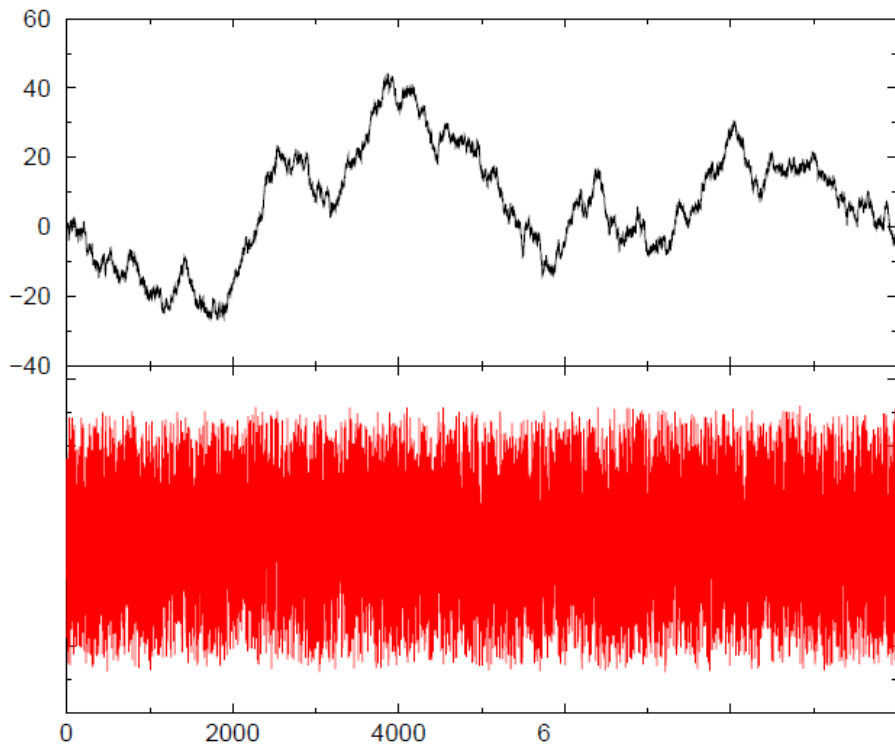


# Old and New, fun to know, « Stylized facts » of financial markets

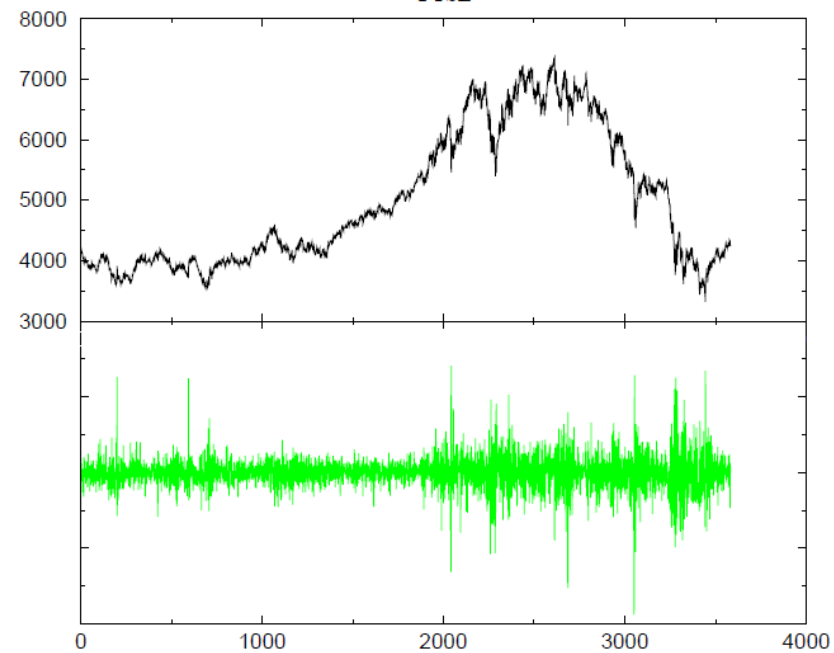
Thanks to:

R. Allez, J.L. Audibert, Ch. Biely, P. Blanc, R.  
Chicheportiche, N. Cosson, L. de Leo, Z. Eisler, S.  
Hardiman, A. Joulin, M. Potters, P.A. Reigner, V.  
Vargas, S. Vazquez

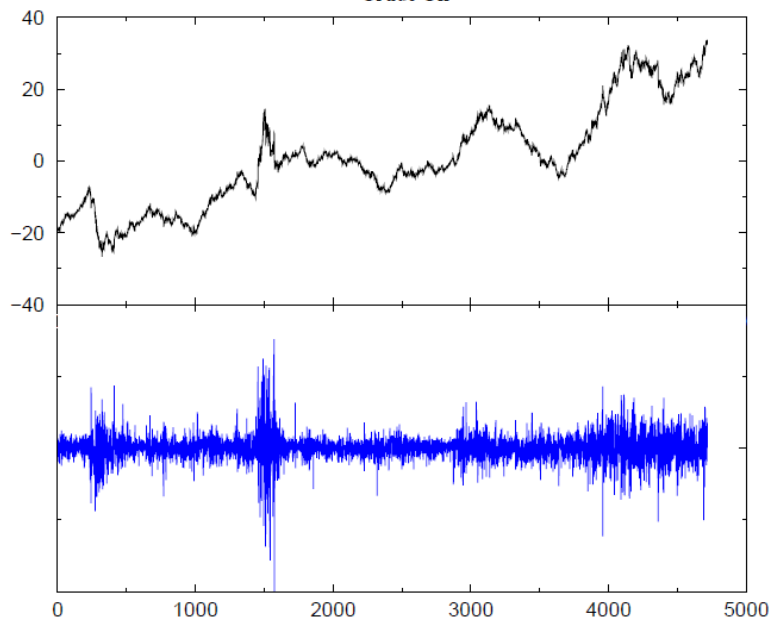
**Bachelier-Einstein Random Walk**



**FTSE**

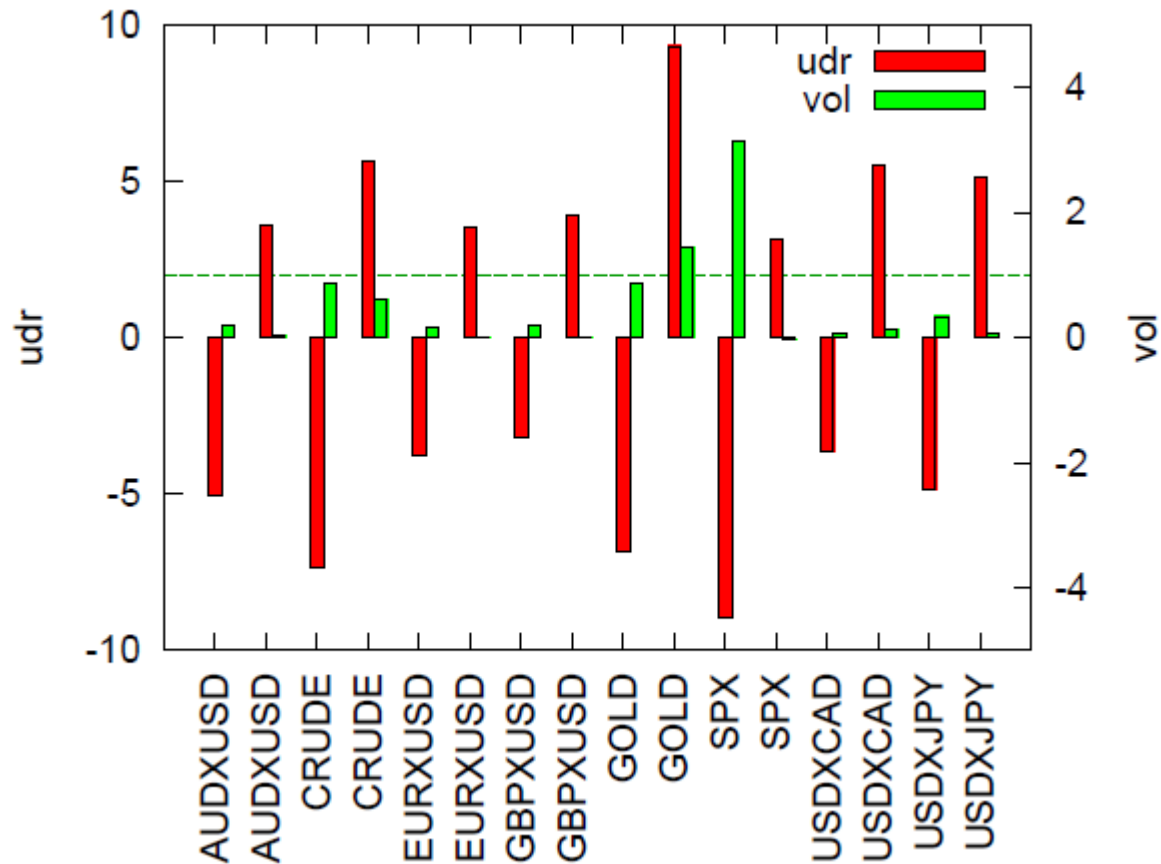


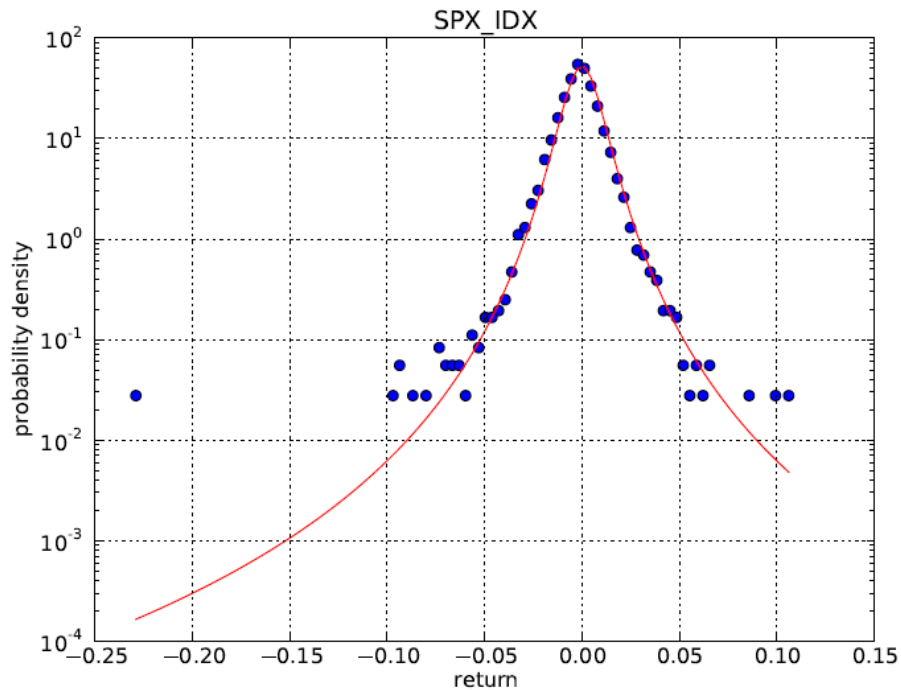
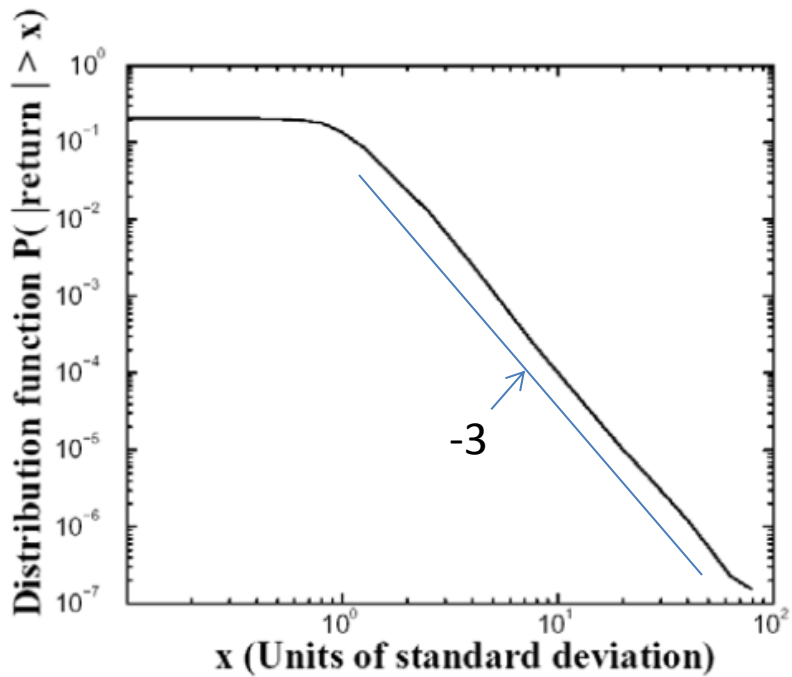
**Crude Oil**



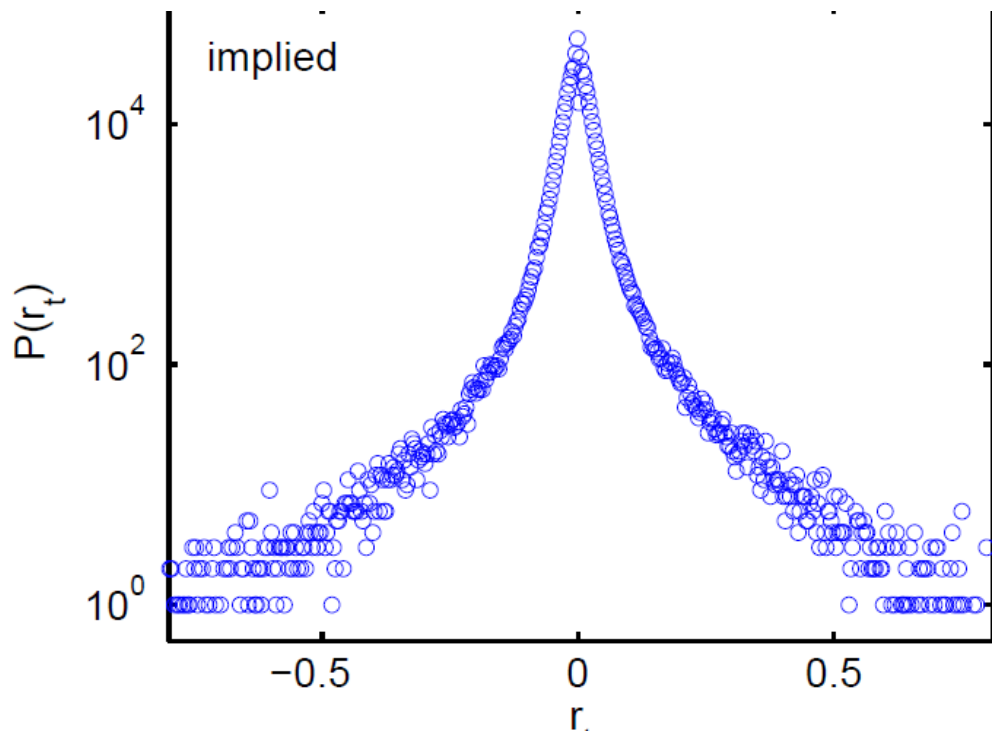
# Largest crashes

Product	Date	$\epsilon$	$\bar{\epsilon}$	Background
Gold	4 Feb. 2000	9.31	1.43	Speculative short squeeze
SPX	19 Oct. 1987	-8.93	3.13	Black Monday
Crude	24 Sep. 2001	-7.35	0.87	OPEC: lack of demand
Crude	17 Jan 1991	-7.00	0.82	US attacks Iraq, trading halt
Gold	28 Sep. 1999	6.93	0.81	Speculative short squeeze

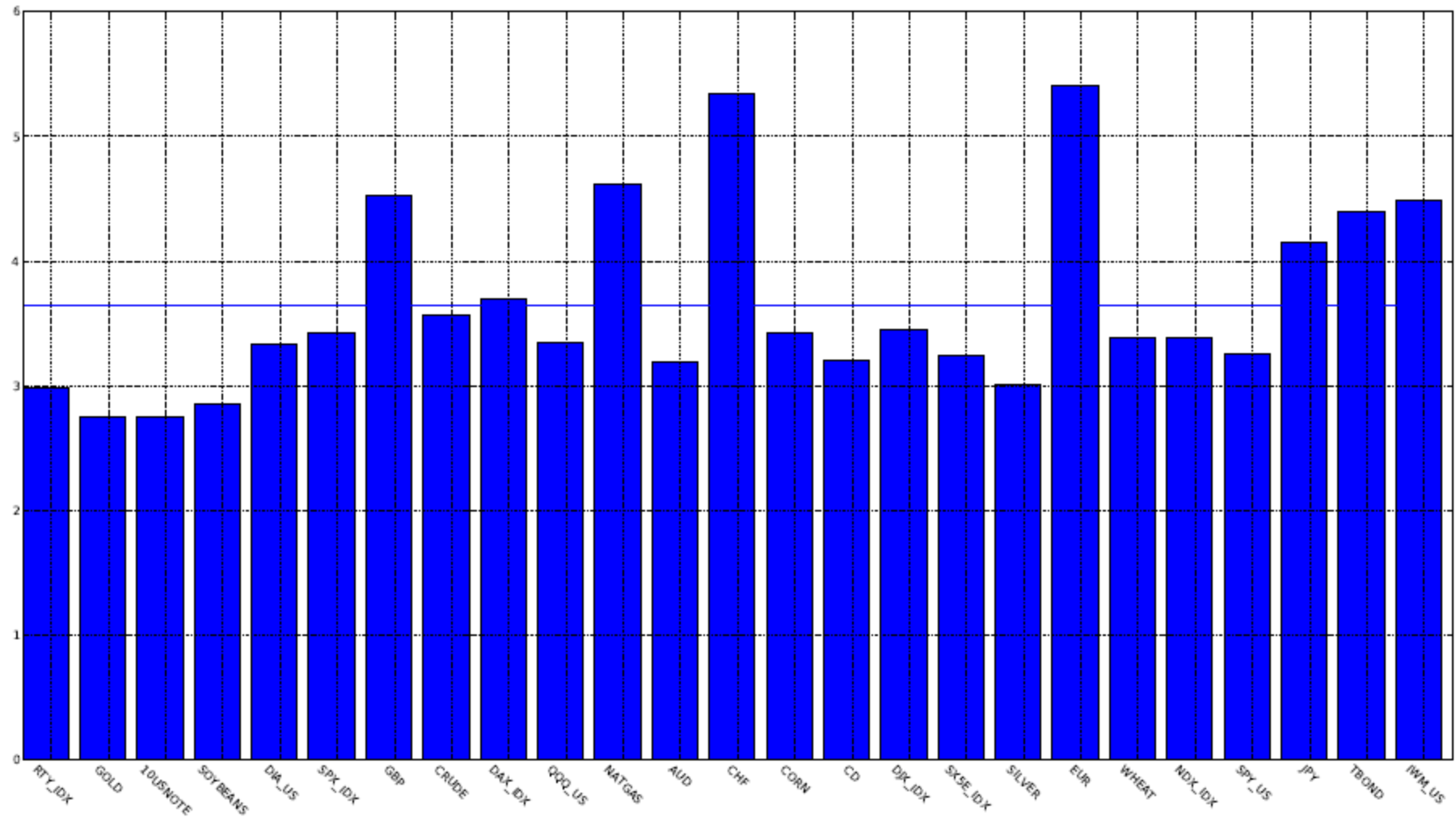




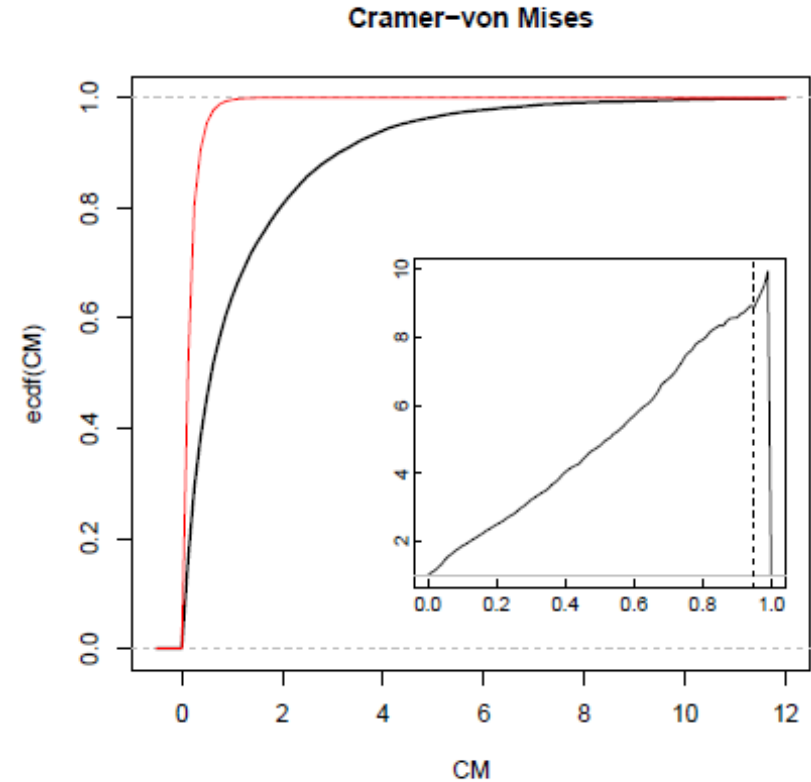
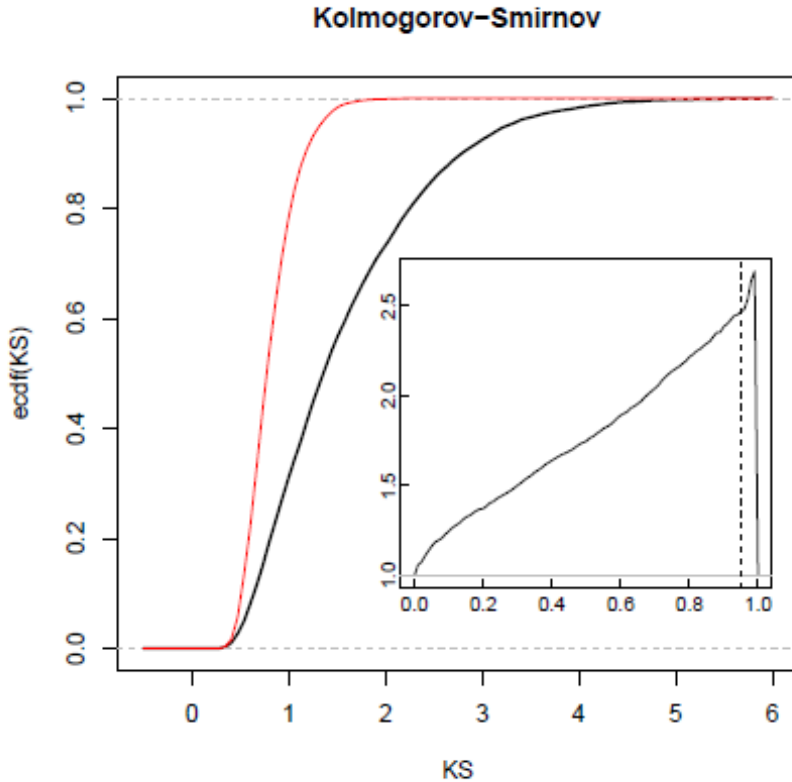
The « inverse cubic law »



# A universal tail exponent ?

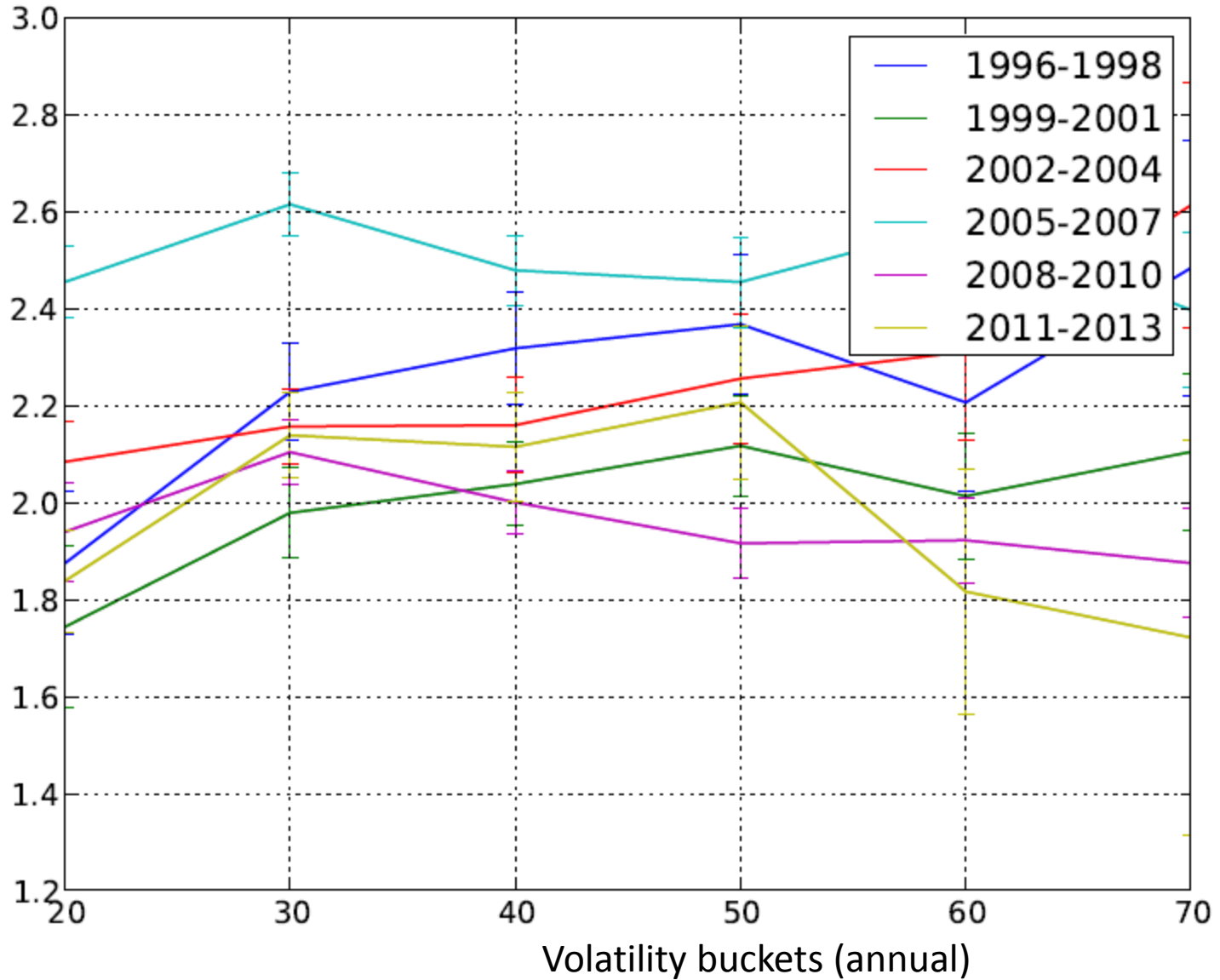


# KS: Beware of dependencies ! (Properties of the Brownian Bridge)

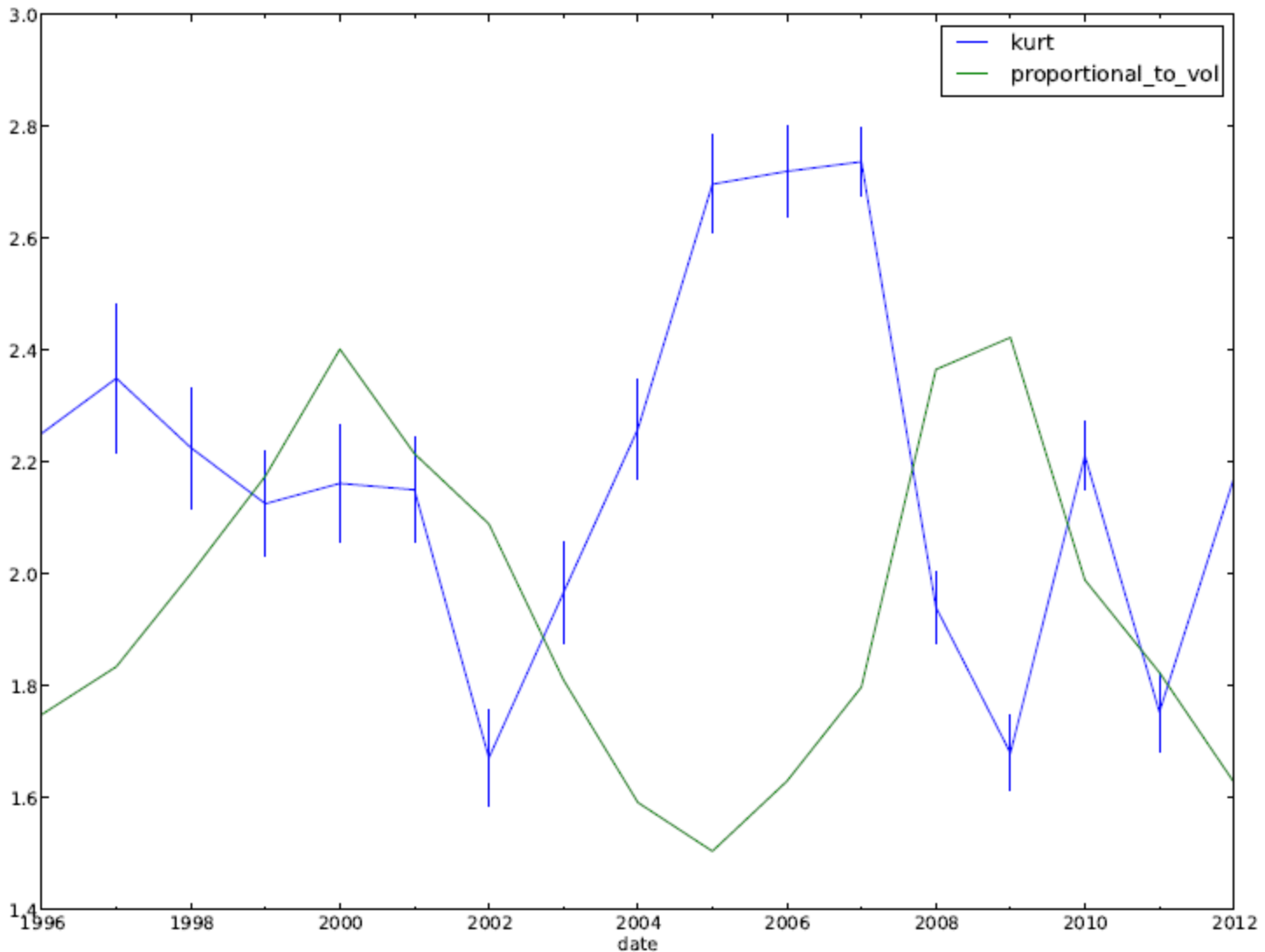


Other extensions: KS with focus on the tails,  
KS for 2 dimensional copulas

# A universal « kurtosis » ?



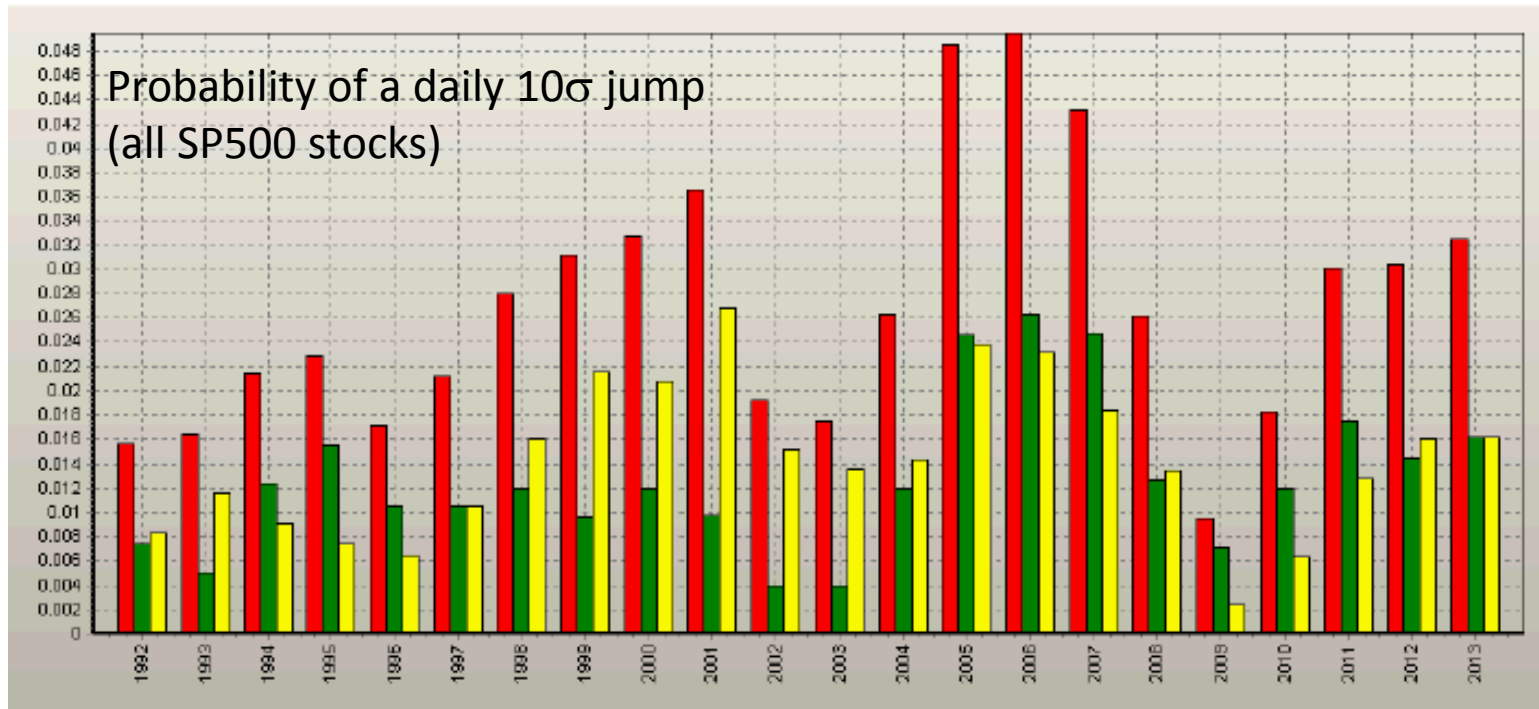
$$\kappa_\alpha(k) = 24 \left( 1 - \sqrt{\frac{\pi}{2}} \frac{\langle |\eta_\alpha(k; t) - \mu_\alpha(k)| \rangle}{\sigma_\alpha(k)} \right)$$



Single factor model:  $X_i = \beta_i I + \varepsilon_i$   
 $\kappa(X)$  increases when  $\rho$  decreases!

