Support Vector Metric Learning

Eddie Xu, **Kilian Q. Weinberger**, Olivier Chapelle, Fei Sha

Washington University in St. Louis

YAHOO! RESEARCH

Warning this talk may contain
small amounts of **neural networks.**

Support Vector Metric Learning

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YAHOO! RESEARCH

What is similarity?

Who is most similar?

Similar gender?

Similar age?

Similar hair cut?

Similarity depends on the context!

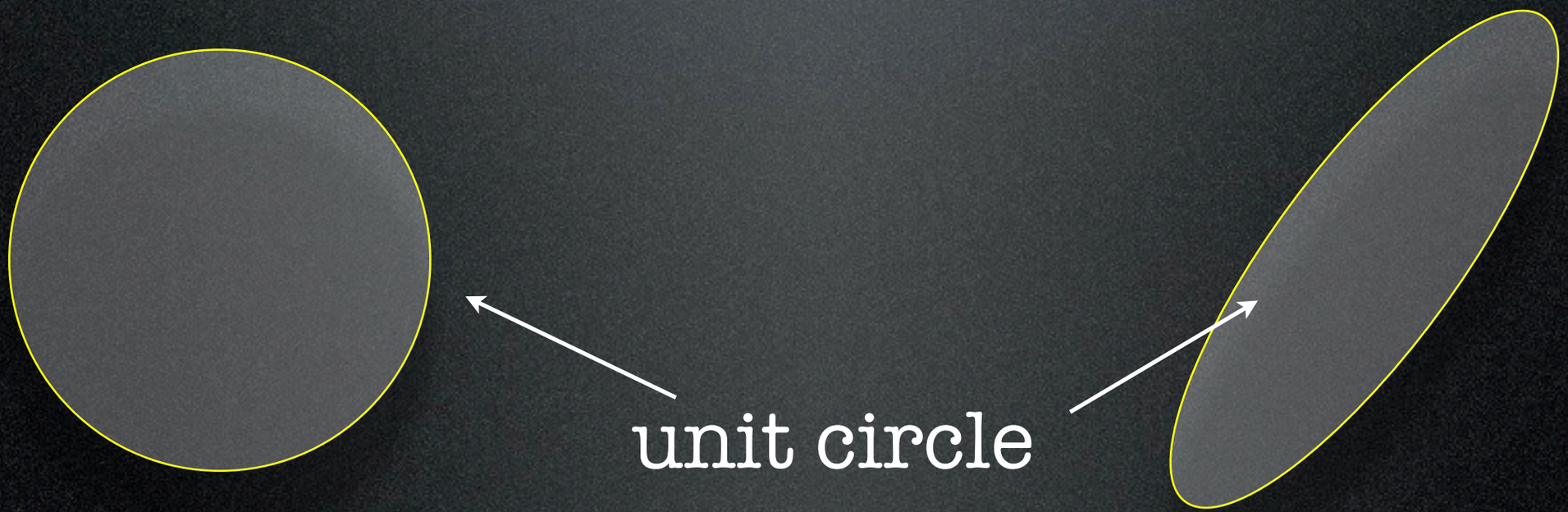
Mahalanobis distance

Euclidean

$$\|\vec{x}_i - \vec{x}_j\|_2 = \sqrt{(\vec{x}_i - \vec{x}_j)^\top (\vec{x}_i - \vec{x}_j)}$$

Mahalanobis

$$\|\vec{x}_i - \vec{x}_j\|_{\mathbf{M}} = \sqrt{(\vec{x}_i - \vec{x}_j)^\top \mathbf{M} (\vec{x}_i - \vec{x}_j)}$$



