat Hurst (now at MSR)

[SDM’07]

T.4 : popularity over time

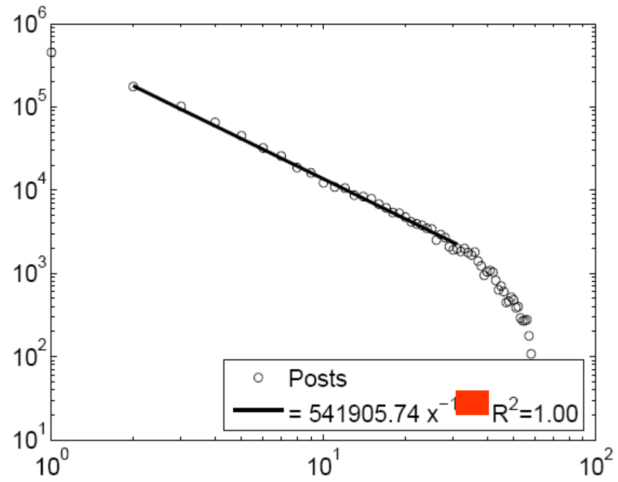


Post popularity drops-off – exponentially?



T.4 : popularity over time

in links (**log**)

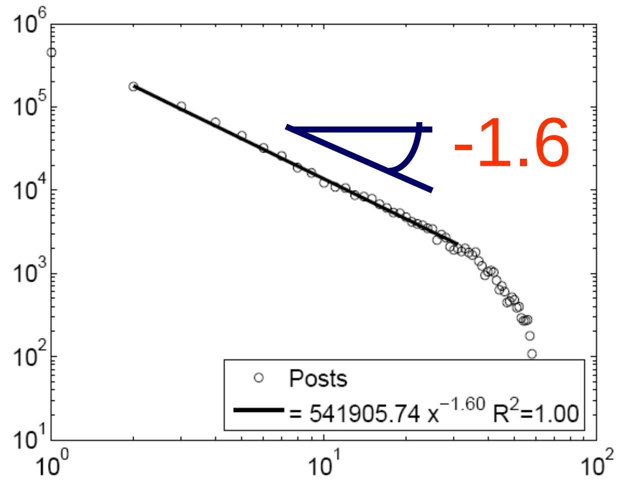


days after post (**log**)

Post popularity drops-off – exponentially?
POWER LAW!
Exponent?

T.4 : popularity over time

in links
(**log**)



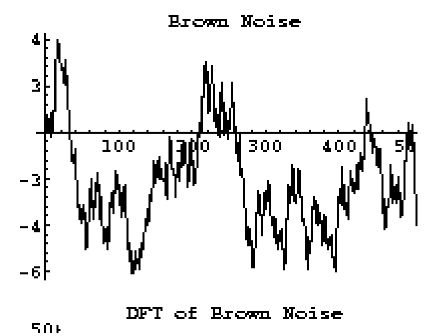
days after post
(**log**)

Post popularity drops-off – exponentially?

POWER LAW!

Exponent? -1.6

- close to -1.5: Barabasi's stack model
- and like the zero-crossings of a random walk



-1.5 slope

J. G. Oliveira & A.-L. Barabási Human Dynamics: The Correspondence Patterns of Darwin and Einstein. *Nature* **437,** 1251 (2005) . [PDF]

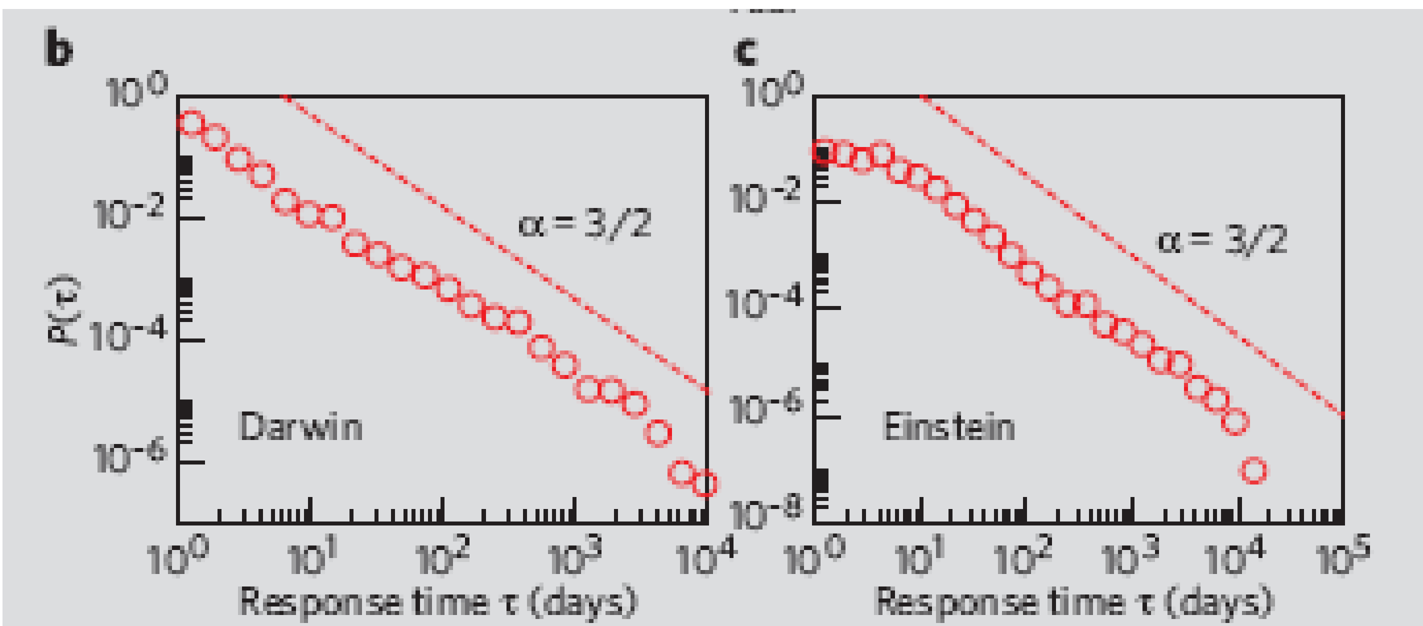


Figure 1 | The correspondence patterns of Darwin and Einstein.

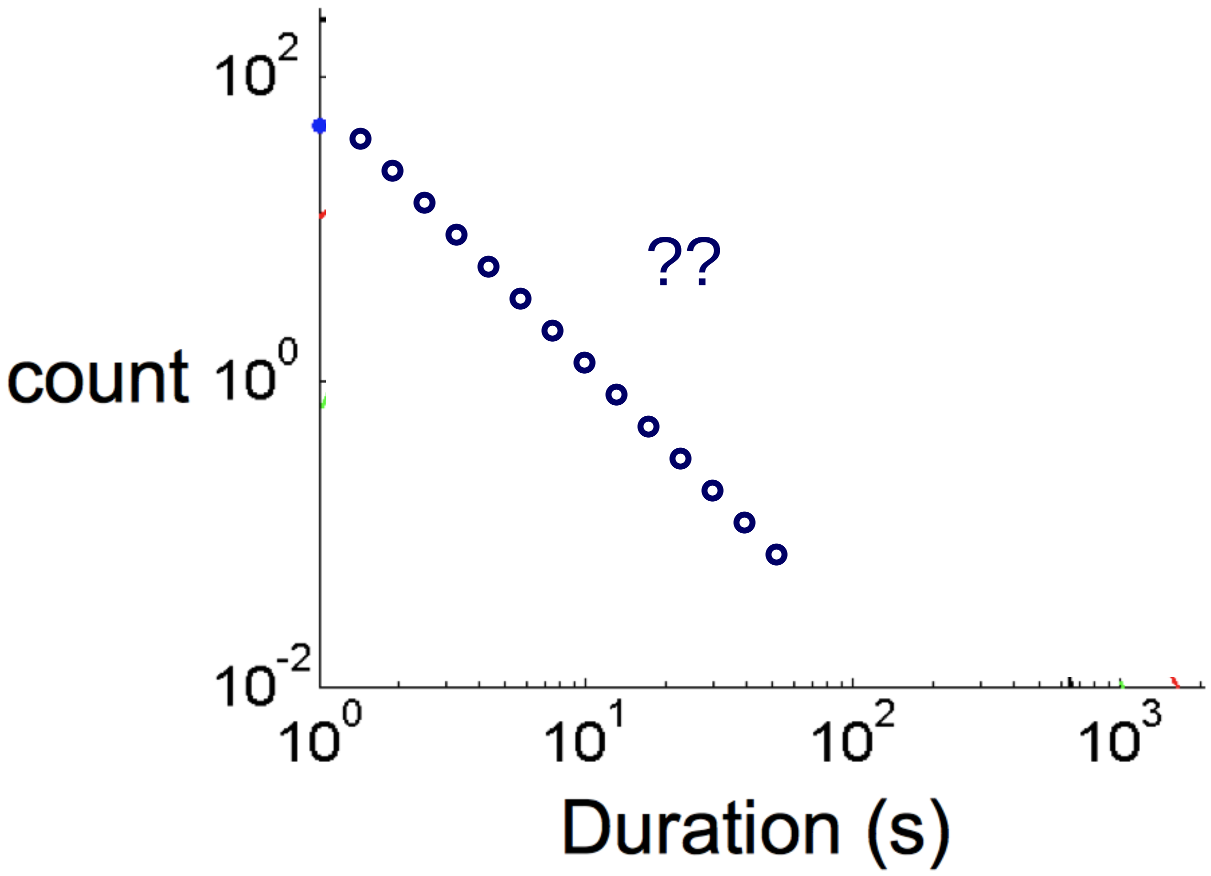
T.5: duration of phonecalls

Surprising Patterns for the Call Duration Distribution of Mobile Phone Users

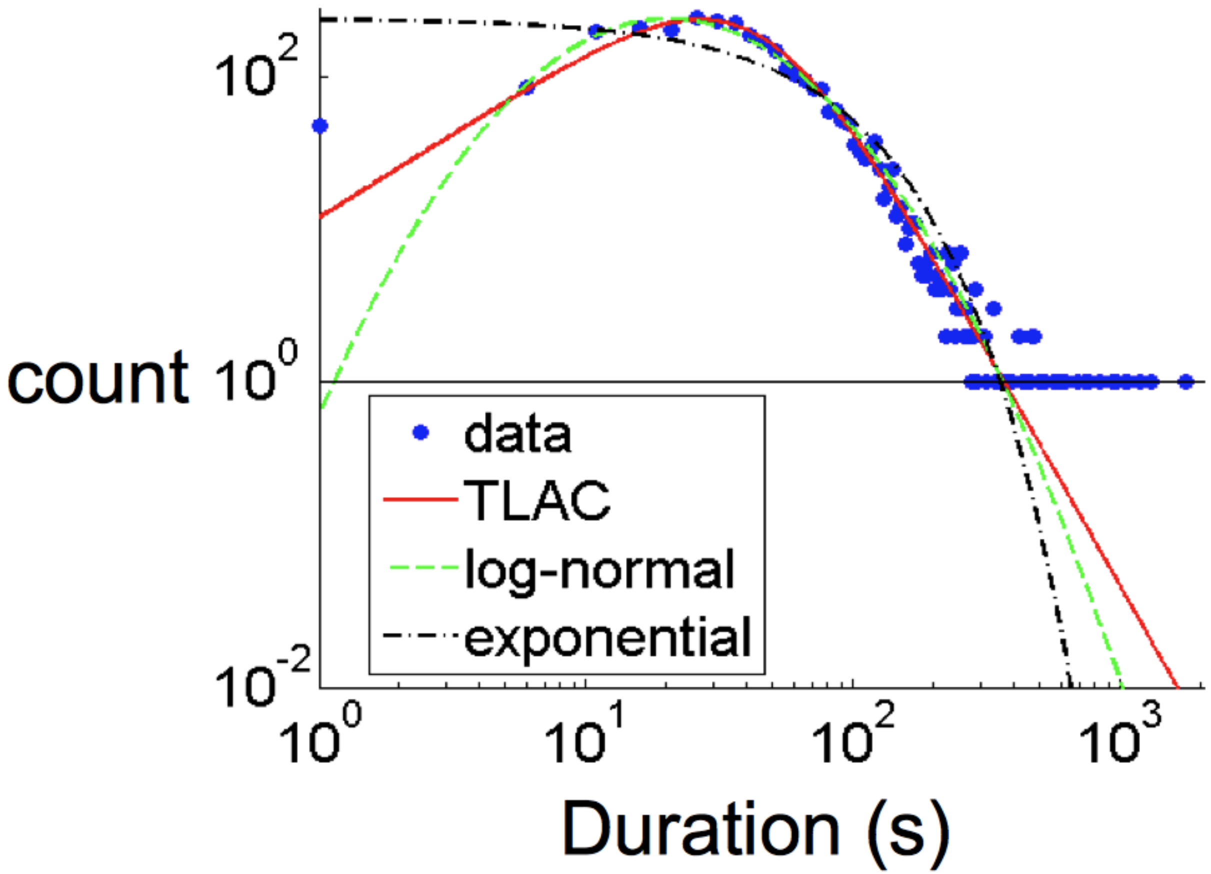
Pedro O. S. Vaz de Melo, Leman Akoglu, Christos Faloutsos, Antonio A. F. Loureiro

PKDD 2010

Probably, power law (?)

