Graph-based Ontology Classification in OWL 2 QL

Domenico Lembo and Valerio Santarelli and Domenico Fabio Savo

Department of Computer, Control and Management Engineering Antonio Ruberti
Sapienza Università di Roma, Italia

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UNIVERSITÀ DI ROMA

10th Extended Semantic Web Conference (ESWC 2013)
Montpellier, France, May 2013

Ontology classification: the problem of computing all subsumption relationships inferred in an ontology between predicate names in the ontology signature, i.e., name concepts (classes), roles (object-properties), and attributes (data-properties).

Classification is a core service for ontology reasoning, and can be exploited for tasks such as:

- ontology navigation
- ontology visualization
- query answering
- explanation

Designing efficient methods for ontology classification is a challenging issue, since in general it is a costly operation.

Classification by ontology reasoners

Popular reasoners for OWL 2 ontologies, such as **FaCT++**, **Hermit**, **Pellet**, **Racer**, offer optimized classification services for expressive DLs, through algorithms based on model construction through tableau (or hyper-tableau).

Other reasoners such as **ELK**, **Snorocket**, and **JCel** are specifically tailored to intensional reasoning over logics of the $\mathcal{EL}$ family (the logical underpinning of OWL 2 EL), and show excellent performances of ontologies in these languages.

The **CB** reasoner is a *consequence-driven* reasoner for the Horn-$\mathcal{SHIQ}$ DL.

So far, no techniques specifically tailored for classification in OWL 2 QL.

The goal: efficient computation of classification in OWL 2 QL

We provide a new method for **ontology classification in OWL 2 QL.**

A simple idea

Encode the ontology TBox into a graph, and compute the transitive closure of the graph to obtain the ontology classification: take advantage of the analogy between simple inference rules in DLs and graph reachability.

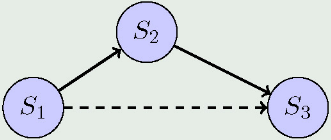
Example

TBox:

- $S_1 \sqsubseteq S_2$
- $S_2 \sqsubseteq S_3$

Inferred inclusion:

- $S_1 \sqsubseteq S_3$



How does graph-based classification work

Classification of an OWL 2 QL ontology:

- for an OWL 2 QL ontology, we show that it is possible to construct a graph whose transitive closure represents the major sub-task for classification of the ontology
- we show that the computed classification only misses "trivial" inclusion assertions inferred by unsatisfiable predicates in the ontology (predicates that always have an empty interpretation in every model of the ontology)
- we provide an algorithm that exploits the transitive closure of the graph, and, through the application of a set of rules, computes all unsatisfiable predicates, allowing to obtain the complete classification of the ontology

Outline

1. Introduction to OWL 2 QL
2. Computation of graph-based ontology classification in OWL 2 QL
3. Implementation and evaluation of the graph-based ontology classification algorithm
4. Conclusions and future works

OWL 2 QL is the "data oriented" profile of OWL 2.

Expressions in OWL 2 QL

$$B \longrightarrow A \mid \exists Q \qquad\qquad Q \longrightarrow P \mid P^-$$
$$C \longrightarrow B \mid \neg B \mid \exists Q.A \qquad R \longrightarrow Q \mid \neg Q$$

Assertions in OWL 2 QL

$B \sqsubseteq C$ *(concept inclusion)*
$Q \sqsubseteq R$ *(role inclusion)*

We call *positive inclusions* axioms of the form $B_1 \sqsubseteq B_2$, $B_1 \sqsubseteq \exists Q.A$, and $Q_1 \sqsubseteq Q_2$, and *negative inclusions* axioms of the form $B_1 \sqsubseteq \neg B_2$, and $Q_1 \sqsubseteq \neg Q_2$.

Theorem

Let $\mathcal{T}$ be an OWL 2 QL TBox containing only positive inclusions, and let S_1 and S_2 be two atomic concepts or two atomic roles. $S_1 \sqsubseteq S_2$ is entailed by $\mathcal{T}$ if and only if at least one of the following conditions holds:

1. a set $\mathcal{P}$ of positive inclusions exists in $\mathcal{T}$, such that $\mathcal{P} \models S_1 \sqsubseteq S_2$;
2. $\mathcal{T} \models S_1 \sqsubseteq \neg S_1$.