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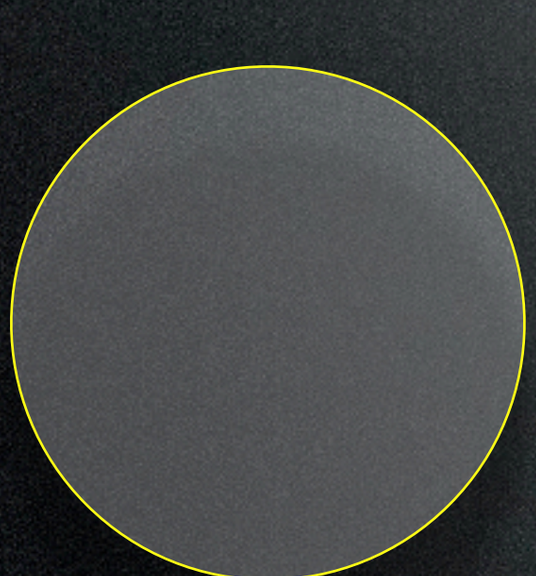
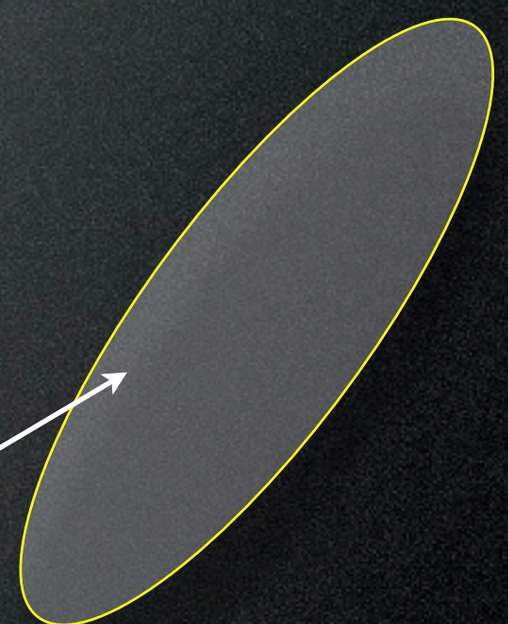
$\mathbf{M} \succeq 0$
positive
semi-definite



unit circle



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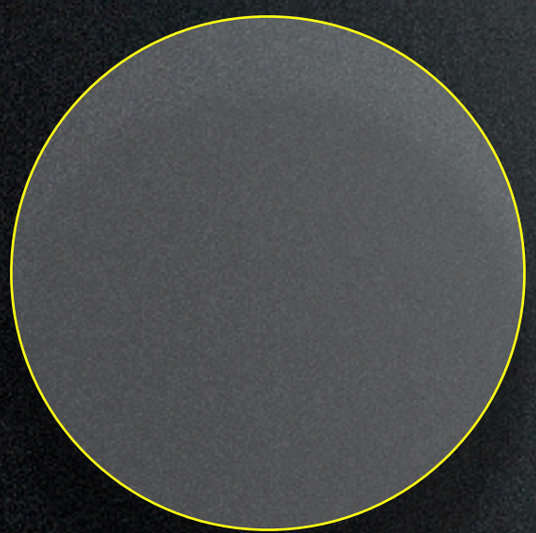