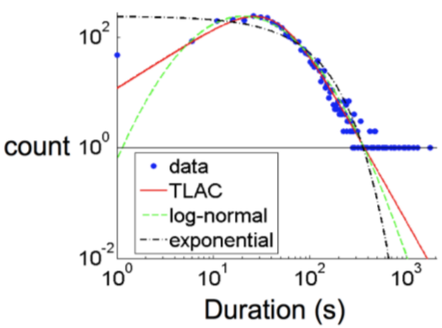


No Power Law!


count
10^2
10^0
10^{-2}
10^0
10^1
10^2
10^3
Duration (s)
data
TLAC
log-normal
exponential

‘TLaC: Lazy Contractor’

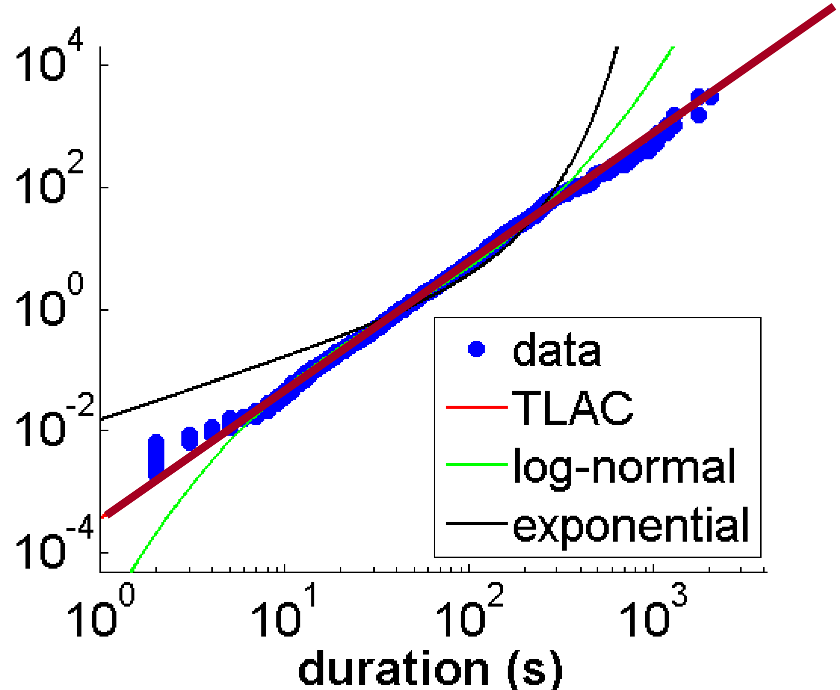
- The longer a task (phonecall) has taken,
- The even longer it will take



Odds ratio=

Casualties(<x):
Survivors(>=x)

== power law



Data Description

- Data from a private mobile operator of a large city
  - 4 months of data
  - 3.1 million users
  - more than 1 billion phone records

Outline

- Introduction – Motivation
- Problem#1: Patterns in graphs
- Problem#2: Tools
  - CenterPiece Subgraphs
  - OddBall (anomaly detection)
  - PEGASUS
- Problem#3: Scalability
- Conclusions

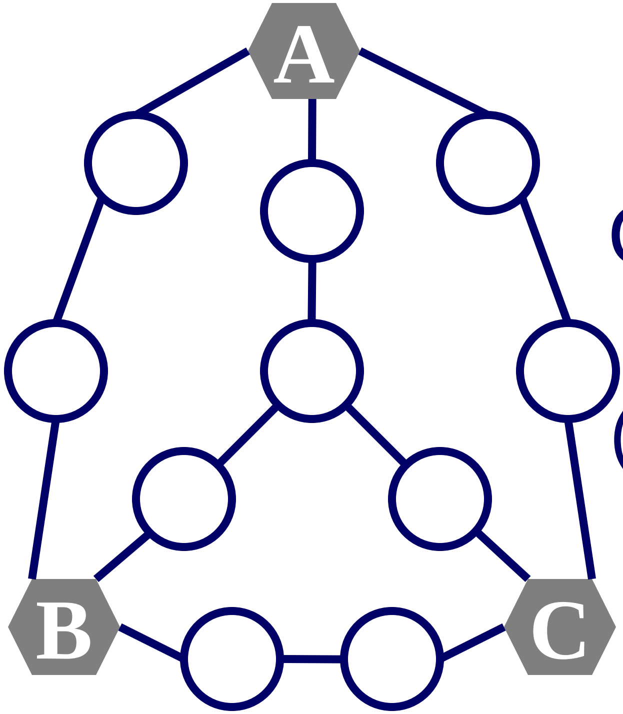
CenterPiece Subgraphs

- Hanghang TONG et al, KDD’06

Center-Piece Subgraph Discovery

[Tong+ KDD 06]

Input



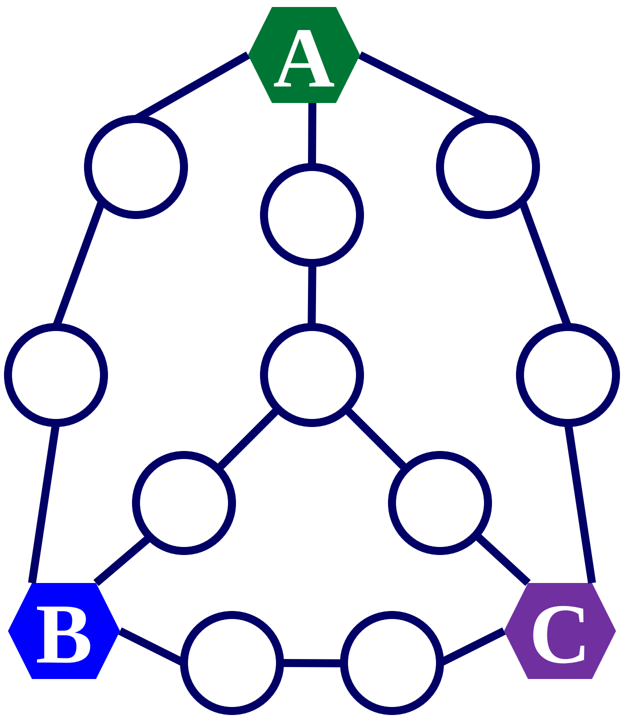
Original Graph

Q: Who is the most central node wrt the black nodes?
(e.g., master-mind criminal, common advisor/collaborator, etc)

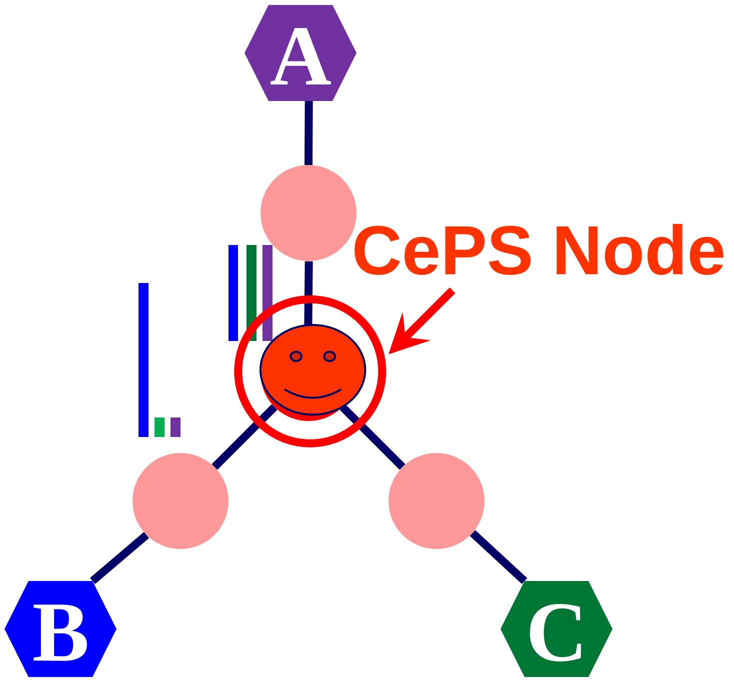
Center-Piece Subgraph Discovery

[Tong+ KDD 06]

Input: original graph



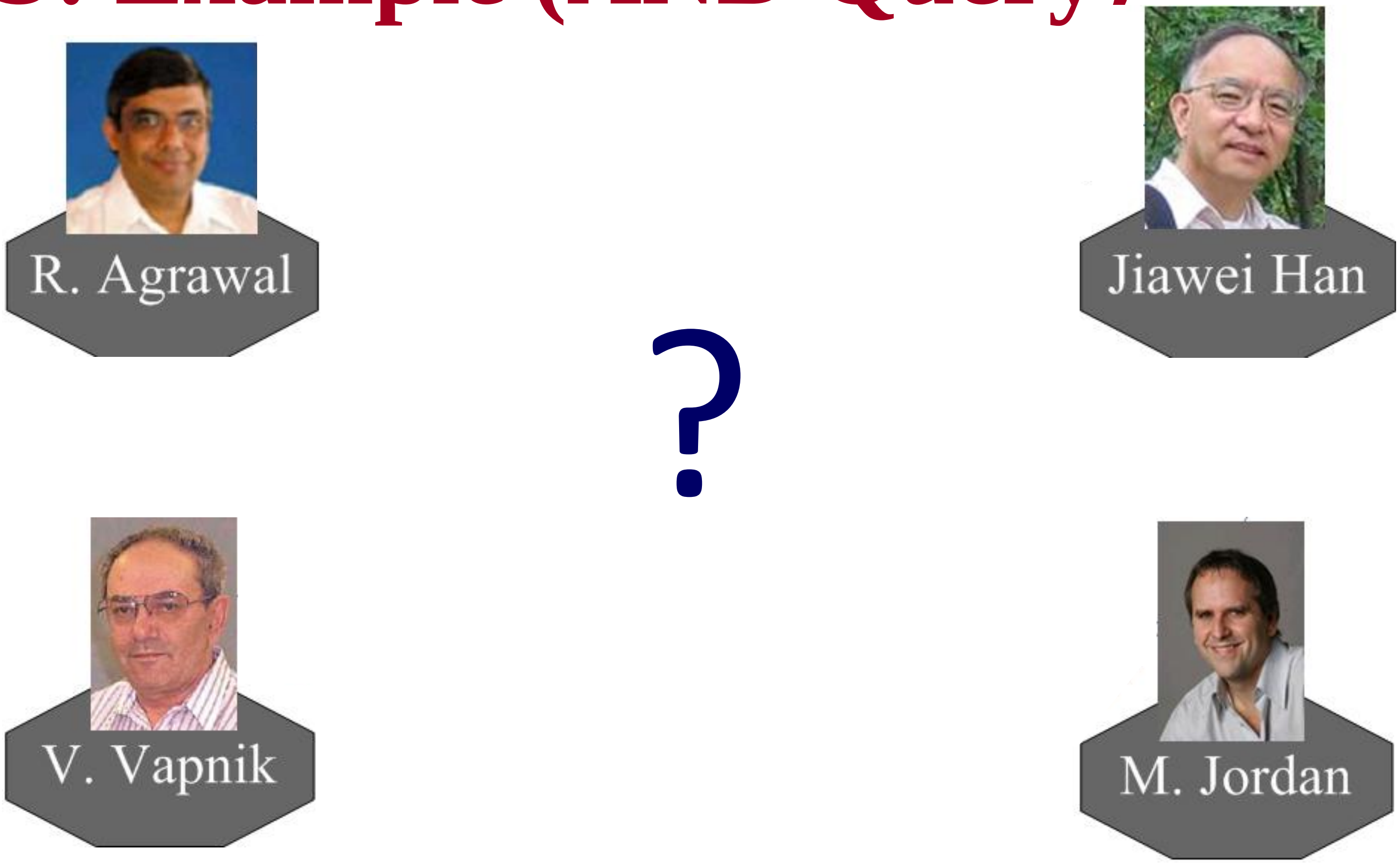
Output: CePS



Q: How to find hub for the query nodes?