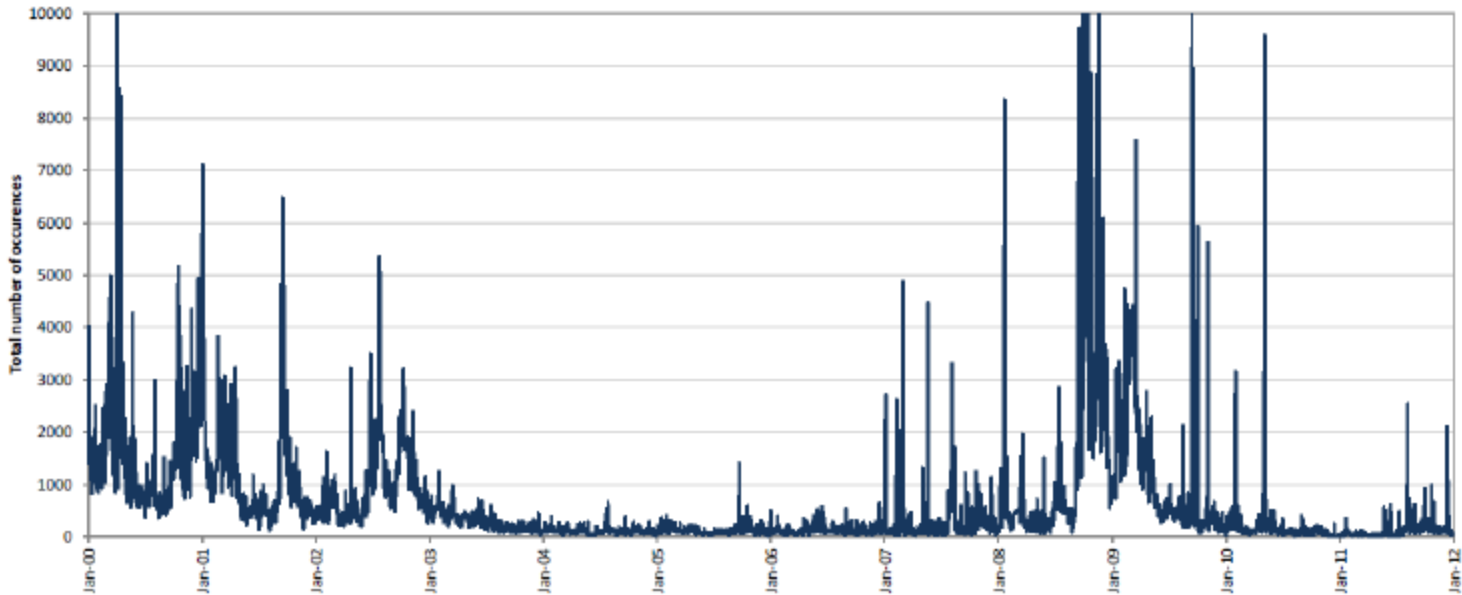


# Jumps: not a new phenomenon !!



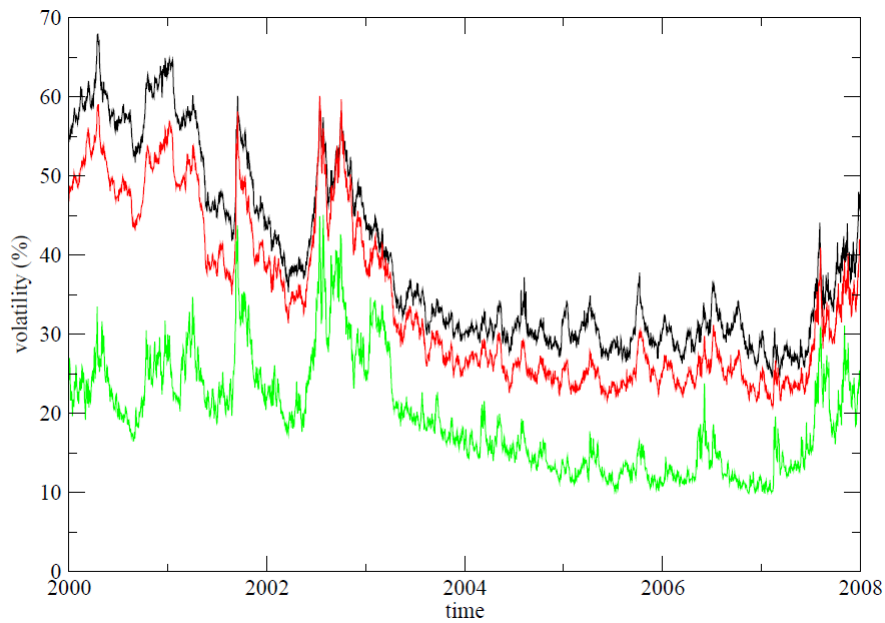
Note: these numbers are  $\sim 8$  times smaller than for  $5\sigma$  jumps  
(the inverse cubic law)

# Jumps: not a new phenomenon !!



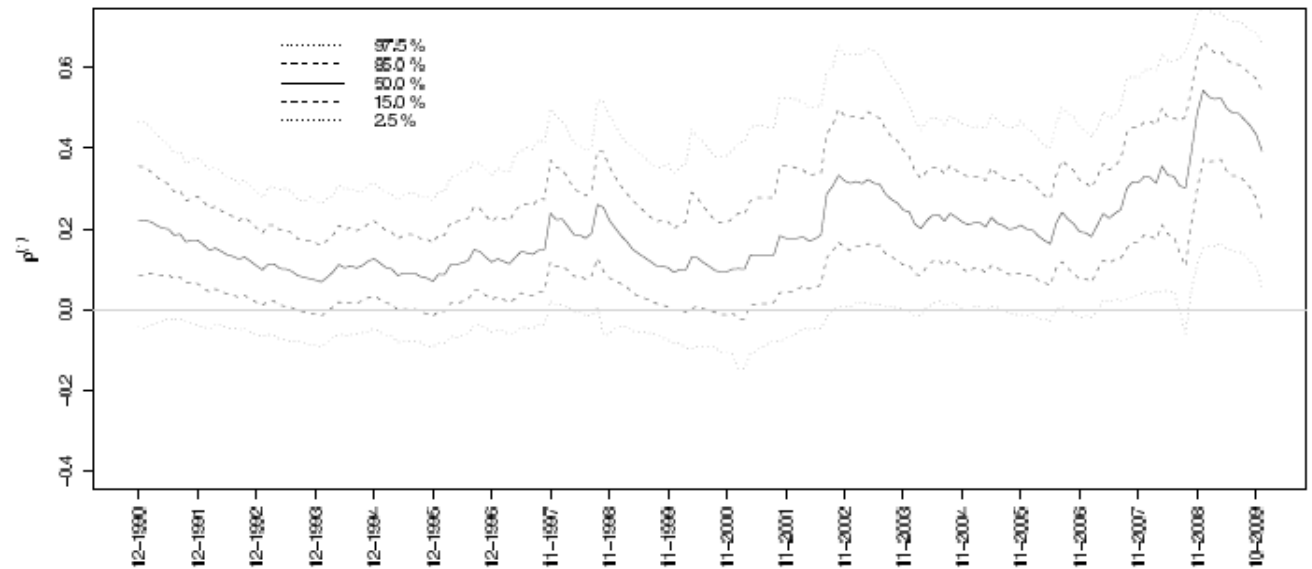
Weekly number of 1% gaps over 1 min for S&P stocks  
(from Credit Suisse)

**Note:** inverse cubic law implies that  
exceedance over a fixed threshold should scale as  $\sigma^3$



The « index » of the vol is not the vol of the index ! (Correlations)

$$\sigma_I^2 \approx \bar{\rho} \overline{\sigma_\alpha^2}$$



# Full multivariate distributions: Copulas

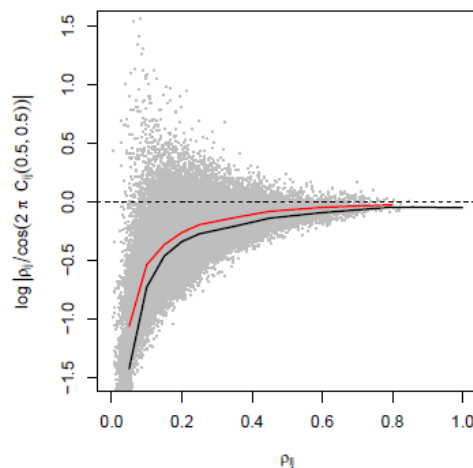
$$C(u_i, u_j) = \mathbb{P} [\mathcal{P}_{<,i}(X_i) \leq u_i \text{ and } \mathcal{P}_{<,j}(X_j) \leq u_j]$$

$$\frac{1 - 2p + C(p, p)}{1 - p} = \tau_{UU}(p)$$

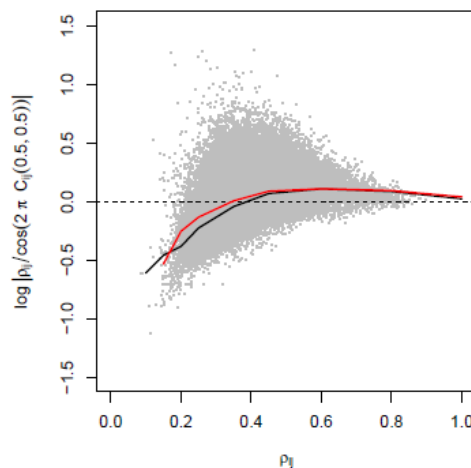
Elliptical multivariate distributions: a **unique volatility** factor (e.g. Student) :  $X_i = \mu_i + \sigma \cdot \epsilon_i$

Copulas' medial point:  $C_G(\frac{1}{2}, \frac{1}{2}) = \frac{1}{4} + \frac{1}{2\pi} \arcsin \rho$

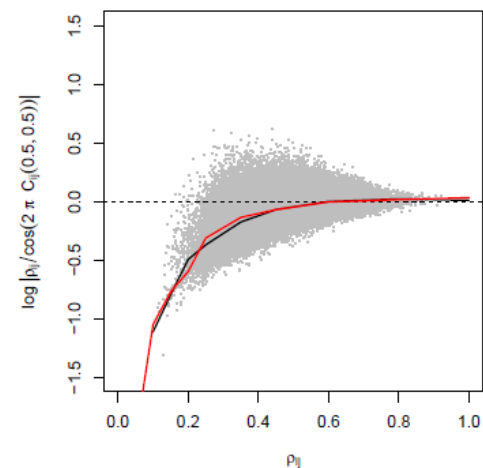
$$P(u = \sigma^2) \propto e^{-\frac{1}{u}} / u^{1+\nu/2}$$



(a) 2000–2004



(b) 2005–2009

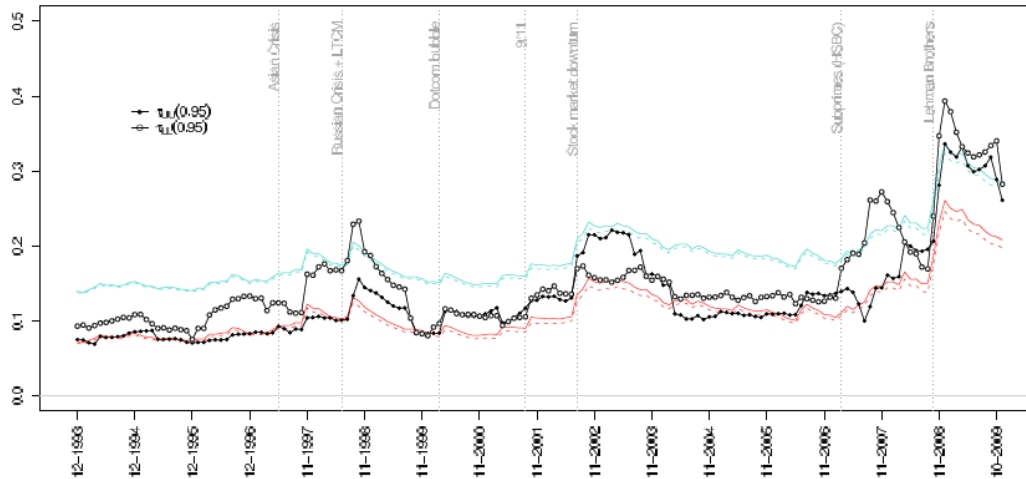


(c) 2009–2012

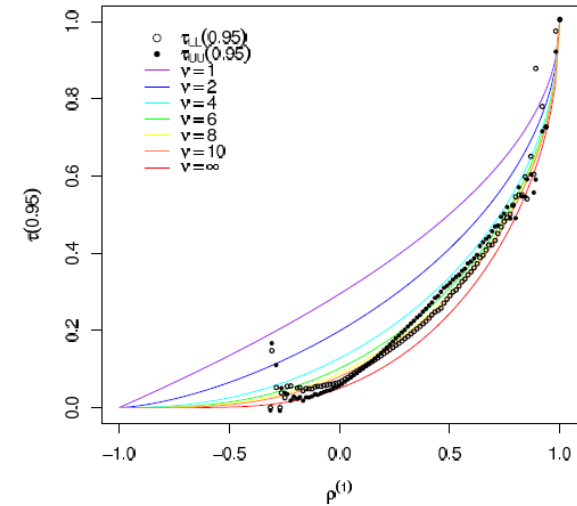
See R. Chicheportiche, JPB for a very recent, multifactor vol model: arXiv 1309:3102

# Tail Correlations

$$\tau_{UU}(p) = \mathbb{P} [X_i > \mathcal{P}_{<,i}^{-1}(p) \mid X_j > \mathcal{P}_{<,j}^{-1}(p)]$$



Student-t family



## « Contagion »

▷ During a market slide, the number  $S$  of stocks that undergo a “large” move is power-law distributed (“avalanche”)

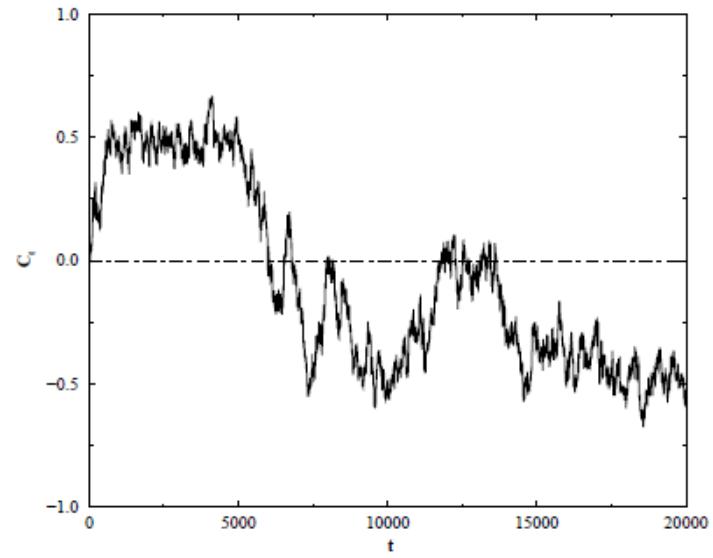
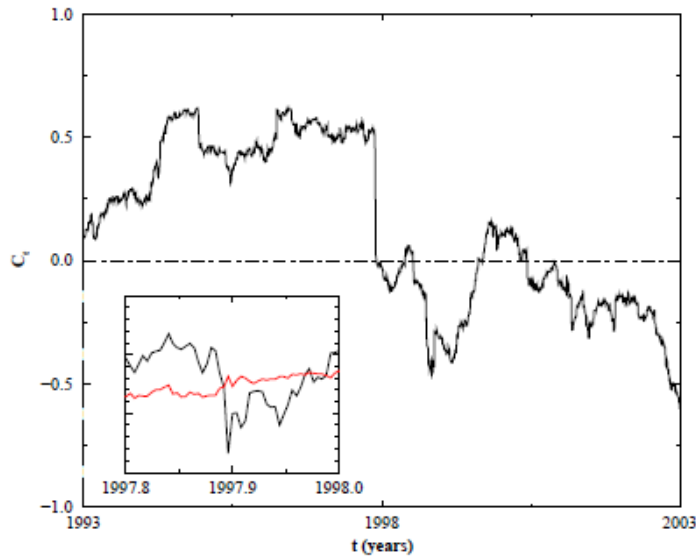
$$P(S) \sim S^{-1-\nu}, \quad \nu \approx 1.25 \rightarrow 1.5$$

(Joulin et al.; Medo et al.)

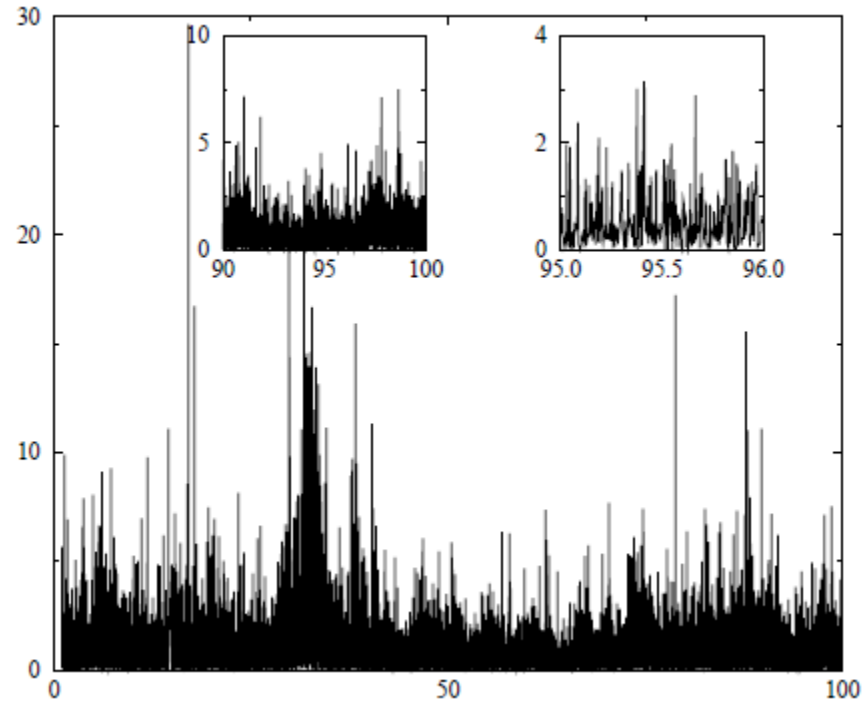
+ Lillo et al. (Recent work)

# An example of a sudden « reversal of the magnetic pole »

## The bond/index correlation

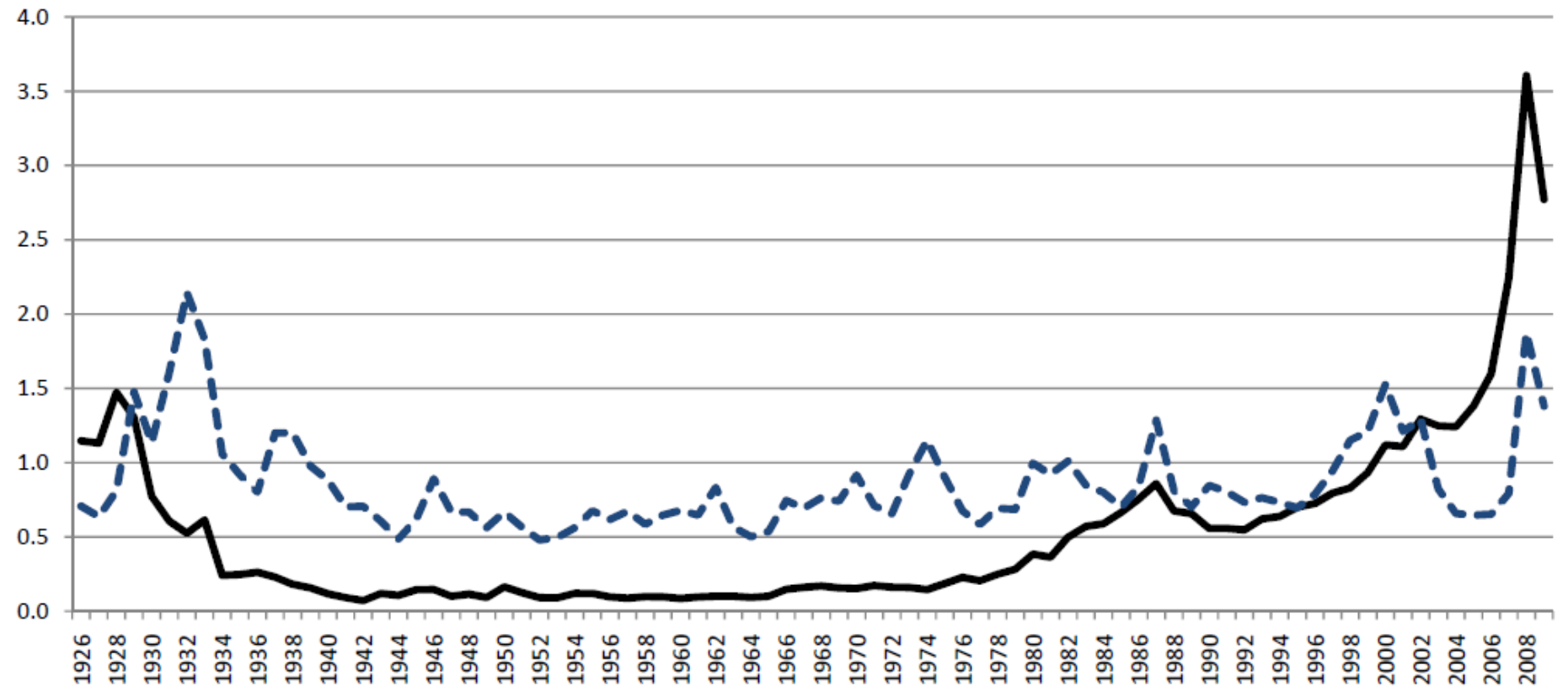


# Volatility: an intermittent, multiscale phenomenon, with long-memory



Dow Jones Index

# Market Volatility: A long term view

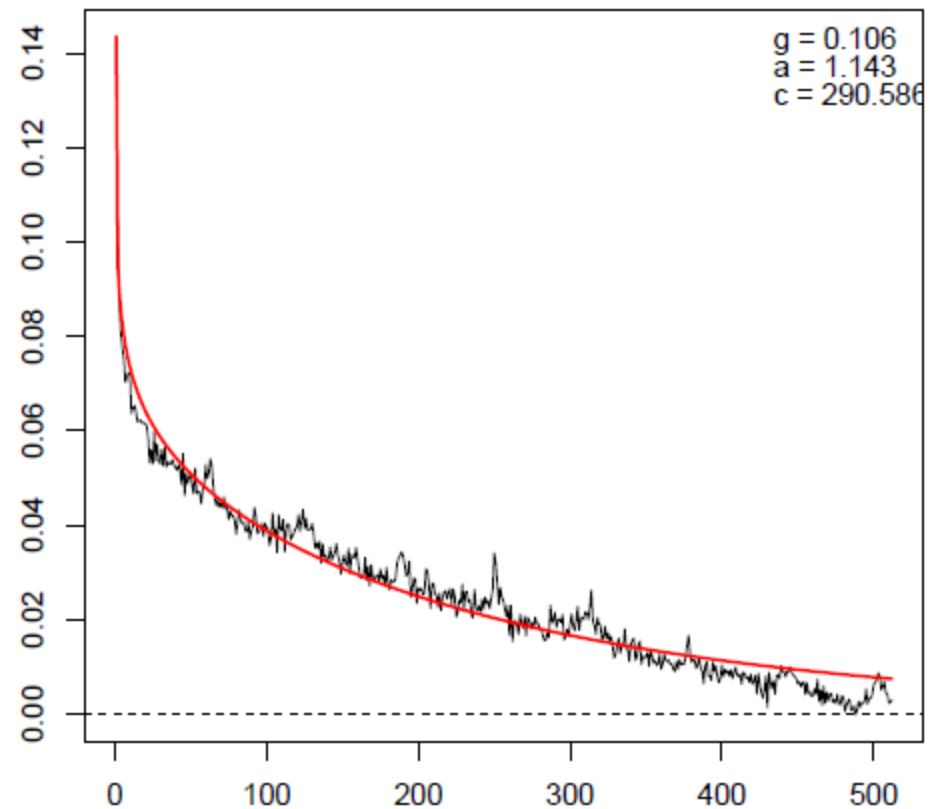


Market Cap weighted average volatility of US single stocks

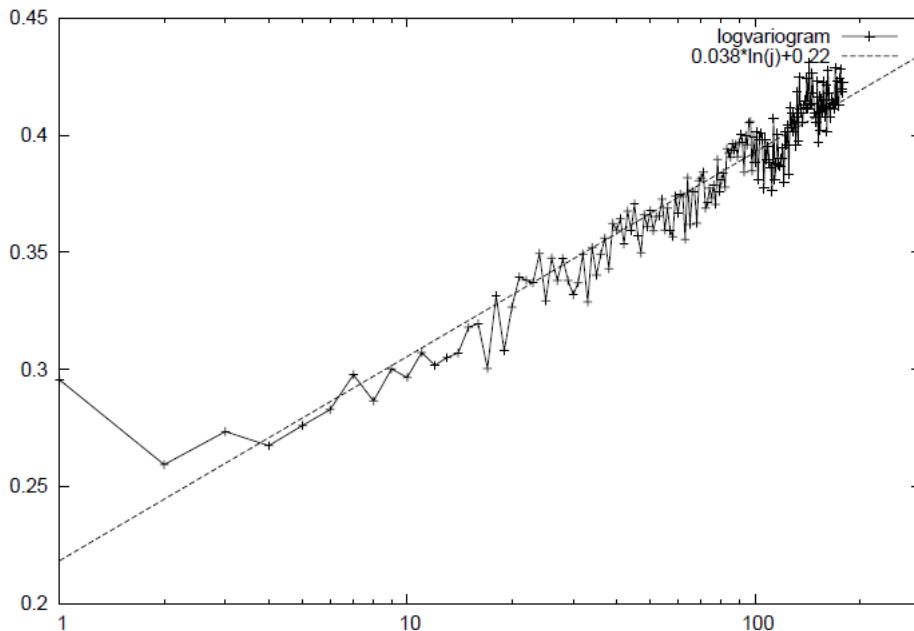


# Long range memory

$$\overline{\sigma_t^2 \sigma_{t+\tau}^2} - \overline{\sigma^2}^2 \propto B\tau^{-\nu},$$



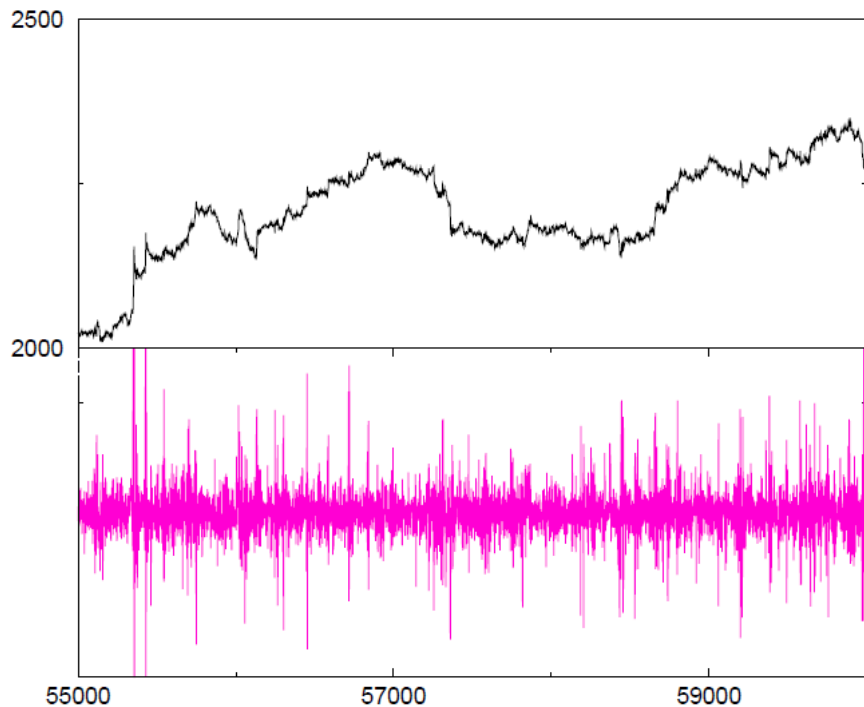
Logvariogram SP500: 2001-2007



$$V_{\ln \sigma}(\tau) = \overline{(\ln \sigma_t - \ln \sigma_{t+\tau})^2} \approx \lambda^2 \ln \tau.$$

A « multifractal » random walk ?

Monoscale model

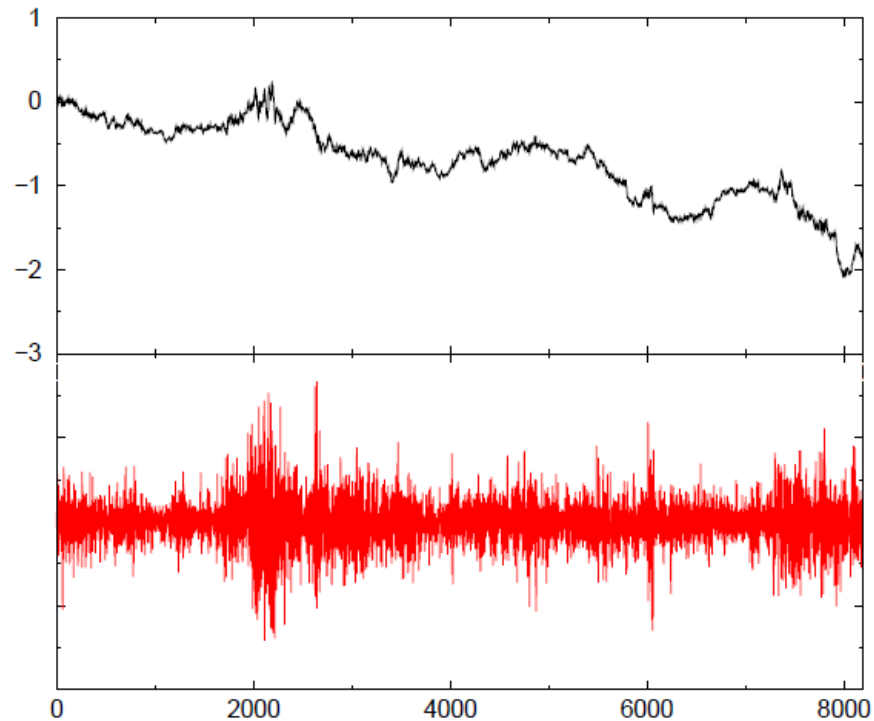



$$\sigma_t^2 = s^2 + \sum_{\tau=1}^{\infty} k(\tau) r_{t-\tau}^2$$

ARCH/GARCH: k short range  
**mono-scale**

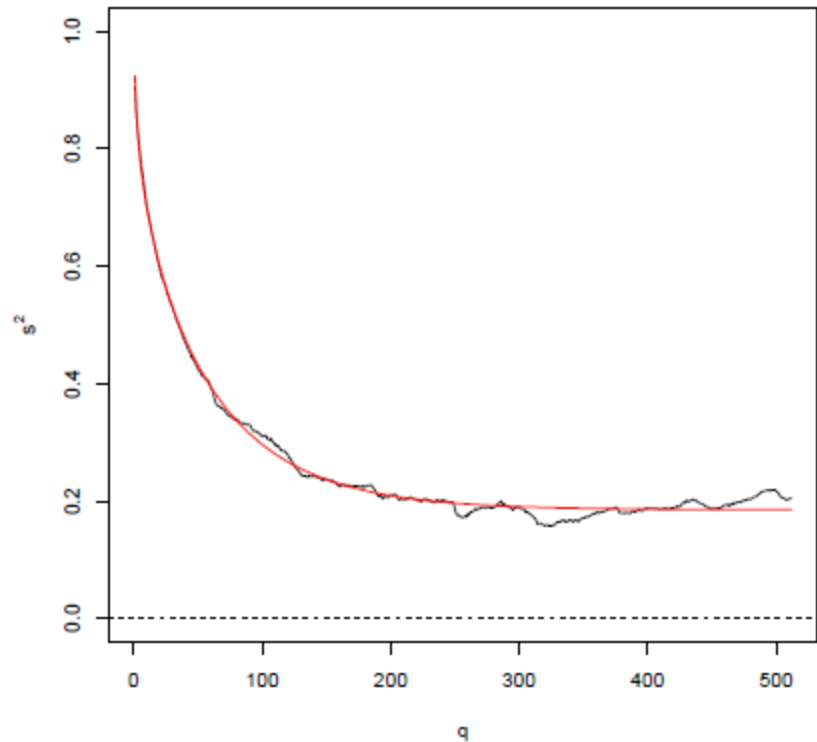
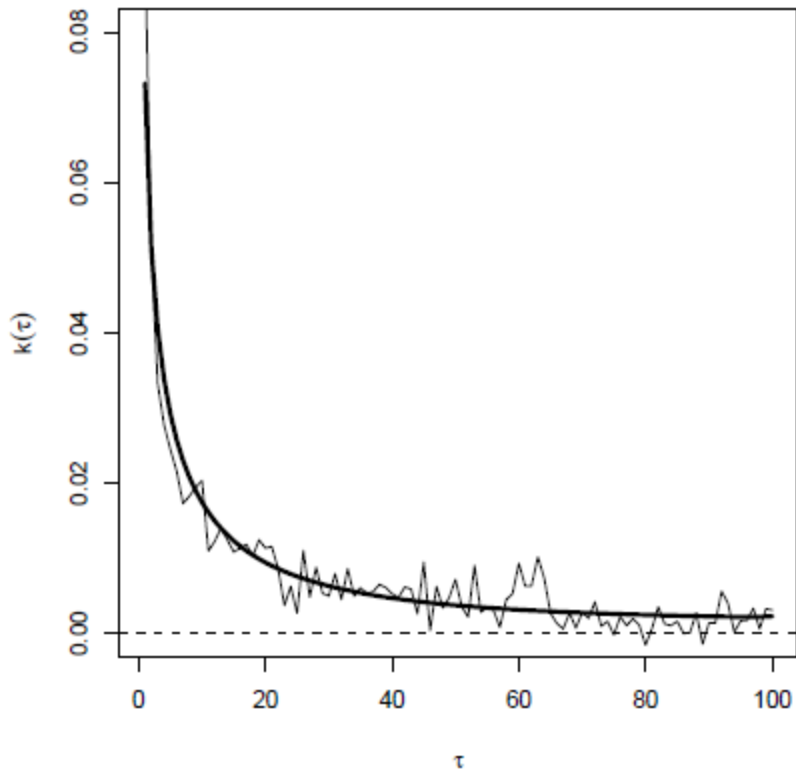


MRW model



Multifractal Random Walk  
Log correlations (**multi-scale**)  
At the border between stat.  
and non stationary! 