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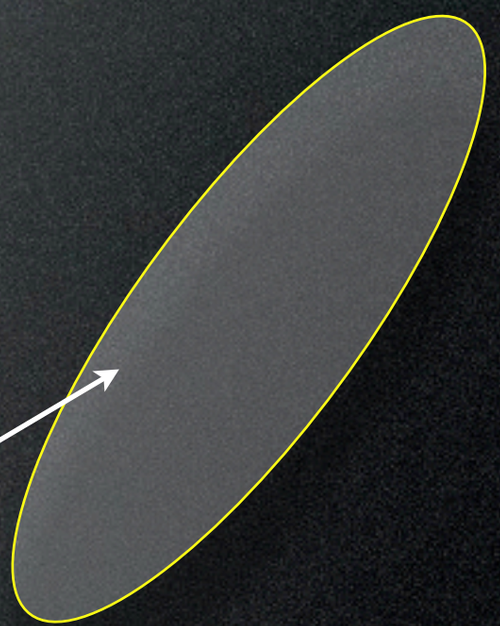
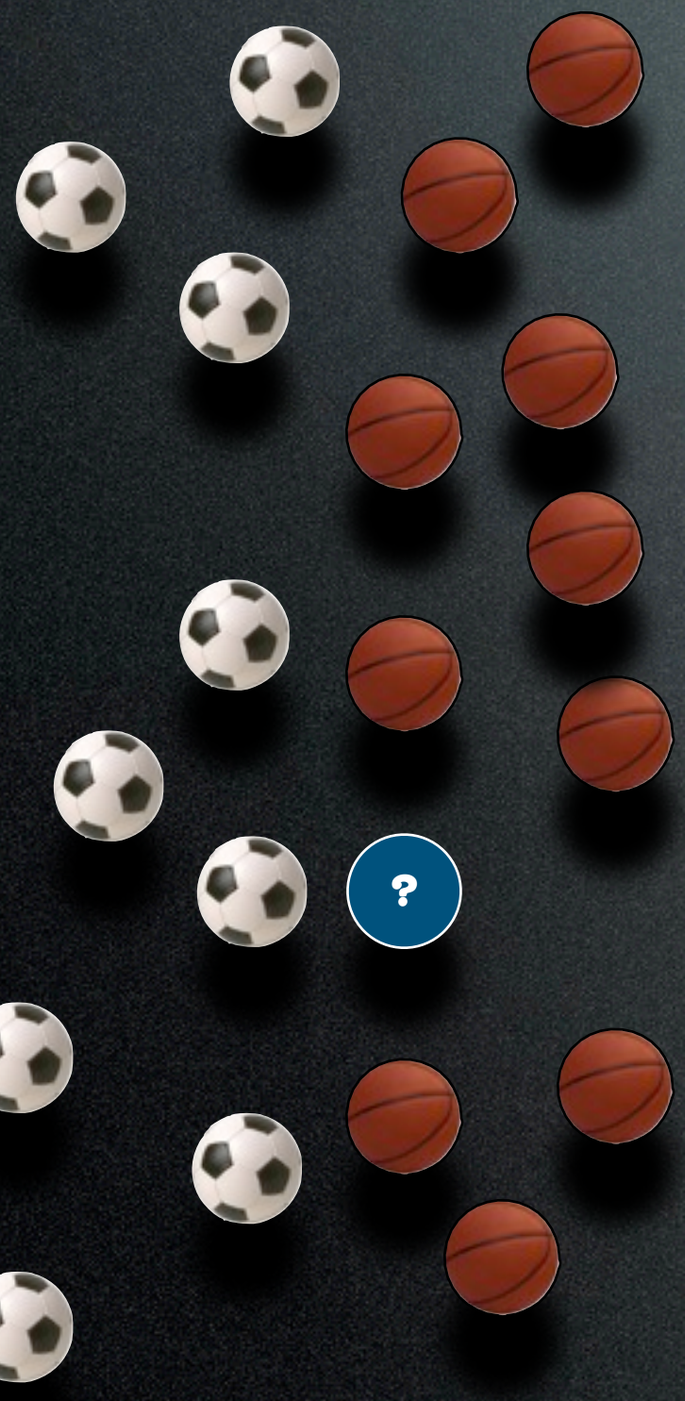


k-NN classification



- Adapt metric to the data
  - Amplify informative directions
  - Squash non-informative directions

k-NN classification

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k-NN classification

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RCA	**Shental et al [eccv 2002]**
MPCK	**Bilenko et al [icml 2004]**
POLAR	**Shalev-Shwartz et al [icml 2004]**
NCA	**Goldberger et al [nips 2005]**
LMNN	**Weinberger et al [nips 2006]**
MLCC	**Globerson and Roweis [nips 2006]**
ITML	**Davis et al [icml 2007]**
mtLMNN	**Parameswaran and Weinberger [nips 2010]**

Support Vector Metric Learning

How about SVMs?

- SVM with RBF Kernel is very similar to nearest neighbor classifier

$$k(\vec{x}_i, \vec{x}) = e^{-\frac{1}{2\sigma^2}(\vec{x}_i - \vec{x})^\top (\vec{x}_i - \vec{x})}$$

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(squared) **Euclidean Distance**

How about SVMs?