A: Combine proximity scores (RWR)

CePS: Example (AND Query)

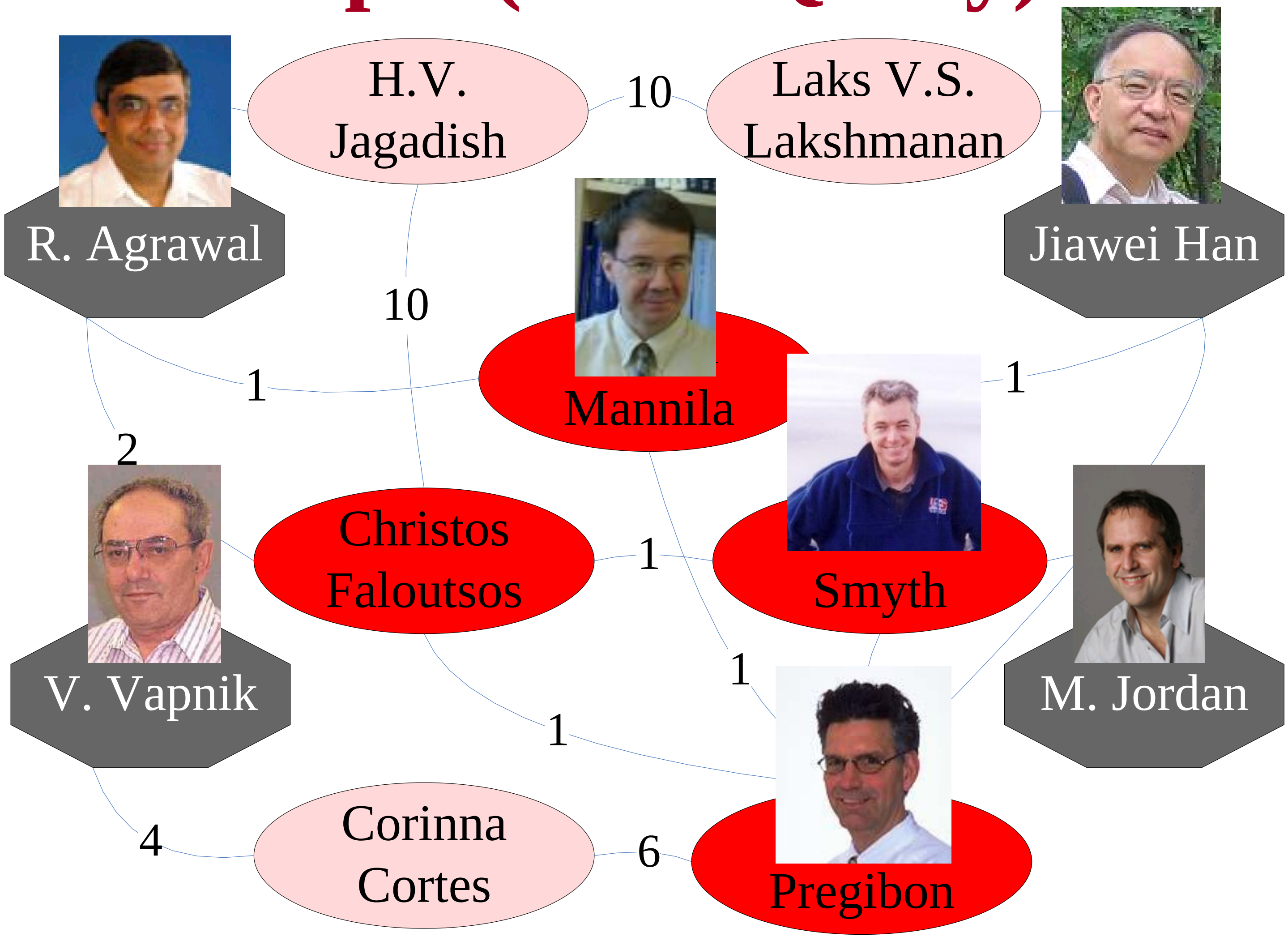


DBLP co-authorship network:

-400,000 authors, 2,000,000 edges

Code at: http://www.cs.cmu.edu/~htong/soft.htm

CePS: Example (AND Query)



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- Conclusions

Carnegie Mellon

OddBall: Spotting Anomalies in Weighted Graphs

Leman Akoglu, Mary McGlohon, Christos Faloutsos

Carnegie Mellon University

School of Computer Science

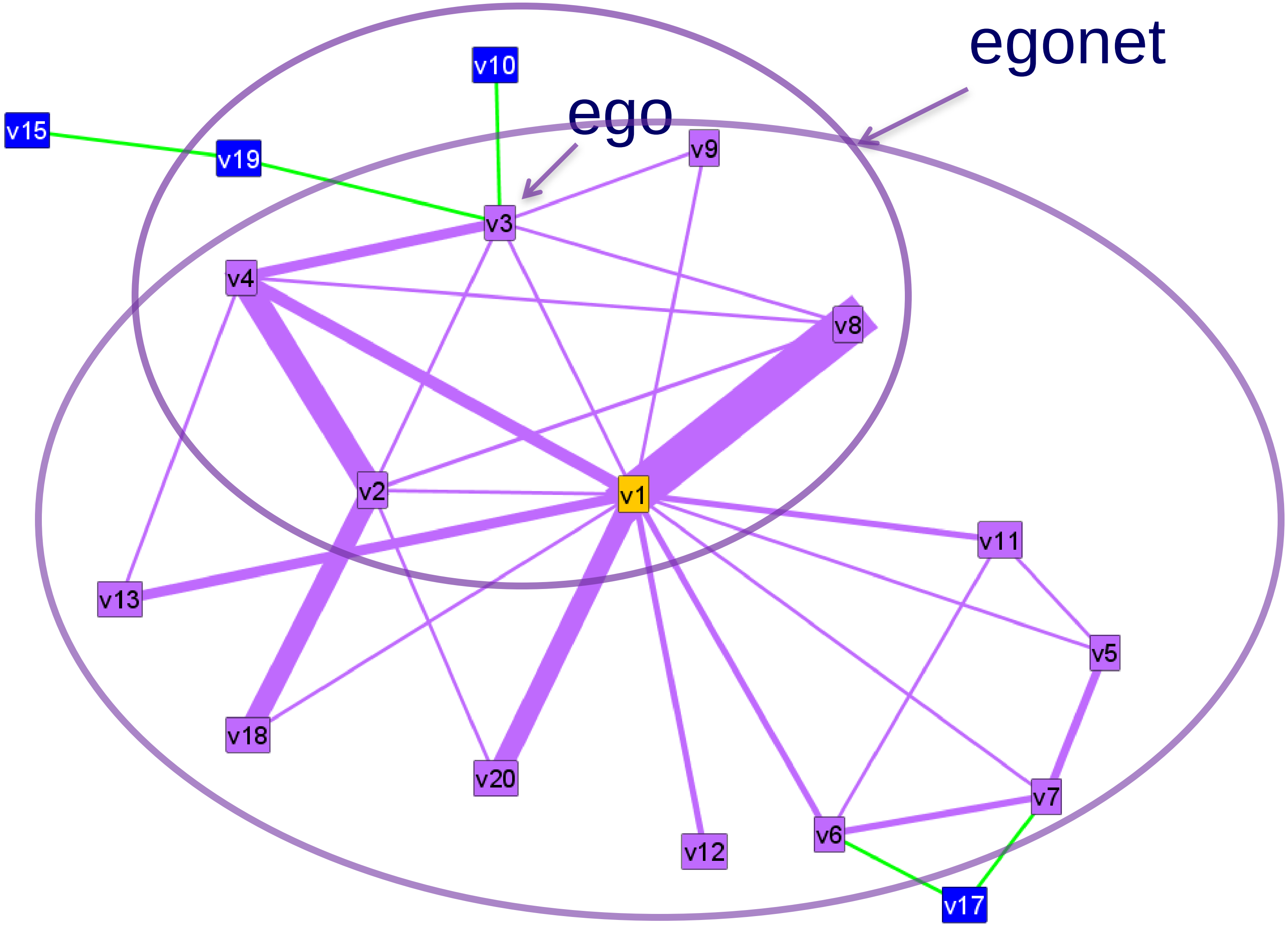
PAKDD 2010, Hyderabad, India

Main idea

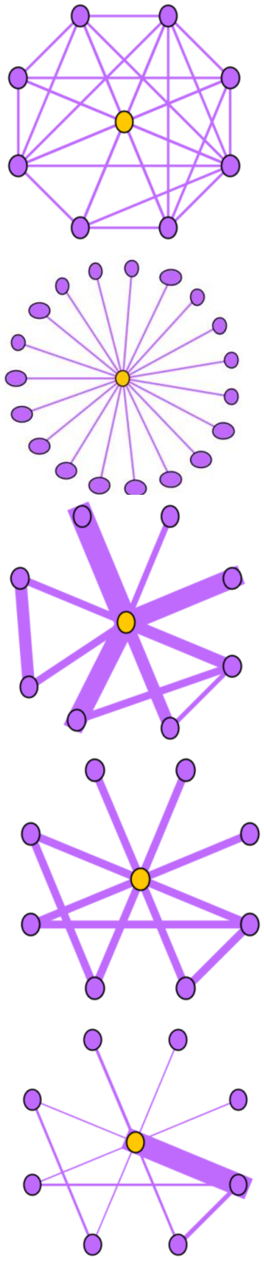
For each node,

- extract ‘ego-net’ (=1-step-away neighbors)
- Extract features (#edges, total weight, etc etc)
- Compare with the rest of the population

What is an egonet?