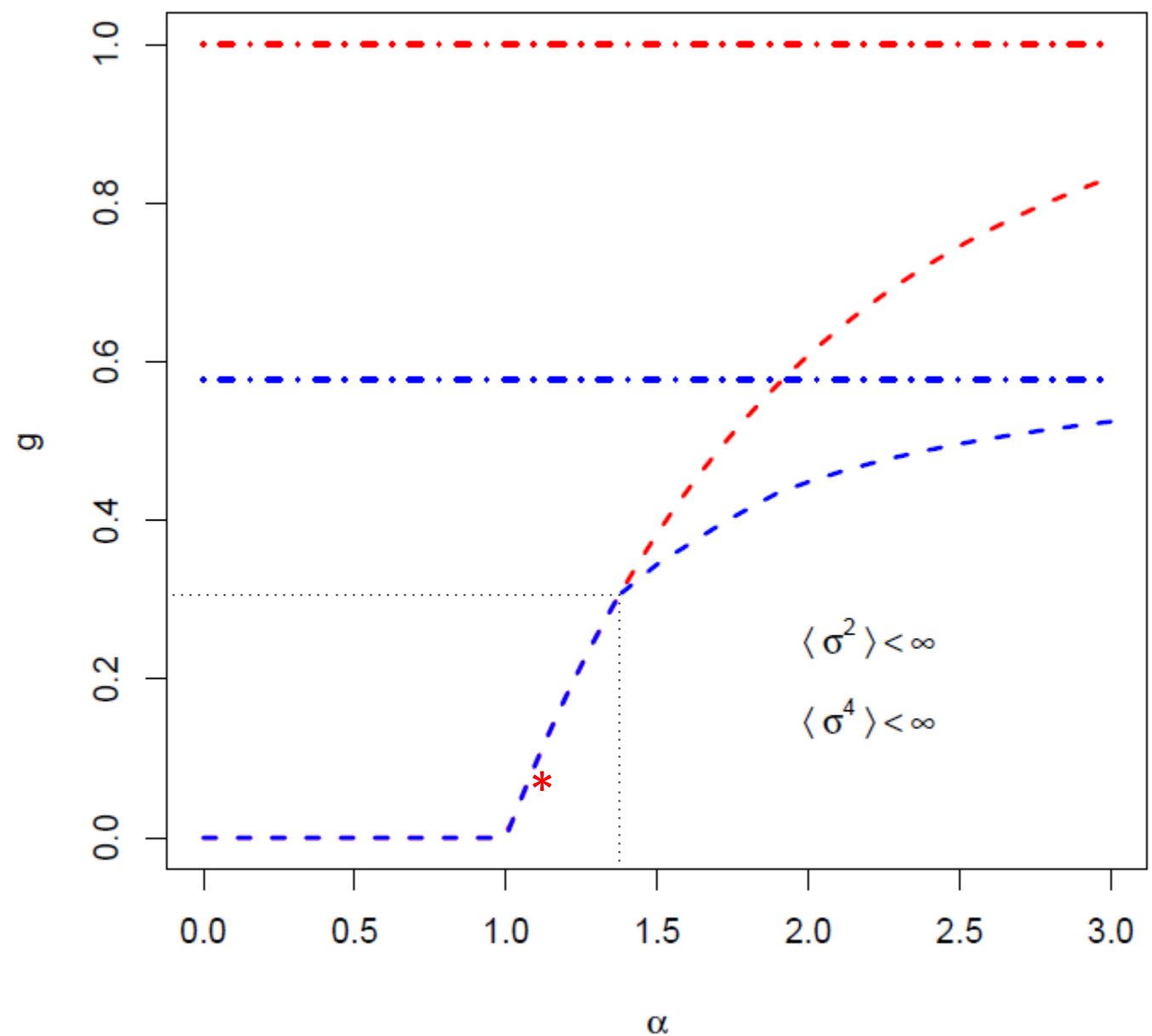
$$\sigma_t^2 = s^2 + \sum_{\tau=1}^{\infty} k(\tau) r_{t-\tau}^2$$



Note 1:  $k$ : power-law ! Note 2:  $s^2$  small

$$\sigma_t^2 = s^2 + \sum_{\tau=1}^{\infty} k(\tau) r_{t-\tau}^2$$

$$k(\tau) \approx g\tau^{-\alpha}$$



$$C^{(2)}(\tau) \equiv \langle (r_t^2 - \langle r^2 \rangle) r_{t-\tau}^2 \rangle_t$$

$$C^{(2)}(\tau) = \sum_{\tau' > 0} k(\tau') C^{(2)}(\tau' - \tau)$$

$$k(\tau) \xrightarrow{\tau \rightarrow \infty} A/\tau^{1+\epsilon}, \quad 0 < \epsilon,$$

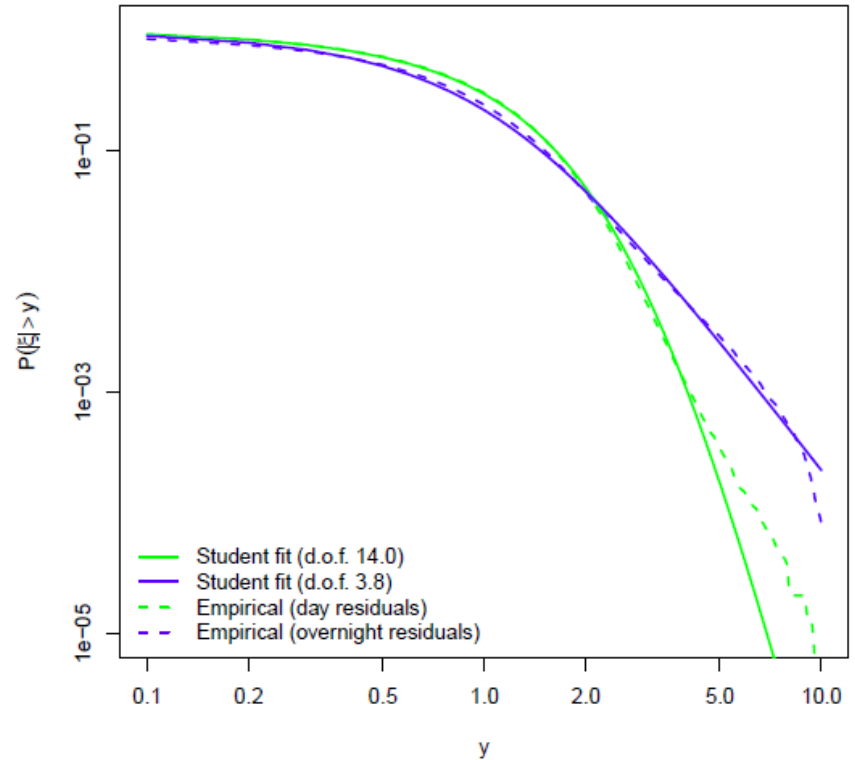
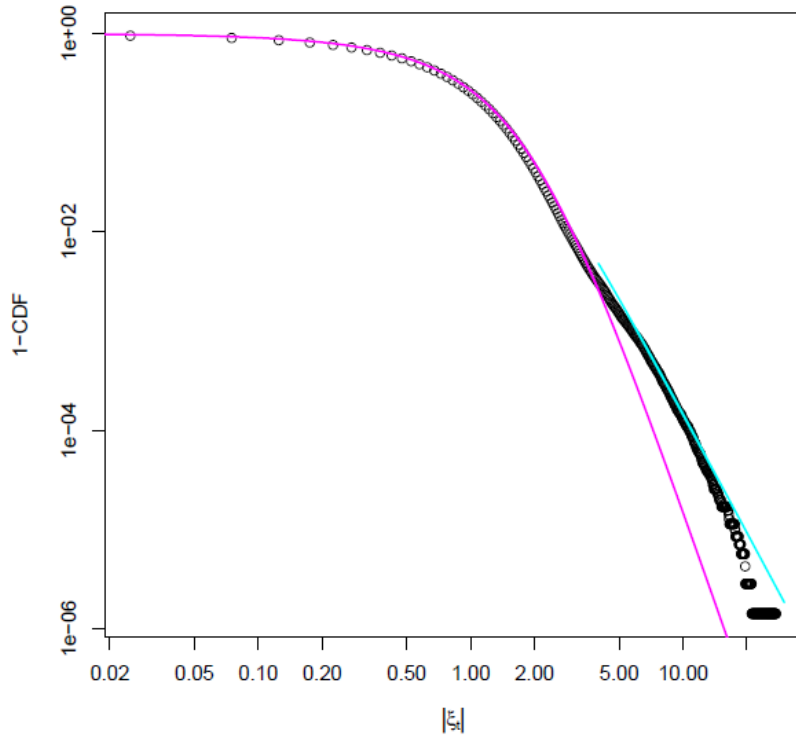
$$C^{(2)}(\tau) \xrightarrow{\tau \rightarrow \infty} B/\tau^\beta, \quad 0 < \beta < 1.$$

$$\boxed{\beta = 1 - 2\epsilon, \quad 0 < \epsilon < \frac{1}{2}.}$$

Exercice: prove this !

Are jumps/tails just a symptom of fluctuating volatility ?  
(An desperate attempt to save the Gaussian paradigm)

*Distribution of rescaled returns (predicted/ implied vol)*

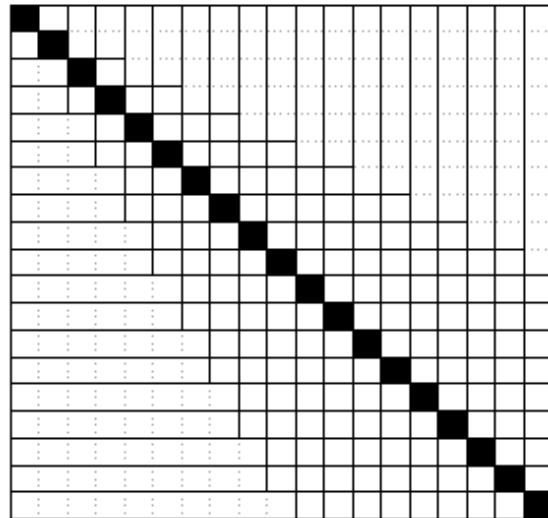
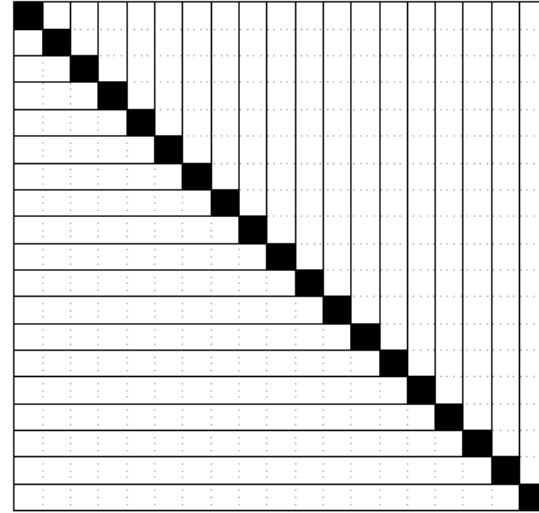
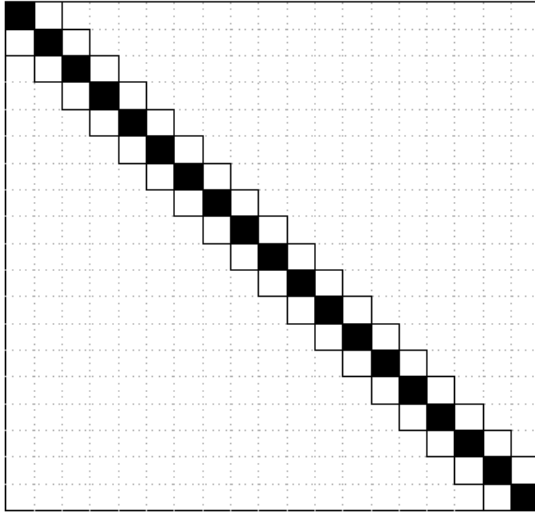


Jumps are still present !!

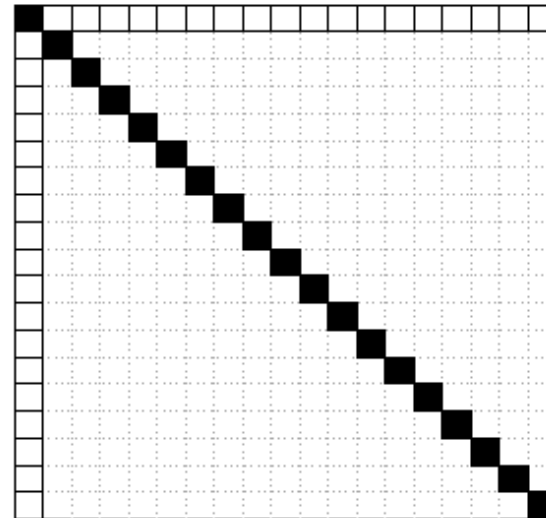
# QARCHs

$$\sigma_t^2 = s^2 + \sum_{\tau=1}^{\infty} L(\tau) r_{t-\tau} + \sum_{\tau, \tau'=1}^{\infty} K(\tau, \tau') r_{t-\tau} r_{t-\tau'}$$

$K(\tau, \tau')$



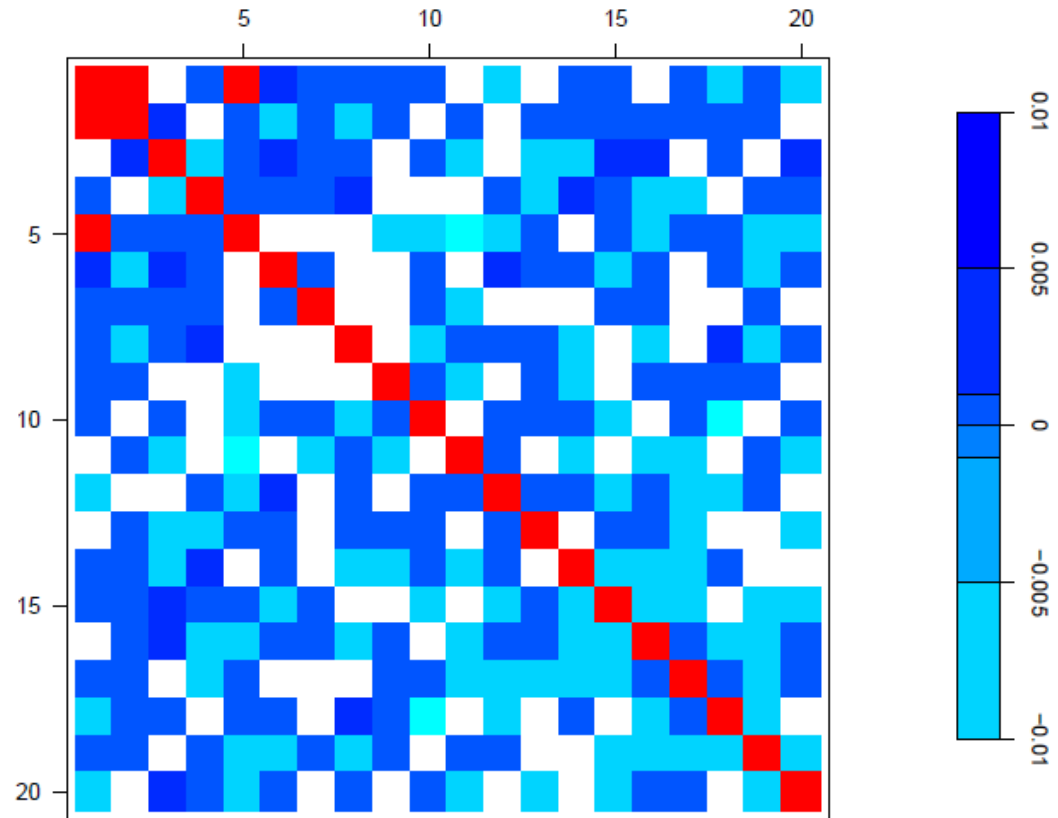
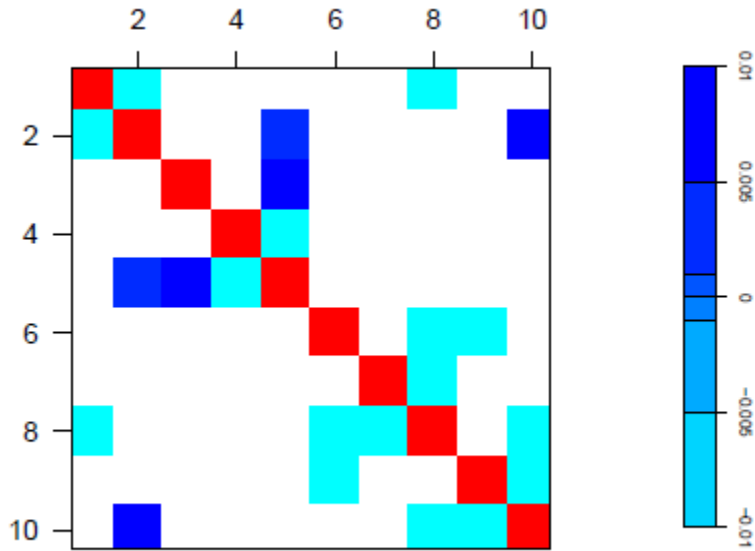
(c)



(d)

# Single stocks (average)

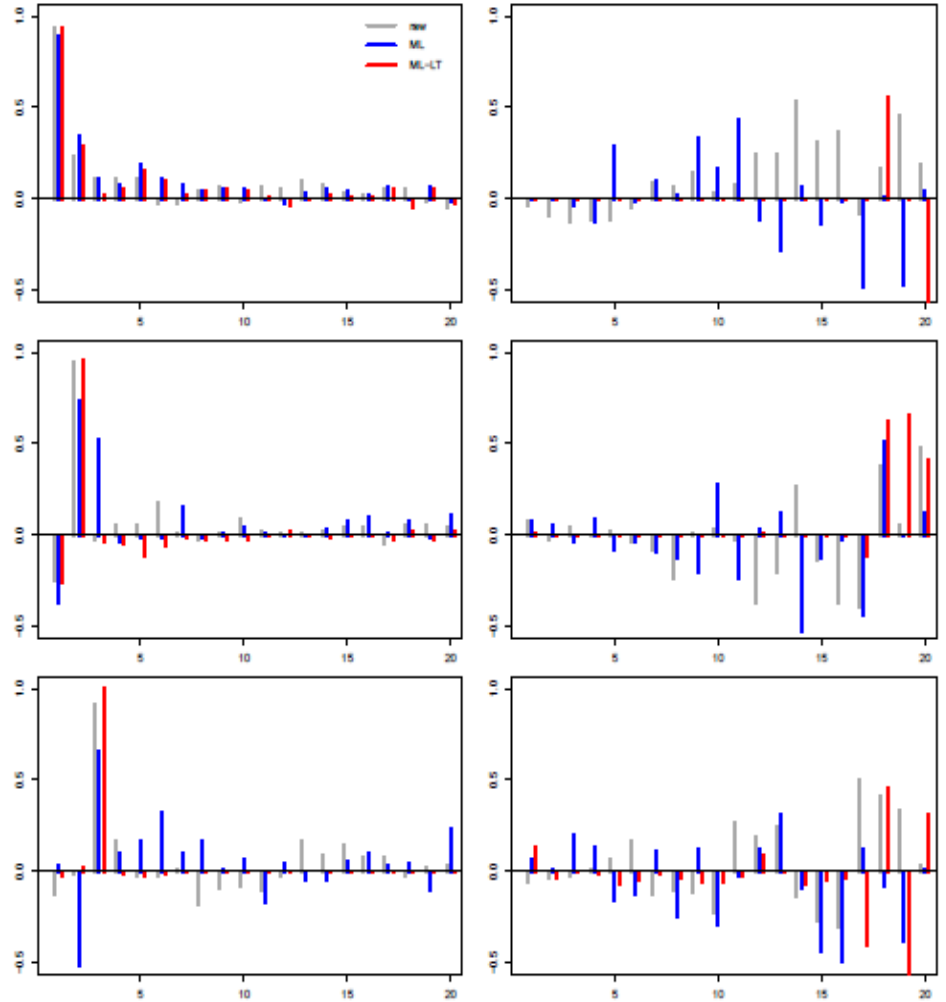
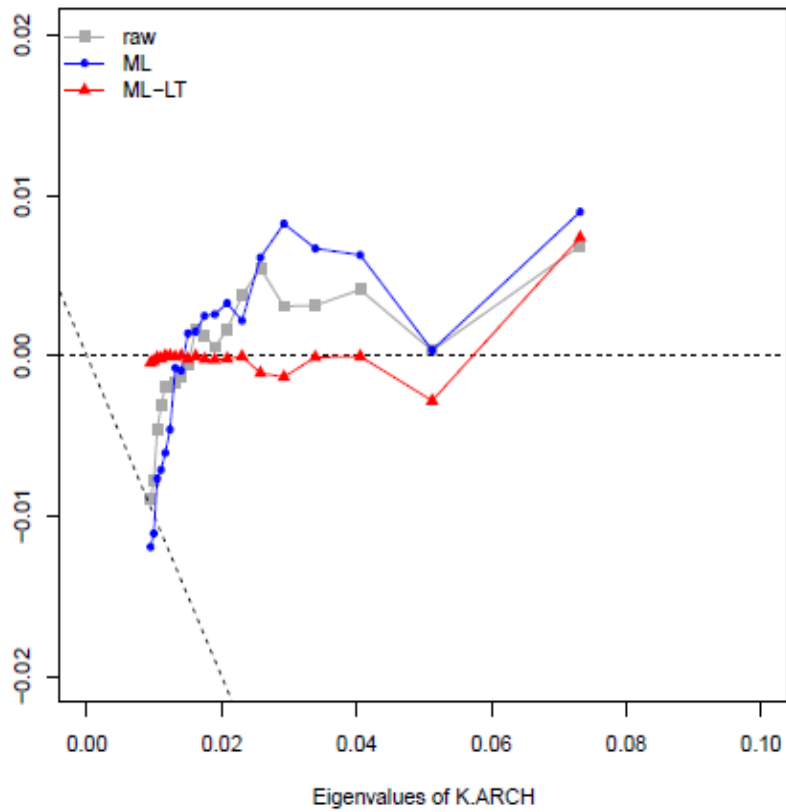
Index



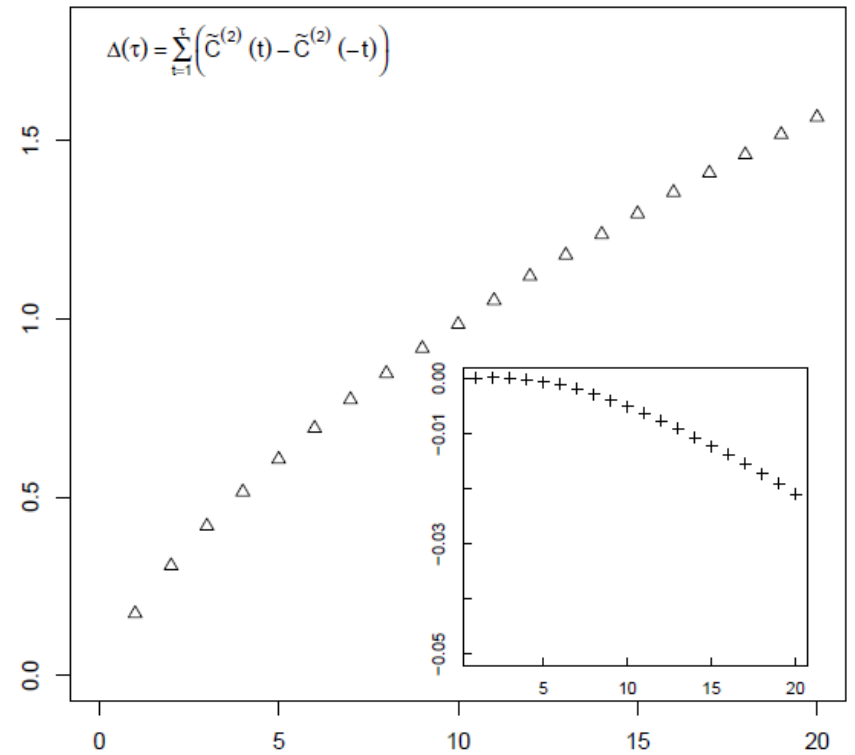
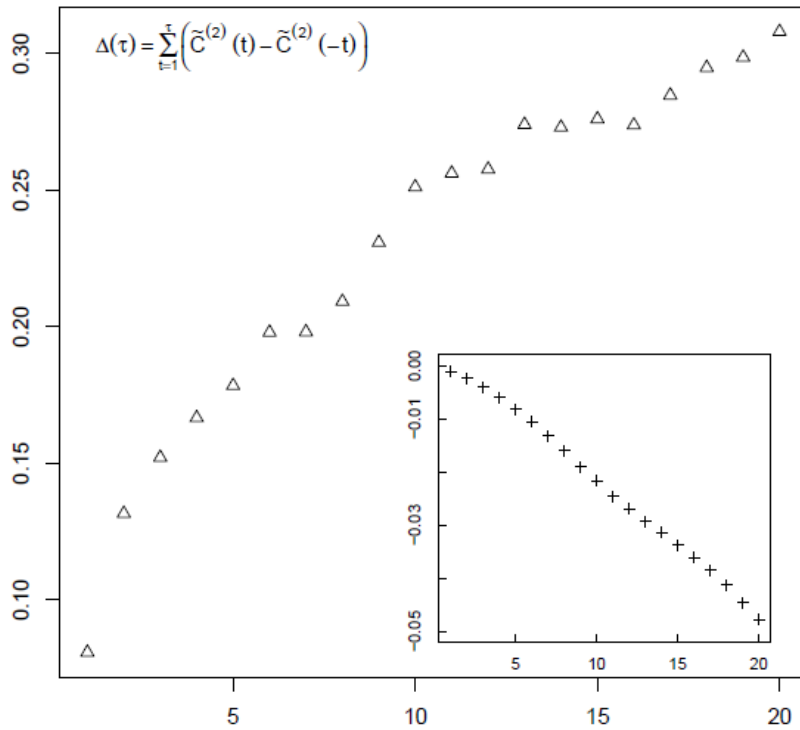
$$\sigma_t^2 = s^2 + \sum_{\tau=1}^{\infty} L(\tau) r_{t-\tau} + \sum_{\tau, \tau'=1}^{\infty} K(\tau, \tau') r_{t-\tau} r_{t-\tau'}$$



# QARCH: Spectrum of K

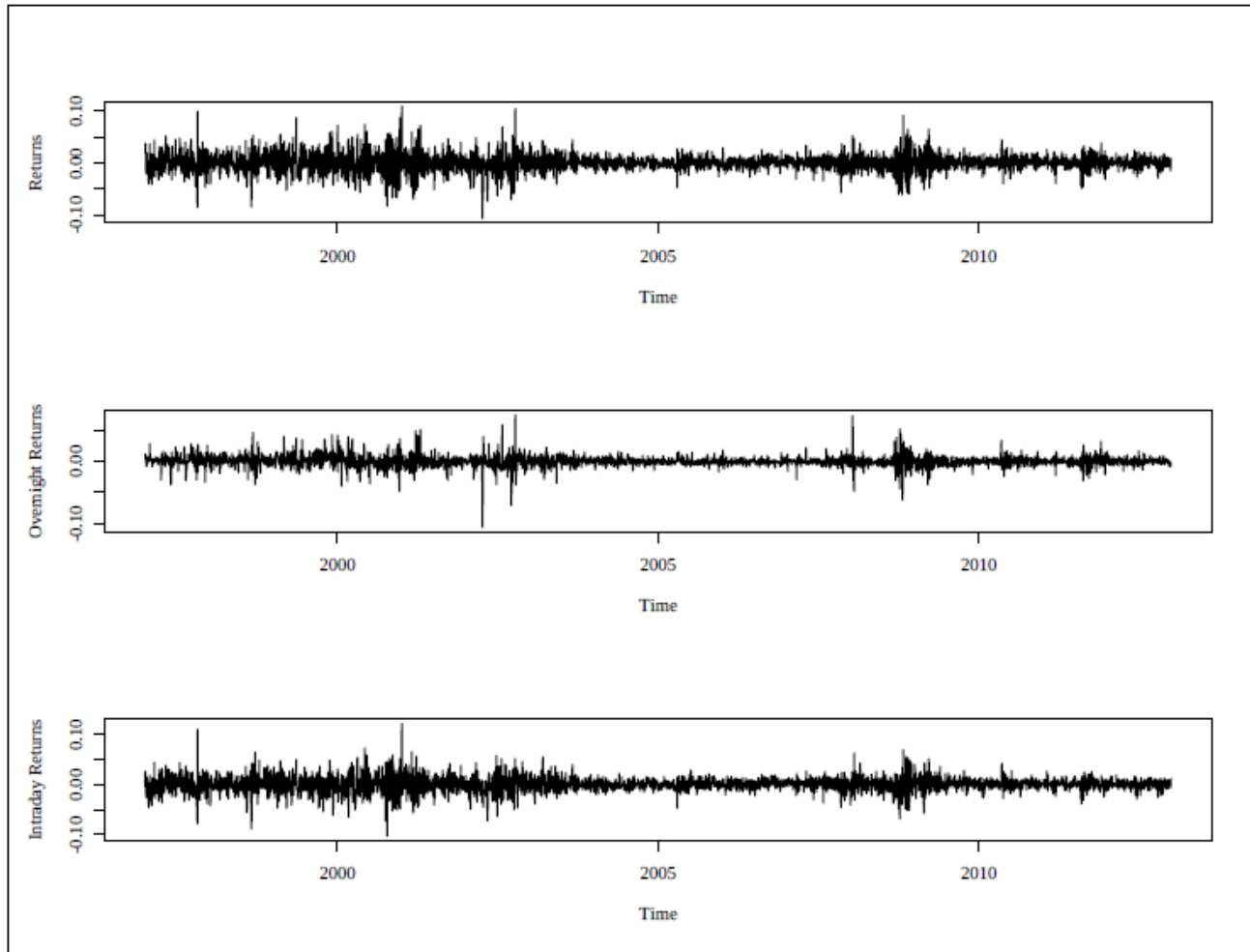


# Violations of time reversal invariance (ARCH)



$$\tilde{C}^{(2)}(\tau) \equiv \langle (\sigma_t^2 - \langle \sigma^2 \rangle) r_{t-\tau}^2 \rangle_t$$

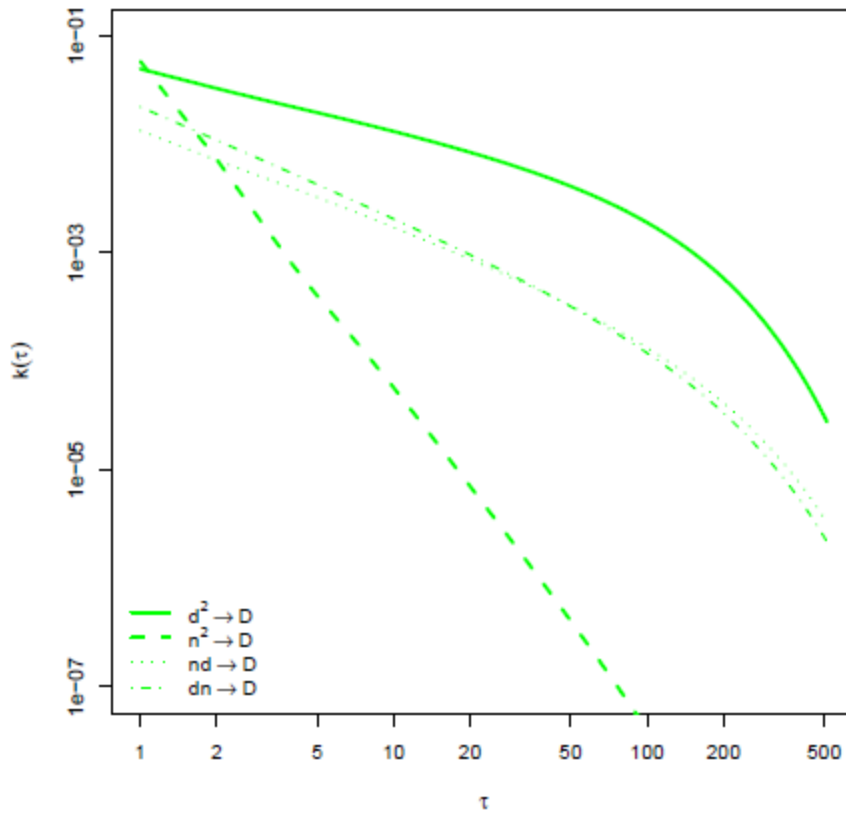
# Intraday and overnights are different beasts



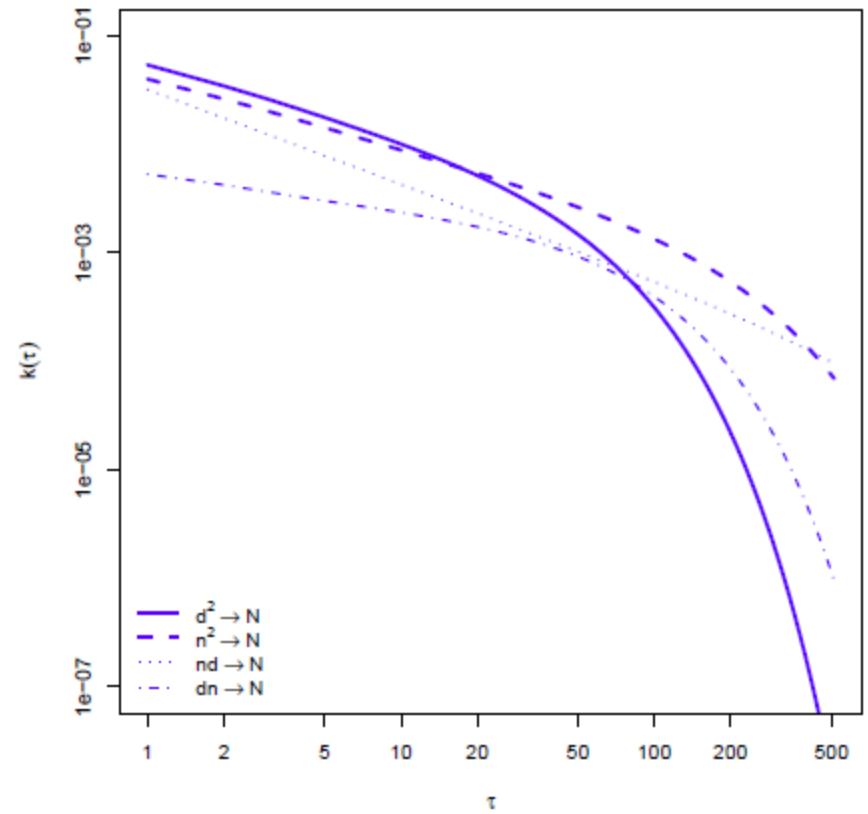
Type of return	$\langle r \rangle$	$\sqrt{\langle r^2 \rangle}$	$\frac{\langle r^3 \rangle}{\langle r^2 \rangle^{\frac{3}{2}}}$	$\frac{\langle r^4 \rangle}{\langle r^2 \rangle^2}$
$r_t$	$7.5 \cdot 10^{-5}$	0.025	-0.61	32.1
$r_{ON,t}$	$1.0 \cdot 10^{-5}$	0.012	-3.46	308.3
$r_{ID,t}$	$6.5 \cdot 10^{-5}$	0.022	-0.13	16.3

$$\sigma_t^2 = s^2 + \sum_{\tau=1}^q k^{\text{dd}}(\tau) r^{\text{D}}_{t-\tau}{}^2 + \sum_{\tau=1}^q k^{\text{nn}}(\tau) r^{\text{N}}_{t-\tau}{}^2 + 2 \sum_{\tau=1}^q k^{\text{nd}}(\tau) r^{\text{N}}_{t-\tau} r^{\text{D}}_{t-\tau}$$

Quadratic kernels (Day)