It follows that $\mathcal{T}$-classification $\equiv \{\Phi_{\mathcal{T}} \cup \Omega_{\mathcal{T}}\}$, where:

- $\Phi_{\mathcal{T}}$ contains only positive inclusions for which statement 1 holds
- $\Omega_{\mathcal{T}}$ contains only positive inclusions for which statement 2 holds

1. Encode positive inclusions in $\mathcal{T}$ into a digraph $\mathcal{G}_{\mathcal{T}}$: each node in $\mathcal{G}_{\mathcal{T}}$ represents a concept or role, and each arc a positive inclusion.

Definition

Let $\mathcal{T}$ be an OWL 2 QL TBox over a signature Σ_P. We call the digraph representation of $\mathcal{T}$ the digraph $\mathcal{G}_{\mathcal{T}} = (\mathcal{N}, \mathcal{E})$ built as follows:

1. for each atomic concept A in Σ_P, $\mathcal{N}$ contains the node A;
2. for each atomic role P in Σ_P, $\mathcal{N}$ contains the nodes P, P^-, $\exists P$, $\exists P^-$;
3. for each concept inclusion $B_1 \sqsubseteq B_2 \in \mathcal{T}$, $\mathcal{E}$ contains the arc (B_1, B_2);
4. for each role inclusion $Q_1 \sqsubseteq Q_2 \in \mathcal{T}$, $\mathcal{E}$ contains the arcs (Q_1, Q_2), (Q_1^-, Q_2^-), $(\exists Q_1, \exists Q_2)$, and $(\exists Q_1^-, \exists Q_2^-)$;
5. for each concept inclusion $B_1 \sqsubseteq \exists Q.A \in \mathcal{T}$, $\mathcal{N}$ contains the node $\exists Q.A$, and $\mathcal{E}$ contains the arcs $(B_1, \exists Q.A)$ and $(\exists Q.A, \exists Q)$;

2. Compute the transitive closure of $\mathcal{G}_{\mathcal{T}}$: $\mathcal{G}^* = (\mathcal{N}, \mathcal{E}^*)$

We denote with $\alpha(\mathcal{E}^*)$ the set of arcs $(S_1, S_2) \in \mathcal{E}^*$ such that both terms S_1 and S_2 denote in $\mathcal{T}$ either two atomic concepts or two atomic roles.

Theorem

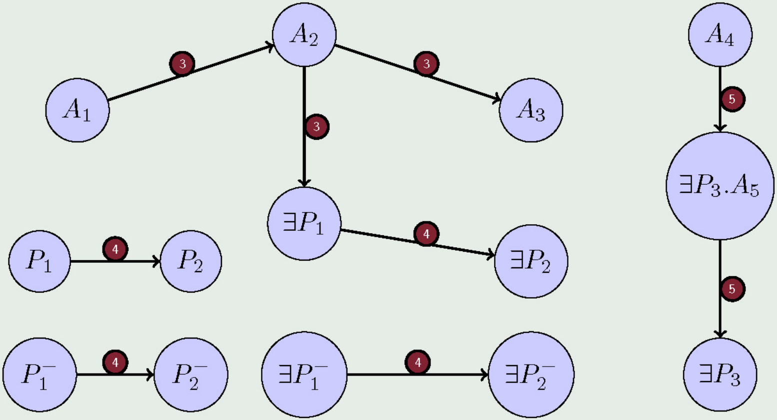
Let $\mathcal{T}$ be an OWL 2 QL TBox and let $\mathcal{G}_{\mathcal{T}} = (\mathcal{N}, \mathcal{E})$ be its digraph representation. Let S_1 and S_2 be two atomic concepts or two atomic roles. An inclusion assertion $S_1 \sqsubseteq S_2$ belongs to $\Phi_{\mathcal{T}}$ if and only if there exists in $\alpha(\mathcal{E}^*)$ an arc (S_1, S_2).

As a consequence of the above theorem, we define algorithm ComputeΦ, that takes as input an OWL 2 QL TBox $\mathcal{T}$, builds $\mathcal{G}_{\mathcal{T}}$, computes $\mathcal{G}^*$, and returns the set $\Phi_{\mathcal{T}}$.