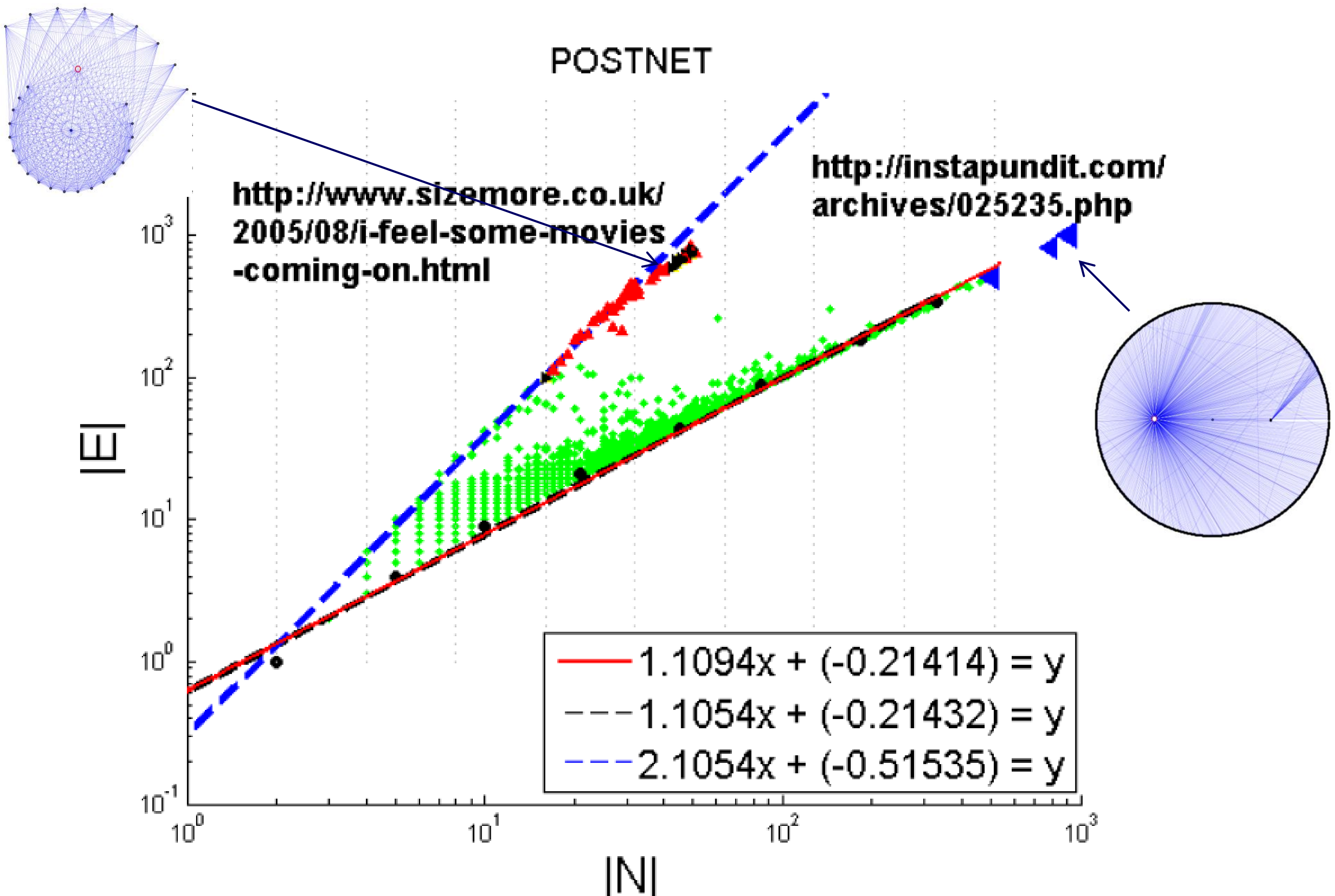


Selected Features



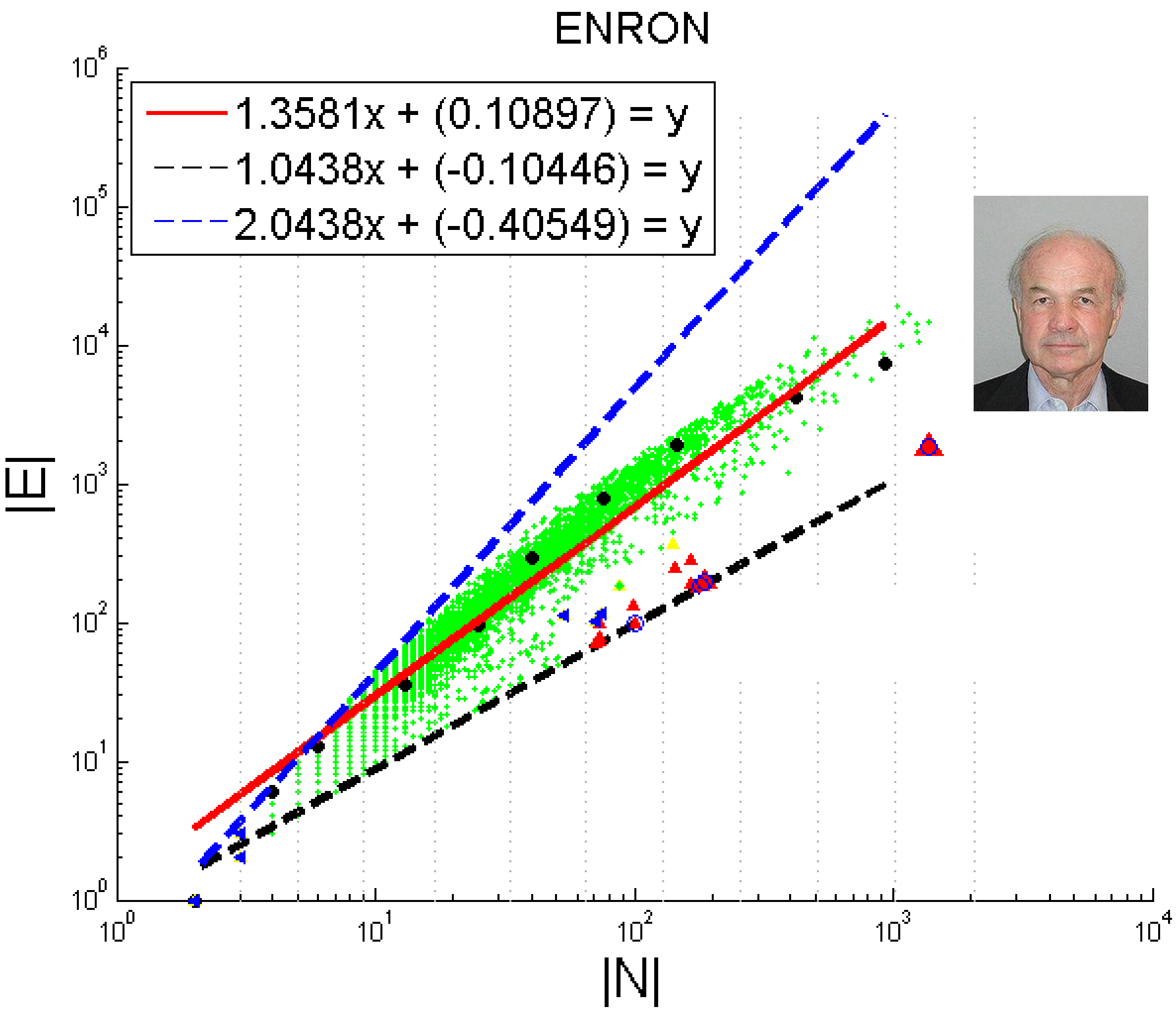
- N_i: number of neighbors (degree) of ego i
- E_i: number of edges in egonet i
- W_i: total weight of egonet i
- $\lambda_{w,i}$: principal eigenvalue of the weighted adjacency matrix of egonet I

Near-Clique/Star



Carnegie Mellon

Near-Clique/Star



Outline

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  - CenterPiece Subgraphs
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- Problem#3: Scalability -PEGASUS
- Conclusions

Scalability

- Google: > 450,000 processors in clusters of ~2000 processors each [Barroso, Dean, Hölzle, *"Web Search for a Planet: The Google Cluster Architecture"* IEEE Micro 2003]
- Yahoo: 5Pb of data [Fayyad, KDD'07]
- Problem: machine failures, on a daily basis
- How to parallelize data mining tasks, then?
- A: map/reduce – hadoop (open-source clone) http://hadoop.apache.org/

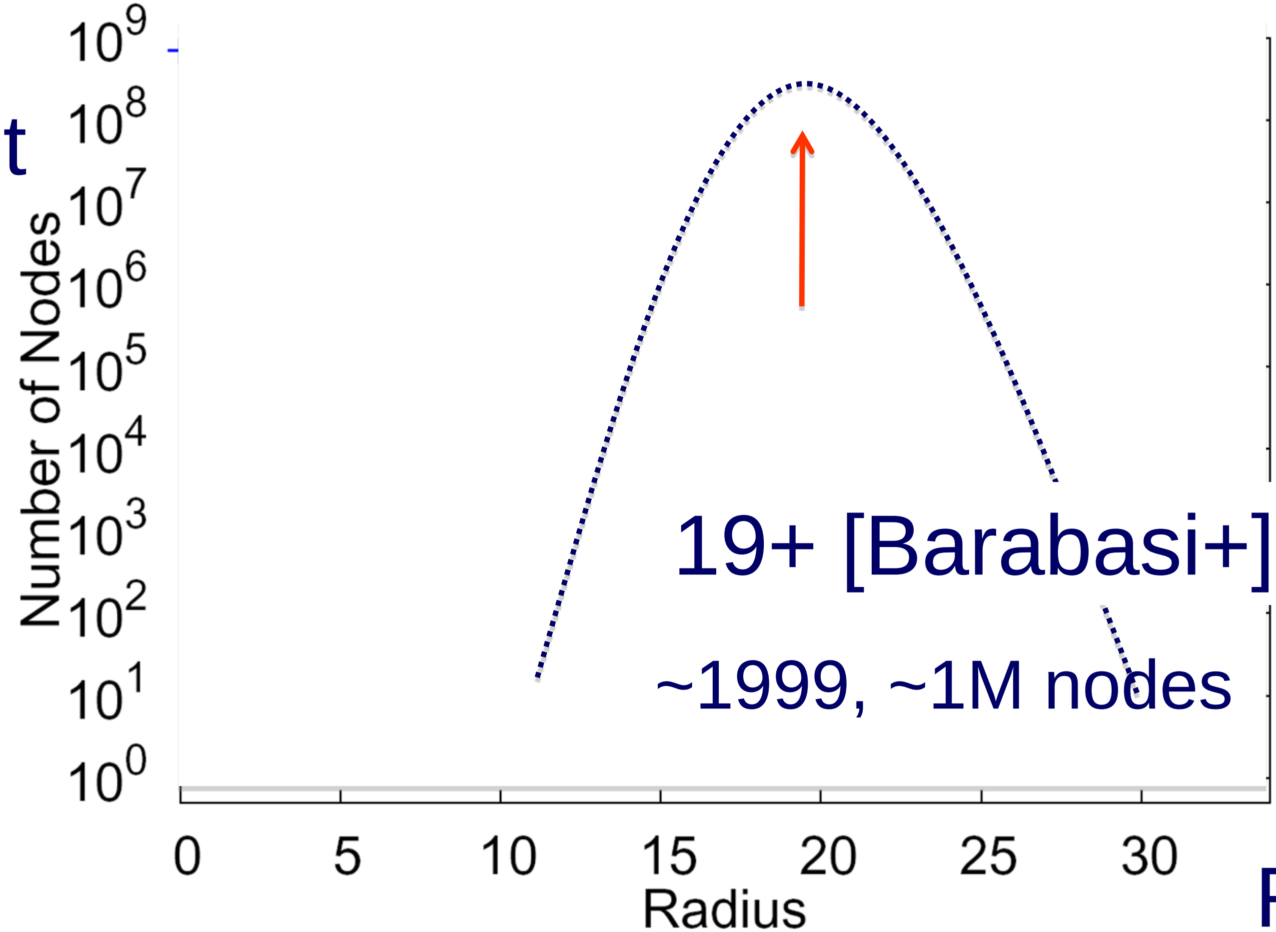
Outline – Algorithms & results

	Centralized	**Hadoop/PEG ASUS**
Degree Distr.	old	old
Pagerank	old	old
Diameter/ANF	old	**HERE**
Conn. Comp	old	**HERE**
Triangles	**done**	
Visualization	**started**	

HADI for diameter estimation

- *Radius Plots for Mining Tera-byte Scale Graphs* **U Kang**, Charalampos Tsourakakis, Ana Paula Appel, Christos Faloutsos, Jure Leskovec, SDM'10
- Naively: diameter needs **O(N**2)** space and up to O(N**3) time – **prohibitive** (N~1B)
- Our HADI: linear on E (~10B)
  – Near-linear scalability wrt # machines
  – Several optimizations -> 5x faster