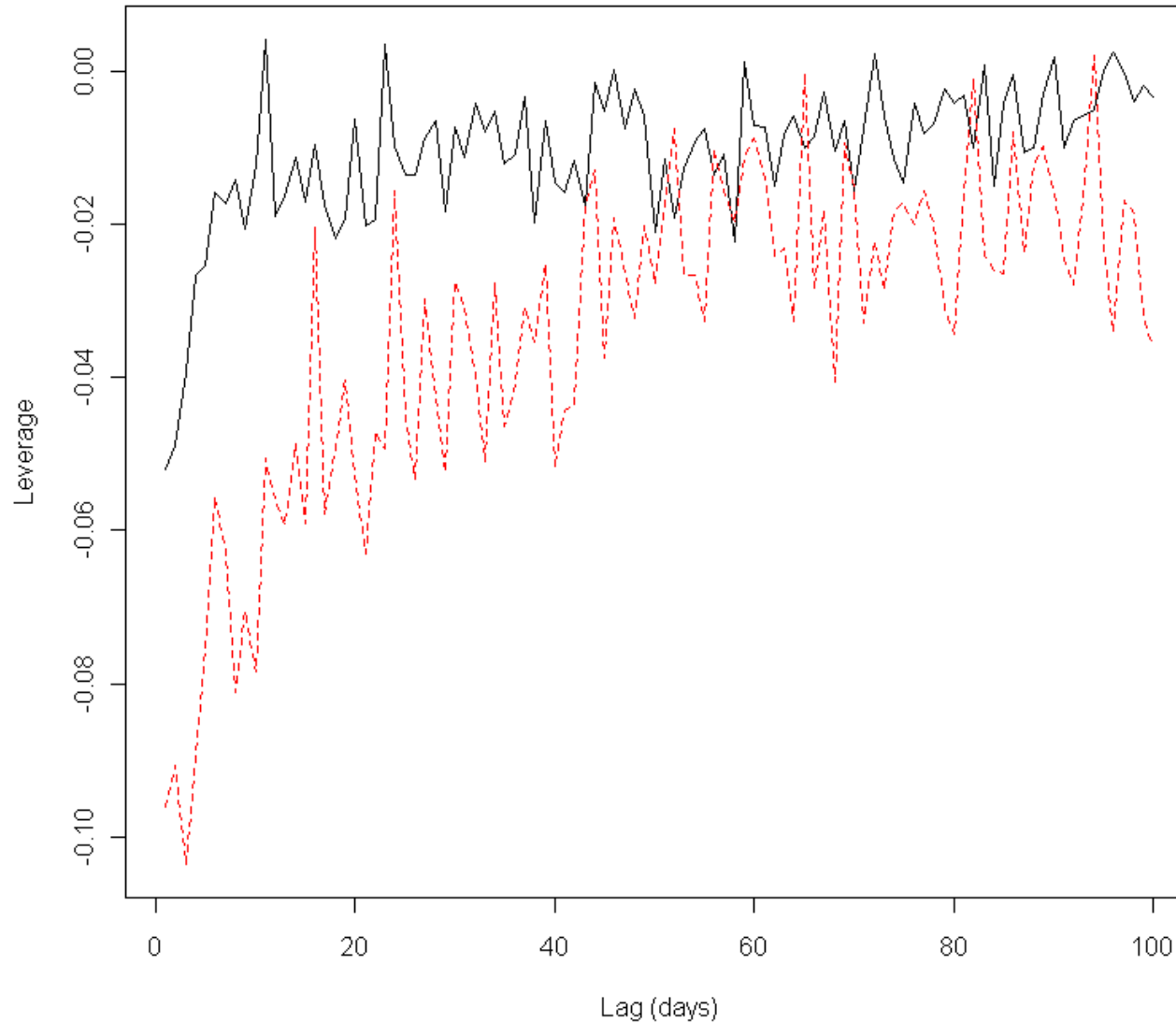


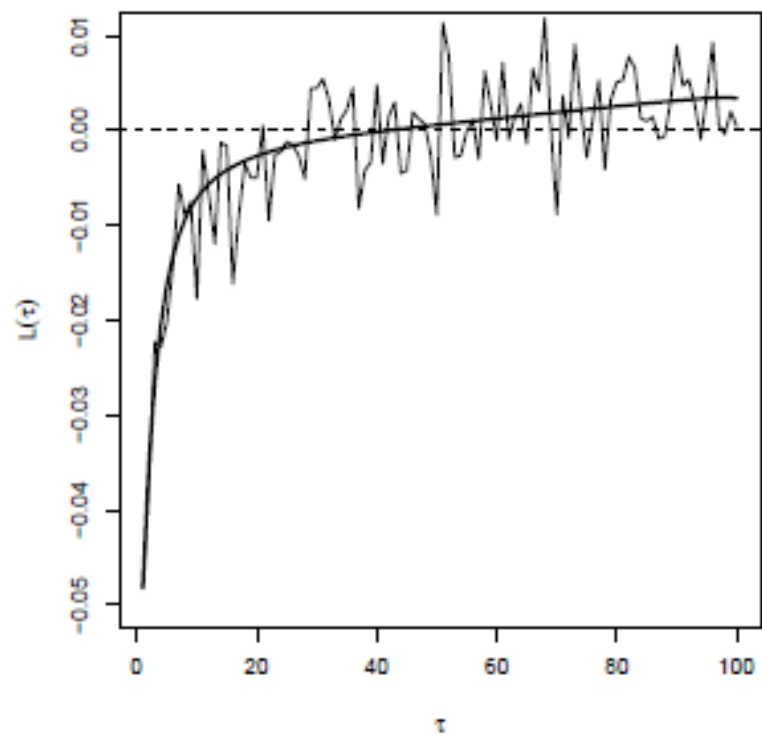
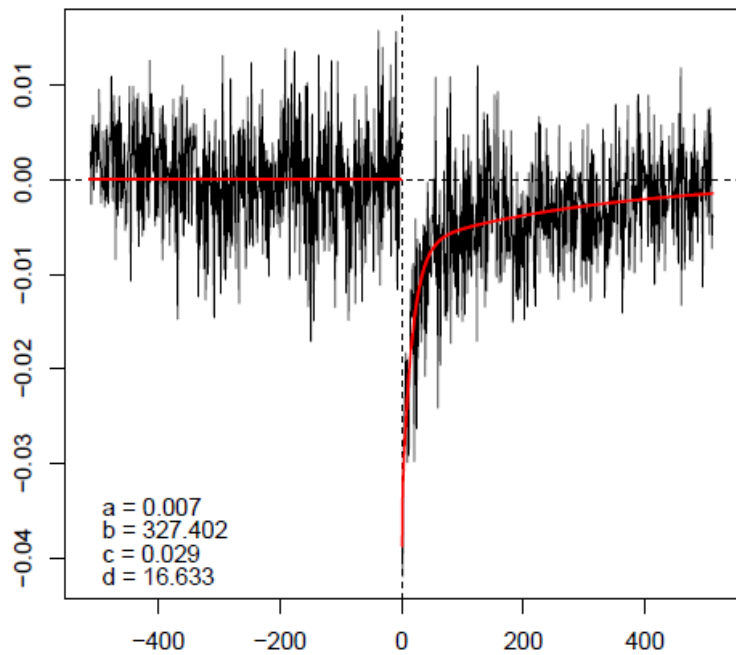
Quadratic kernels (Night)



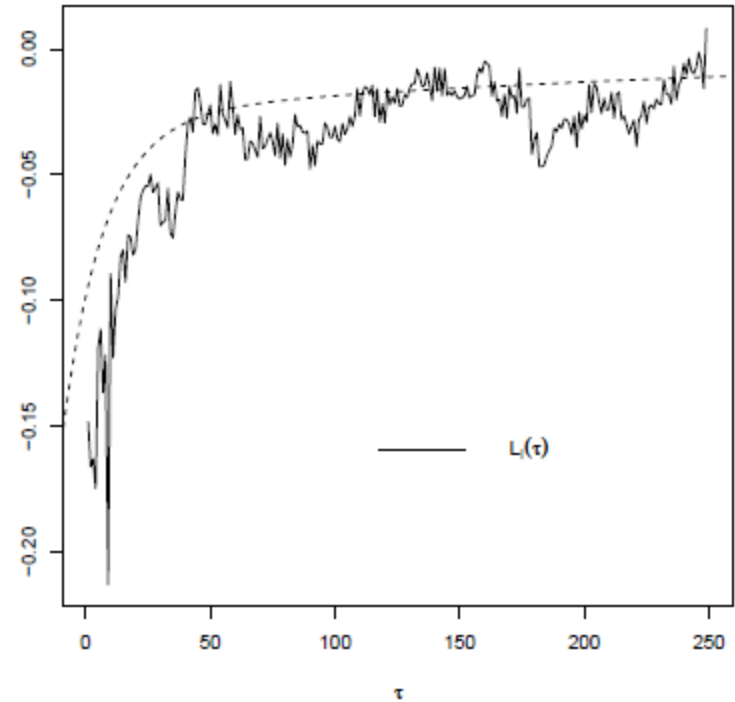
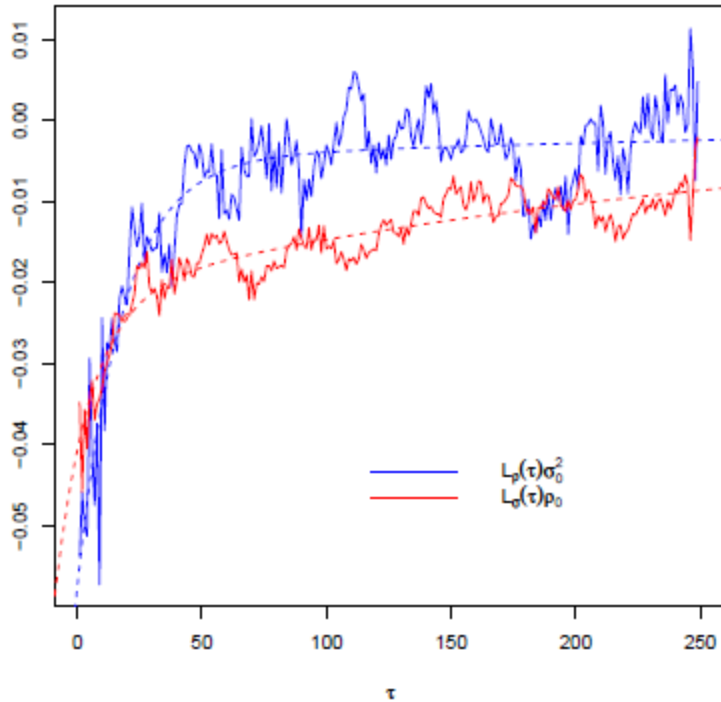
# Index and currency (carry signed) leverage

$$I_e < r_t r_{t+t}^2 >$$





# Correlation leverage

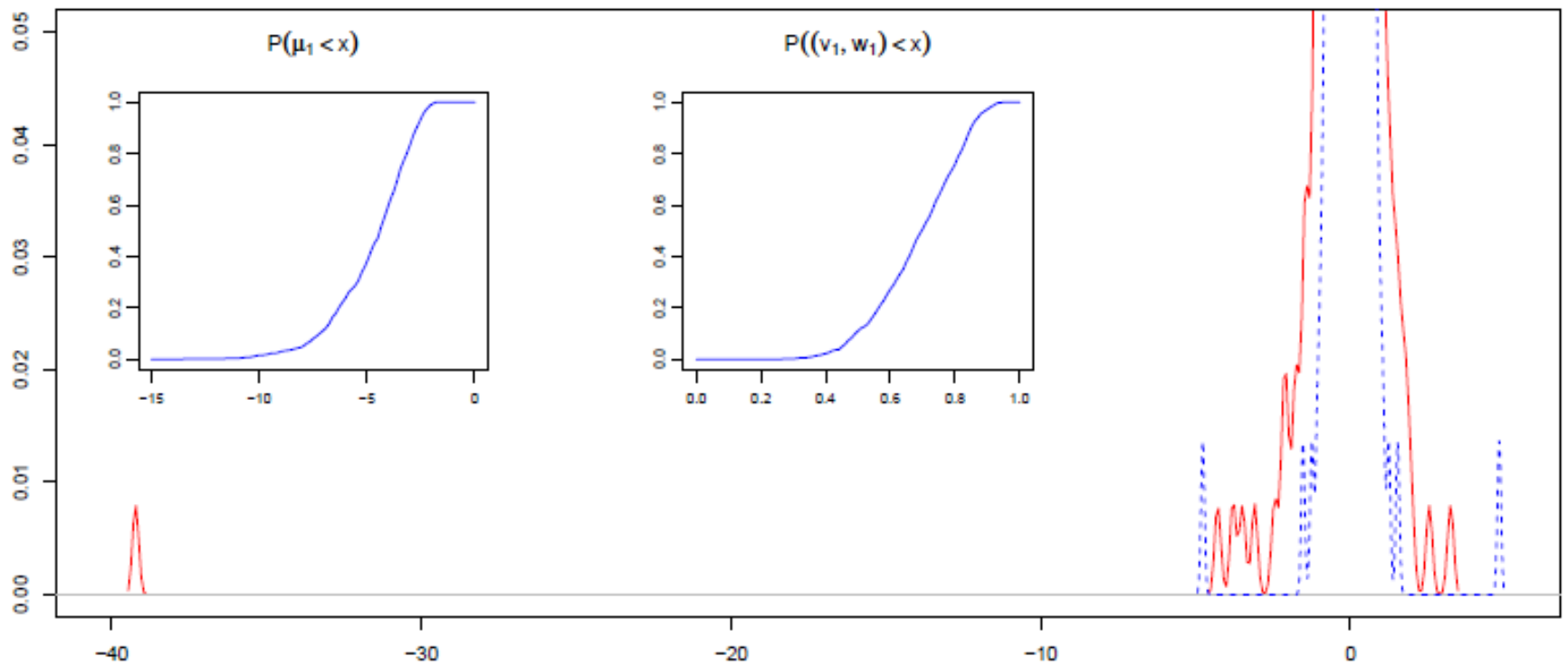


$$\mathcal{L}_I(\tau) = \frac{\langle I(t-\tau)I(t)^2 \rangle}{\langle I(t)^2 \rangle}$$

$$\mathcal{L}_\sigma(\tau) = \frac{\langle I(t-\tau)\sigma(t)^2 \rangle}{\langle I(t)^2 \rangle}, \quad \mathcal{L}_\rho(\tau) = \frac{\langle I(t-\tau)\rho(t) \rangle}{\langle I(t)^2 \rangle}.$$

# « Principal regression analysis »

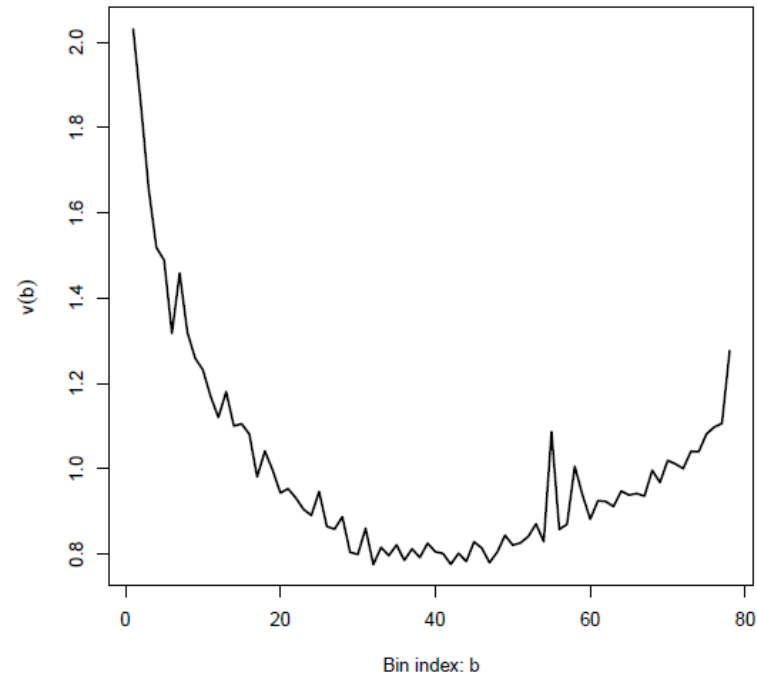
$$\hat{\eta}_\alpha(t)\hat{\eta}_\beta(t) := C_{\alpha,\beta} + D_{\alpha,\beta}(\tau)I(t - \tau) + \varepsilon_{\alpha,\beta}(t, \tau).$$





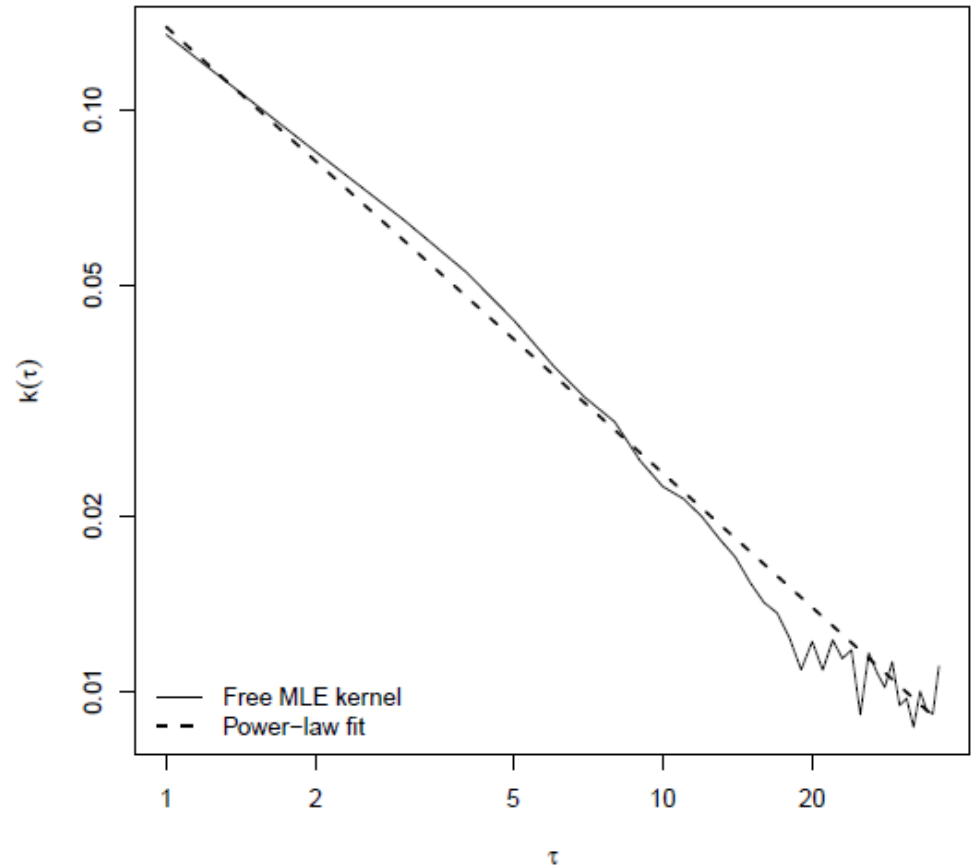
# Intraday effects

$$\sigma_t^2 = s^2 + \sum_{\tau=1}^{\infty} k(\tau) r_{t-\tau}^2$$

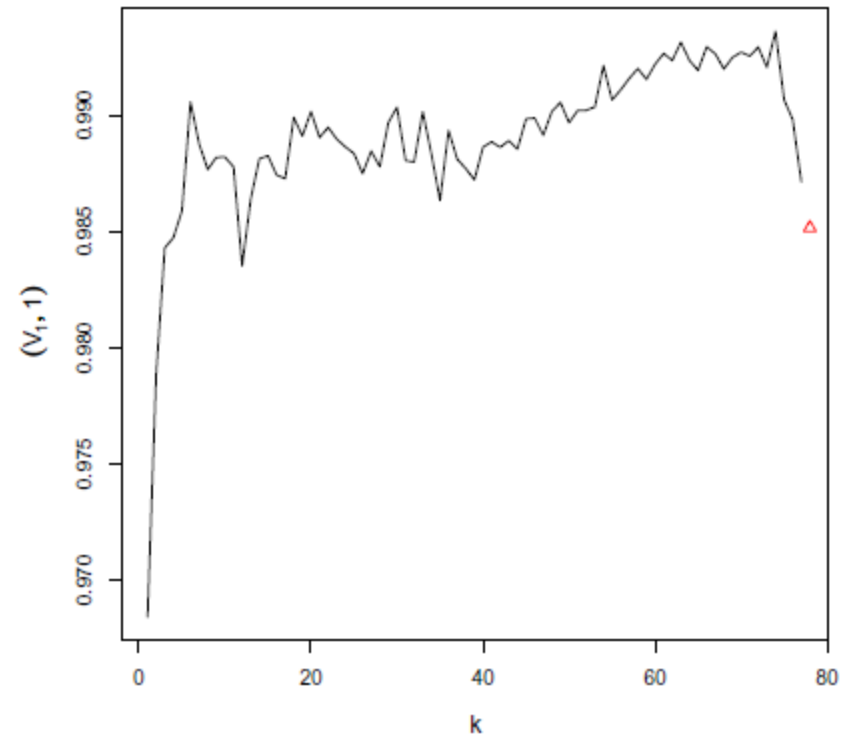
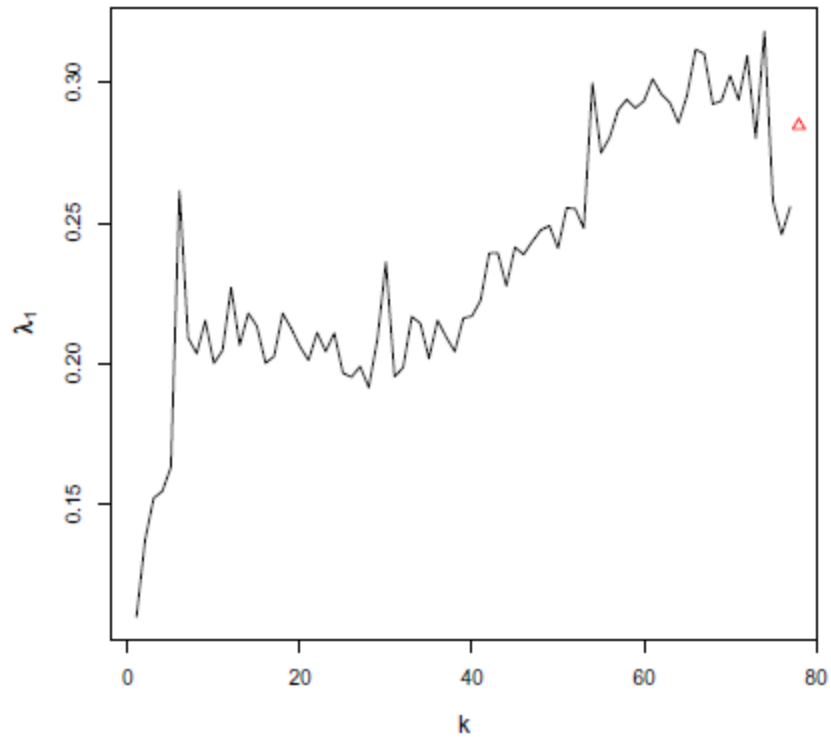


$g$	$\alpha$
0.164	0.894

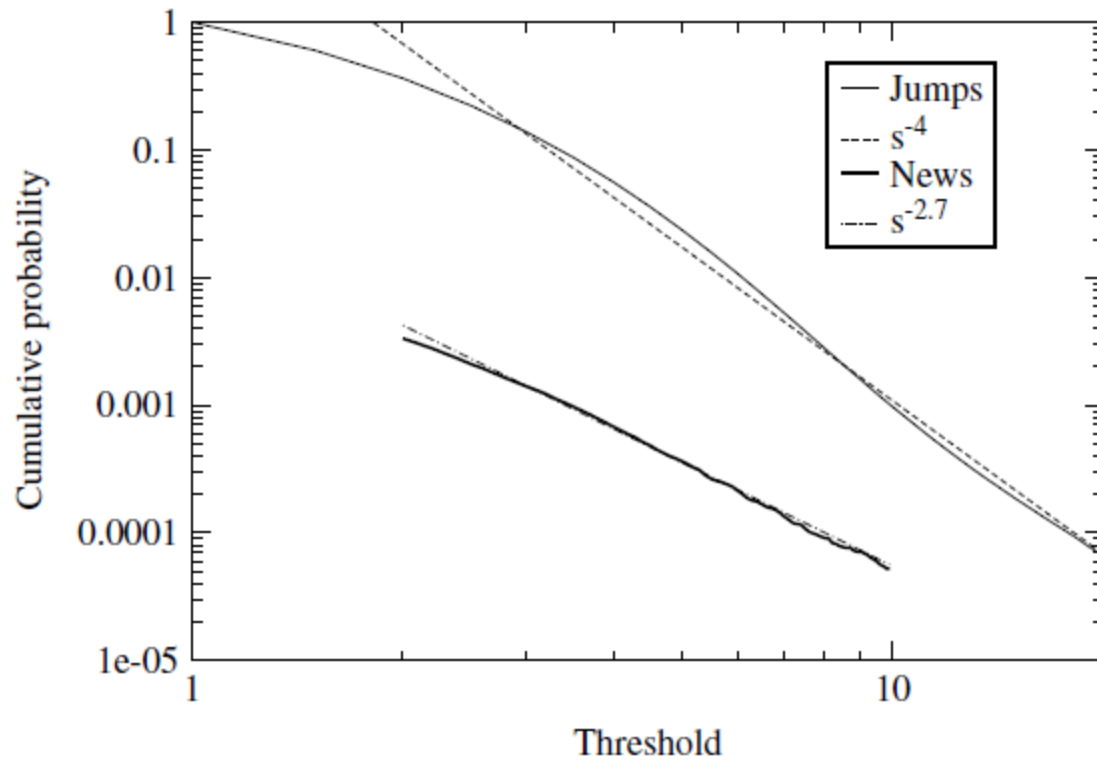
$\nu$	$s^2$
4.93	0.0629



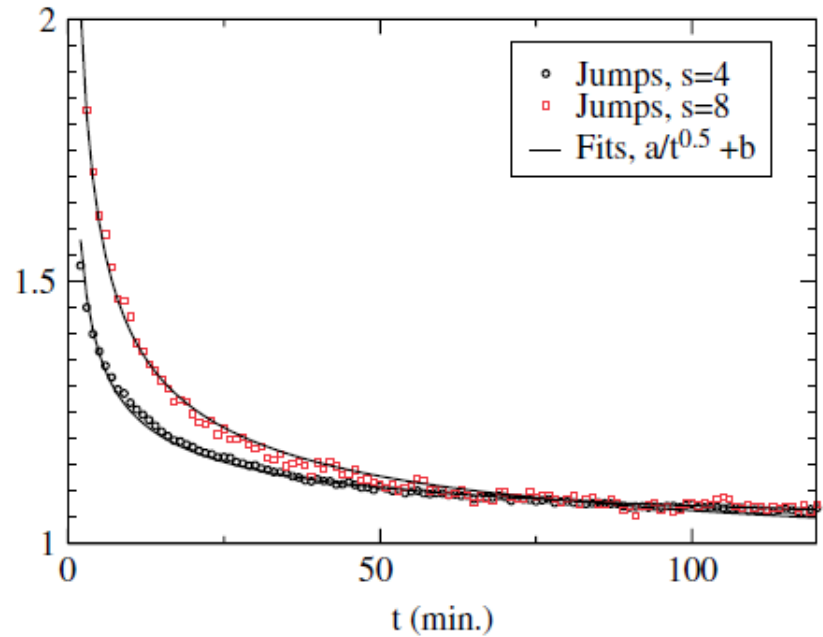
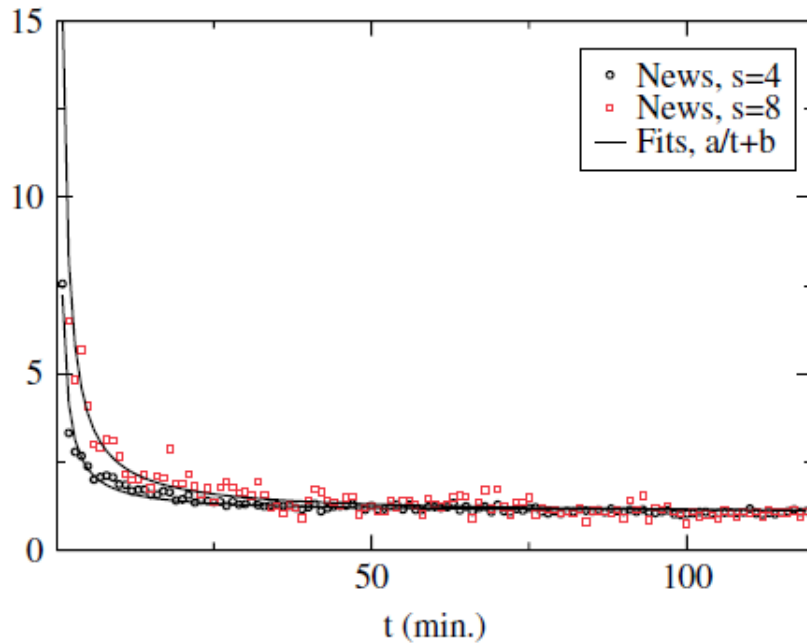
# Intraday effects on correlations



# Jumps: News or No-News ?



# « Aftershocks »

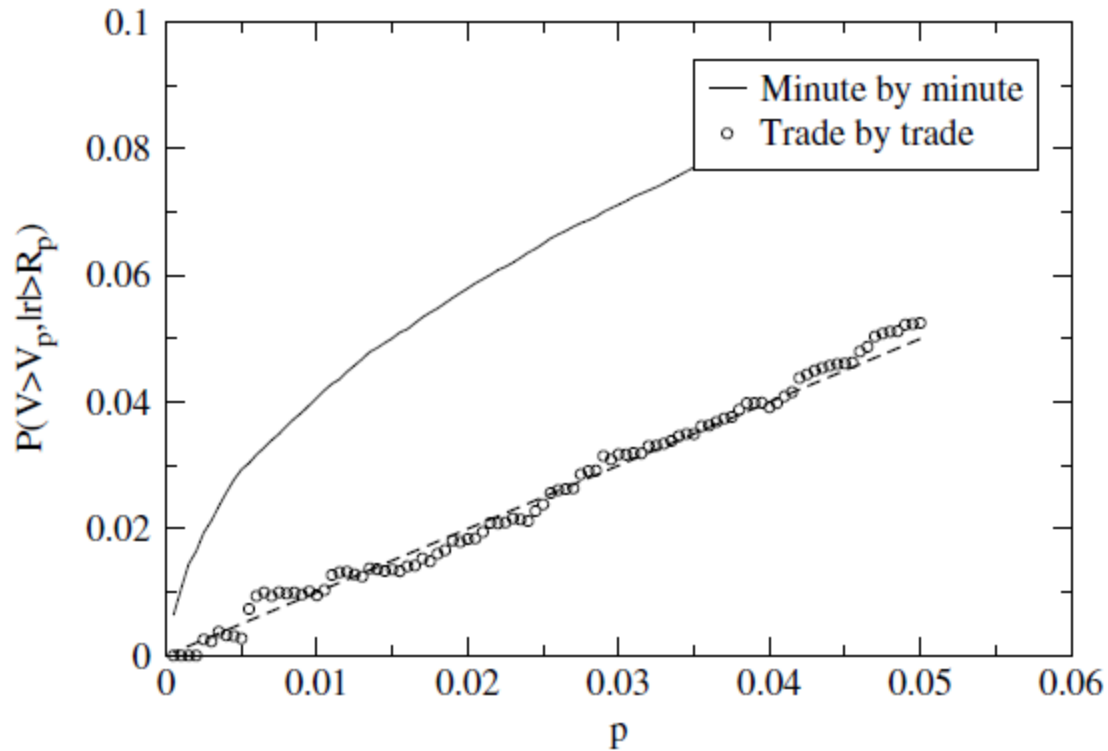


Different types of events !!

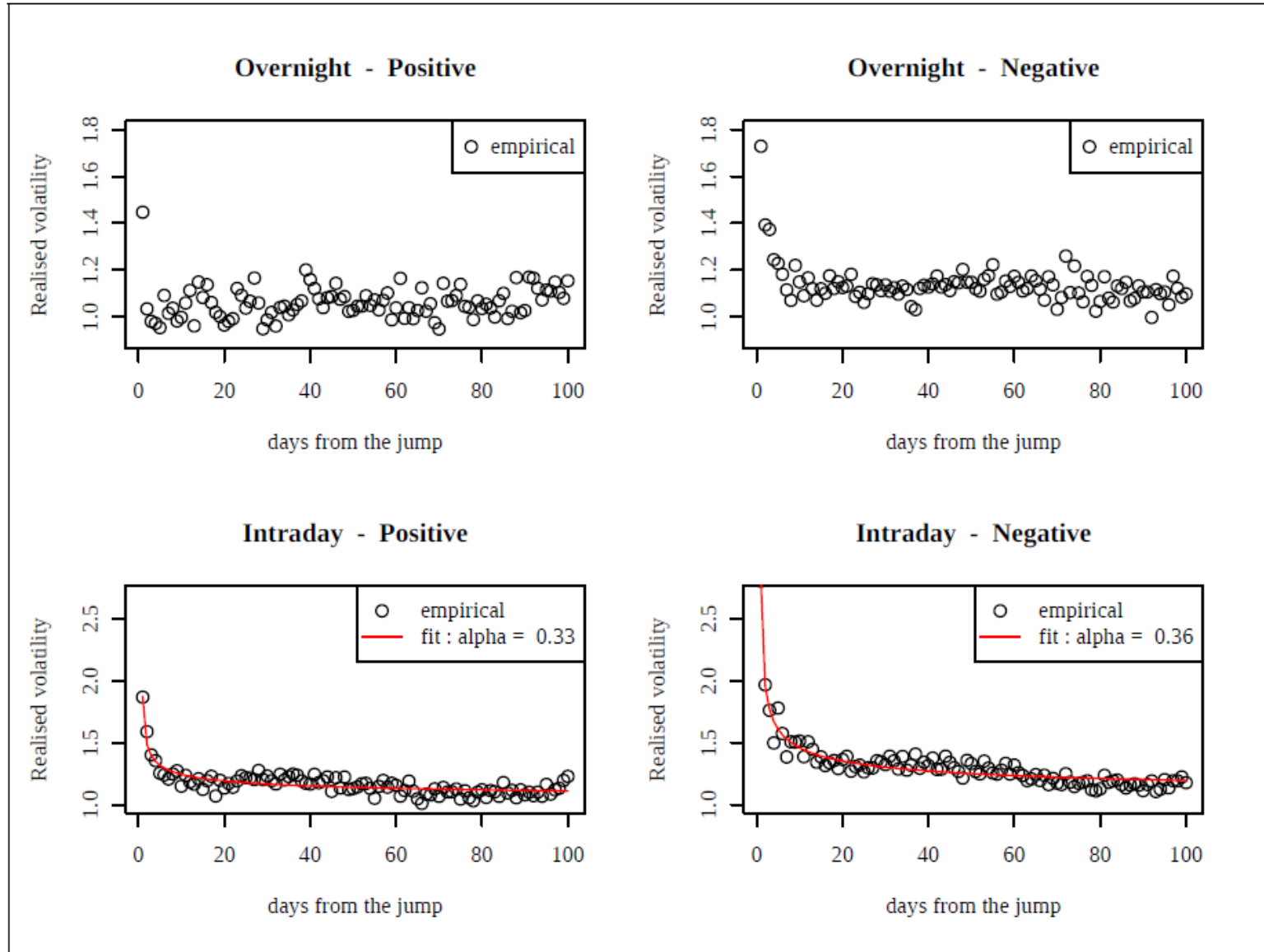
Exogenous (rare)

Endogenous (frequent)

Are Jumps induced by large volumes ?