Computation of $\Phi_{\mathcal{T}}$: Example

Example

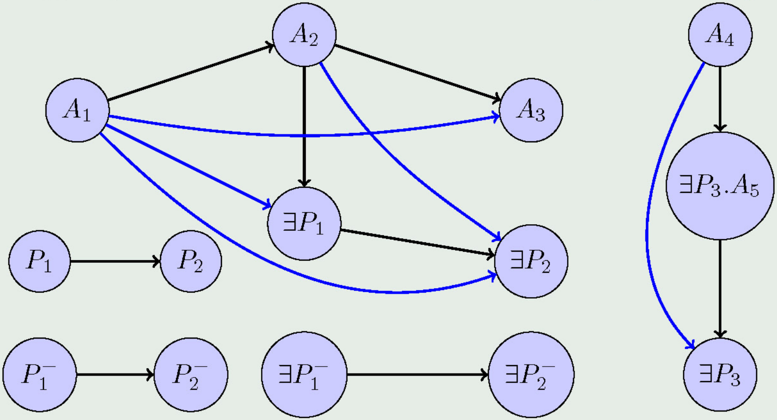
TBox: $A_1 \sqsubseteq A_2$ $\quad A_2 \sqsubseteq A_3$ $\quad A_2 \sqsubseteq \exists P_1$ $\quad A_4 \sqsubseteq \exists P_3.A_5$ *(concept inclusions)*
$P_1 \sqsubseteq P_2$ *(role inclusion)*



Example

TBox: $A_1 \sqsubseteq A_2$ $A_2 \sqsubseteq A_3$ $A_2 \sqsubseteq \exists P_1$ $A_4 \sqsubseteq \exists P_3.A_5$ *(concept inclusions)*

$P_1 \sqsubseteq P_2$ *(role inclusion)*



Computation of $\Omega_{\mathcal{T}}$: algorithm computeUnsat

Algorithm: computeUnsat
Input: an OWL 2 QL TBox $\mathcal{T}$
Output: a set of concept and role expressions

```
Emp ← ∅;
foreach negative inclusion S1 ⊑ ¬S2 ∈ T do
    Emp ← Emp ∪ {predecessors(S1, G*_T) ∩ predecessors(S2, G*_T)}  /* step 1 */
    foreach n1 ∈ predecessors(S1, G*_T) do                          /* step 2 */
        foreach n2 ∈ predecessors(S2, G*_T) do
            if (n1 = ∃Q⁻ and n2 = A) or (n2 = ∃Q⁻ and n1 = A)
            then Emp ← Emp ∪ {∃Q.A};
Emp′ ← ∅;
while Emp ≠ Emp′ do
    Emp′ ← Emp;
    foreach S ∈ Emp′ do
        foreach n ∈ predecessors(S, G*_T) do
            Emp ← Emp ∪ {n};                                          /* step 3 */
            if n = P or n = P⁻ or n = ∃P or n = ∃P⁻                  /* step 4 */
            then Emp ← Emp ∪ {P, P⁻, ∃P, ∃P⁻};
            if there exists B ⊑ ∃Q.n ∈ T
            then Emp ← Emp ∪ {∃Q.n};
return Emp.
```

- The set **predecessors**$(n, \mathcal{G}^*)$ contains n and all n' s.t. $\mathcal{G}^*$ contains (n', n).

Computation of $\Omega_{\mathcal{T}}$: algorithm computeUnsat

For each $S_1 \sqsubseteq \neg S_2$, computes **predecessors**$(S_1, \mathcal{G}^*_{\mathcal{T}})$ and **predecessors**$(S_2, \mathcal{G}^*_{\mathcal{T}})$:

(Step 1) all predicates whose corresponding nodes occur in both **predecessors**$(S_1, \mathcal{G}^*_{\mathcal{T}})$ and **predecessors**$(S_2, \mathcal{G}^*_{\mathcal{T}})$ are unsatisfiable;

(Step 2) all qualified existential roles $\exists Q.A$ whose node $\exists Q^-$ occurs in **predecessors**$(S_1, \mathcal{G}^*_{\mathcal{T}})$ (resp. **predecessors**$(S_2, \mathcal{G}^*_{\mathcal{T}})$**)** and node A in **predecessors**$(S_2, \mathcal{G}^*_{\mathcal{T}})$ (resp. **predecessors**$(S_1, \mathcal{G}^*_{\mathcal{T}})$) are unsatisfiable.

Further unsatisfiable predicates are identified through a cycle, in which:

(Step 3) if $S \in$ Emp, then all expressions corresponding to the nodes in **predecessors**$(S, \mathcal{G}^*_{\mathcal{T}})$ are in Emp;

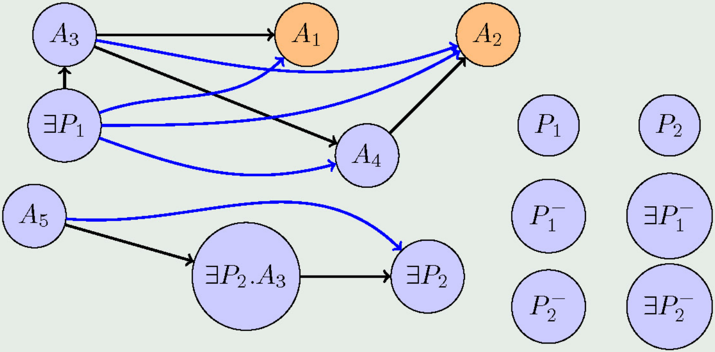
(Step 4)

1. if at least one of the expressions $P, P^-, \exists P, \exists P^-$ is in Emp, then all four expressions are in Emp;
2. for each expression $\exists Q.A$ in $\mathcal{N}$, if $A \in$ Emp, then $\exists Q.A \in$ Emp.