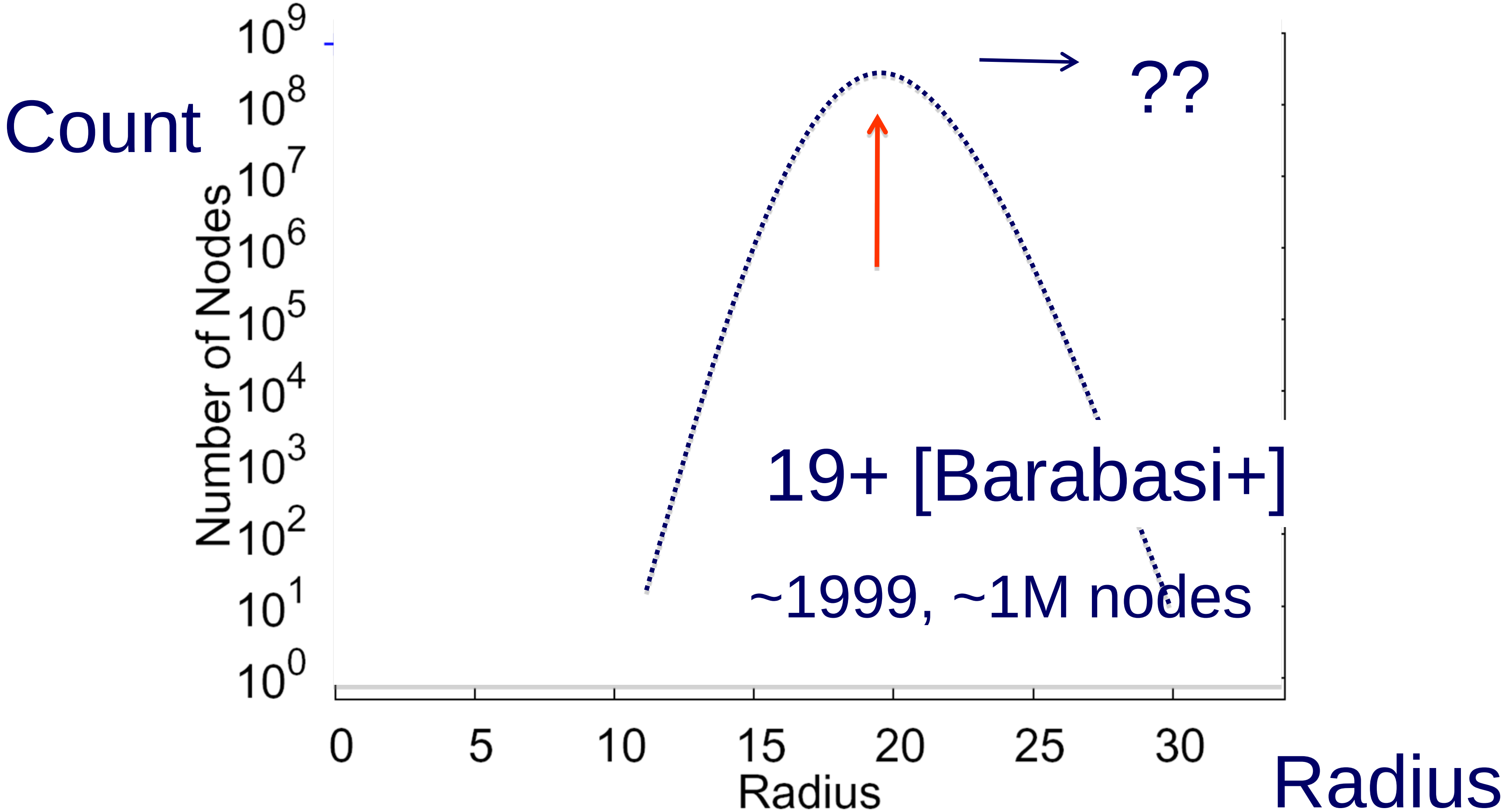
Count



19+ [Barabasi+]

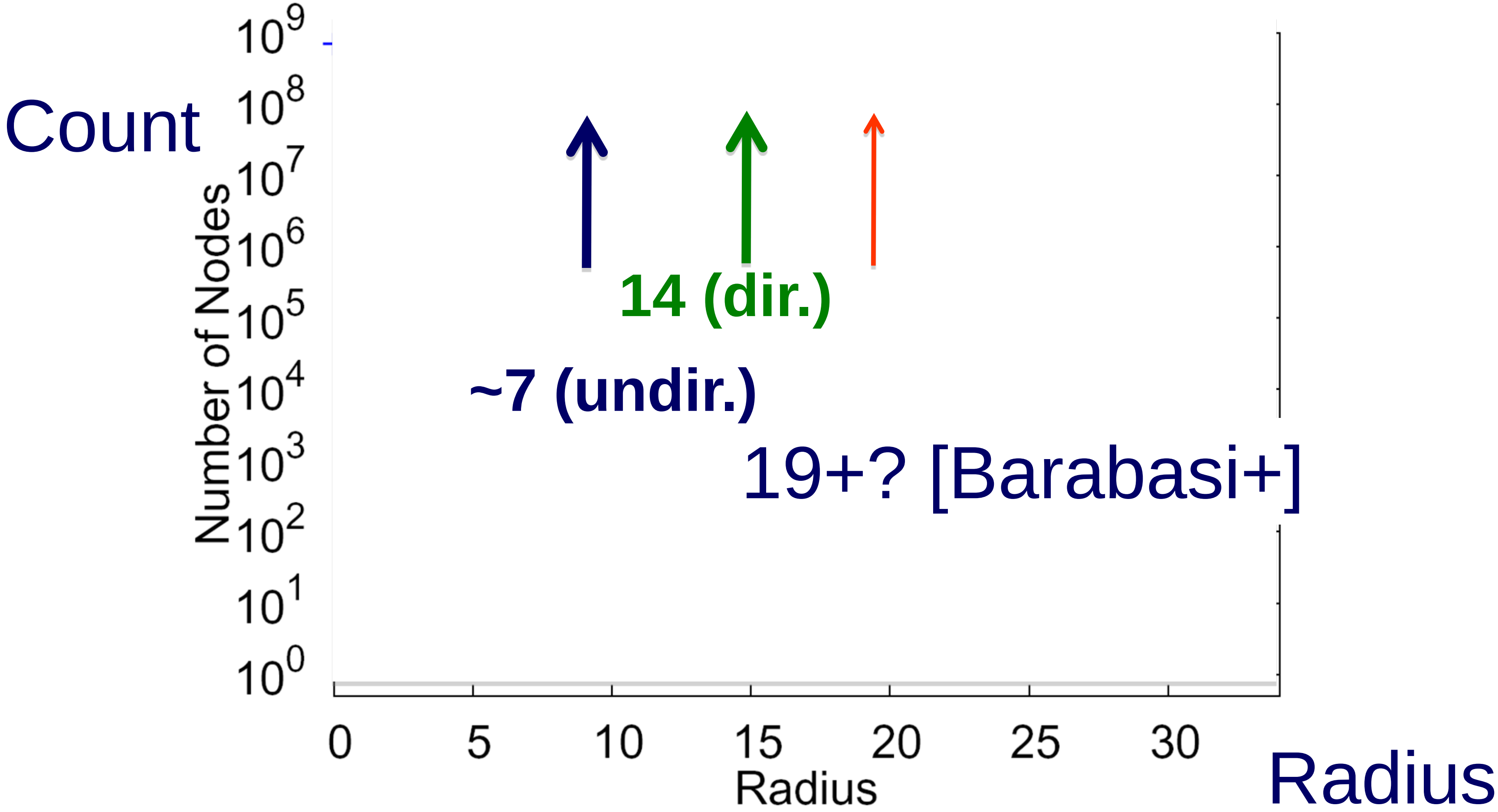
~1999, ~1M nodes

Radius



YahooWeb graph (120Gb, 1.4B nodes, 6.6 B edges)

- Largest publicly available graph ever studied.



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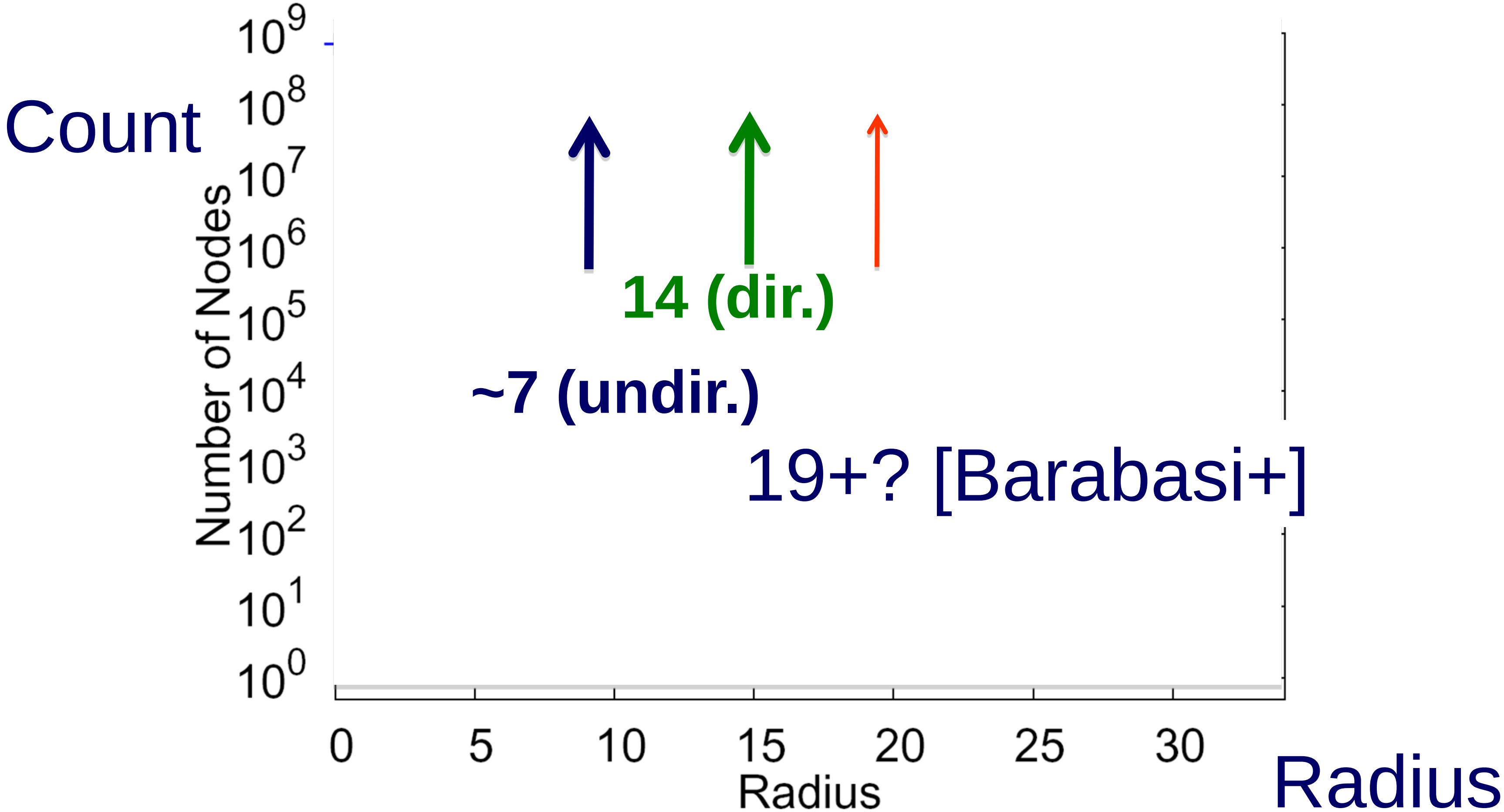
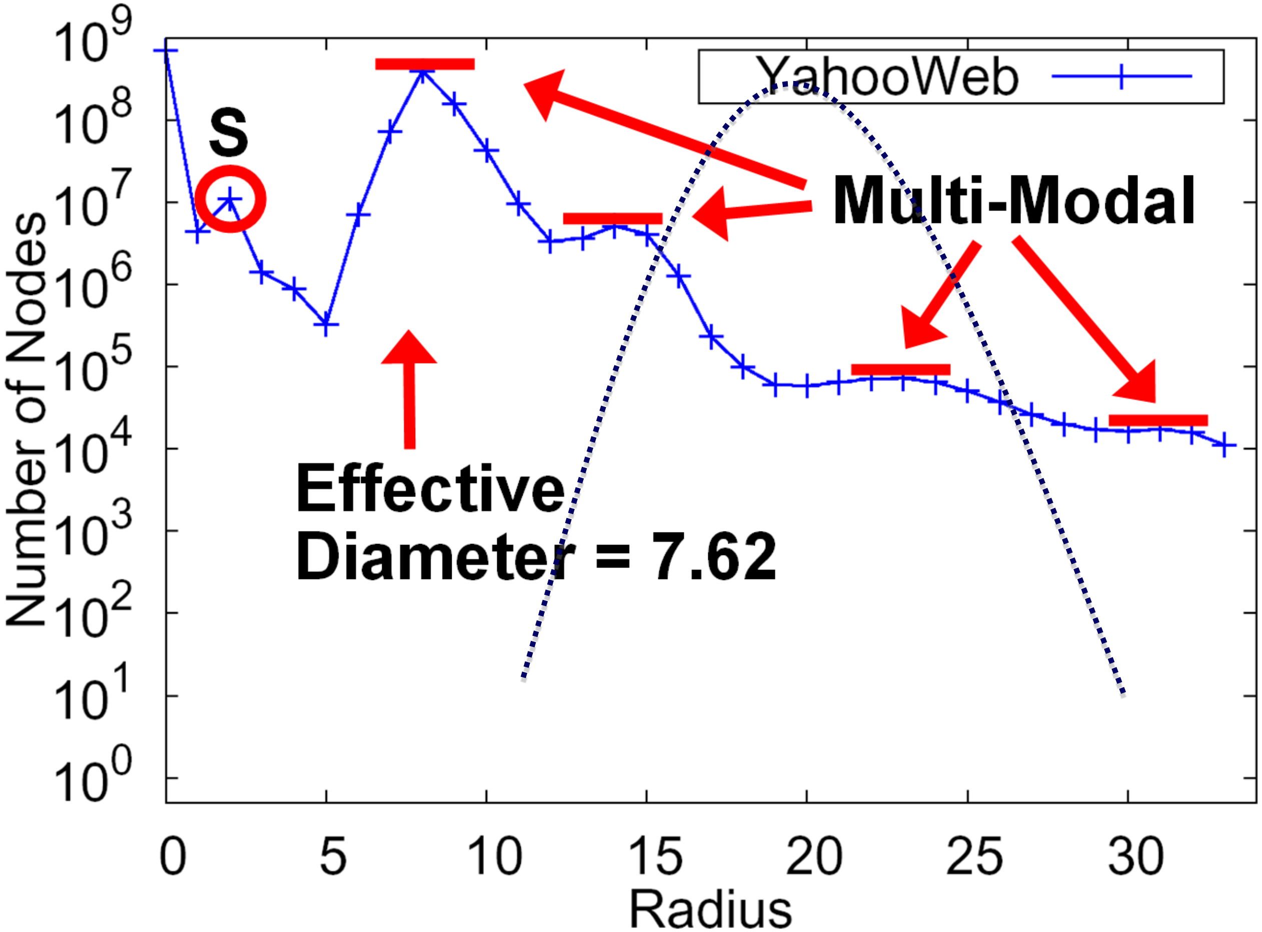