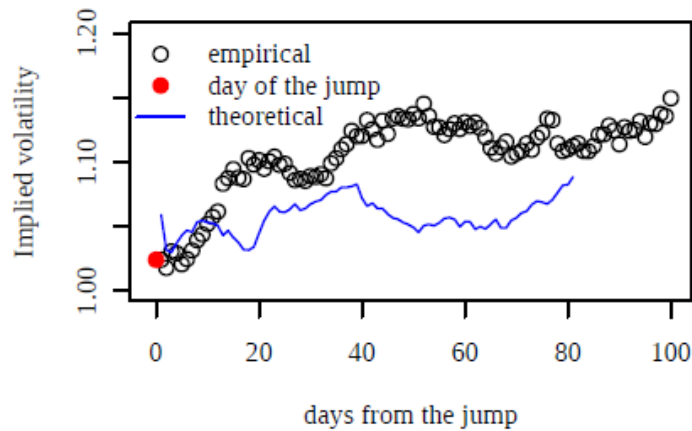


# A full picture of Aftershocks...

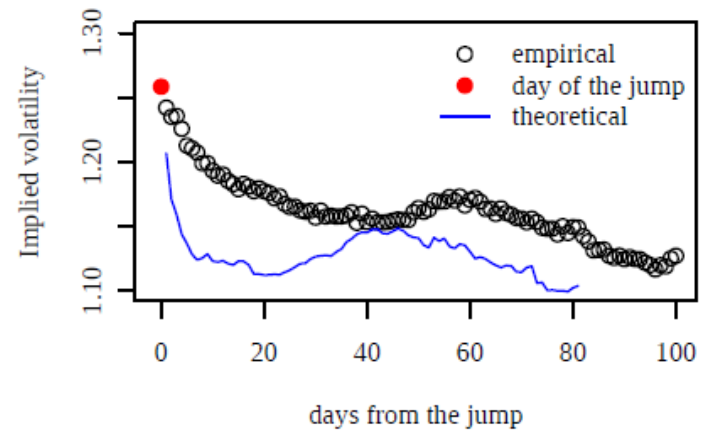


Overnight = news = rapid decay of the vol thereafter

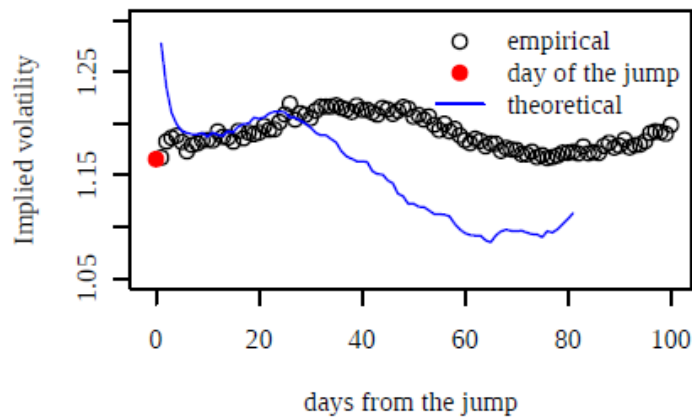
**Overnight - Positive**



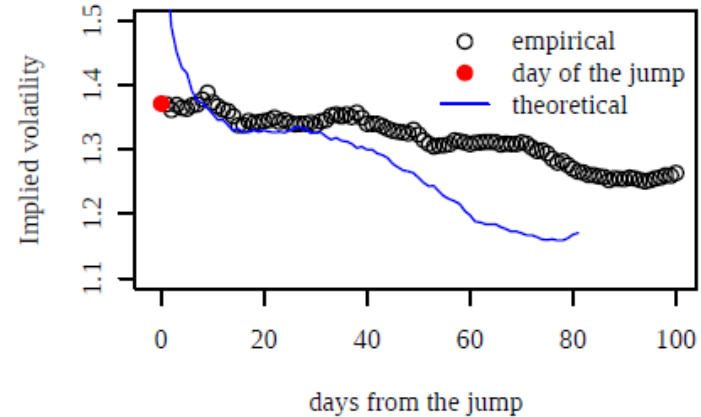
**Overnight - Negative**



**Intraday - Positive**



**Intraday - Negative**



$$\sigma_{imp}^{th}(t) = \sqrt{\frac{1}{T} \int_t^{t+T} \sigma_{rea}^2(t') dt'}$$

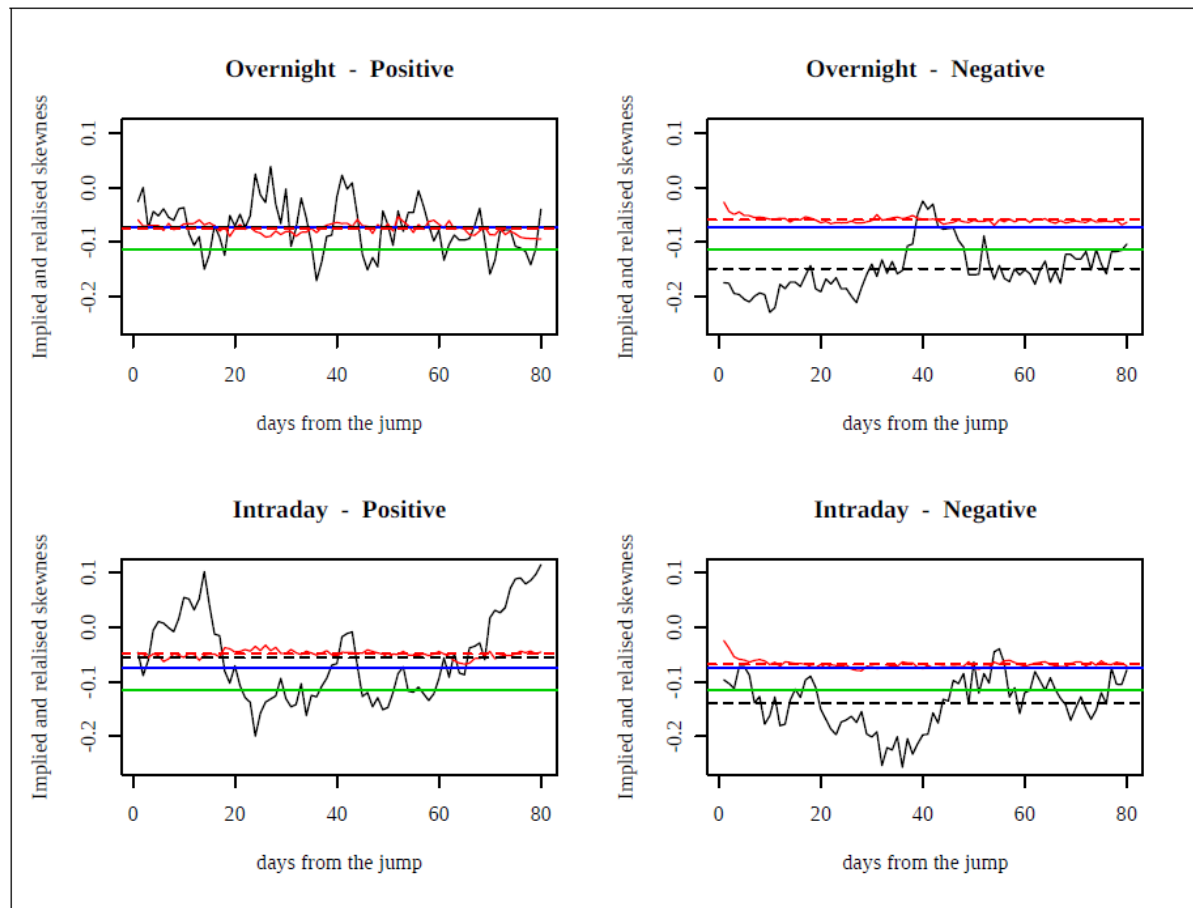


Figure 4.5: Implied and realized skewness of stocks around price jumps for different types of jumps for all periods.

Black : realized skewness and its mean, Red : implied skewness and its mean

Green : realized skewness over all data, Blue : implied skewness over all data

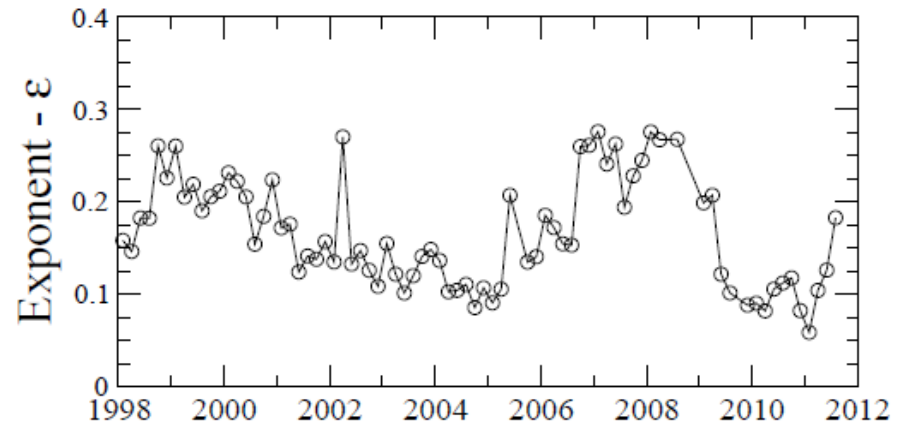
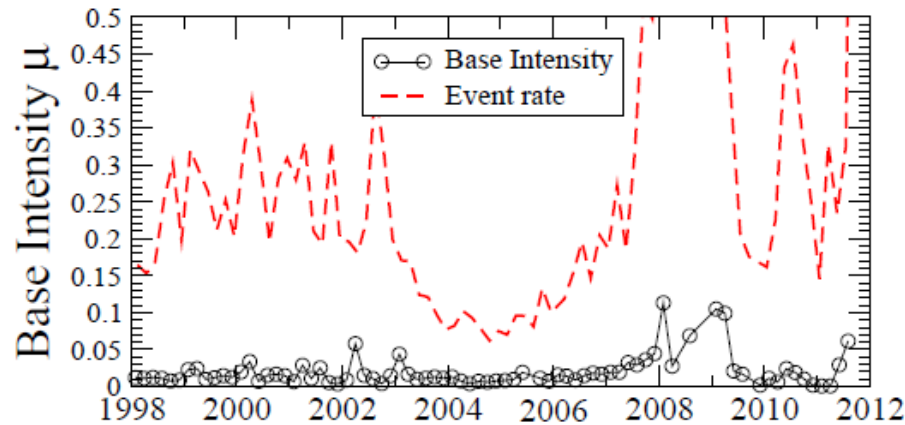
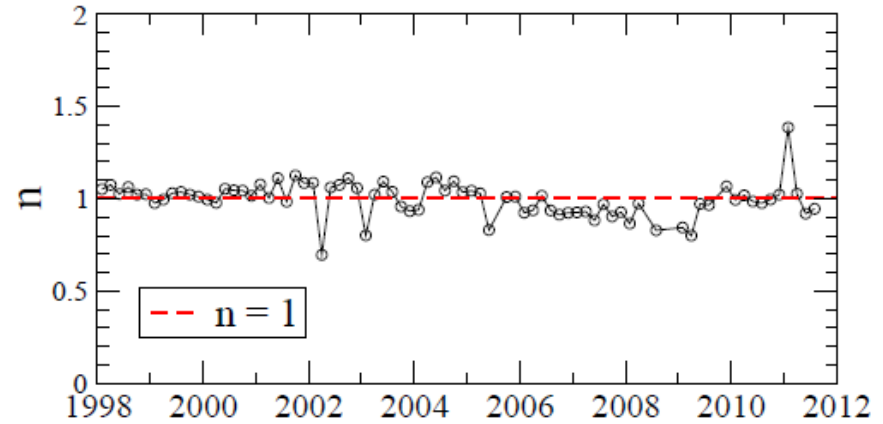
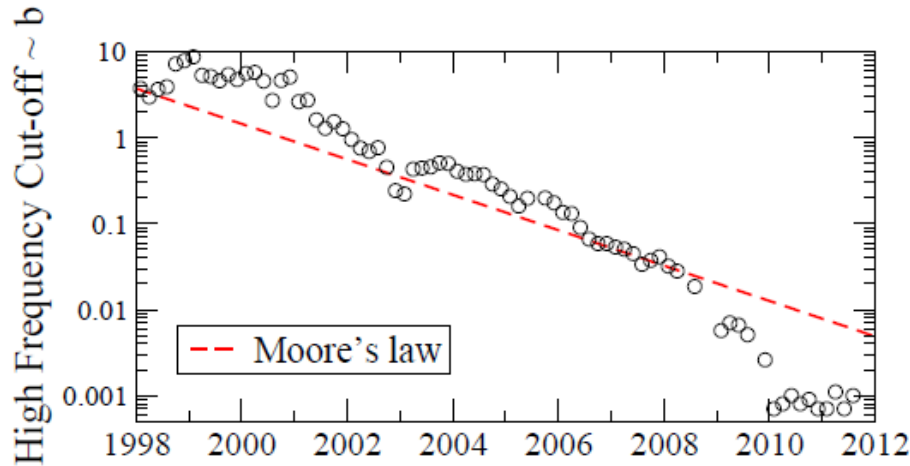
$$\sigma_B = \sigma(\alpha_T + \beta_T \mathcal{M} + \gamma_T \mathcal{M}^2 + O(\mathcal{M}^3)),$$

$$\beta = \sqrt{\frac{\pi}{2}} [1 - 2P(u_T > 0)],$$

$$\gamma_T = \sqrt{\frac{\pi}{2}} p_T(0) - \frac{1}{2\alpha_T}$$



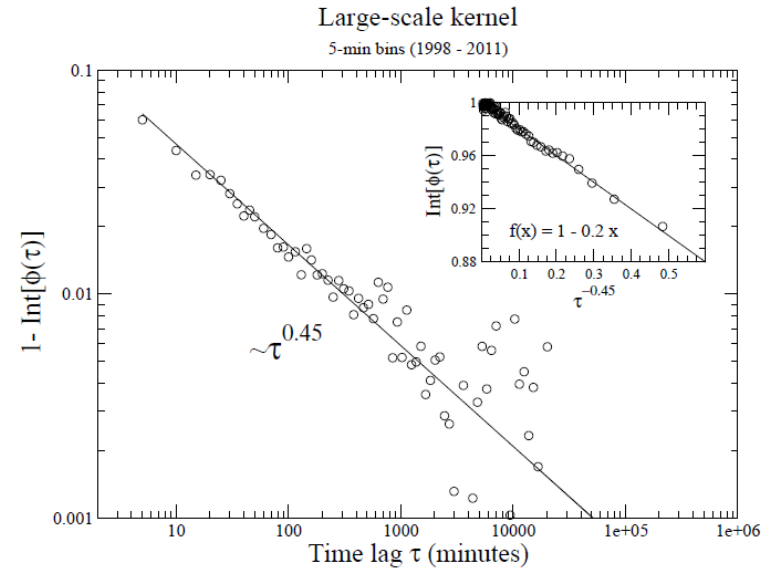
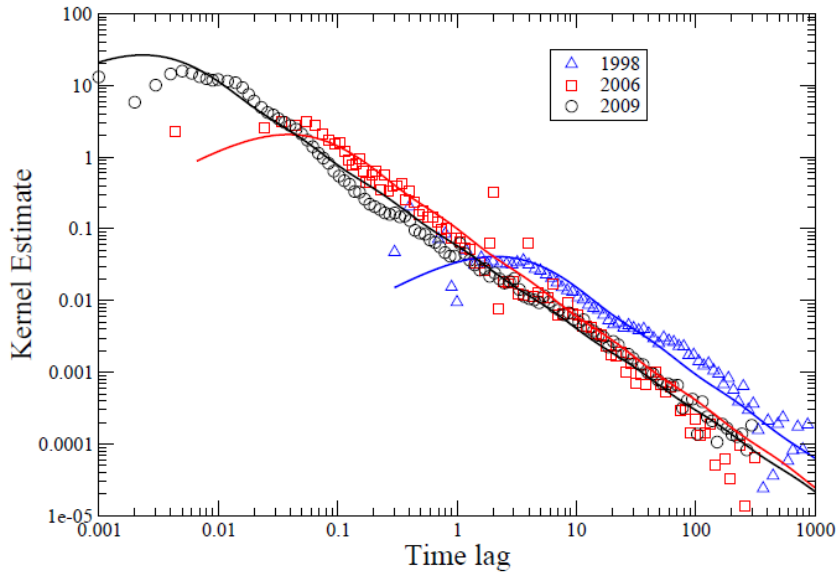
# Hawkes model of the activity feedback



$$\lambda(t) = \mu + \int_{-\infty}^t \phi(t-s) dN(s)$$

$$\phi(\tau) = \varphi_0 \Theta(\tau - \tau_0) \frac{\tau_0^\epsilon}{\tau^{1+\epsilon}}, \quad \epsilon > 0.$$

# A Clear empirical proof of near-criticality (fragility)



$$\nu(\tau) \propto \tau^{-\alpha}$$

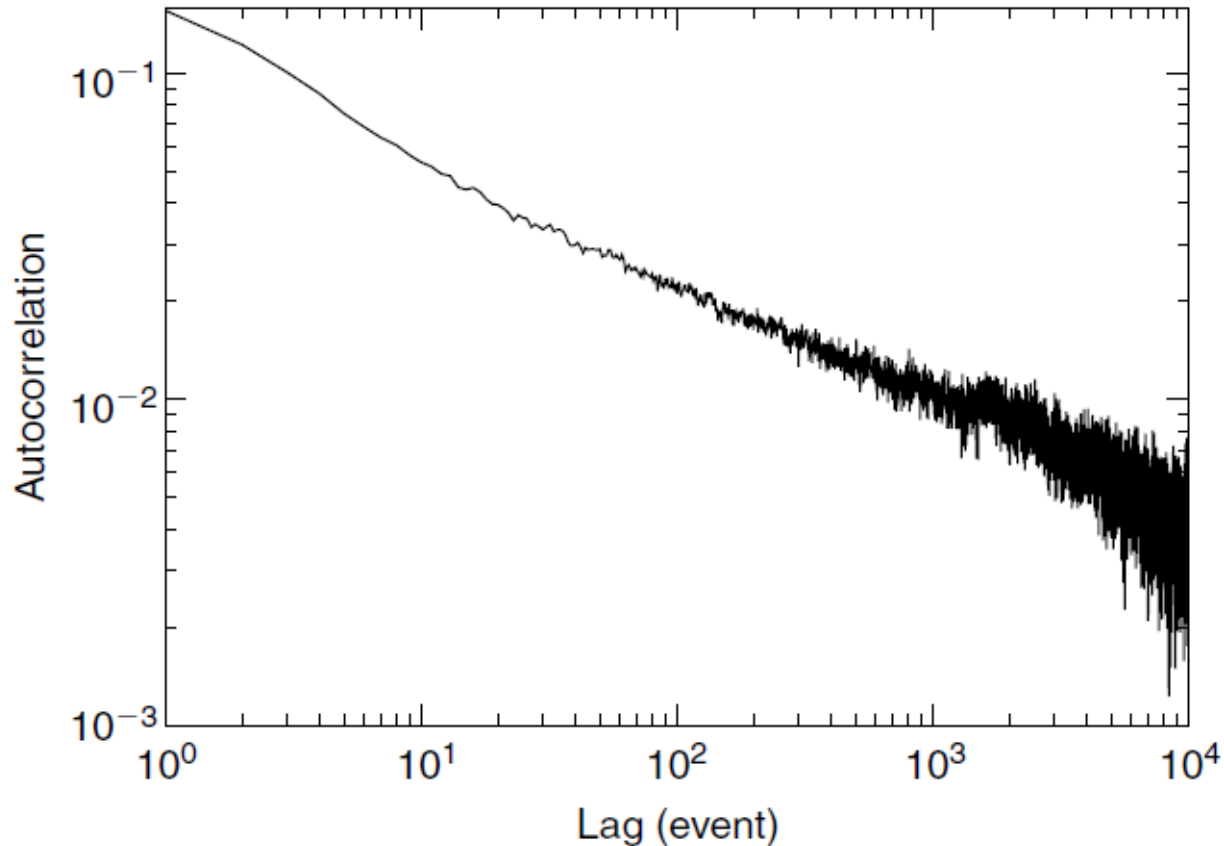
$$\alpha = 1 - 2\epsilon$$

$$\int_{-\infty}^{\infty} \nu(\tau) d\tau = \Lambda \left( \frac{1}{1-n} \right)^2$$

Note 1: flash crash prediction ???

Note 2: causal « feedback » or mere correlations ???

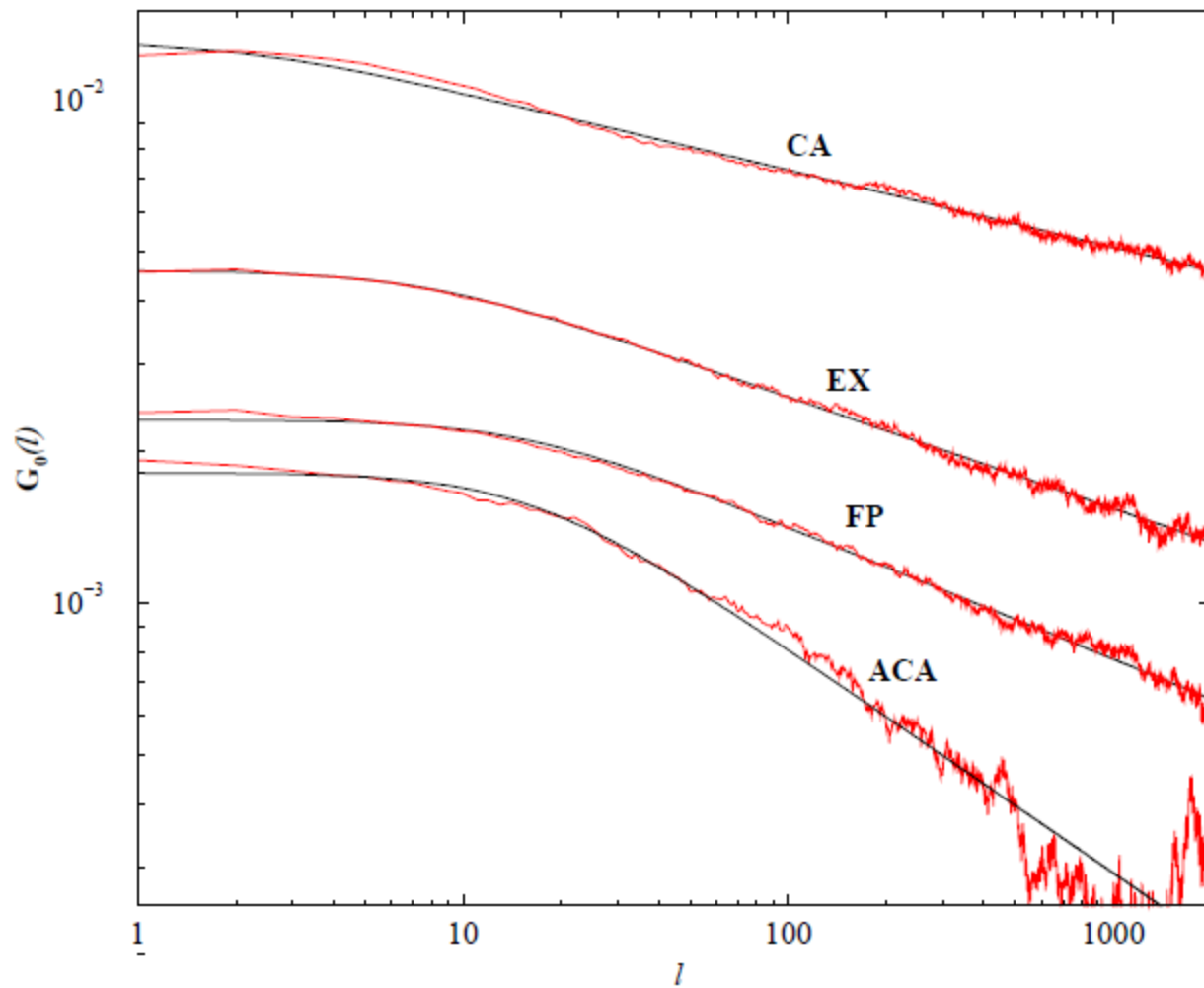
## Another « stylized fact » : Long memory in order flow



### Questions:

- 1) Where does this long memory come from? → small outstanding liquidity
- 2) How is it that returns are uncorrelated but order flow that impacts prices long-range correlated?? → impact is a decaying function of time

# Impact decay

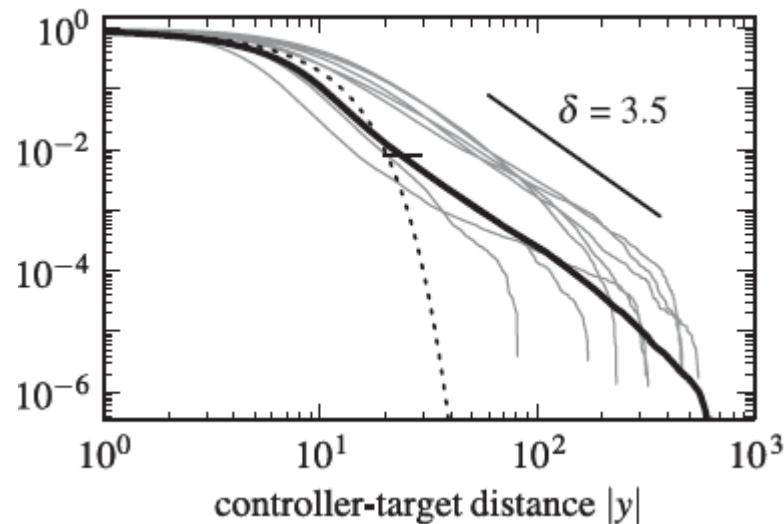


Criticality → Long range memory

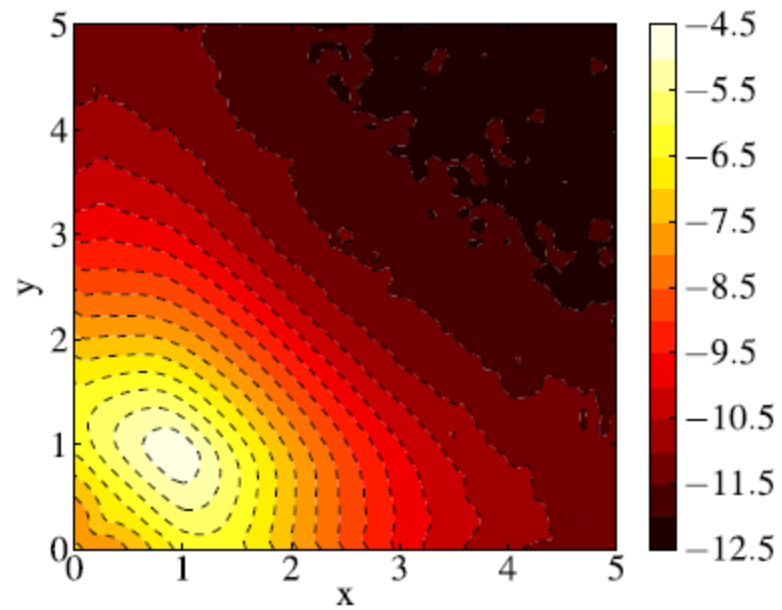
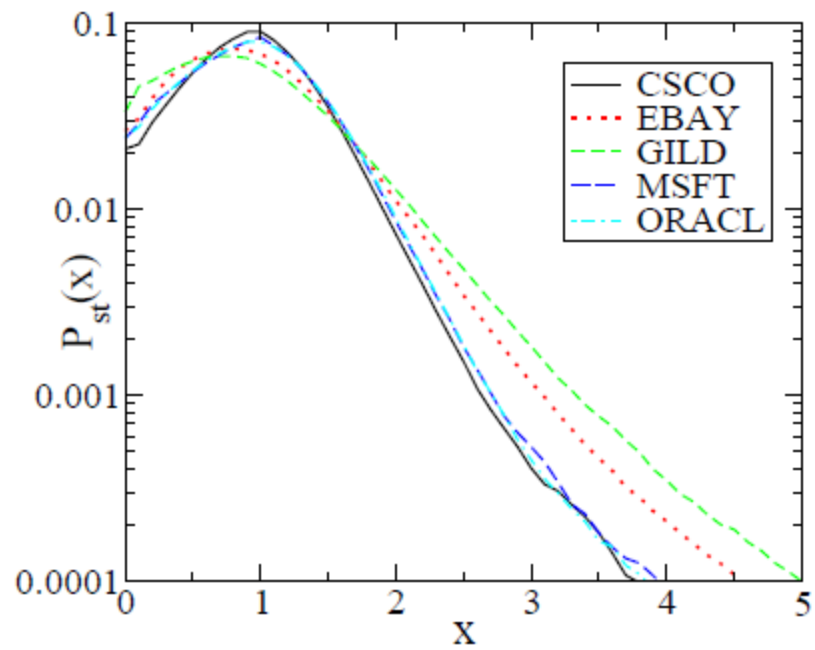
Criticality → Power-law returns ? An open question

### Criticality of Adaptive Control Dynamics

Felix Patzelt and Klaus Pawelzik



An interesting (relevant?) scenario for SOC



# Risk of Optimized Portfolios

- Let  $\mathbf{E}$  be an noisy estimator of  $\mathbf{C}$  such that  $\langle \mathbf{E} \rangle = \mathbf{C}$
- “In-sample” risk

$$R_{\text{in}}^2 = \mathbf{w}_E^T \mathbf{E} \mathbf{w}_E = \frac{G^2}{\mathbf{g}^T \mathbf{E}^{-1} \mathbf{g}}$$

- True minimal risk

$$R_{\text{true}}^2 = \mathbf{w}_C^T \mathbf{C} \mathbf{w}_C = \frac{G^2}{\mathbf{g}^T \mathbf{C}^{-1} \mathbf{g}}$$

- “Out-of-sample” risk

$$R_{\text{out}}^2 = \mathbf{w}_E^T \mathbf{C} \mathbf{w}_E = \frac{G^2 \mathbf{g}^T \mathbf{E}^{-1} \mathbf{C} \mathbf{E}^{-1} \mathbf{g}}{(\mathbf{g}^T \mathbf{E}^{-1} \mathbf{g})^2}$$

- Using convexity arguments, and for large matrices:

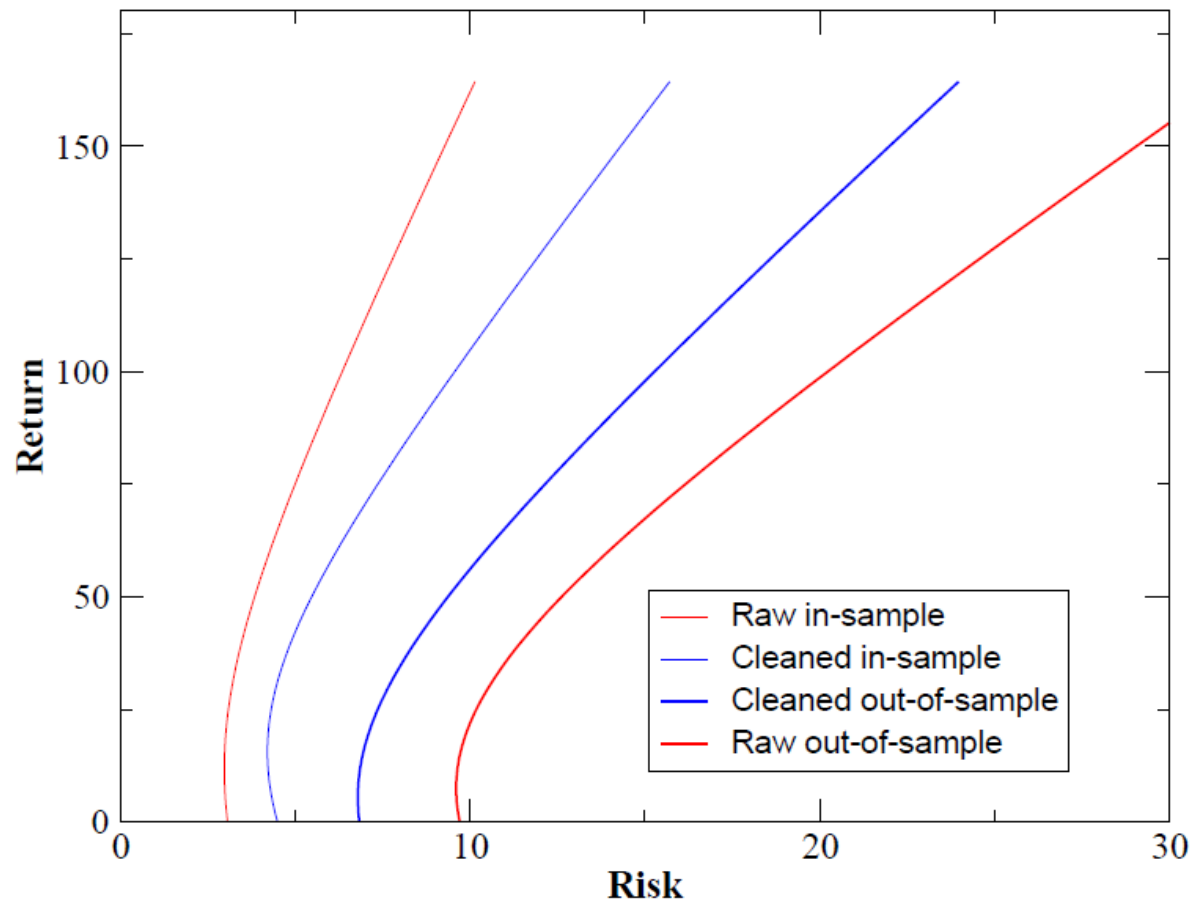
$$R_{\text{in}}^2 \leq R_{\text{true}}^2 \leq R_{\text{out}}^2$$

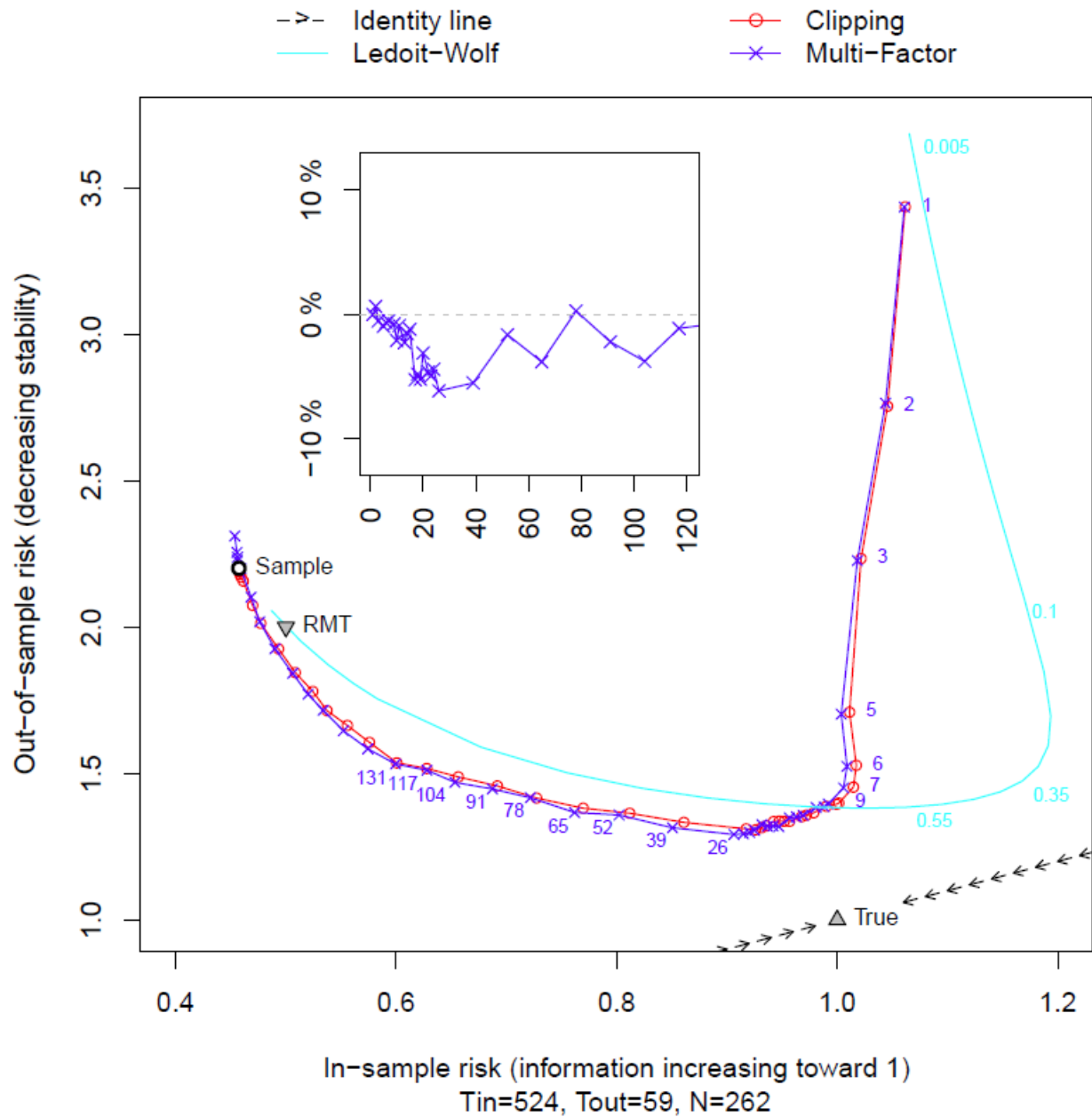
- Importance of eigenvalue cleaning:

$$w_i \propto \sum_{kj} \lambda_k^{-1} V_i^k V_j^k g_j = g_i + \sum_{kj} (\lambda_k^{-1} - 1) V_i^k V_j^k g_j$$

- Eigenvectors with  $\lambda > 1$  are suppressed,
- Eigenvectors with  $\lambda < 1$  are enhanced. Potentially very large weight on small eigenvalues.
- Must determine which eigenvalues to keep and which one to correct to avoid over-allocation on pseudo-low risk modes









CAPITAL FUND MANAGEMENT

# **LIQUIDITY, MARKET IMPACT, HFT : THE COMPLEX ECOLOGY OF FINANCIAL MARKETS**

**JP Bouchaud**, with: M. Wyart, J. Kockelkoren, M. Potters, B. Toth, S. Hardiman, Y. Lemperière, J. de la Taillade, R. Chicheportiche, N. Bercot, I. Mastromatteo..