Computation of $\Omega_{\mathcal{T}}$: Example

Example

TBox: $A_3 \sqsubseteq A_4$ $\quad A_4 \sqsubseteq A_2$ $\quad A_3 \sqsubseteq A_1$ $\quad \exists P_1 \sqsubseteq A_3$ $\quad A_5 \sqsubseteq \exists P_2.A_3$ $\quad A_1 \sqsubseteq \neg A_2$

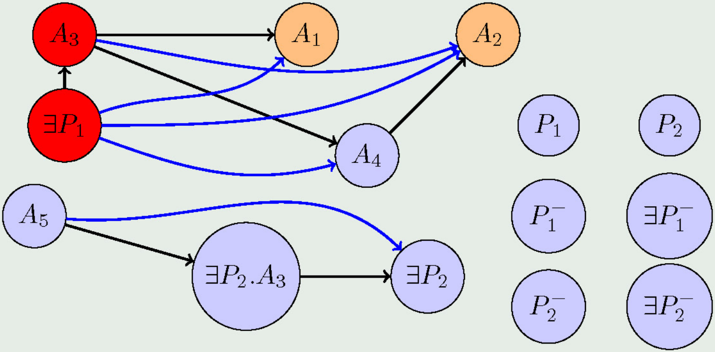


predecessors$(A_1, \mathcal{G}^*_{\mathcal{T}}) = \{A_1, A_3, \exists P_1\}$
predecessors$(A_2, \mathcal{G}^*_{\mathcal{T}}) = \{A_2, A_4, A_3, \exists P_1\}$

Computation of $\Omega_{\mathcal{T}}$: Example

Example

TBox: $A_3 \sqsubseteq A_4$ $A_4 \sqsubseteq A_2$ $A_3 \sqsubseteq A_1$ $\exists P_1 \sqsubseteq A_3$ $A_5 \sqsubseteq \exists P_2.A_3$ $A_1 \sqsubseteq \neg A_2$

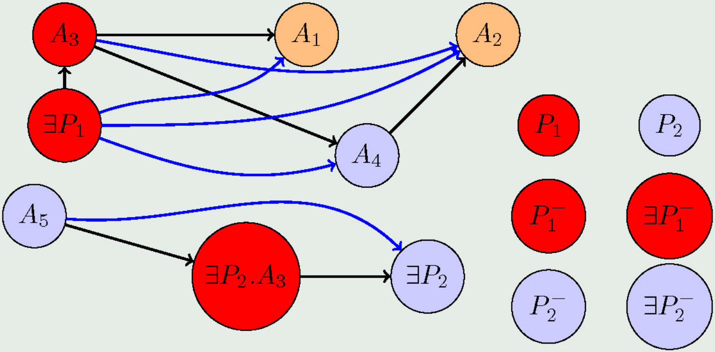


Emp $= \{A_3, \exists P_1\}$

Computation of $\Omega_{\mathcal{T}}$: Example

Example

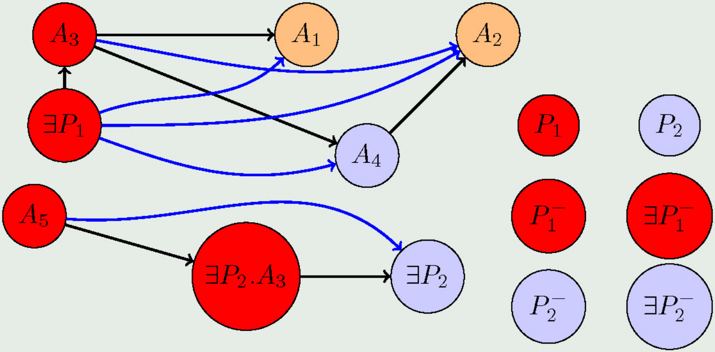
TBox: $A_3 \sqsubseteq A_4$ $A_4 \sqsubseteq A_2$ $A_3 \sqsubseteq A_1$ $\exists P_1 \sqsubseteq A_3$ $A_5 \sqsubseteq \exists P_2.A_3$ $A_1 \sqsubseteq \neg A_2$