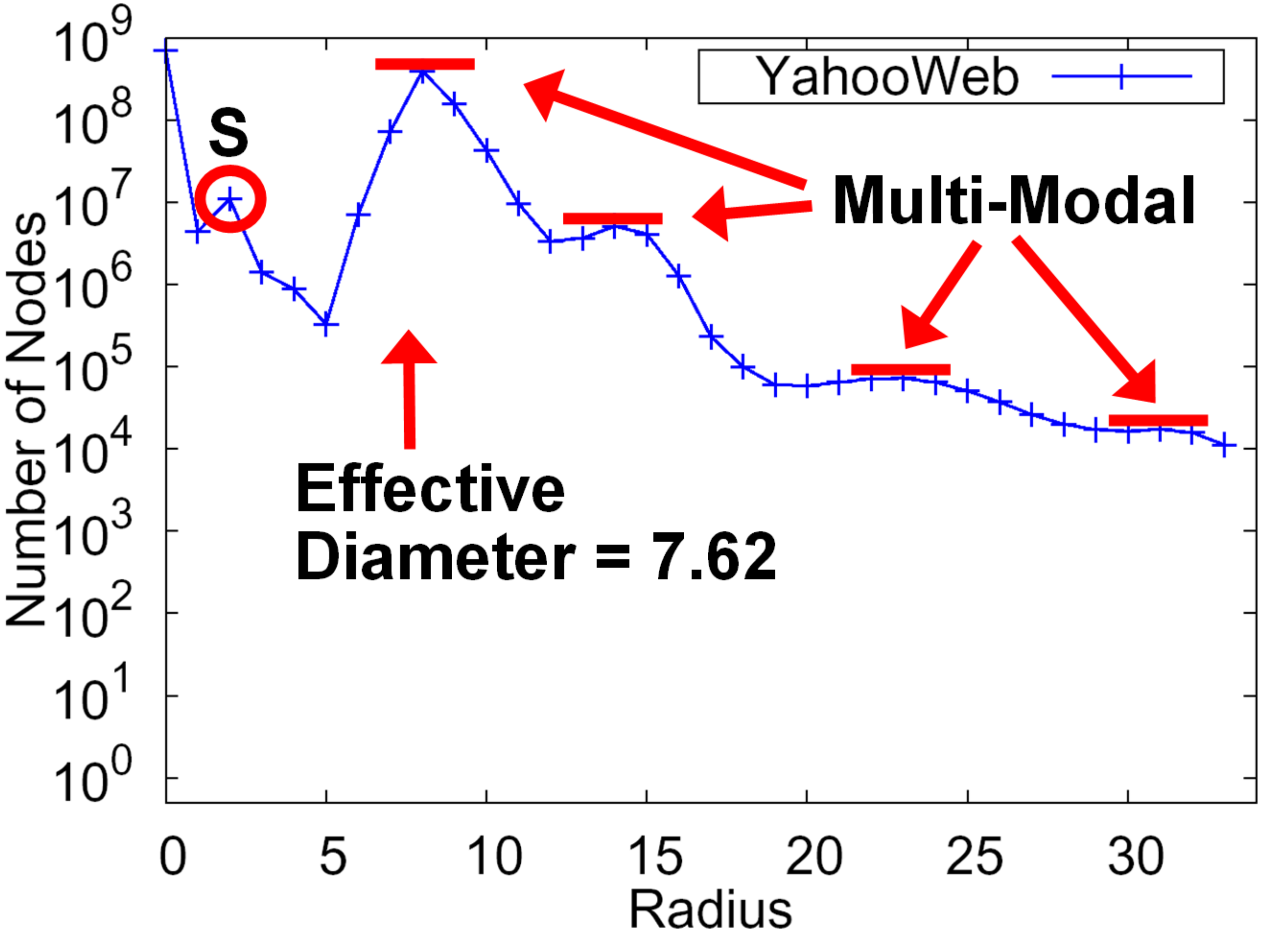
YahooWeb graph (120Gb, 1.4B nodes, 6.6 B edges)

- 7 degrees of separation (!)
- Diameter: shrunk



YahooWeb graph (120Gb, 1.4B nodes, 6.6 B edges)

- effective diameter: surprisingly small.
- Multi-modality: probably mixture of cores .



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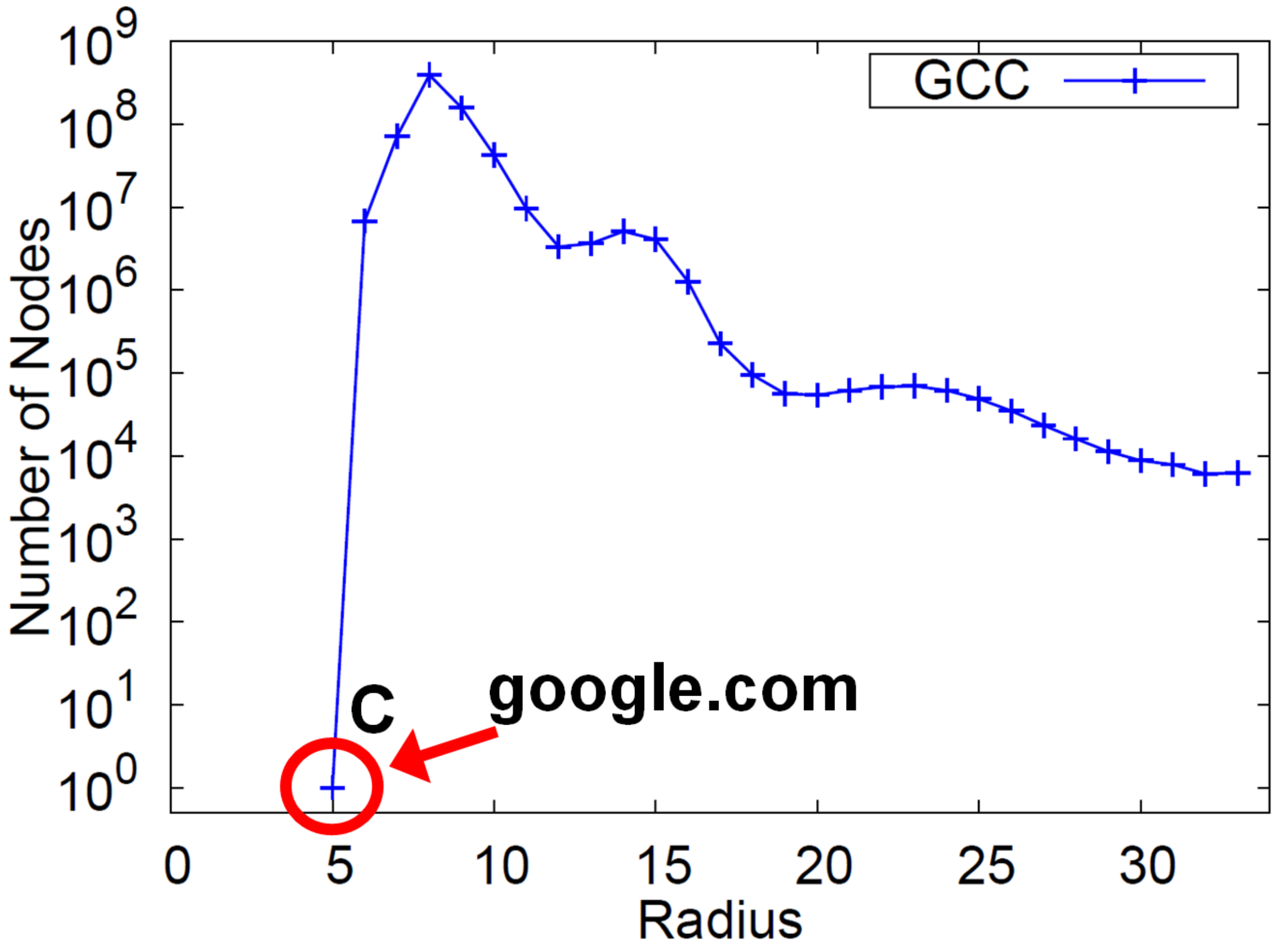
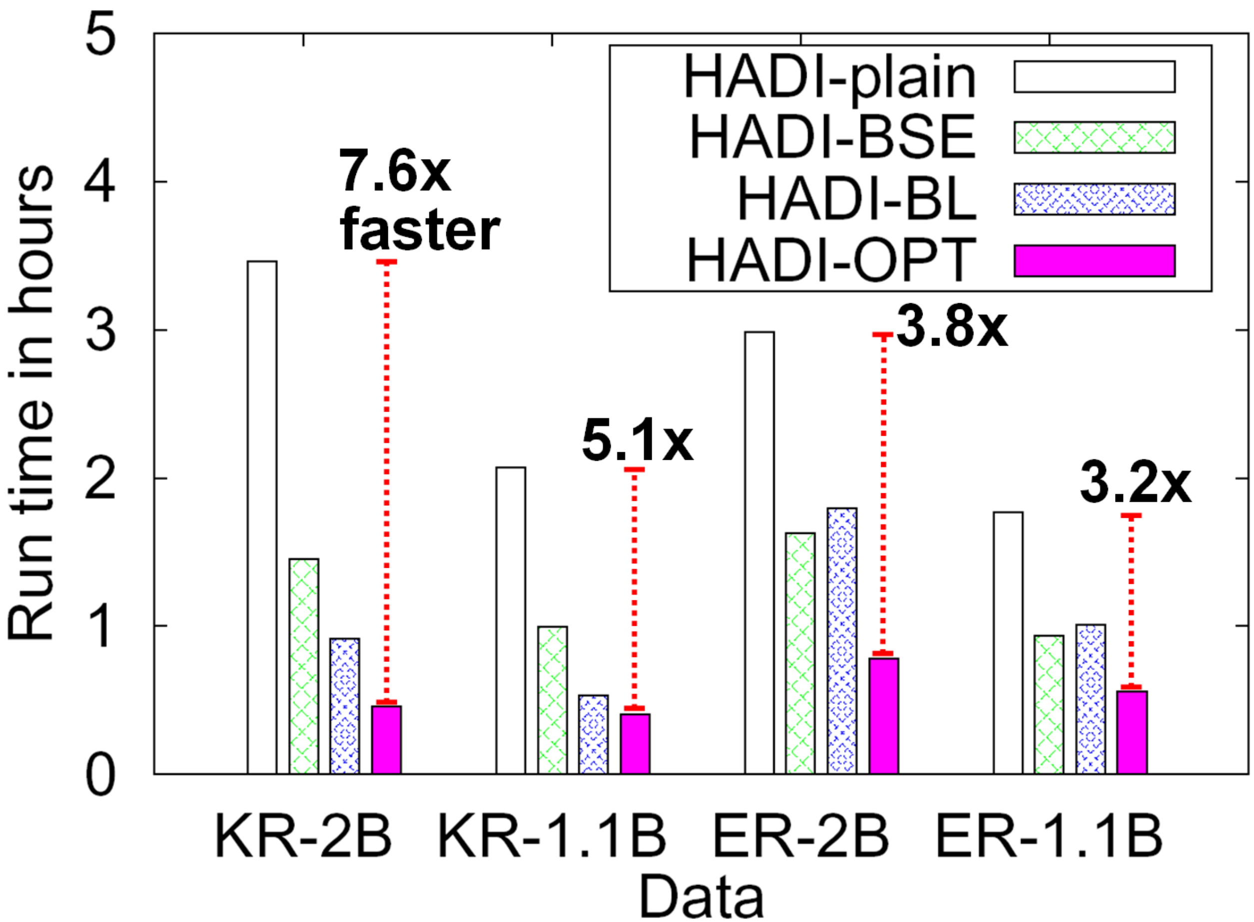