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$$k(\vec{x}_i, \vec{x}) = e^{-\frac{1}{2\sigma^2}(\vec{x}_i - \vec{x})^\top \mathbf{M} (\vec{x}_i - \vec{x})}$$

(squared) **Mahalanobis Distance**

First Idea

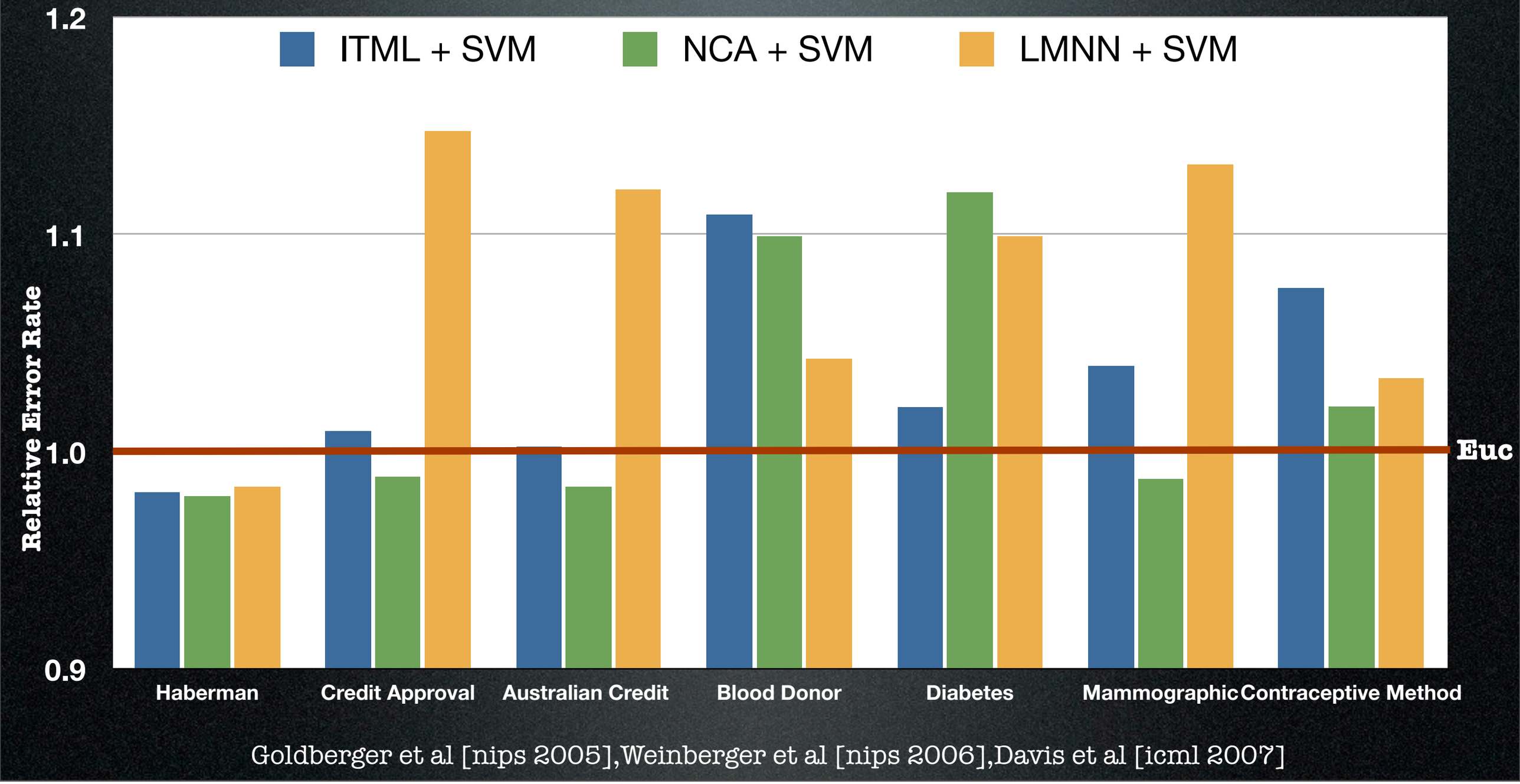
First Metric Learning, then SVM.

Baseline:
RBF Kernel with cross validation
(Features normalized)

Euc

Naive Experiments

First Metric Learning, then SVM.