# 1. WHAT'S GOING ON IN MARKETS? (BASICS)

- A « *pot-pourri* » of things we understood at CFM since 2002 from an empirical, practical and theoretical standpoints
- → An introduction to microstructure, price impact, HFT, FTT and all that..
- → The background « *Ariadne thread* »: **Are markets stable?**
- References
  - J.-P. Bouchaud, J. D. Farmer, and F. Lillo, **How Markets Slowly Digest Changes in Supply and Demand**, in Handbook of Financial Markets: Dynamics and Evolution (Elsevier: Academic Press, New York, 2008)
  - B. Toth, Y. Lempérière, C. Deremble, J. de Lataillade, J. Kockelkoren, and J.-P. Bouchaud, **Anomalous Price Impact and the Critical Nature of Liquidity in Financial Markets** PHYSICAL REVIEW X 1, 021006 (2011)
  - S. Hardiman, N. Bercot, J.P. Bouchaud, **Critical reflexivity in financial markets: a Hawkes process analysis**, arXiv:1302.1405

# 1. WHAT'S GOING ON IN MARKETS? (BASICS)

- Markets are the place where buyers meet sellers...and the price adapts so market clears
- But is it *that simple*? How does it work *really*?

The buyer: *How much is it?*

The seller: *\$1,50*

The buyer: *OK I'll take it*

The seller: *It's \$1,60*

The buyer: *What, you just said \$1,50?*

The seller: *That was before I knew you wanted it*

The buyer: *You can't do that!*

The seller: *It's my stuff*

The buyer: *But I need a 100 of these!*

The seller: *A 100? It's \$1,70.*

The buyer: *This is insane!*

The seller: *It's the law of offer and demand, buddy. You want it or not?*

# 1. WHAT'S GOING ON IN MARKETS? (BASICS)

- **Markets are the place where buyers meet sellers...** Is it *that simple*?
- At any instant in time, **there is no reason why supply and demand should match – or worse: buyers (sellers) want to hide how much they way to buy (sell)**
- Without a liquidity buffer, trading either stops or price make huge swings – or both...
- Tâtonnement and price formation: Efficient market theory tells us that any up- or down-tick away from the « fundamental » price should attract sellers/buyers so that **markets should function and be stable on their own....really ?**
- **Market operators have long realized the need of intermediaries: Market Makers/liquidity providers (spared by all FTT taxes to date)**

# 1. WHAT'S GOING ON IN MARKETS?

- Market Impact: orders to buy/sell, even **uniformed/random**, **impact prices** up/down – this is an empirical fact
- Trading/impact by itself can trigger more orders and *cascade*: **An important piece of information appears to be *trading itself***
  - self-reflexivity, endogenous dynamics, *excess volatility* – quite far from the EMT picture based on “fundamentals”
  - Understanding the **determinants of impact** is crucial:
    - **From a theoretical point of view**: why do price changes? How much do they reflect some underlying fundamental price (if at all)?
    - **From a practical point of view**: price impact can be a large fraction of transaction costs (see below)



## 2. MARKET MICROSTRUCTURE

- Microstructure: Bid/Ask quotes (Limit Orders) and trades (Market Orders), **Ask-Bid=Spread S**

- S is the **cost** of an immediate roundturn

- Market Makers post quotes, pocket that spread + fees but face adverse selection, or rather: **adverse impact** from MOs

- In the « old days » (1900 – 1980):

$$S \sim 70 \text{ bp} = 0.7\%$$

- In present electronic markets:

$$S \sim \text{a few bps (see below)}$$



LAST MATCH		TODAY'S ACTIVITY	
Price	25.1290	Orders	67,212
Time	11:42:15.597	Volume	12,778,400

BUY ORDERS		SELL ORDERS	
SHARES	PRICE	SHARES	PRICE
<u>600</u>	25.1240	<u>500</u>	25.1470
<u>3,200</u>	25.1230	<u>400</u>	25.1470
<u>3,200</u>	25.1220	<u>600</u>	25.1480
<u>4,000</u>	25.1220	<u>100</u>	25.1500
<u>100</u>	25.1210	<u>3,200</u>	25.1520
<u>100</u>	25.1200	<u>4,000</u>	25.1520
<u>3,200</u>	25.1200	<u>4,000</u>	25.1530
<u>9,600</u>	25.1130	<u>7,200</u>	25.1530
<u>4,000</u>	25.1130	<u>3,200</u>	25.1550
<u>400</u>	25.1130	<u>4,000</u>	25.1570
<u>4,000</u>	25.1130	<u>4,000</u>	25.1570
<u>8,000</u>	25.1120	<u>100</u>	25.1590
<u>5,000</u>	25.1110	<u>800</u>	25.1680
<u>3,000</u>	25.1100	<u>8,000</u>	25.1680
<u>1,000</u>	25.1100	<u>5,000</u>	25.1690

(237 more)

(119 more)



## 2. MARKET MICROSTRUCTURE

- Markets are “hide and seek” games (cf. little dialogue) – liquidity is inherently fragile (more below)

- Small revealed liq.** vs. large « latent » liq.

- Total turnover in a day: 0.5% of Mcap

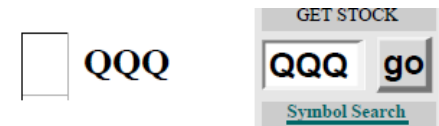
- But total volume in OB: **only  $10^{-5}$  of Mcap**

« **Hide and Seek** »:

\* Adverse selection → low liquidity and *signs of market orders show long memory*

« **Tit for tat** »:

\* Dynamics of bid/ask/market order volume is approx. scale invariant: mismatches create instabilities



LAST MATCH		TODAY'S ACTIVITY	
Price	25.1290	Orders	67,212
Time	11:42:15.597	Volume	12,778,400

BUY ORDERS		SELL ORDERS	
SHARES	PRICE	SHARES	PRICE
<u>600</u>	25.1240	<u>500</u>	25.1470
<u>3,200</u>	25.1230	<u>400</u>	25.1470
<u>3,200</u>	25.1220	<u>600</u>	25.1480
<u>4,000</u>	25.1220	<u>100</u>	25.1500
<u>100</u>	25.1210	<u>3,200</u>	25.1520
<u>100</u>	25.1200	<u>4,000</u>	25.1520
<u>3,200</u>	25.1200	<u>4,000</u>	25.1530
<u>9,600</u>	25.1130	<u>7,200</u>	25.1530
<u>4,000</u>	25.1130	<u>3,200</u>	25.1550
<u>400</u>	25.1130	<u>4,000</u>	25.1570
<u>4,000</u>	25.1130	<u>4,000</u>	25.1570
<u>8,000</u>	25.1120	<u>100</u>	25.1590
<u>5,000</u>	25.1110	<u>800</u>	25.1680
<u>3,000</u>	25.1100	<u>8,000</u>	25.1680
<u>1,000</u>	25.1100	<u>5,000</u>	25.1690

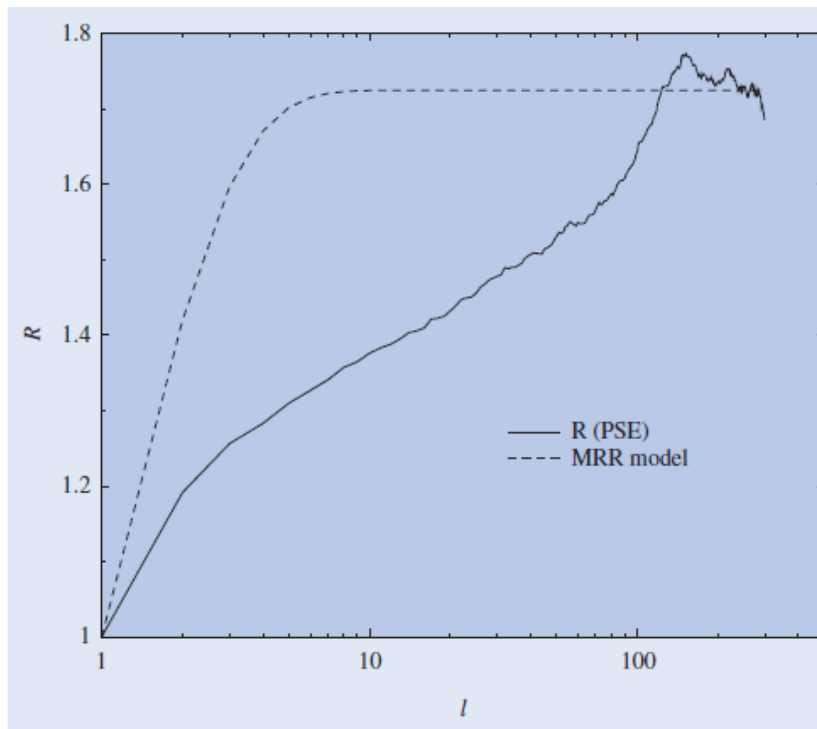
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## 2. MARKET MAKING IS HARD

- Market makers attempt to earn the spread but lose **adverse impact**
- How does the **average impact** (response) of *single trades* look like ?

$$\mathcal{R}_\ell(v) = \langle \epsilon_i \cdot (m_{\ell+i} - m_i) \rangle |_{v_i=v}$$



- Slow: note the **log scale** in x
- Impact increases by a factor  $\sim 2$  with lag (due to persistence of market orders)
- Dotted line: Markovian order flow (MRR model)

## 2. MARKET MAKING IS HARD

- Gains of a **SIMPLE** market making strategy, **with inventory control**, but **no privileged information** ( $\varepsilon$ -participation to all trades)

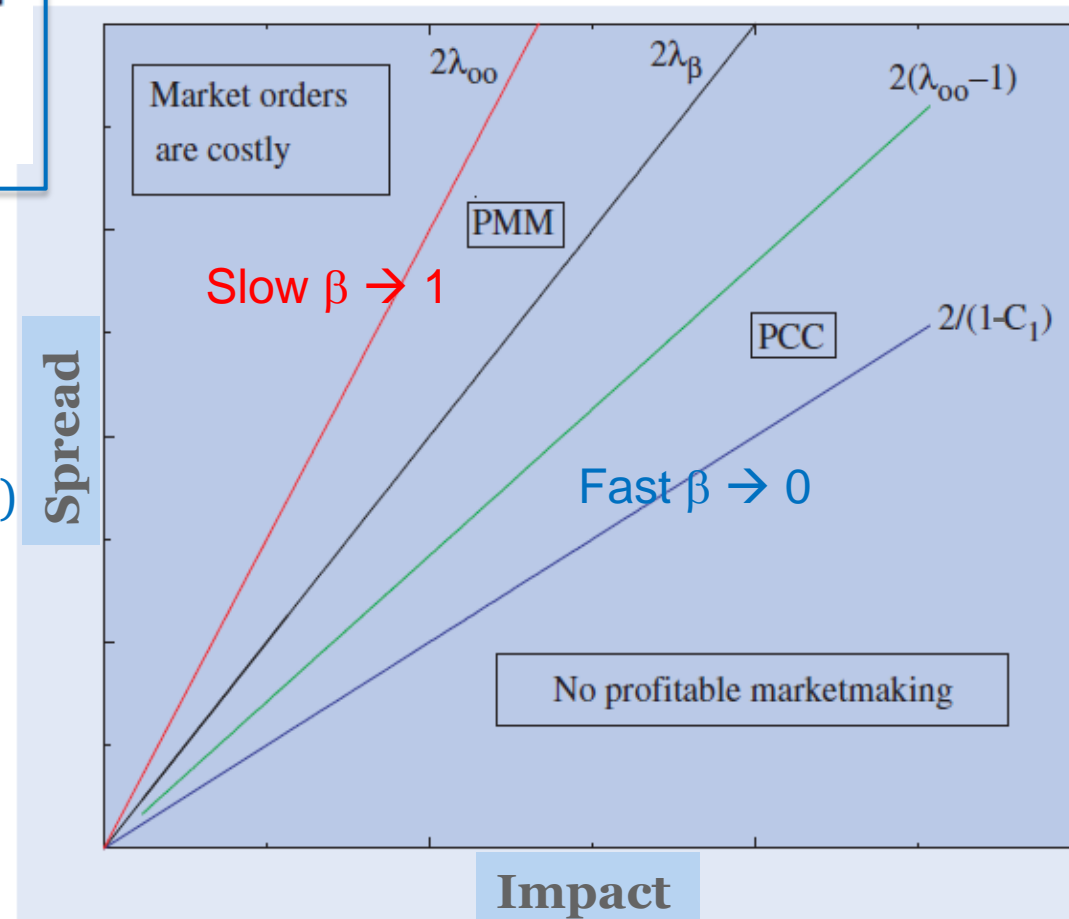
$$\frac{\mathcal{G}_L(\beta)}{T\varphi_0\langle v \rangle} = \frac{\langle vS \rangle}{2\langle v \rangle} \left[ 1 - \frac{1-\beta}{\beta} \sum_{\ell=1}^{\infty} \beta^{\ell} C(\ell) \right] - \frac{1-\beta}{\beta} \sum_{\ell=1}^{\infty} \beta^{\ell} \frac{\langle v\mathcal{R}_{\ell}(v) \rangle}{\langle v \rangle},$$

$\beta$ : sets the round turn frequency

**Note 1:** HF ( $\beta \rightarrow 0$ ) mechanically increases MM profits and decreases inventory risk

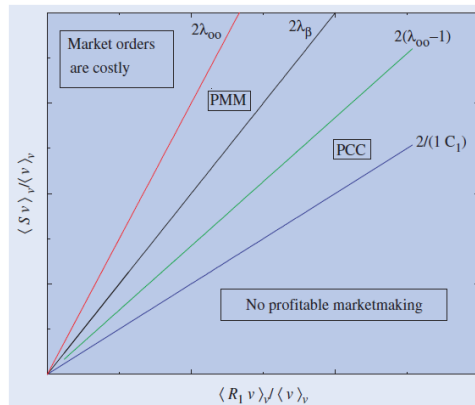
**Note 2:** In « equilibrium »:  $S=R/(1-C_1)$

**Note 3:** More info about short-term price moves helps bringing S down

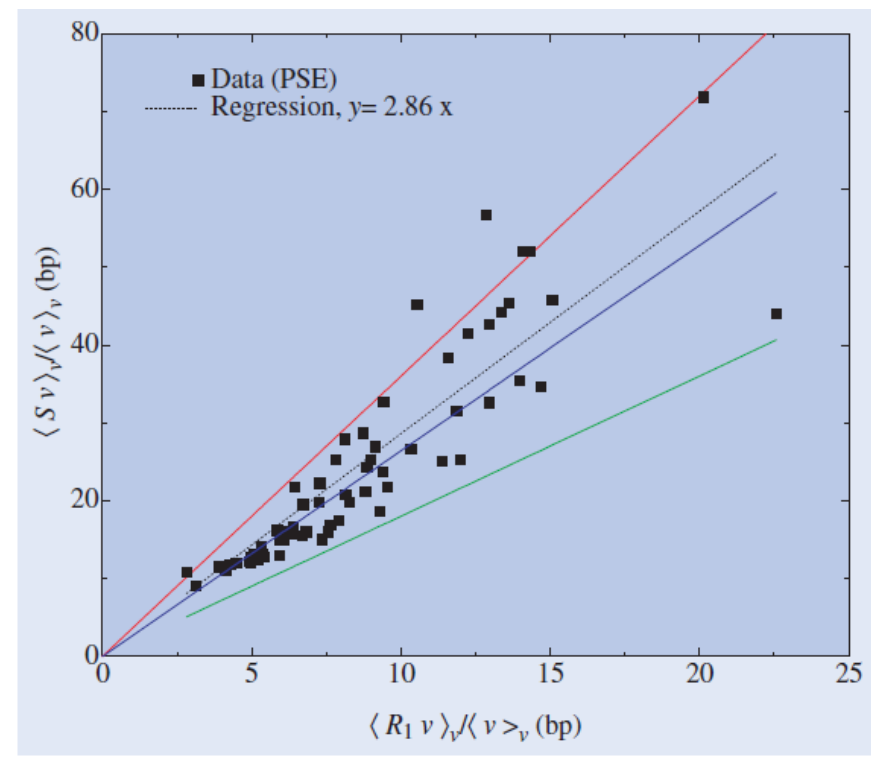
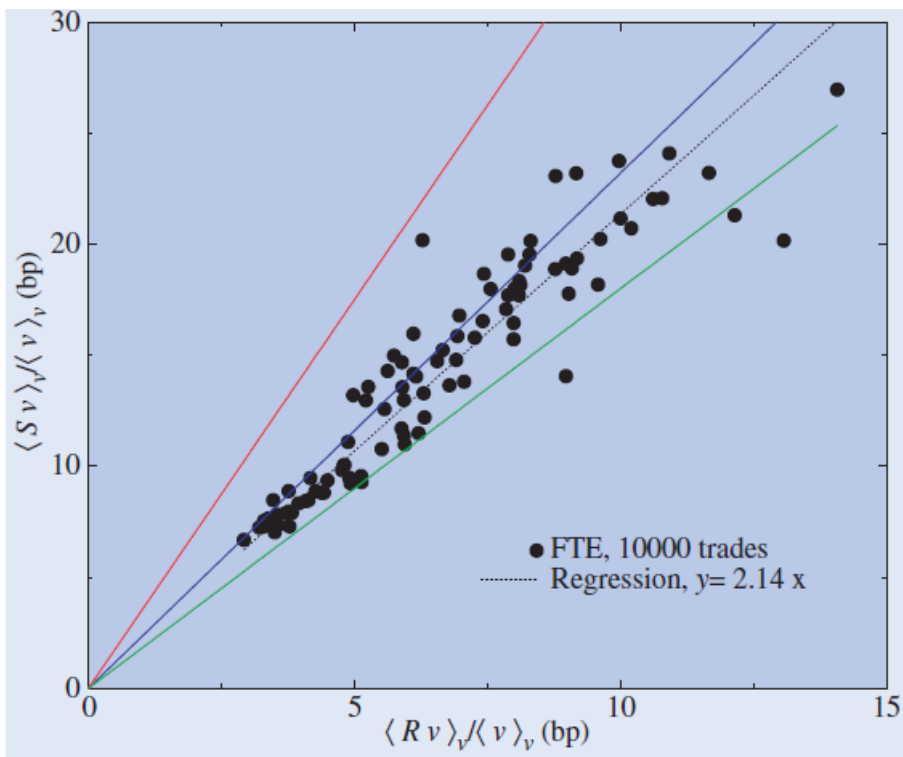


## 2. MARKET MAKING IS HARD

- Gains of a (simple) market making strategy with inventory control

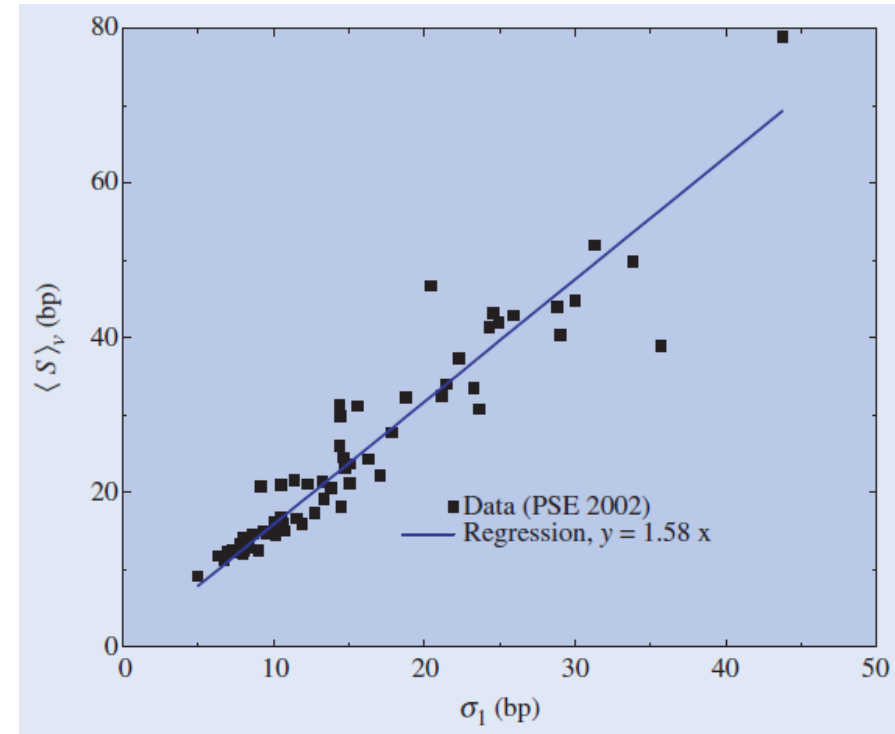
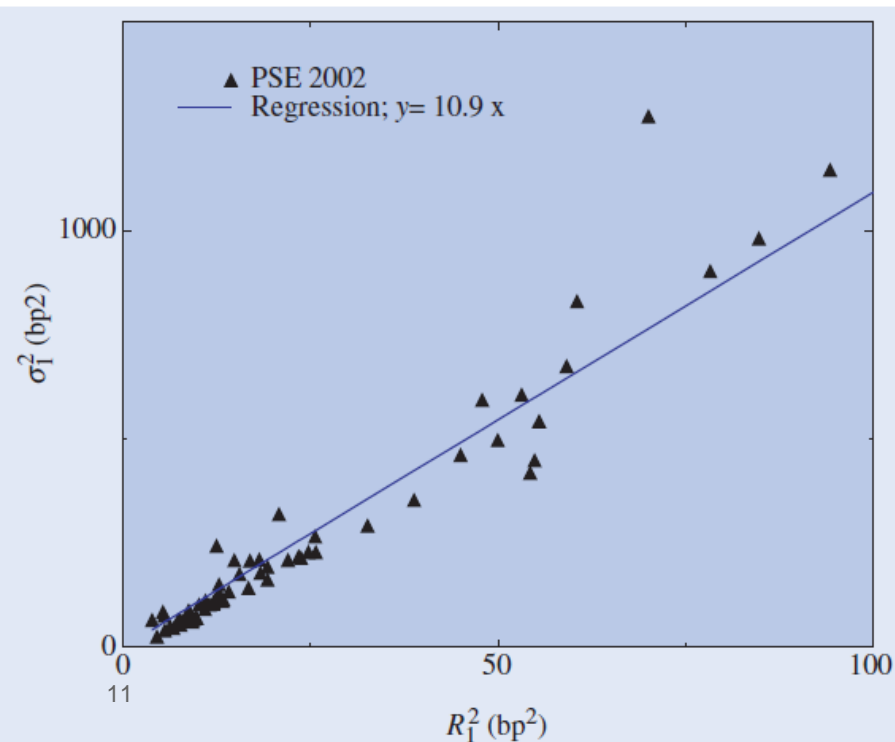


The « blue » line: an equilibrium market ecology



## 2. SPREADS MATTER

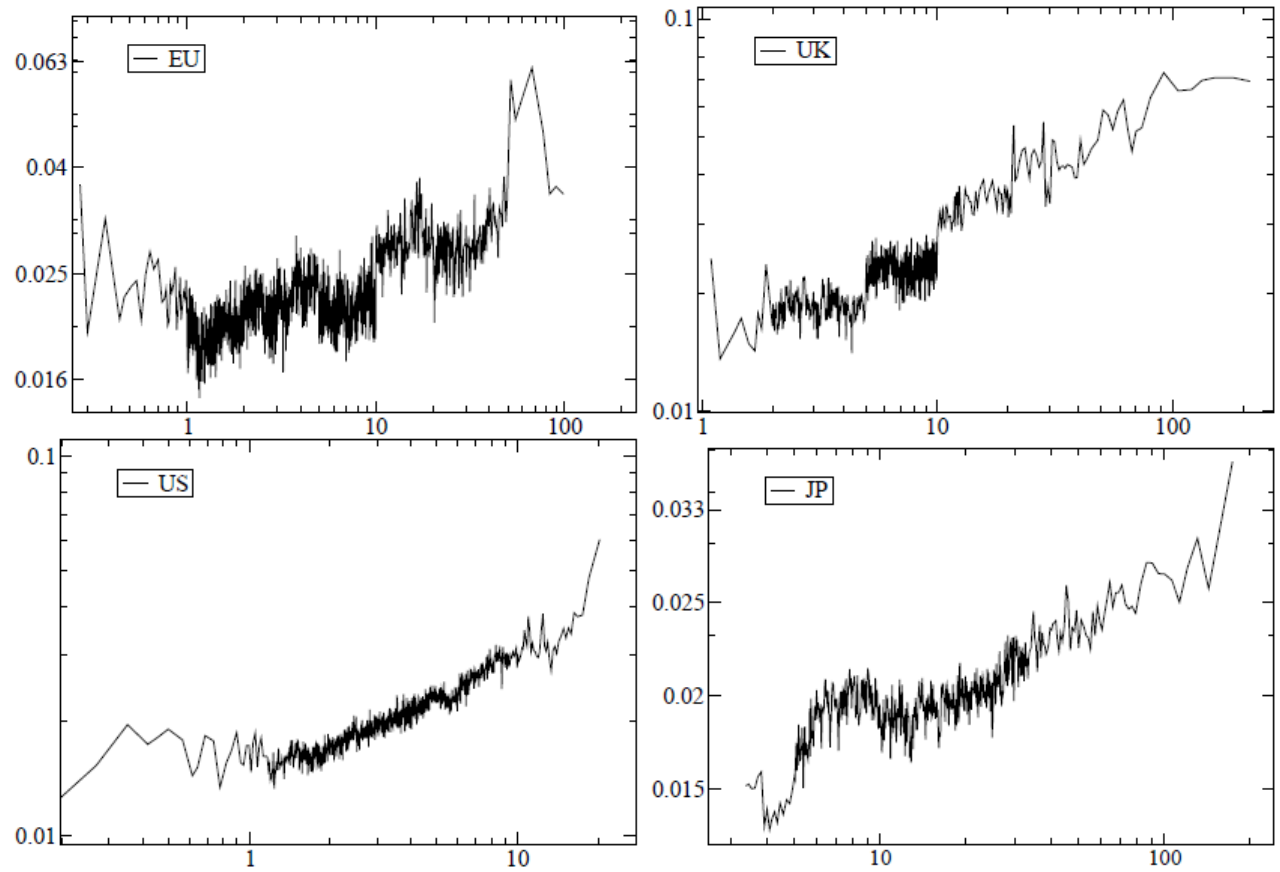
- Spread and Response (impact) are indeed proportionnal
- → Vol. *per trade* and Response must be as well
- Vol. is due to trade impact, little from news jumps (cf Joulin et al.)
- → A new « law »: **vol. per trade  $\sigma_1 = \alpha S$**  ✓



## 2. SPREADS MATTER!

Spreads (or taxes!) also influence vol per unit time:

- CFM's data:
- Larger spreads  
→ Larger impact  
of single trades
- Would FTT  
*really* reduce vol?  
**Note:** an FTT of 10bp  
is ~20 x the profits  
of (passive) HFT/MM
- Agrees with other  
Studies (Hau 2002)



## 2. MARKET MAKING IS HARD

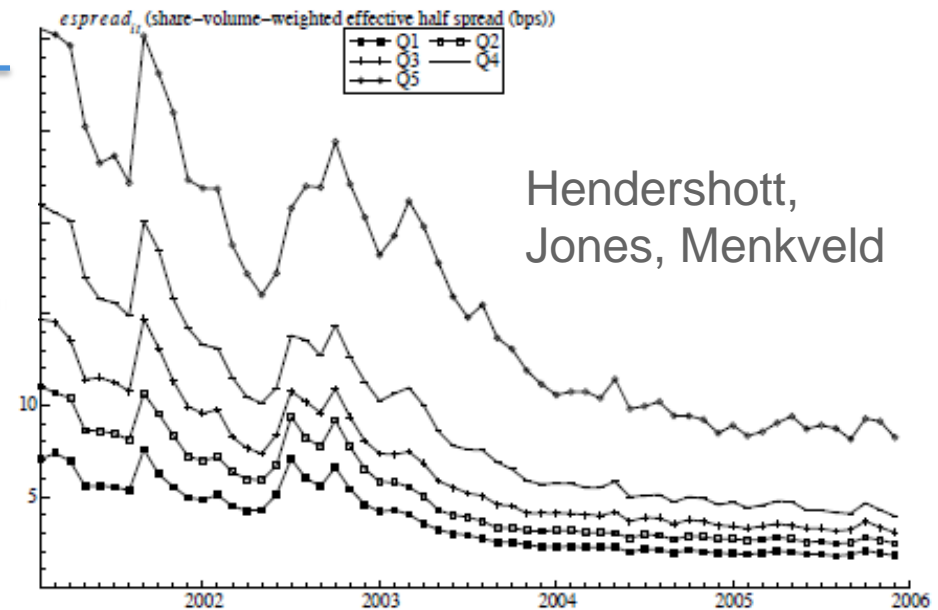
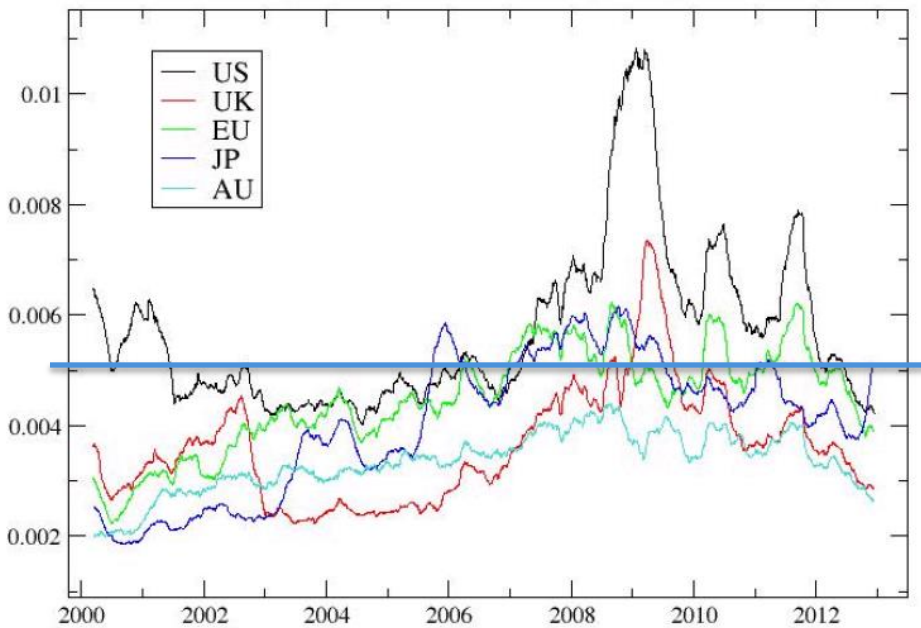
- **So** – Market makers attempt to earn the spread but lose adverse impact: profits are *necessarily small* in a competitive liquidity providing setting (i.e. not 30 years ago...), and highly skewed
- Liquidity in the book is SMALL! (ditto)
- *Liquidity is inherently fragile: any blip in perceived risk scares MM/liquidity providers/HFT away and leads to increased spreads (this may cascade: spread is itself a risk gauge)*
- **Liquidity is essential to markets** – but **how much should we pay for it ?**
- Estimated profits of MM: 1 bp/daily transactions = 2 B\$/year for US stocks (ballpark in 2009, and decreases fast); compare with Kerviel/London whale.
- Is HFT/MM stable ? Probably not but was it ever before? Are today's markets more unstable than in the past?



# HFT & MARKET STABILITY: SOME EMPIRICAL FACTS

- Stocks turnover has (only) increased by a factor  $\sim 2$  since 95
- Spreads have noticeably decreased (70 bp  $\rightarrow$  a few bps)

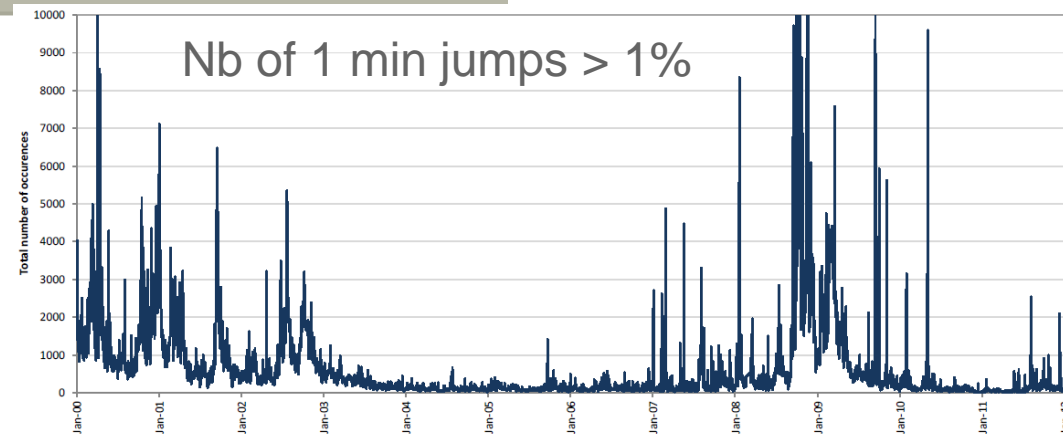
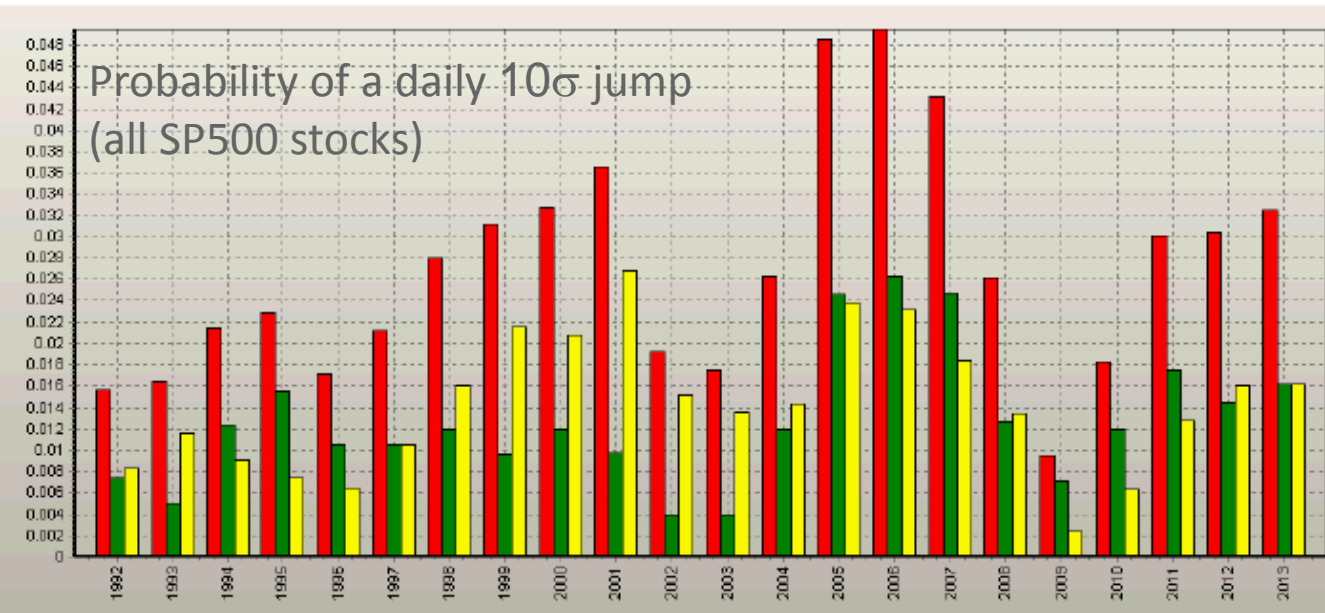
fraction of market cap traded per day  
ratio of total daily turnover and sum of marketcaps, averaged over 100 days





# HFT & MARKET STABILITY: SOME EMPIRICAL FACTS

- Jump statistics' tail stable ( $1/x^3$ ); jump frequency has not increased (but scales as  $\sigma^3$ ...)