Radius Plot of **GCC** of YahooWeb.

details



Running time - Kronecker and Erdos-Renyi Graphs with billions edges.

Outline – Algorithms & results

	Centralized	**Hadoop/PEG ASUS**
Degree Distr.	old	old
Pagerank	old	old
Diameter/ANF	old	**HERE**
Conn. Comp	old	**HERE**
Triangles		done
Visualization	**started**	

Generalized Iterated Matrix Vector Multiplication (GIMV)

PEGASUS: A Peta-Scale Graph Mining System - Implementation and Observations.
U Kang, Charalampos E. Tsourakakis, and Christos Faloutsos.
(ICDM) 2009, Miami, Florida, USA.
Best Application Paper (runner-up).

details

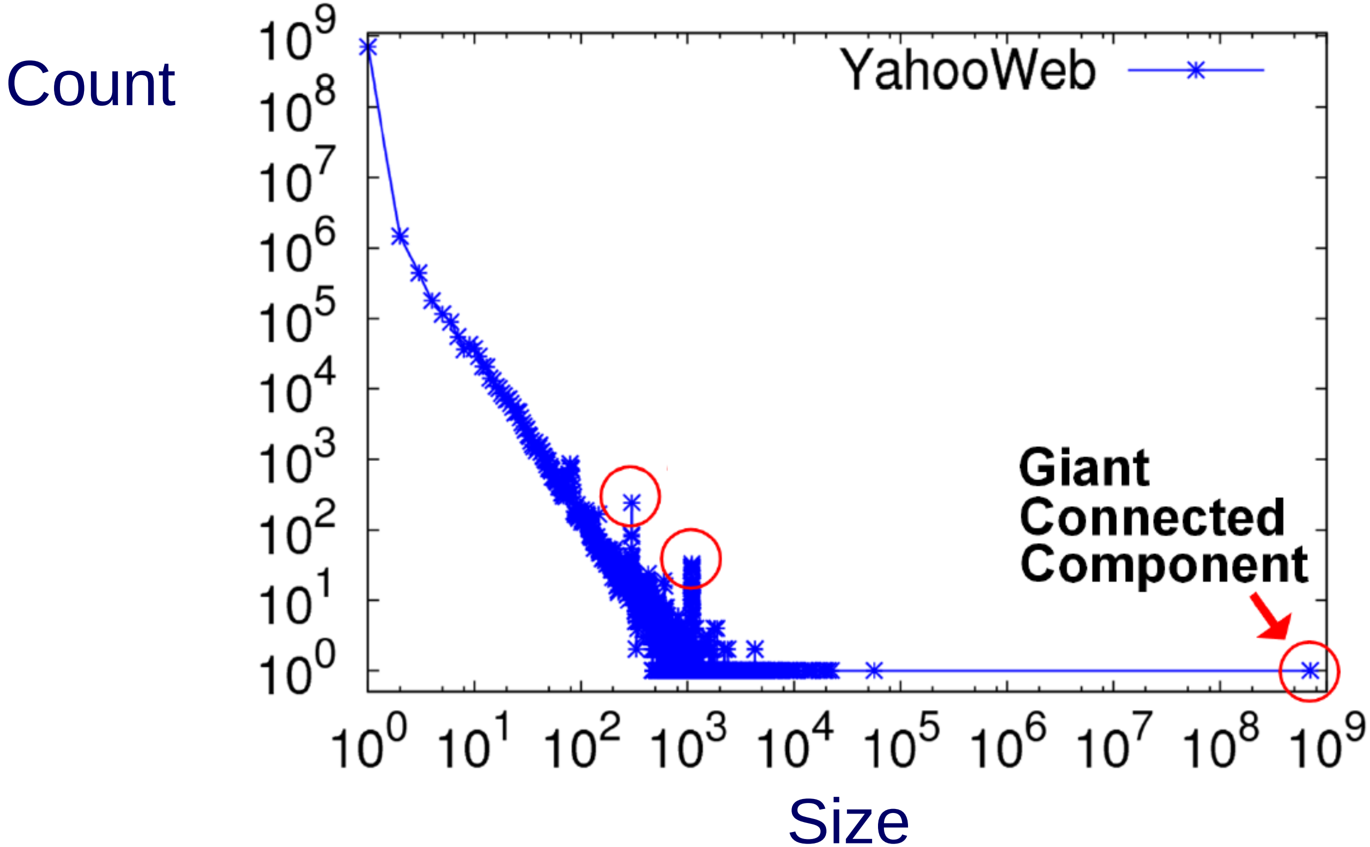
Generalized Iterated Matrix Vector Multiplication (GIMV)

- PageRank
- proximity (RWR)
- Diameter
- Connected components
- (eigenvectors,
- Belief Prop.
- …)

Matrix – vector Multiplication (iterated)