Goldberger et al [nips 2005],Weinberger et al [nips 2006],Davis et al [icml 2007]

SVM

$$\min_{\alpha_1,\ldots,\alpha_n} \sum_i \alpha_i - \frac{1}{2} \sum_{i,j} \alpha_i \alpha_j y_i y_j \mathbf{K}_{ij}$$

subject to:

$$\sum_i \alpha_i y_i = 0$$

$$\alpha_i \geq 0$$

SVML

$$\min_{\mathbf{M}} \text{Validation Error}$$

$$\min_{\alpha_1,\ldots,\alpha_n} \sum_i \alpha_i - \frac{1}{2} \sum_{i,j} \alpha_i \alpha_j y_i y_j \mathbf{K}_{ij}$$

subject to:

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Chapelle et al 2002, Kloft et al 2010

SVML

$$\min_{\mathbf{M}} \frac{1}{|V|} \sum_{(\mathbf{x},y)\in V} 0.5(\text{sign}(-y \sum_i \alpha_i y_i \mathbf{K}^{\mathbf{M}}(\mathbf{x}_i, \mathbf{x}) + b) + 1)$$

$$\min_{\alpha_1,\ldots,\alpha_n} \sum_i \alpha_i - \frac{1}{2} \sum_{i,j} \alpha_i \alpha_j y_i y_j \mathbf{K}^{\mathbf{M}}_{ij}$$

subject to:

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SVML

$$\min_{\mathbf{M}} \frac{1}{|V|} \sum_{(\mathbf{x},y)\in V} s_a(-y \sum_i \alpha_i y_i \mathbf{K}^{\mathbf{M}}(\mathbf{x}_i, \mathbf{x}) + b)$$

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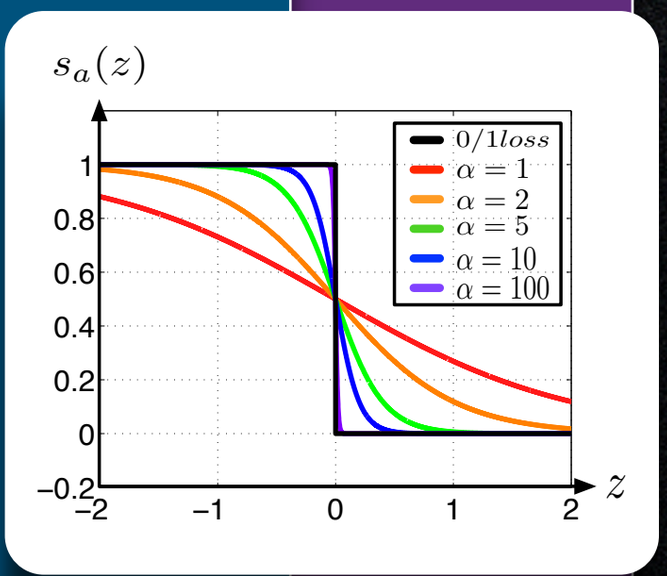

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Two side notes:

1. We use the squared hinge-loss
2. We learn C together with $\mathbf{M}$

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$$\mathbf{K}^{\mathrm{M}} \leftarrow \mathbf{K}^{\mathrm{M}} + \frac{1}{C}\mathbf{I}$$

Removes slack variables.

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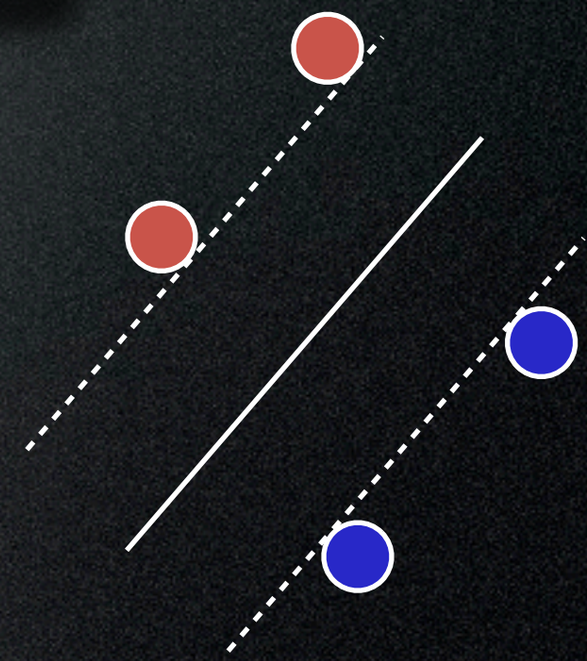
Why not?