Emp $= \{A_3, \exists P_1, P_1, P_1^-, \exists P_1^-, \exists P_2.A_3\}$

Example

TBox: $A_3 \sqsubseteq A_4$ $A_4 \sqsubseteq A_2$ $A_3 \sqsubseteq A_1$ $\exists P_1 \sqsubseteq A_3$ $A_5 \sqsubseteq \exists P_2.A_3$ $A_1 \sqsubseteq \neg A_2$



Emp $= \{A_3, \exists P_1, P_1, P_1^-, \exists P_1^-, \exists P_2.A_3, A_5\}$

Computation of $\Omega_{\mathcal{T}}$

The following theorem shows that algorithm computeUnsat can be used for computing the set containing all the unsatisfiable concepts and roles in $\mathcal{T}$.

Theorem

Let $\mathcal{T}$ be an OWL 2 QL TBox and let S be either an atomic concept or an atomic role in Σ_P. $\mathcal{T} \models S \sqsubseteq \neg S$ if and only if $S \in$ computeUnsat$(\mathcal{T})$.

The following theorem states that the graph-based technique is sound and complete with respect to the problem of classifying an OWL 2 QL TBox.

Theorem

Let $\mathcal{T}$ be an OWL 2 QL TBox and let S_1 and S_2 be either two atomic concepts or two atomic roles. $\mathcal{T} \models S_1 \sqsubseteq S_2$ if and only if $S_1 \sqsubseteq S_2 \in \mathsf{Compute}\Phi(\mathcal{T}) \cup \mathsf{Compute}\Omega(\mathcal{T})$.

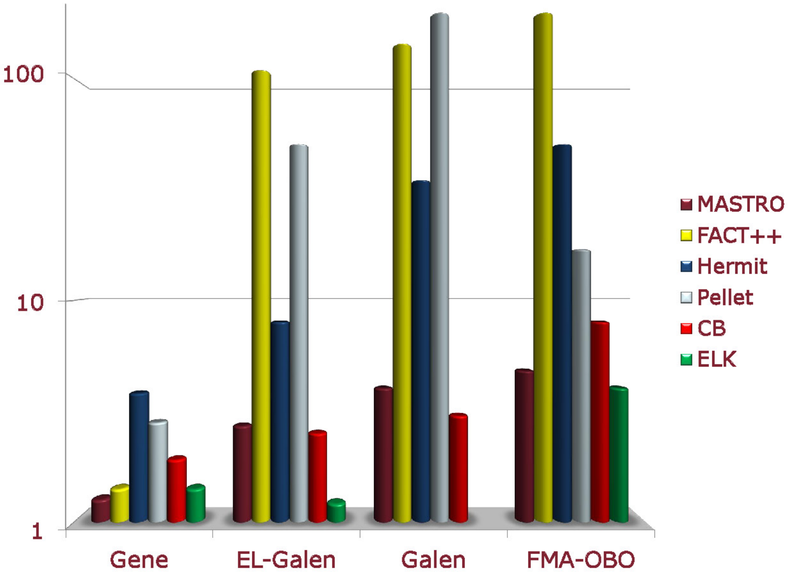
By exploiting these theoretical results, we have developed a Java-based OWL 2 QL classification module for the MASTRO reasoner for Ontology-Based Data Access (OBDA). In this implementation, the transitive closure of the digraph $\mathcal{G}_\mathcal{T}$ is based on a breadth first search through $\mathcal{G}_\mathcal{T}$.

We have performed comparative experiments on a suite of 20 ontologies, testing MASTRO against several popular ontology reasoners:

- the **FaCT++**, **Hermit**, **Pellet** OWL 2 reasoners
- the **CB** Horn-$\mathcal{SHIQ}$ reasoner
- the **ELK** OWL 2 EL reasoner

Each benchmark ontology was preprocessed through an approximation procedure prior to classification in order to fit OWL 2 QL expressivity.

Classification test results (seconds)



Conclusions and future work

We have presented a technique for efficiently computing classification of OWL 2 QL ontologies, based on the idea of encoding the ontology TBox into a directed graph, and reducing core reasoning to computation of the transitive closure of the graph.

Even though the current implementation relies on a naive algorithm for computation of transitive closure, test results on benchmark ontologies offer promising results.

Future Work:

- development of more efficient technique for transitive closure
- optimization of procedure for identification of unsatisfiable predicates
- extension of technique to computation of all inclusions inferred by the TBox
- extention of graph-based classification to more expressive languages

Thank you!

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