Example: GIM-V At Work

- Connected Components

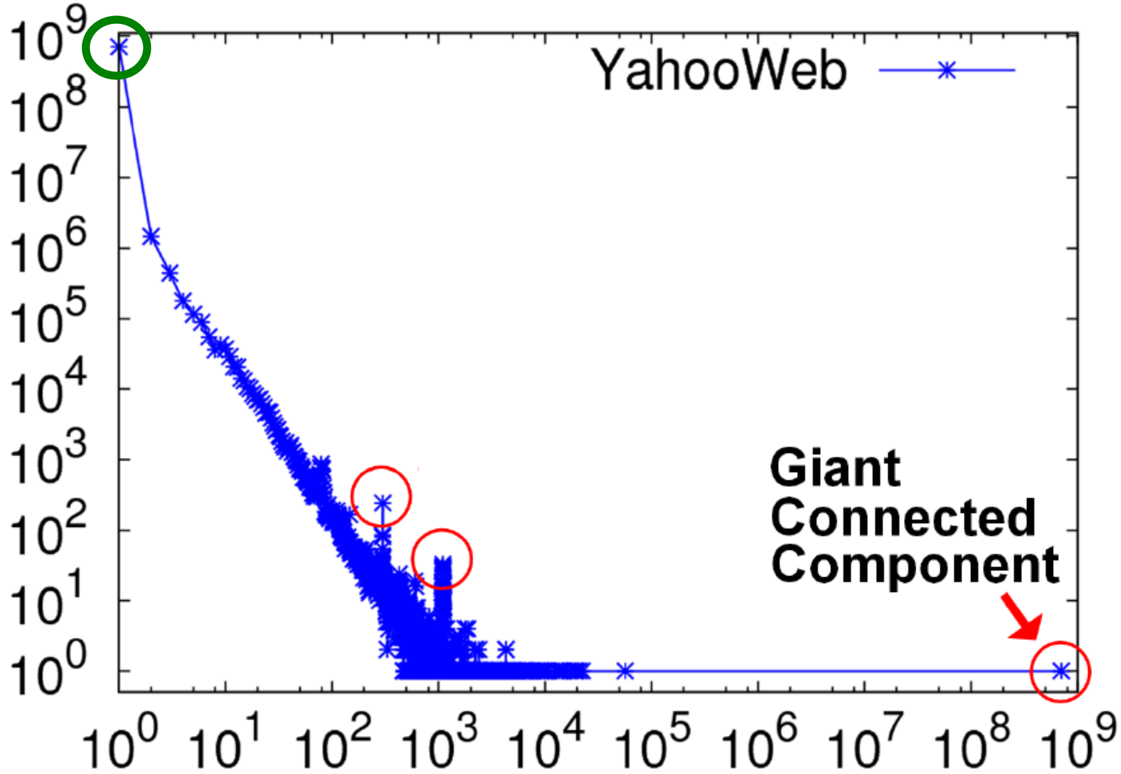


Example: GIM-V At Work

- Connected Components

Count

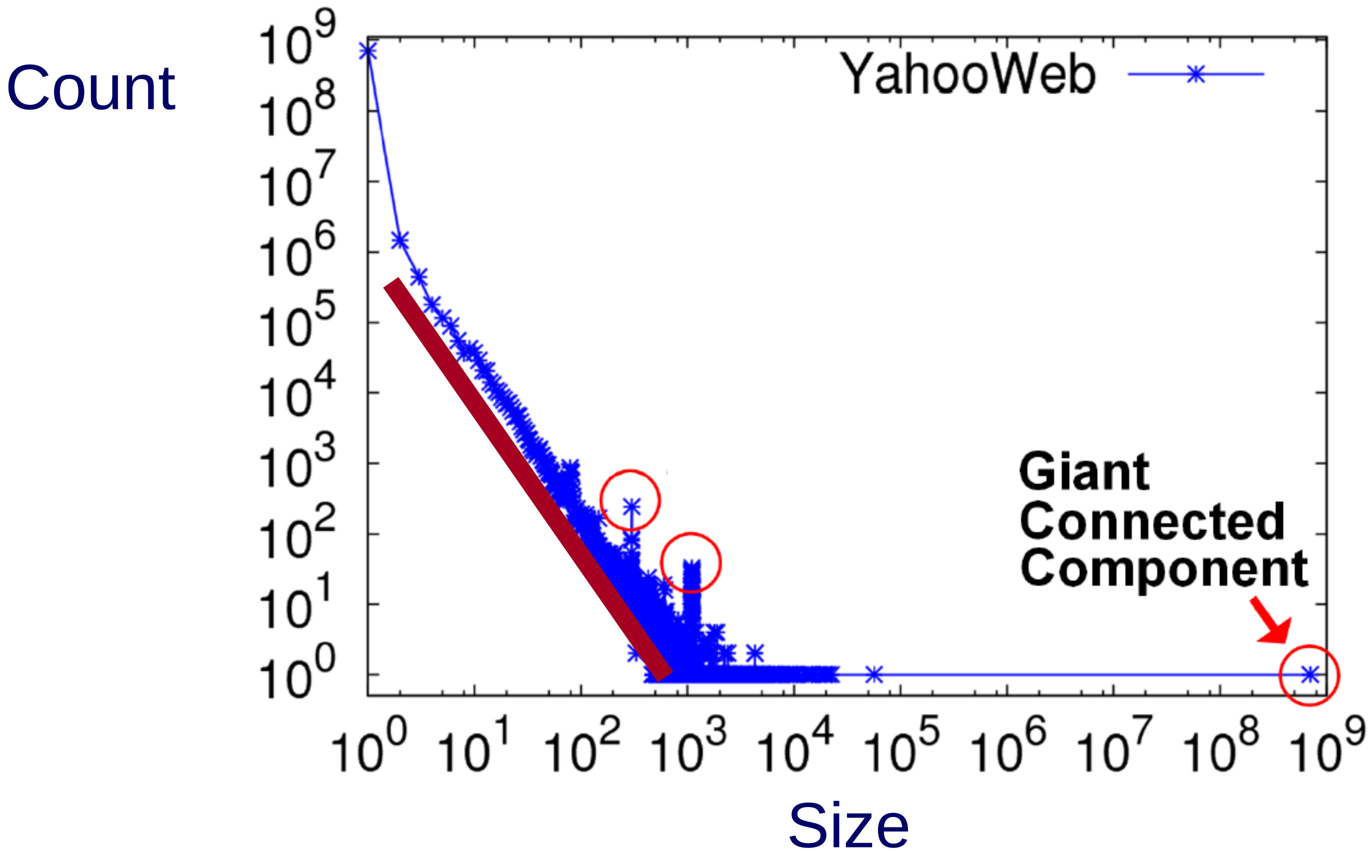
~0.7B singleton nodes



Size

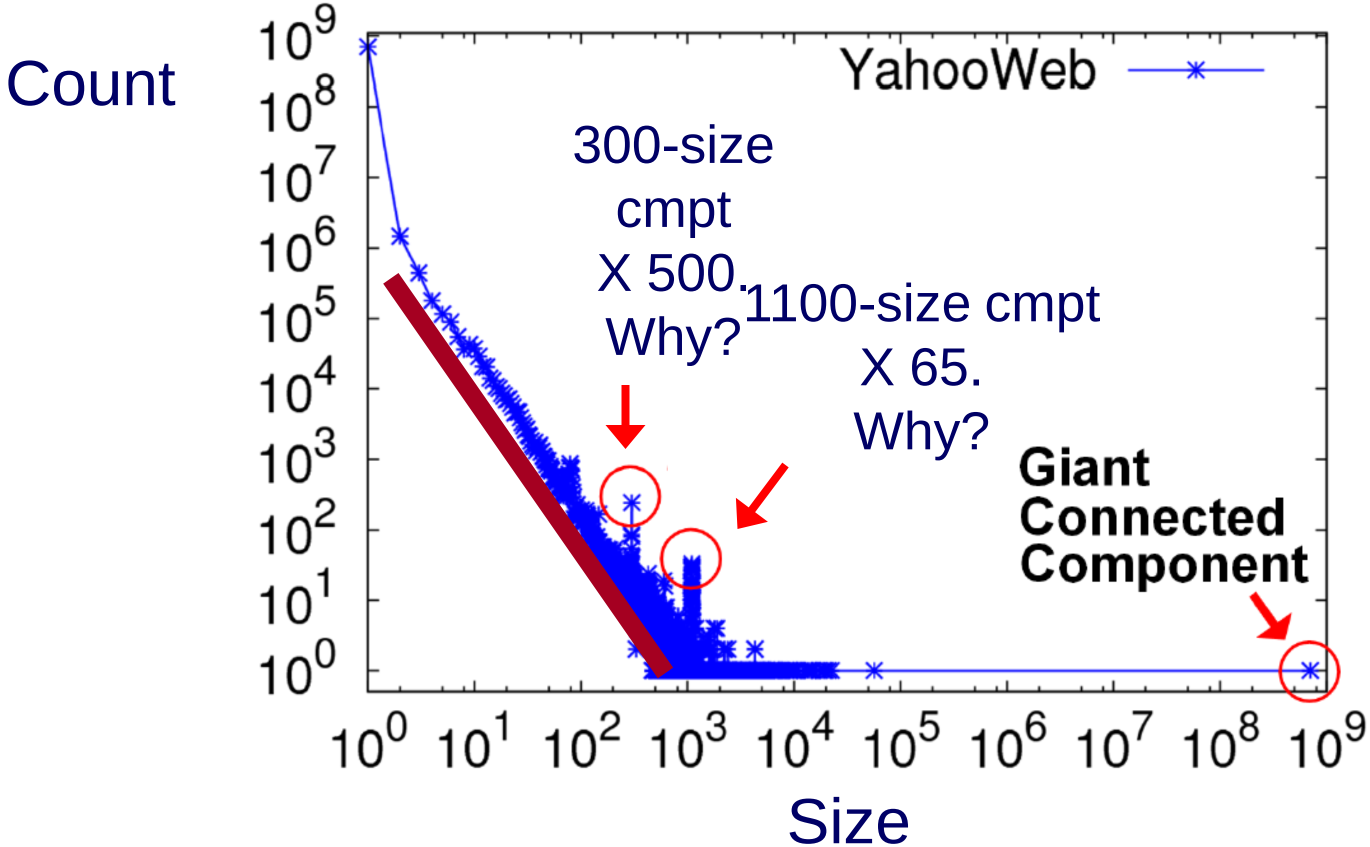
Example: GIM-V At Work

- Connected Components



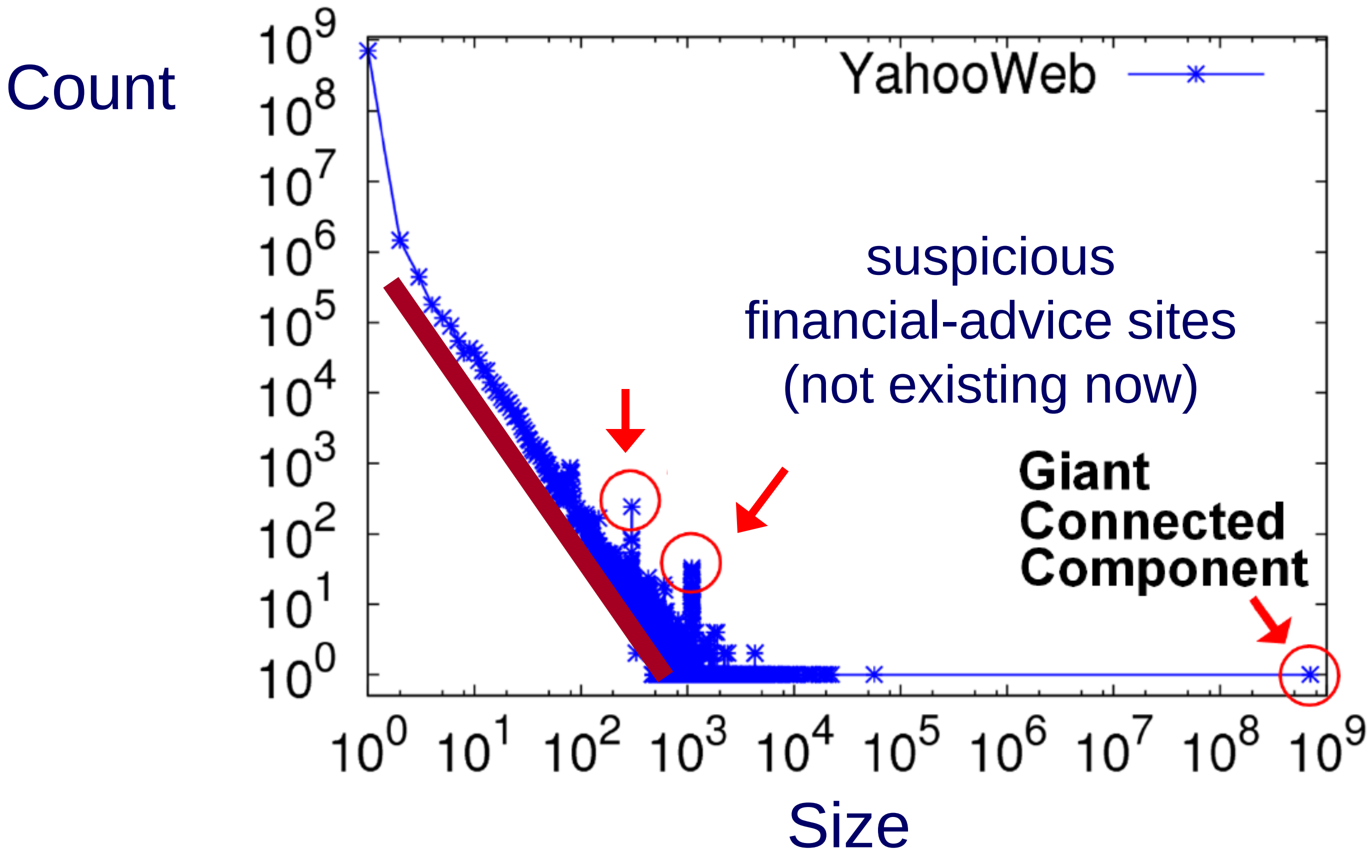
Example: GIM-V At Work

- Connected Components



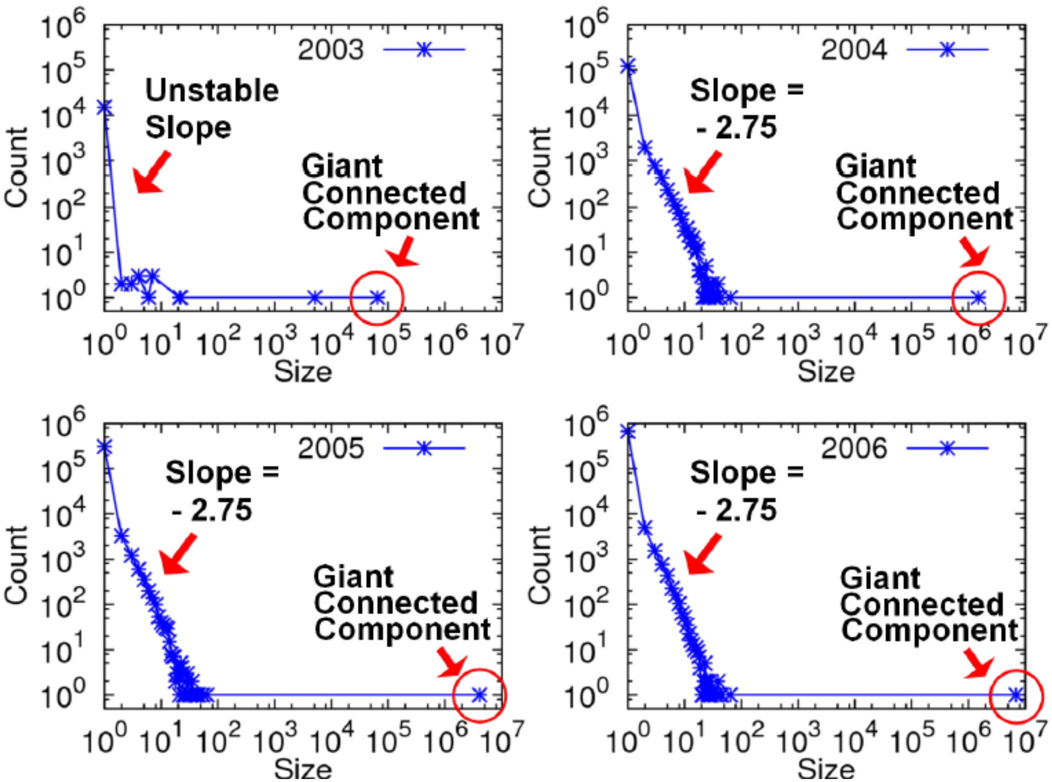
Example: GIM-V At Work

- Connected Components



GIM-V At Work

- Connected Components over Time
- **LinkedIn: 7.5M nodes and 58M edges**



Stable tail slope
after the gelling point

Outline

- Introduction – Motivation
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- Problem#3: Scalability
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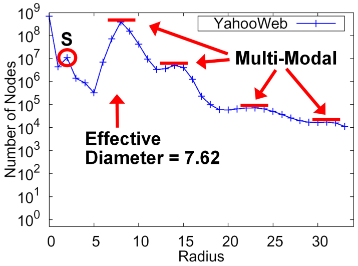
OVERALL CONCLUSIONS – low level:

- Several new **patterns** (fortification, triangle-laws, conn. components, etc)
- New **tools**:
  - CenterPiece Subgraphs, anomaly detection (OddBall)
- **Scalability**: PEGASUS / hadoop

OVERALL CONCLUSIONS – high level

- **Large** datasets reveal patterns/outliers that are invisible otherwise



- Terrific opportunities
  – Large datasets, easily(*) available PLUS
  – s/w and h/w developments

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