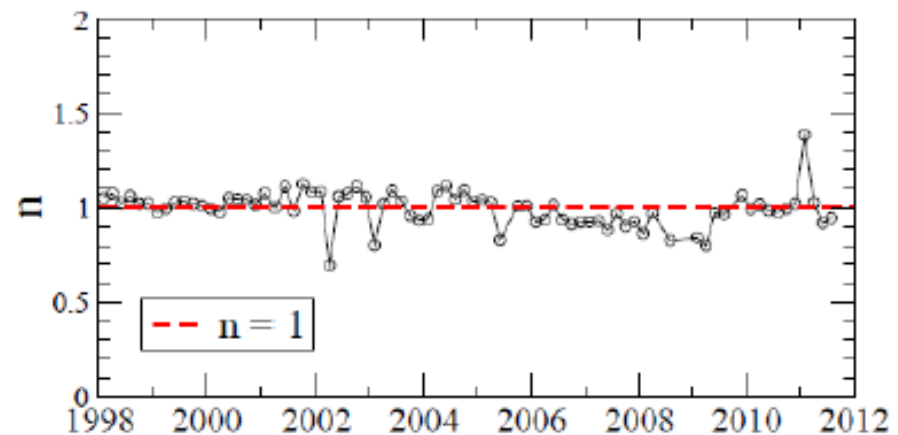
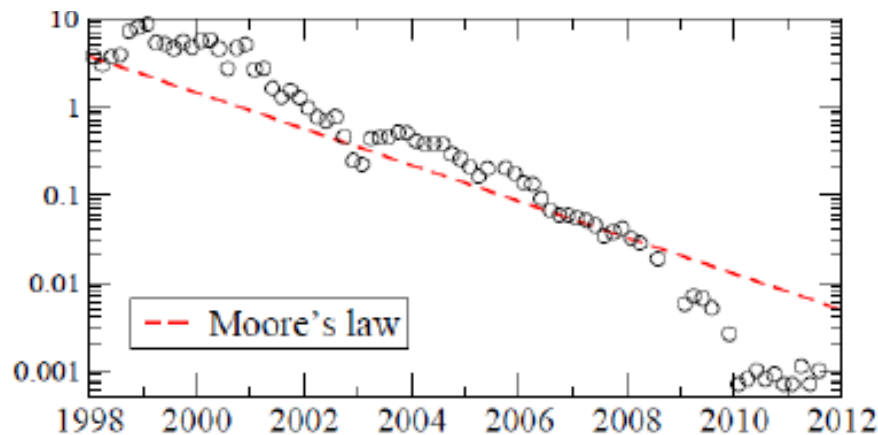


# HFT & MARKET STABILITY: SOME EMPIRICAL FACTS

- “Self-reflexivity” has not increased (at variance with Filimonov-Sornette)

Hawkes process description of mid point changes:

$$\lambda(t) = \mu + \int_{-\infty}^t \phi(t-s) dN(s) \quad n \equiv \int_0^{\infty} \phi(\tau) d\tau : \text{Branching ratio}$$



« Self reflexivity » in markets is CRITICAL, and always has been:  
**Same movie, just played faster**

### 3. MARKET IMPACT AND THE **TRUE** COST OF TRADING

- What is the true cost of trading ?
- Naïve answer from above: a fraction of the bid-ask spread
- True for **small** trades – but as soon as size Q is substantial, **impact costs become dominant** (and  $\gg$  than naively thought!)

$$I(Q) = Y\sigma_T \sqrt{\frac{Q}{V_T}} \quad [\text{BARRA 97(!), Almgren, Engle, JPM, DB, LH, CFM...}]$$

- Impact of “metaorders” is dominated by “latent liquidity” and very little by microstructure (at variance with single orders) – little dependence on spreads, LO/MO, **HFTs (!)**, etc.
- Order of magnitude: for  $Q=2\%$  of daily volume:

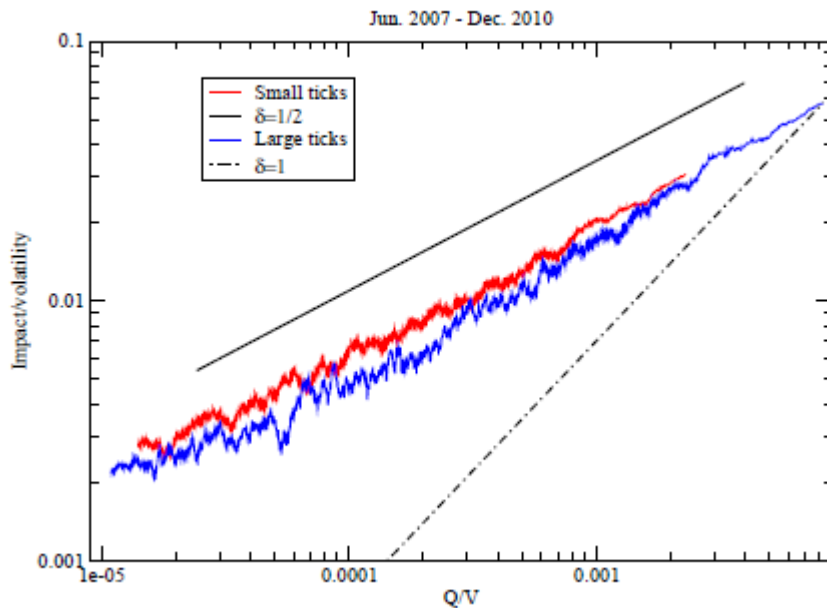
$$\text{Cost} = 1 \text{ bp} + 0.5 * 2\% * \text{sqrt}(2\%) = 1 + 14 \text{ bp}$$

- Impact is **unavoidable** and much larger than spreads

# IMPACT OF METAORDERS (CFM DATA)

Empirical result: a square-root (non additive) impact!

Results from 500,000 CFM trades:



$$I(Q) = Y \sigma_T \sqrt{\frac{Q}{V_T}}$$

Independent of: Markets, epoch, style of trading (LO/MO), execution time, data treatment... (!!)

Still true in 2013 with constant Y factor!

- ▷ Note 1:  $I(Q) \ll \sigma_T$  whenever  $Q \ll V_T$
- ▷ Note 2: **Singular for  $Q \rightarrow 0$** : Trading  $\sim 1\%$  of average daily volume moves the price by  $\sim 10\%$  of daily volatility!

# IMPACT OF METAORDERS

## Why is impact a square-root ? (Latent vs revealed liquidity)

After having traded  $Q/2$ , the next  $Q/2$  will have less impact  
 $\Rightarrow$  this means that there must be increasing volume available deeper in the book

However the typical order book that one **observes** is not like that.

There has to be some **latent** volume that only appears when we push the price.

Suppose the latent volume grows linearly with depth:

$$V_{\pm}(p) \propto |p - p_0|$$

$$Q = \int_{p_0}^{p_0+I} V_{+}(p) dp \longrightarrow p = p_0 + a\sqrt{Q} \quad (2)$$

Note:  $V_{\pm}(p \rightarrow p_0) \rightarrow 0\dots$

## Small liquidity $\rightarrow$ Market fragility

## Why is « latent volume » linear in distance from price?

A dynamical theory of liquidity for  $\varepsilon$ -intelligence markets (analytical):

Let  $\rho(u)$  be the density of the book at a distance  $u$  away from the best price. Let's assume that the price process is **diffusive**.  
In the (moving) reference frame of the mid price,  $u = p - p(t)$ :

$$\frac{\partial \rho}{\partial t} = \frac{\sigma^2}{2} \frac{\partial^2 \rho}{\partial u^2} + \lambda - \nu \rho \quad (3)$$

$$\Rightarrow \rho_{st}(u) = \rho_\infty \left(1 - e^{-u/u^*}\right); \quad u^* = \sqrt{\frac{\sigma^2}{2\nu}} \quad (4)$$

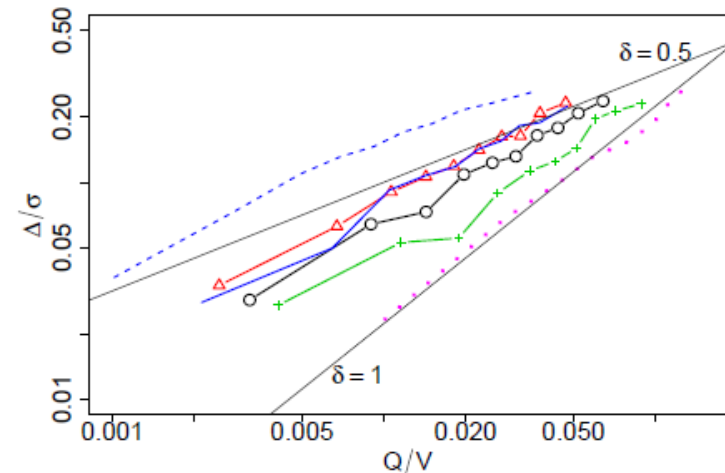
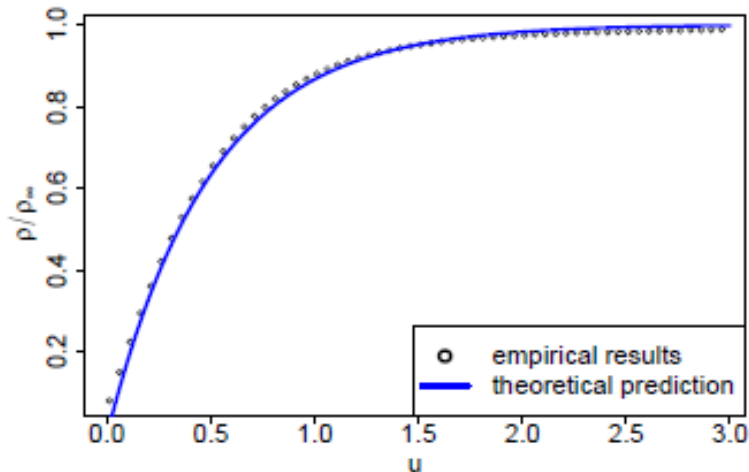
Close to the current price ( $u \rightarrow 0$ ):  $\rho(u) \approx \rho_\infty \frac{u}{u^*}$ .

Generic behaviour, valid whenever  $\lambda, \nu$  are regular around  $u = 0$ , but provided prices are diffusive

# IMPACT OF METAORDERS

Why is « latent » volume linear in distance from price?

A numerical « agent based » model of liquidity ( $\epsilon$ -intelligence)



# CONCLUSION: INHERENT MARKET FRAGILITY

## The critical nature of liquidity

- Local liquidity is vanishingly small by necessity! (it is eaten by the diffusive motion of prices)
- This imposes a splitting up of metaorders and long-range memory in the sign of trades...
- ...and leads to a breakdown of linear response and an anomalously large impact for small trades ( $\Rightarrow$  concave impact)
- Liquidity fluctuations are bound to play a crucial role  $\rightarrow$  microcrises and jumps in prices **without news**



The endogenous dynamics of  
markets:  
price impact, feedback loops  
and instabilities

J.P. Bouchaud



*<http://www.cfm.fr>*

# The Sacred Lore of Efficient Markets

- Why and how do market prices move?
- Efficient market theory:
  - ▷ Rational Agents and Market in “Equilibrium”
  - ▷ Prices reflect faithfully the Fundamental Value of assets and only move because of exogenous unpredictable news.
- Platonian markets that merely reveal fundamental values without influencing them
  - ▷ or is it a mere tautology??
  - ▷ If we had a way to check, we would not need markets!

# The Sacred Lore of Efficient Markets

- **Markets are fundamentally stable:** any mispricing is arbitrated away by those who “know”

▷ but who exactly is supposed to know the price??

(An efficient market is such that prices are correct within a factor  $2\sqrt{}$  (F. Black))

- Crashes can only be **exogenous**, not induced by markets dynamics itself – oh really??
- *Market stability is trivial and not even an interesting question* (M. Friedman) – when **feedback loops and instabilities** are everywhere!

# Eyes Wide Shut

- I think that calls for a radical reworking of the field go too far. [...] The financial crisis did not discredit the usefulness of economic research and analysis by any means,

still: The crisis should motivate economists to think further about their modeling of HUMAN BEHAVIOUR. Most economic researchers continue to work within the classical paradigm that assumes rational, self-interested behavior and the maximization of expected utility,

and: Another issue brought to the fore by the crisis is the need to better understand the determinants of LIQUIDITY in financial markets. The notion that financial assets can always be sold at prices close to their fundamental values is built into most economic analysis...

– Ben Bernanke, Princeton, September 2010

# Quite, Pr. Bernanke...(Human behaviour)

- Let's face it: we, humans, are lost in the dark –

swamped by noisy/superabundant information and *radical uncertainty*.

We make **errors**, are subject to **biases**\*, have regrets...we do not behave as game theorists would like us to behave...

- **Animal spirits**

*Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as the result of **animal spirits** – a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities. (Keynes)*

\*see: **D. Kahneman**, *Thinking fast and slow*

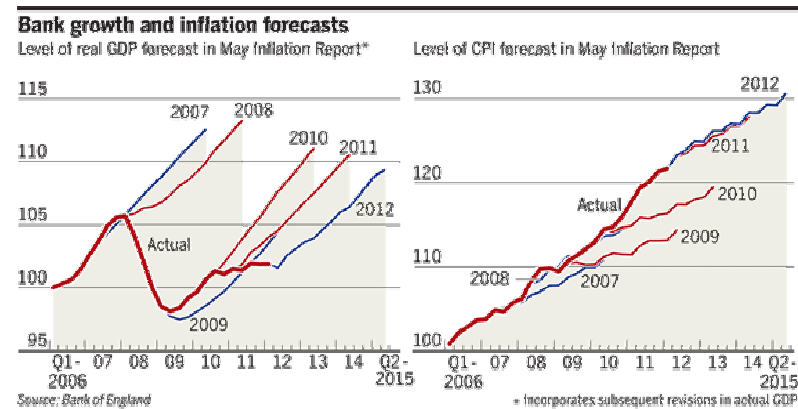
# Quite, Pr. Bernanke...(Human behaviour)

- We rely on “fast and frugal” rules to make suboptimal decisions (Gigerenzer)
  - ▷ We are strongly influenced by the behaviour of others (who might have more information) – panic feeds panic
  - ▷ We are strongly influenced by past patterns (that might repeat) – trends feed trends
  - ▷ We are strongly risk adverse and intuitively cope with unknown unknowns – Survival instinct and risk limits
- Theories that treat these effects consistently are still at an early stage
  - ▷ see e.g. JPB, *Crises and collective socio-economic phenomena: cartoon models and challenges*, arXiv:1209.0453

# Can we be rational? Easy shots at “forecasting”

- A 2002 International Monetary Fund study looked at consensus forecasts (the forecasts of large groups of economists) that were made in advance of 60 different national recessions in the 90s: in 97% of the cases the economists did not predict the contraction a year in advance. On those rare occasions when economists did successfully predict recessions, they significantly underestimated their severity
- Earning forecasts are overoptimistic on average, worse than just repeating last year's result, and 10 times less dispersed than wrong! (Guedj & JPB)
- A. Bénassy-Quéré reports that FX experts fare worse than 50-50 in predicting next year's move of the \$!
- Bloomberg.com on the 2008 performance of Wall Street analysts: none predicted a down year and the average forecast was for a gain of 11 percent. Instead, the S&P 500 tumbled 38 percent and \$29 trillion was erased from global markets.

# BoE forecasts



Macro models failed to predict the crisis and seemed incapable of explaining what was happening to the economy in a convincing manner. As a policy-maker during the crisis, I found the available models of limited help. In fact, I would go further: in the face of the crisis, we felt abandoned by conventional tools. JC Trichet



# Quite, Pr. Bernanke...(Liquidity)

- Liquidity and impact of trades
  - ▷ Empirical fact: Trading, even with relatively small volumes in usual market conditions, moves prices in a measurable way – see below
  - ▷ This is called PRICE IMPACT
- Impact transforms trades into price changes: this is a key ingredient to understand *market dynamics and stability*
- Impact also contributes to costs and limits the size of trading strategies

## Quite, Pr. Bernanke...(Liquidity)

- **Efficient market story:** Informed agents successfully forecast short term price movements and trade accordingly. This results in correlations between trades and price changes, but **uninformed trades should have no price impact** – prices must stick to “Fundamental Values”
- **An empirically rooted story:** since there is no easy way to distinguish “informed” from “non informed” traders, **all trades do statistically impact prices** (✓)
  - ▷ Agents believe/fear that trades **might** contain useful information they don't have
  - ▷ Even silly/random trades do impact market prices: **a transmission belt for feedback loops and avalanches**

# Endogenous crashes: Impact-induced instabilities

- Impact-induced feedback loops that can and do lead to crises
  - ▷ Pattern following: trends feed trends
  - ▷ Crowd following: panic feeds panic
  - ▷ The risk aversion/liquidity feedback loop and flash crash(es)
  - ▷ Model induced feedback loops: e.g. the BS feedback loop in 1987, the CDO feedback loop in 2008,...
  - ▷ Risk/Regulation induced feedback loops: mark to market, risk limits, margin calls, deleveraging...
  - ▷ Intermarket contagion: spillovers, induced correlations

# Some questions with empirical answers

- Financial markets offer **Terabytes of information** (daily) to try to investigate why and how prices move, and offer an **ideal test bed** for some fundamental questions in economics/finance:

- **A) Exogenous vs. Endogenous dynamics**

**Are news really the main determinant of volatility?**

- **B) How do trades impact prices?**

**How sensitive is the market to trades?**

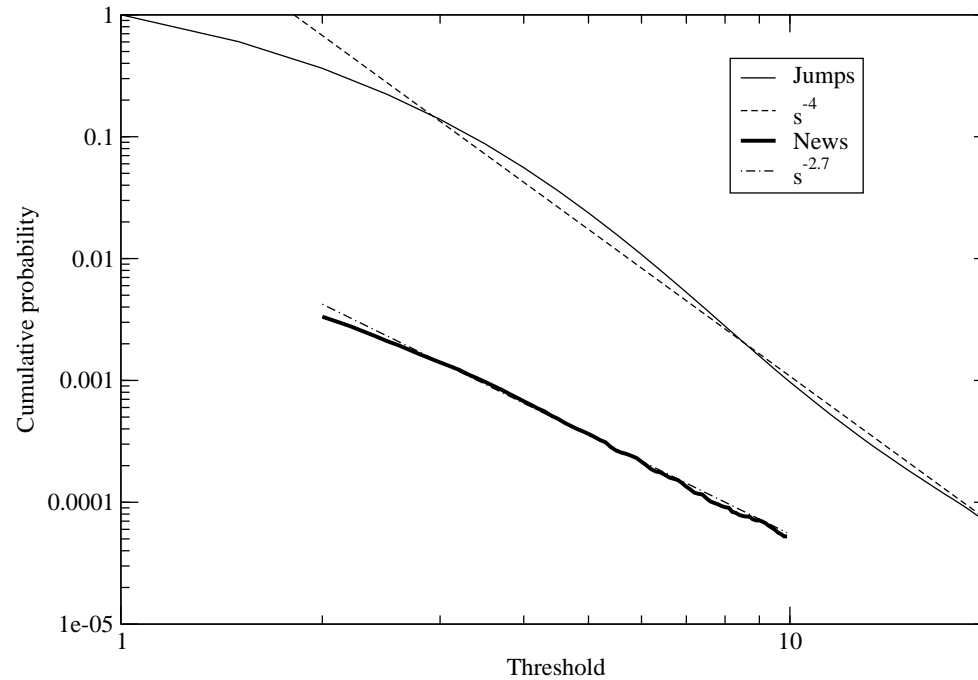
# A) Exogenous or endogenous dynamics?

- Accumulating body of observations
  - ▷ Power-law distribution of jump sizes: crises of all scales (like earthquakes)
  - ▷ Most jumps are unrelated to news and look endogenous
  - ▷ Excess volatility, with long range memory – looks like endogenous intermittent noise in complex systems (turbulence, Barkhausen noise, earthquakes, etc.)
- To a large extent: Universal observations in time, space & assets – details may evolve, but main features remain

## A) Exogenous or endogenous dynamics?

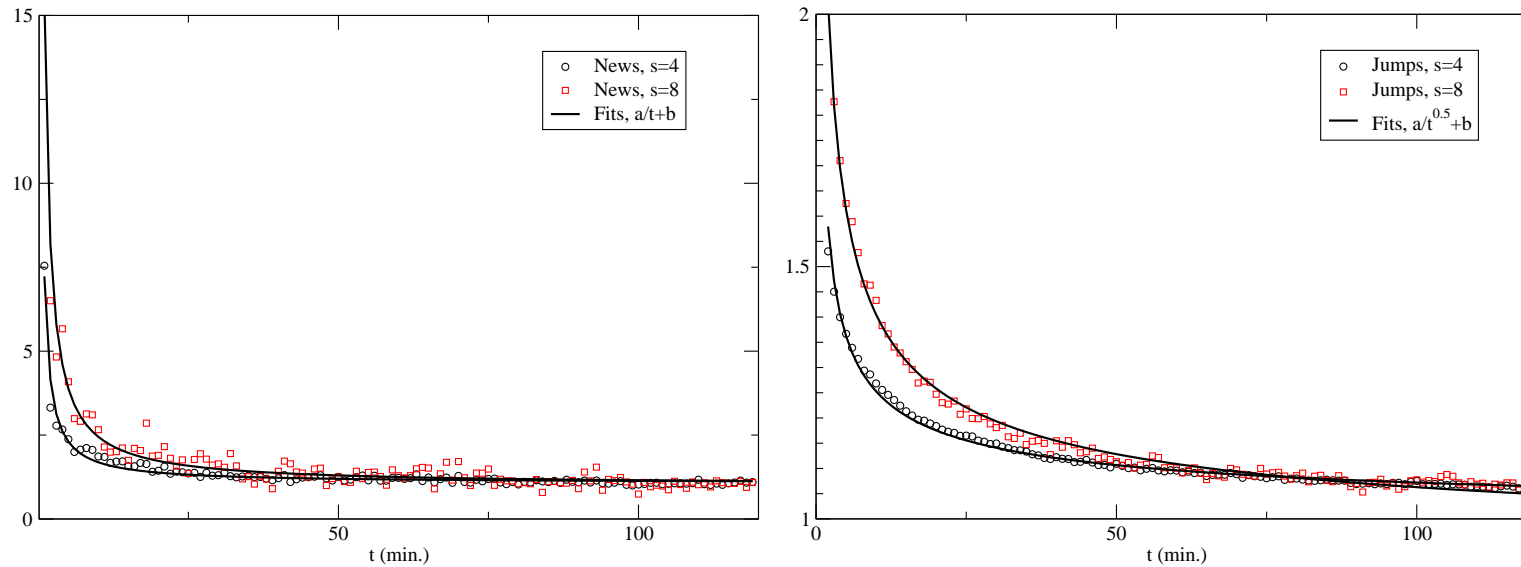
- Yes, **some** news make prices jump, sometimes a lot, but **jump freq. is much larger than news freq.**
- On stocks, **only  $\sim 5\%$**  of  $4 - \sigma$  jumps can be attributed to news, most jumps appear to be **endogenous**
  - ▷ Similar conclusions on daily data in seminal papers (**Cutler, Poterba, Summers; Shiller; Fair**)
  - ▷ **Private information should not induce jumps!** (**Kyle**)
  - ▷ Jumps are NOT triggered by big volumes, but are *followed* by large activity
- **Return distributions** and **'aftershocks'** (volatility relaxation) are markedly distinct

# Jump frequencies



Power-law distribution of news jumps and no-news jumps. With  
A. Joulin, D. Grunberg, A. Lefevre

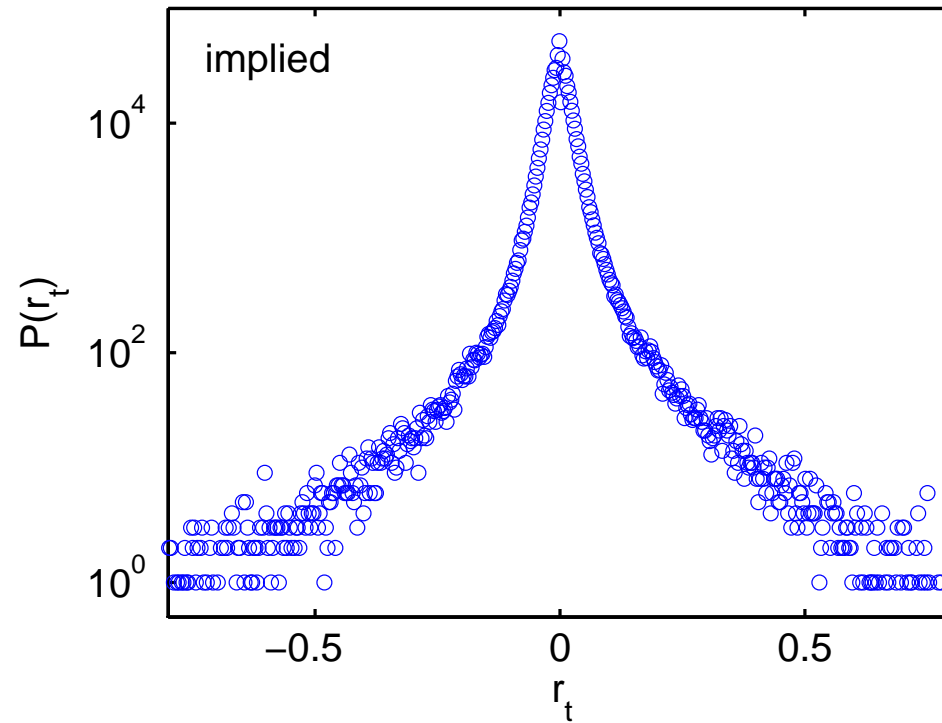
# Two jump types: Aftershocks



Volatility relaxation after news ( $t^{-1}$ , left) and endogenous jumps ( $t^{-1/2}$ , right). With [A. Joulin](#), [D. Grunberg](#), [A. Lefevre](#)

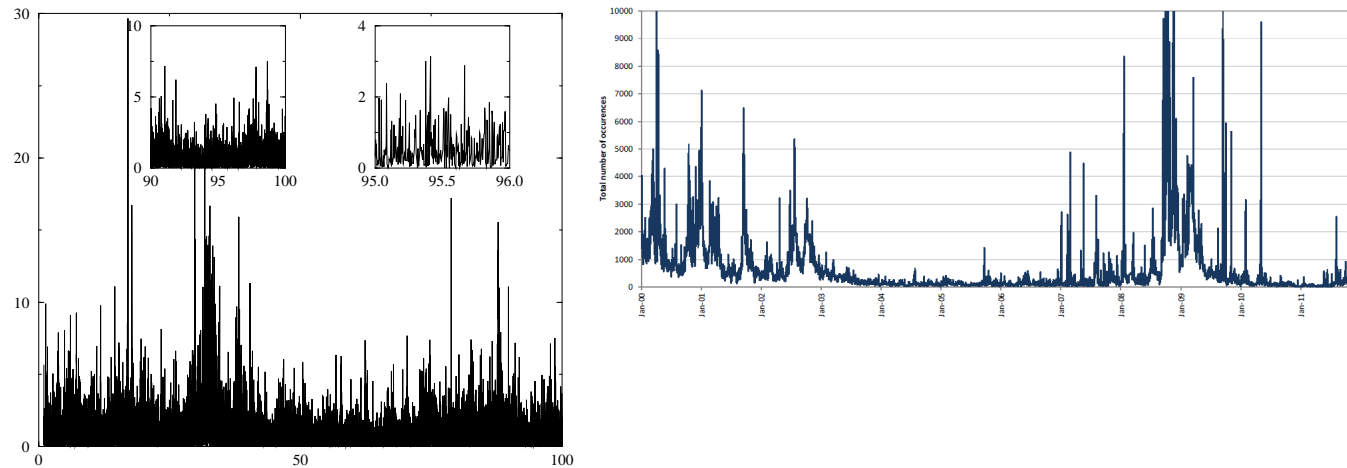


# Power-law tails



Distribution of daily volatility moves on option markets or *any other traded stuff*:  $\approx$  inverse cubic law

# Multiscale intermittency

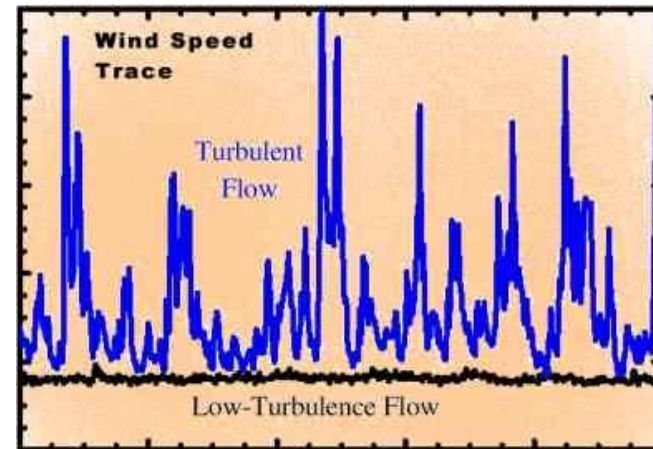
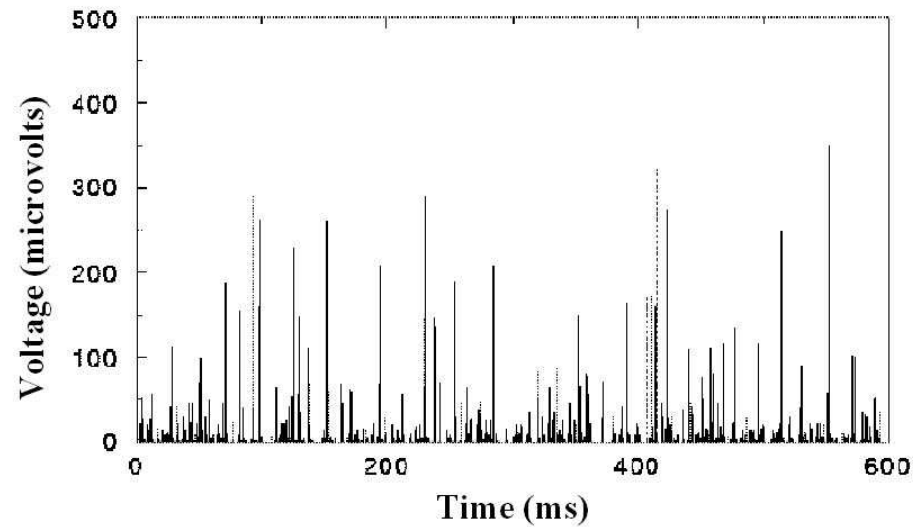


Excess volatility, with long range memory

– looks a lot like **endogenous noise** in complex systems

(Right: number of 1% jumps/min on S&P stocks)

# Intermittency: Barkhausen noise, Turbulence



Slow, regular and featureless exogenous drive →  
Intermittent endogenous dynamics

## A) Exogenous or endogenous dynamics?

- Excess volatility, with long range memory – looks like endogenous intermittent noise in complex systems (turbulence, Barkhausen noise, earthquakes, etc.)
- To a large extent: Universal observations in time, space & assets
  - ▷ details may evolve, but main features remain
- These observations and analogies strongly suggest that endogenous dynamics is the solution to the excess volatility puzzle – NOT DUE TO FUNDAMENTALS
  - ▷ We need models for endogenous crises and spontaneous discontinuities – EDH rather than EMH !

## A) Exogenous or endogenous dynamics?

- Calibration of models indeed suggest that  $\approx 80\%$  of volatility is due to **self-reflexive feedback of activity** onto itself!

- **QARCH-q**:  $r_t = \sigma_t \xi_t$  with

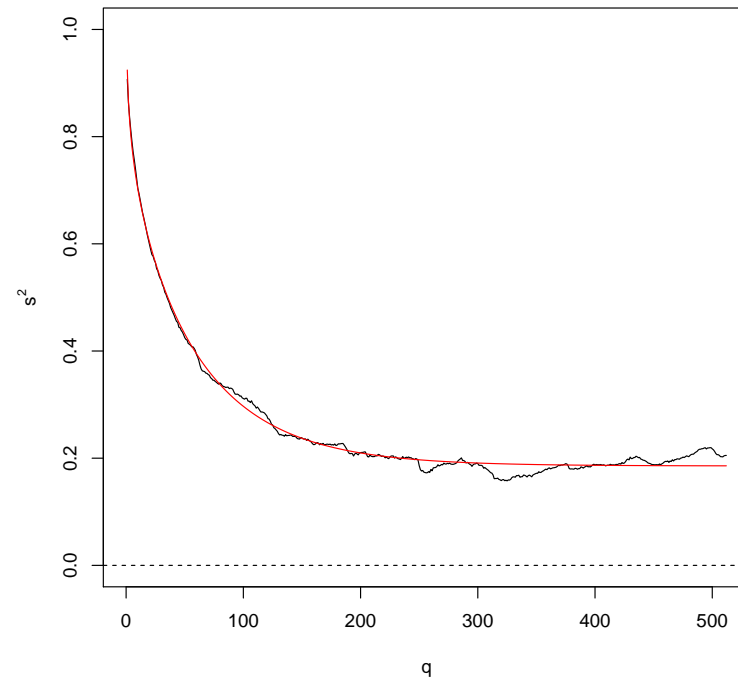
$$\sigma_t^2 = s^2 + \sum_{\tau=1}^q L(\tau) r_{t-\tau} + \sum_{\tau=1}^q K(\tau) r_{t-\tau}^2$$

▷  $s^2(q \rightarrow \infty) \approx 0.2$

▷  $\xi_t$  still has tails (unexpected jumps)

▷  $s^2 \rightarrow 0$  intraday !

# Exogeneous base volatility level



US stocks (with [R. Chicheportiche](#), [P. Blanc](#))

## A) Exogenous or endogenous dynamics?

- Calibration of **Hawkes process** suggest an even larger contribution of self-reflexivity:

$$\lambda(t) = \mu + \int_{t' < t} K(t - t') dN(t')$$

- Each event has  $n = \int_0^\infty du K(u)$  “child events”
  - ▷  $\bar{\lambda} = \frac{\mu}{1-n}$
  - ▷ **When  $n > 1$ , the process is unstable**
  - ▷ **Calibration suggests  $n \approx 1$  (with S. Hardiman)**

## A) Exogenous or endogenous dynamics?

- Many other interesting “stylized facts” suggesting **contagion and self-referential dynamics**

▷ During a market slide, the number  $S$  of stocks that undergo a “large” move is power-law distributed (“avalanche”)

$$P(S) \sim S^{-1-\nu}, \quad \nu \approx 1.25 \rightarrow 1.5$$

(Joulin et al.; Medo et al.)

▷ The index “leverage” effect is due to an increased correlation after down moves:

$$r_\alpha(t)r_\beta(t) := C_{\alpha,\beta} + D_{\alpha,\beta}(\tau)I(t - \tau) + \varepsilon_{\alpha,\beta}(t).$$

(Principal Regression Analysis)

Note:  $\sigma_I^2 \approx \bar{\rho} \overline{\sigma_\alpha^2}$



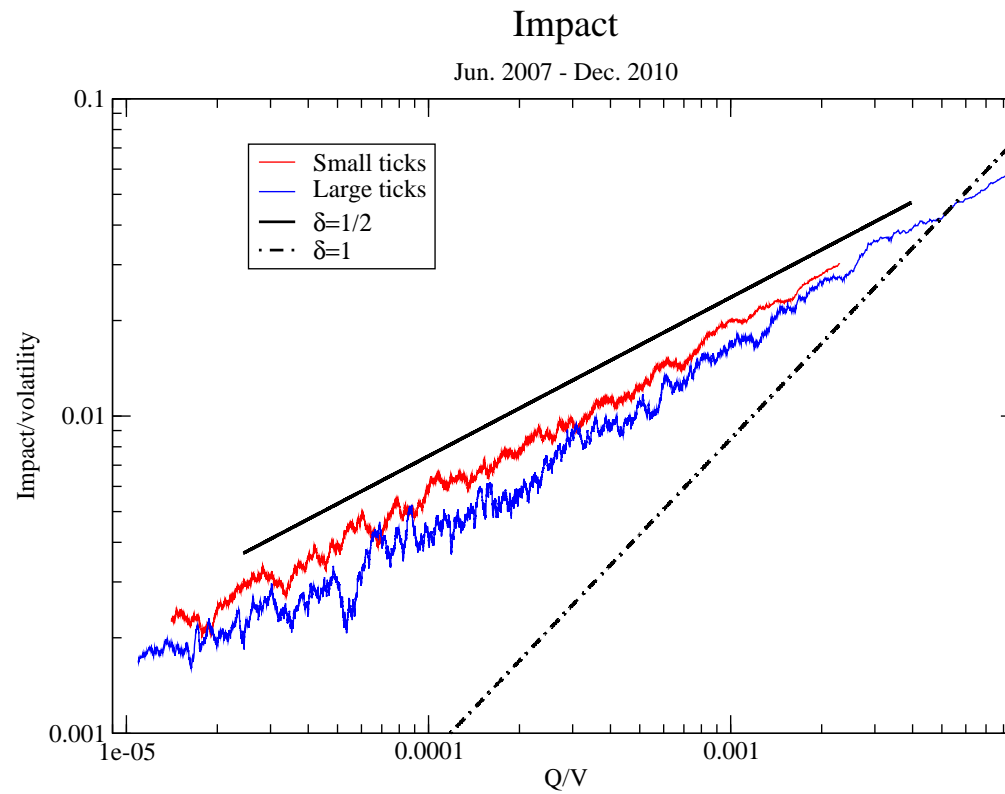
## B) How do trades impact prices?

- The fundamental paradox of liquid markets: very small instantaneous liquidity but rather large daily volume
  - ▷ Total liquidity immediately accessible on large US stocks:  $\sim 10^{-6}$  of market cap.
  - ▷ Total daily traded volume: 5,000 times larger!
  - ▷ Trades must be executed incrementally → “metaorders”
- The (average) impact of a metaorder of size  $Q$  is singular

$$I(Q) \sim \sigma \sqrt{\frac{Q}{V}}$$

- ▷ Again: A universal observation (BARRA, Almgren, Engle, JPM, DB, LH, CFM): different strategies, markets, tick sizes, periods (1995 – 2012)...

# The square-root impact law



From ca. 500,000 CFM trades on futures markets – with [B. Toth et al.](#)

## B) How do trades impact prices?

- A non trivial impact law:
  - ▷ Impact is concave (not additive):  $1 + 1 = 1.4142 < 2$
  - ▷ Anomalously large impact of small trades: 1% of ADV pushes the price by 10% of its vol!
  - ▷ Important: impact is usually small compared to volatility itself
- Why is impact so large (singular) and liquidity so small?

## B) How do trades impact prices?

- Why is impact so large (singular) and liquidity so small?

- A statistical theory of liquidity:

▷ Even with “zero-intelligence” agents: provided the price makes a **random walk**, and for **generic order flow**, the probability to have unexecuted orders close to the current price is **linearly small**

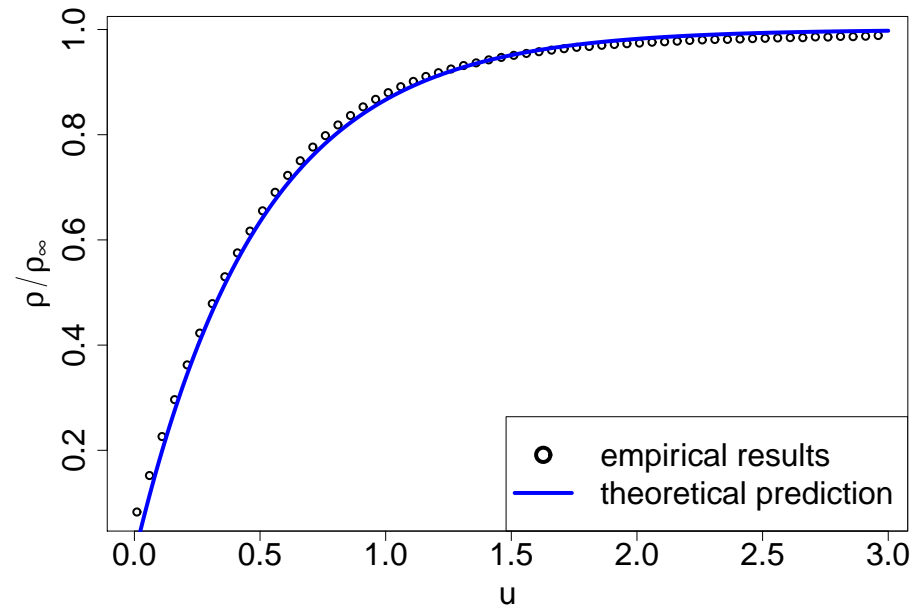
▷ **Analytical result**

$$\frac{\partial \rho}{\partial t} = \sigma^2 \frac{\partial^2 \rho}{\partial u^2} + \lambda(u) - \nu(u)\rho$$

$u$  : distance from current price

▷ + **Agent-based numerical simulations**

# A linear liquidity profile



A generic result – with B. Toth et al.

## B) How do trades impact prices?

- Why is impact so large (singular) and liquidity so small?
- A statistical theory of liquidity:
  - ▷ The probability to have unexecuted orders close to the current price is **linearly small**
  - ▷ Consequence: square-root impact!

$$Q = \int_p^{p+I} \alpha u \, du = \frac{\alpha}{2} I^2 \rightarrow I \propto \sqrt{Q}$$

## B) How do trades impact prices?

- Critical liquidity and **Intrinsic Market Fragility**
  - Markets are NOT obviously stable, Pr. Friedman
- Liquidity around current price is vanishingly small (eaten by the diffusive motion of prices): **Market makers are needed!**
  - ▷ **Liquidity fluctuations** are bound to play a crucial role: **Micro-crises and jumps in prices without news** (cf. above)
  - ▷ Regulation must engineer **stabilizing feedback loops**
    - **favoring liquidity** when it is most needed (cf. debate about HFT)

# Endogenous crashes: Impact-induced instabilities

- **Impact-induced feedback loops** that can and do lead to crises:
  - ▷ **Model induced feedback loops:** e.g. the BS feedback loop in 1987, the CDO feedback loop in 2008,...
  - ▷ **Risk/Regulation induced feedback loops:** mark to market, risk limits, margin calls, deleveraging...
  - ▷ **Contagion, spill-over:** the quant crunch



# Endogenous crashes and financial engineering

- Derivatives can be good (insurance, loans to people with low credit rating, etc.), but sadly a large part of “Financial Engineering” is devoted to engineer information asymmetry, and not care too much about toxicity and systemic risk
- Many well known models are absurdly remote from reality – famous example: Black-Scholes for options
  - ▷ Little effort to understand underlying mechanisms, phenomena and the *intuition* behind abstract equations
  - ▷ Rush to write down workable models, even if wrong and dangerous, provided they allow to sell – e.g. CDOs

*The salesman knows nothing about what he is selling, save that he is charging a great deal too much for it. (Wilde)*

# Endogenous crashes and financial engineering

*But against those who warned, most were convinced that banks knew what they were doing. They believed that the financial wizards had found new and clever ways of managing risks. (Letter to the Queen)*

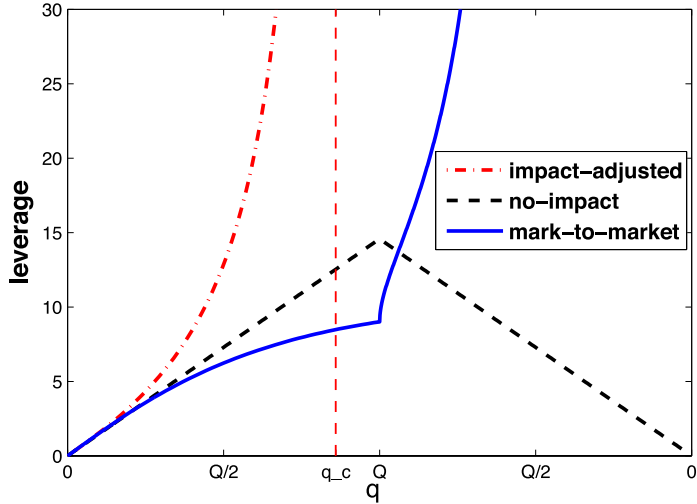
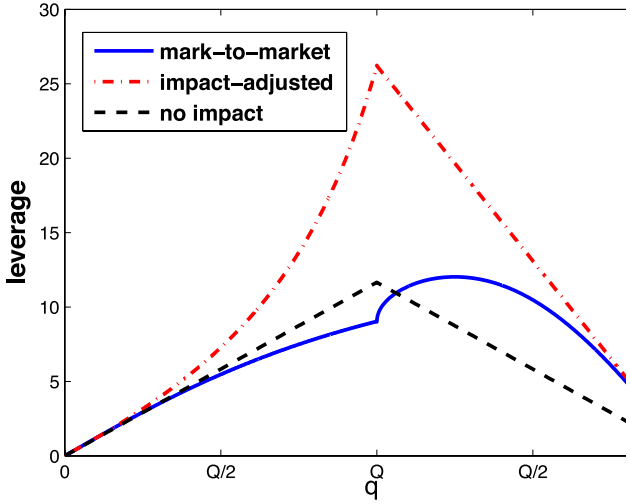
# Endogenous crashes and financial engineering

- “Portfolio insurance” and the BS crash of 1987
  - ▷ Perfect  $\Delta$  hedge of puts = Sell when market goes down
  - ▷ 80 B\$ “insured” like this when ADV was 5 B\$ → Impact did the rest
- Wrong models are **dangerous**
  - ▷ Models underestimate risks significantly, while giving a false sense of security and promoting dangerous practice
  - ▷ Models can be worse than no-models!

# Endogenous crashes: danger of mark-to-market

- Square-root impact means that liquidation first *increases* leverage → possible deleveraging trap when the initial leverage is too high
- Liquidity discount to marked-to-market pricing:  $\sigma \sqrt{\frac{Q}{V}}$ .

# Impact adjusted mark-to-market



with F. Caccioli, D. Farmer

# Endogenous crashes: the 'quant-crunch'

- August 2007: the quant-crunch as an invisible crash
- Stat-arb portfolio: market and sector neutral, i.e.:

$$\sum_{\alpha} U_{\alpha}^k q_{\alpha} = 0, \quad \forall k \in [1, n]$$

▷  $k = 1$ : the market mode (the index)  $U_{\alpha}^1 \approx N^{-1/2}$  – say  $q_{\alpha} = S_{\alpha} g V_{\alpha}$

▷ The portfolio is immune against market moves

▷ The market is immune against portfolio deleveraging

▷ ...same for sectors...

# Endogenous crashes: the 'quant-crunch'

- Now imagine that one manager  $i$  has to deleverage his book (due to losses elsewhere) in  $T$  days

▷ The impact on stock  $\alpha$  is  $\Delta p_\alpha = -S_\alpha^i \sigma \sqrt{g/T}$ , very small for each stock

▷ But another manager  $j$  will see the value of his portfolio impacted as:

$$\Delta W(i \rightarrow j) = \sum_{\alpha} q_{\alpha} \Delta p_{\alpha} = -\sigma \sum_{\alpha} g^{3/2} T^{-1/2} V_{\alpha} S_{\alpha}^i S_{\alpha}^j$$

▷ The quantity  $C_{ij} = \sum_{\alpha} V_{\alpha} S_{\alpha}^i S_{\alpha}^j$  is the volume weighted **similarity** between managers

# Endogenous crashes: the 'quant-crunch'

- August 2007: the quant-crunch as an invisible crash
- $C_{ij} = \sum_{\alpha} V_{\alpha} S_{\alpha}^i S_{\alpha}^j$ 
  - ▷ For uncorrelated positions,  $C_{ij} \propto \sqrt{N}$  and the impact adds to the background noise
  - ▷ For correlated strategies,  $C_{ij} \propto N$  and this becomes *dominant* over usual volatility
  - ▷ Remember:  $\Delta W(i \rightarrow j) \propto -C_{ij}$ : the deleveraging of  $i$  hurts  $j$ , up to a point  $j$  might choose to deleverage as well
  - ▷ This may create an avalanche if the branching ratio is  $> 1$  (for similar ideas: Caccioli et al., Cont & Wagalath, F. Lillo et al.)



# A cartoon model for self-referential behaviour

- People do not make decision in isolation but rely on the choice/opinion of others: many direct empirical evidence.

*When men are in close touch with each other, they no longer decide randomly and independently of each other, they each react to the others. Multiple causes come into play which trouble them and pull them from side to side, but there is one thing that these influences cannot destroy and that is their tendency to behave like Panurges sheep*

(Poincaré 1900, on Bachelier's thesis!)

# Love-locks on Pont Des Arts

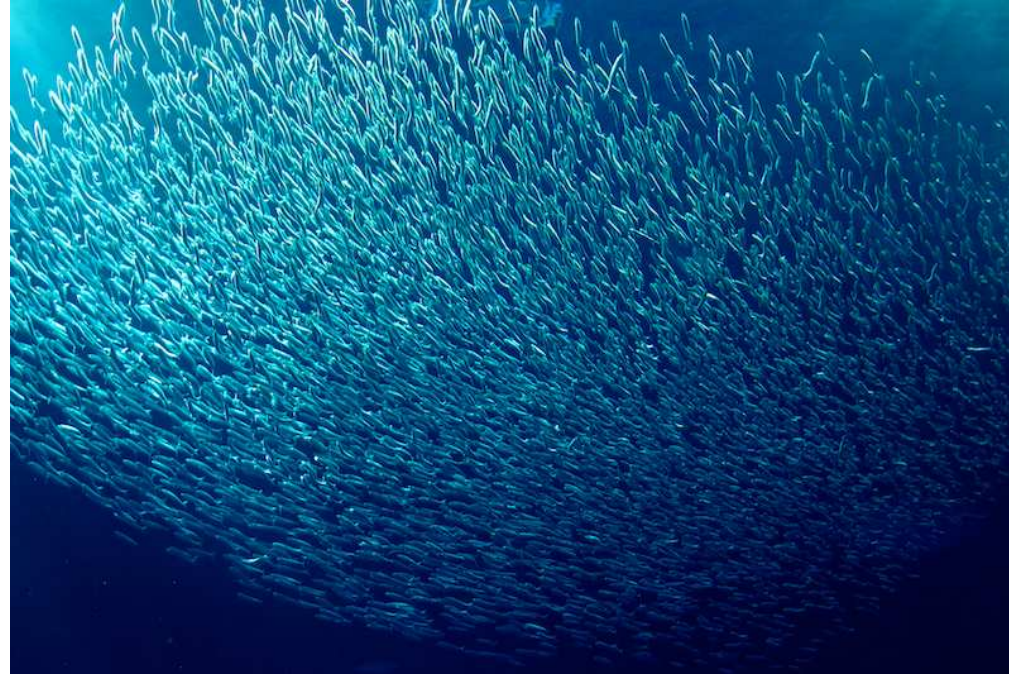


The madness of crowds ([Newton](#))

# A cartoon model for self-referential behaviour

- **Very strong distortion/amplification phenomena** due to imitation: fads & fashion (e.g. love-locks), bubbles & crashes
  - ▷ Difficult to understand without imitation
- **Many important situations in practice:** vaccines, hygiene, fertility, driving, crime, tax evasion, technology, etc.

# Starlings in Rome and Fish Schools



A. Cavagna et al.

# Cartoon models for collective behaviour

- **A simple framework:**  $N$  homogeneous agents with “mean-field” interactions (i.e. only influenced by the aggregate decision)
  - ▷ **Binary decision** of agent  $i$ :  $D_i = 0, 1$  (to buy/lend/trust or not to buy/lend/trust, etc.)
  - ▷ **Aggregate decision:**  $\phi = N^{-1} \sum_i D_i$

# Cartoon models for collective behaviour

- Relative benefit of the switch  $D_i = 0 \longrightarrow D_i = 1$ :

$$v(0 \rightarrow 1) = -\alpha + \beta\phi$$

- ▷  $\alpha > 0$ : cost of leaving the “0” state (e.g.: pay one’s taxes, cost of a cell phone, cost of moving to solar heating, etc.)
- ▷  $\beta > 0$ : incentive due to either social/peer pressure, true benefits due to improved infrastructure/social welfare/technology or reduced costs, increased usefulness (phones), etc.
- ▷ Note: **homogeneous agents**, with all the same  $\alpha, \beta$ .

# Model of choices and dynamics

- **Choice Theory:** allowing for non-deterministic (non-rational) choices:

$$P(0 \rightarrow 1) = \mu \frac{e^{\zeta v}}{1 + e^{\zeta v}}; \quad P(1 \rightarrow 0) = \mu \frac{1}{1 + e^{\zeta v}}$$

▷  $\mu$ : rate of individual choices per unit time

▷  $\zeta$ : parameter allowing for **non-rational choices**

$\zeta = 0 \rightarrow$  random choice;  $\zeta \rightarrow \infty$ : rational choice

- **Evolution of  $N_1 =$  number of agents in “state” 1:**

$$\pi(N_1 \rightarrow N_1 + 1) = \mu \frac{z}{1 + z} (1 - \phi); \quad \pi(N_1 \rightarrow N_1 - 1) = \mu \frac{1}{1 + z} \phi;$$

with  $\phi = N_1/N$  and  $z = \exp(\zeta v)$  (depends on  $\phi$ ).



# Model of choices and dynamics

- Taking the appropriate **continuous time limit**, one derives the following SDE for  $\phi$ :

$$d\phi = F(\phi)dt + \sigma(\phi)\frac{dW}{\sqrt{N}}$$

with:

$$F(\phi) = \mu\left[\frac{z}{1+z} - \phi\right]; \quad \sigma^2(\phi) = \mu\frac{(1-\phi)z + \phi}{1+z}$$



# Equilibrium states and “free-energy”

- The “free-energy”  $V(\phi)$  of the problem is defined as:

$$F(\phi) = -\frac{dV}{d\phi} \Rightarrow V(\phi) = \frac{\phi^2}{2} - \frac{\ln(1 + e^{-\zeta\alpha + \zeta\beta\phi})}{\beta\zeta}$$

- The stationary points are the “equilibrium” states:

$$F(\phi) = 0 \rightarrow \frac{z}{1+z} = \phi$$

or:

$$m^* = 2\phi^* - 1 = \tanh\left[\frac{\zeta\beta m^*}{4} + \frac{\zeta h}{2}\right]$$

with  $h = \beta/2 - \alpha$ .

# Equilibrium states and “free-energy”

$$m^* = 2\phi^* - 1 = \tanh \left[ \frac{\zeta\beta m^*}{4} + \frac{\zeta h}{2} \right]$$

▷ Mean-field equation for the ferromagnetic Ising model (Curie-Weiss; Brock & Durlauf; etc.)

▷ When  $\zeta\beta < 4$  (strong irrationality, weak collective effects):

a unique solution describing a “mixed” population, constantly evolving between the two choices

▷ When  $\zeta\beta > 4$ :

two “polarized” solutions; in equilibrium the population should jump discontinuously from  $\phi^* \approx 0$  for  $\beta < 2\alpha$  to  $\phi^* \approx 1$  for  $\beta > 2\alpha$ .

# Beyond equilibrium: Dynamics

- When  $\zeta\beta > 4$  (and  $\beta > \alpha$ ):

The free-energy  $V(\phi)$  has **two local minima** around  $\phi \approx 0$  and  $\phi \approx 1$ , separated by an **energy barrier**

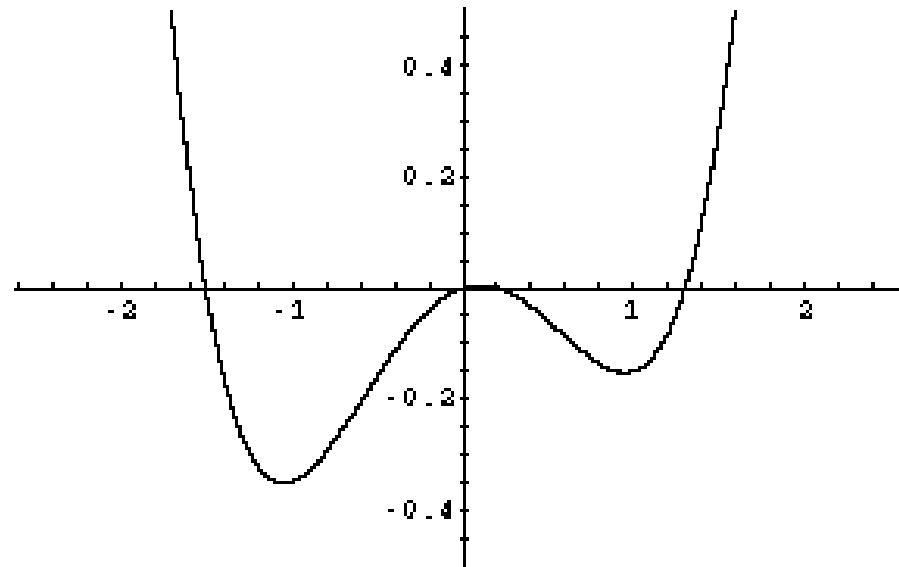
▷ located at  $\phi_c \approx \alpha/\beta$

▷ of height (from 0  $\rightarrow$  1):

$$V_c = V(\phi_c) = \frac{\alpha^2}{2\beta^2}$$

(Note:  $V_c \rightarrow 0$  when  $\alpha \rightarrow 0$ )

# Double well potential



# Beyond equilibrium: Dynamics

- Recall the dynamical equation for  $\phi$ :

$$d\phi = F(\phi)dt + \sigma(\phi)\frac{dW}{\sqrt{N}}$$

- ▷ The time needed to cross the barrier “spontaneously” is given by:

$$\tau_c \approx \exp\left(\frac{N\alpha^2}{2\beta^2\sigma^2(\phi_c)}\right)$$

- ▷ Unless  $\alpha$  is really small or  $\beta$  really large, **this never happens in a large population!**
- ▷ **The tragedy of the commons:** everybody knows that  $\phi = 1$  is a better social optimum, but it is individually too costly for everyone to make the move

# Beyond equilibrium: Dynamics

- Solutions to escape from a “bad” minimum:
  - ▷ Enforce  $\alpha \rightarrow 0$  (e.g. make the cost of cheating high)
  - ▷ Finite range (non mean field) interactions: nucleation of the “good” state around a certain point in space (cities)
  - ▷ Synchronization of the moves (instead of independent decisions): advertizing, word of mouth

# Cartoon model for collective decisions: RFIM

- **The RFIM**: a unifying framework for many phenomena, for example Barkhausen noise – Sethna et al. “Crackling Noise”, Birth rates, Cell phones, Clapping...(with Q. Michard)
- Now the agents are both **heterogeneous** and **influenced** by the behaviour of others, and for simplicity  $\zeta \rightarrow \infty$  (quasi-rational decisions)

⇒ **the RFIM update rule** (mean-field):

$$D_i(t) = \frac{1}{2} (1 + \text{sign} [\psi_i - \alpha(t) + \beta\phi(t)]),$$

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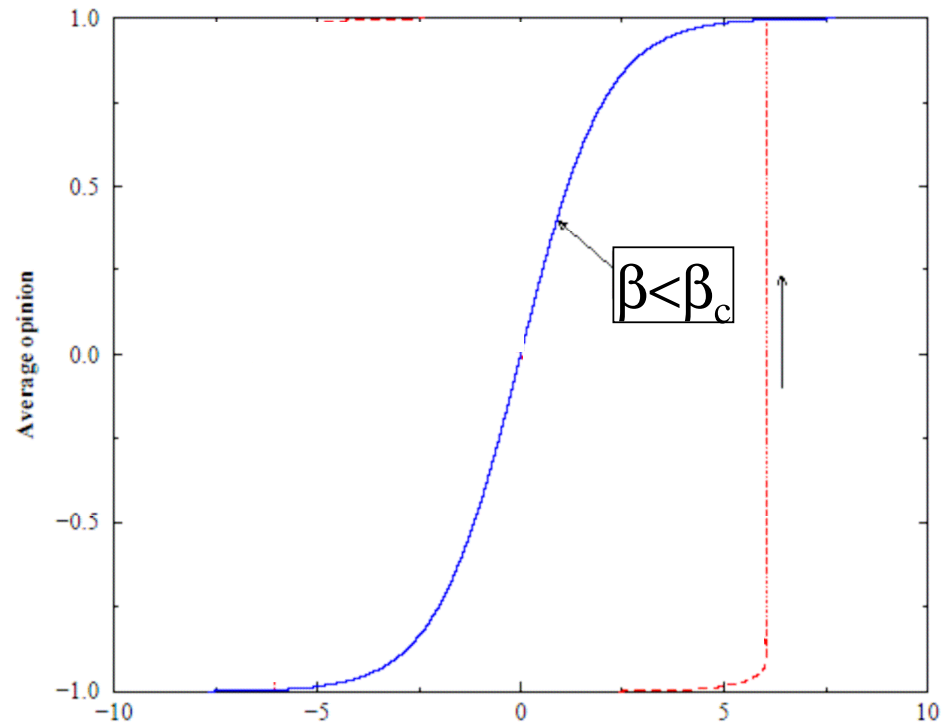
- ▷ **personal opinion**, propensity or utility  $\psi_i$  – heterogeneous with probability  $P$  with rms  $\sigma^*$
- ▷ **public information** (price, technology level, news, zeitgeist)  $-\alpha(t)$  (for illustration purposes, **smooth** in time)
- ▷ **social pressure** or imitation effects  $\beta\phi(t)$

\*Previous case:  $\psi_i = \text{const.}$



# Soft landing or crash?

Fraction of ‘pessimists’ as a function of time

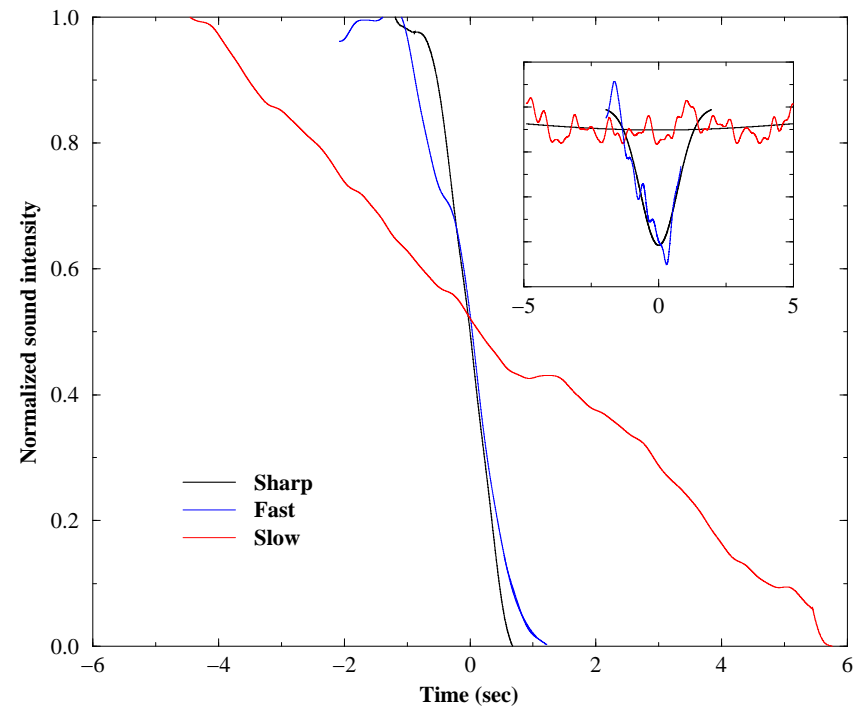


Breakdown of Representative Agent; Spontaneous discontinuities

# Cartoon model of self-referential behaviour

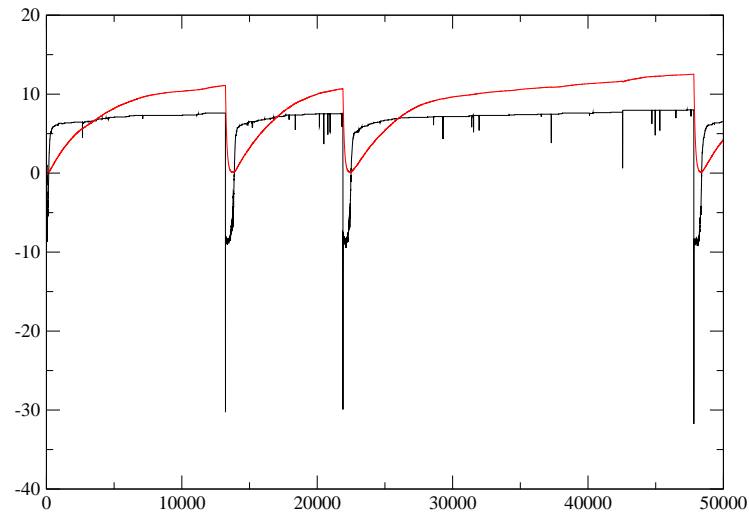
- $\beta < \beta_c$ : personal choices dominate, smooth demand curve
- $\beta > \beta_c$ : herding dominates, strong distortion/amplification of the fundamental demand curve: discontinuities appear at the macro level – imitations induced panic/crashes/mistrust, rushing for the exit...
  - ▷ Hysteresis in and out of the crisis
- $\beta \approx \beta_c$ : avalanche dynamics (power-law distribution of sizes)
- $\beta_c \propto \sigma$ : More dispersed opinions avoid polarisation and stabilizes the system

# Cartoon model of self-referential behaviour



End of clappings

# Application: spontaneous evaporation of trust



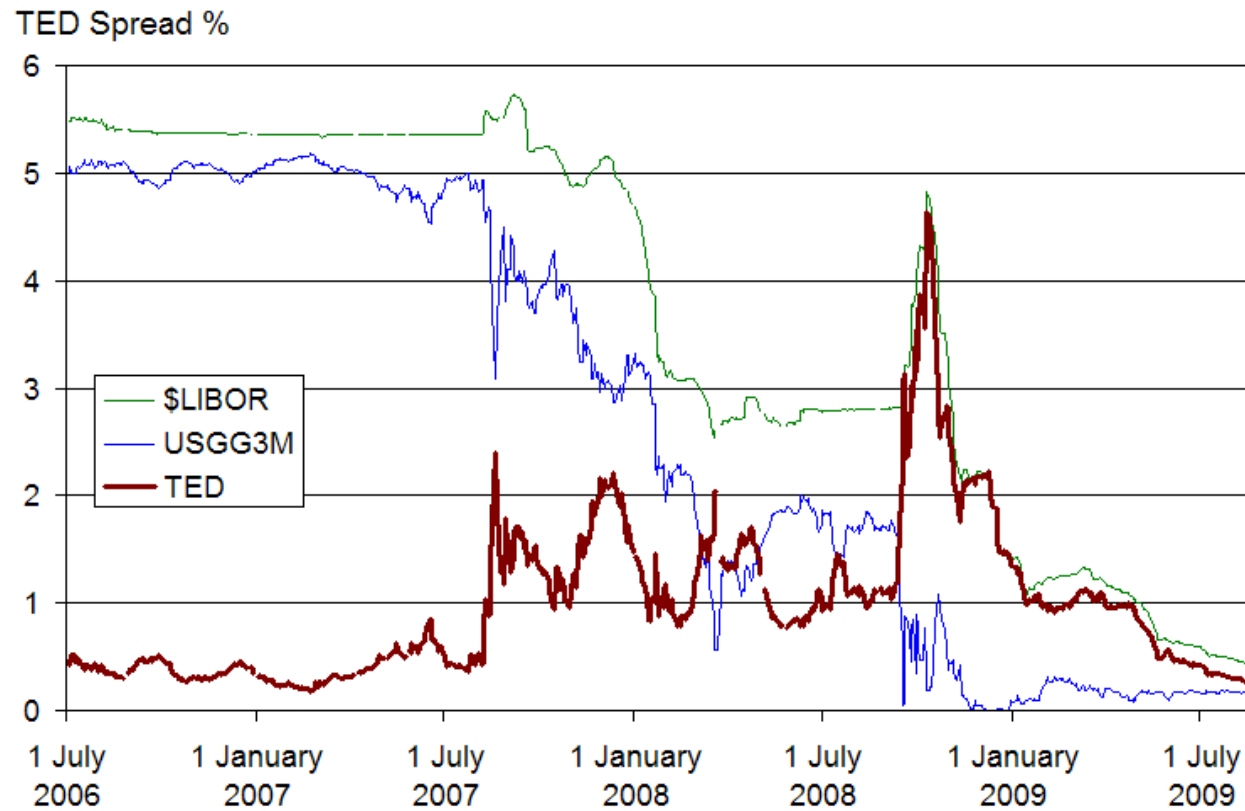
Number of trust bonds and average trust as a function of time

I trust you because he trusts you because I trust you

As trust builds up, the system becomes more fragile

Battiston et al., Kirman et al., Batista et al.

# Evaporation of trust and money market freeze



# Conclusion – Endogenous crises?

- Financial markets, the economy, many other social phenomena exhibit **crises, ruptures, sudden discontinuities** that resemble far-from-equilibrium phenomena in complex systems
  - ▷ Accumulating empirical evidence for positive feedback loops, self-reflexivity and endogenous crises
    - Most price jumps appear unrelated **to any news at all**
    - Market statistics share features with **slowly driven, heterogeneous interacting systems with many equilibria**

# Conclusion – Endogenous crises?

- Financial markets seem to operate close to criticality, making them particularly **fragile**
- Many “tensions” in markets try to equilibrate in a “tug of war”
  - ▷ Buyers vs. Sellers
  - ▷ Trend followers vs. mean reverters (Lux-Marchesi, Giardina-JPB)
  - ▷ Liquidity providers vs. liquidity takers (Wyart et al.) →? Liquidity is **critical**
- Recent theoretical ideas: Minority games, stick balancing task, etc.

## Conclusion – Endogenous crises?

- A major scientific program: infer “macro behaviour from micro-motives” (Schelling)
  - ▷ Ideas & methods from statistical physics and numerical simulations of ABM (multiple equilibria, collective behaviour, hysteresis, avalanches, etc. ) are promising and provide interesting insights about complex systems...

*So in summary, Your Majesty, the failure to foresee the timing, extent and severity of the crisis and to head it off, while it had many causes, was principally a failure of the collective imagination of many bright people, both in this country and internationally, to understand the risks to the system as a whole. (Letter to the Queen)*

- ▷ ...but still a long way to go before old dogmas are abandoned....



# References

- This talk is based on the following papers:
  - ▷ J.-P. Bouchaud, *The Endogenous Dynamics of Markets: Price Impact, Feedback Loops and Instabilities*, in *Lessons from the 2008 Crisis*, edited by A. Berd (Risk Books, Incisive Media, London, 2011).
  - ▷ J.-P. Bouchaud, J. D. Farmer, F. Lillo, *How markets slowly digest changes in supply and demand*, in: *Handbook of Financial Markets: Dynamics and Evolution*, North-Holland, Elsevier, 2009
  - ▷ B. Tòth, Y. Lempérière, C. Deremble, J. De Lataillade, J. Kockelkoren, and J.-P. Bouchaud. *Anomalous price impact and the critical nature of liquidity in financial markets*. *Physical Rev. X*, 1:021006, (2011)

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- ▷ R. Chicheportiche, J.-P. Bouchaud, *The fine structure of volatility feedback*, arXiv:1206.2153
- ▷ A. Joulin, A. Lefevre, D. Grunberg, J.-P. Bouchaud, *Stock price jumps: news and volume play a minor role*, Wilmott Mag., Sept/Oct 2008; arXiv:0803.1769
- ▷ J.-P. Bouchaud, *Crises and collective socio-economic phenomena: cartoon models and challenges*, arXiv:1209.0453