Gradient Descent

$$\frac{\partial h}{\partial \mathbf{M}} = \frac{\partial h}{\partial \alpha}\frac{\partial \alpha}{\partial \mathbf{M}} + \frac{\partial h}{\partial \mathbf{K}}\frac{\partial \mathbf{K}}{\partial \mathbf{M}} + \frac{\partial h}{\partial b}\frac{\partial b}{\partial \mathbf{M}}$$



Chapelle et al [Machine Learning, 2002]

Gradient Descent

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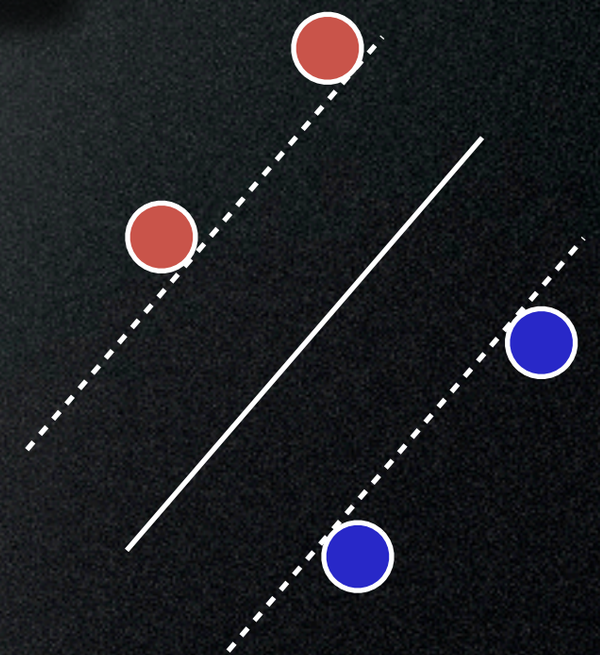
$$\underbrace{\begin{pmatrix} \bar{\mathbf{K}} & \mathbf{y} \\ \mathbf{y}^\top & 0 \end{pmatrix}}_{\mathbf{H}} \begin{pmatrix} \alpha \\ b \end{pmatrix} = \begin{pmatrix} \mathbf{1} \\ 0 \end{pmatrix}$$



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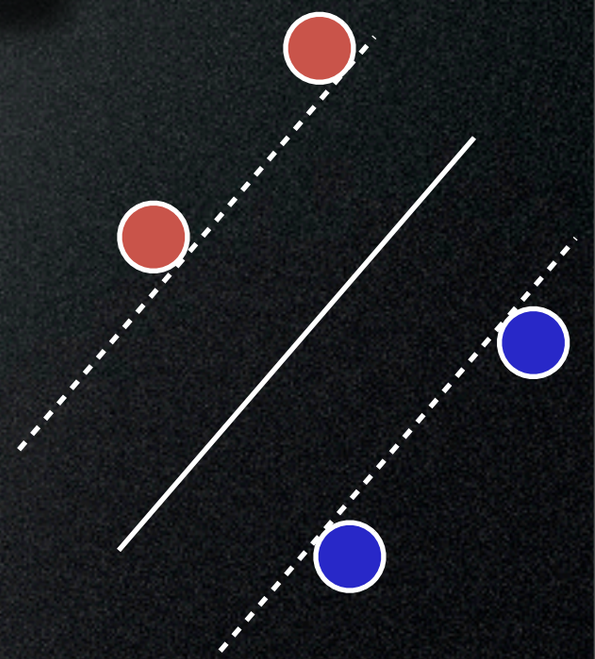
$$(\alpha, b)^\top = \mathbf{H}^{-1}(1 \cdots 1, 0)^\top$$



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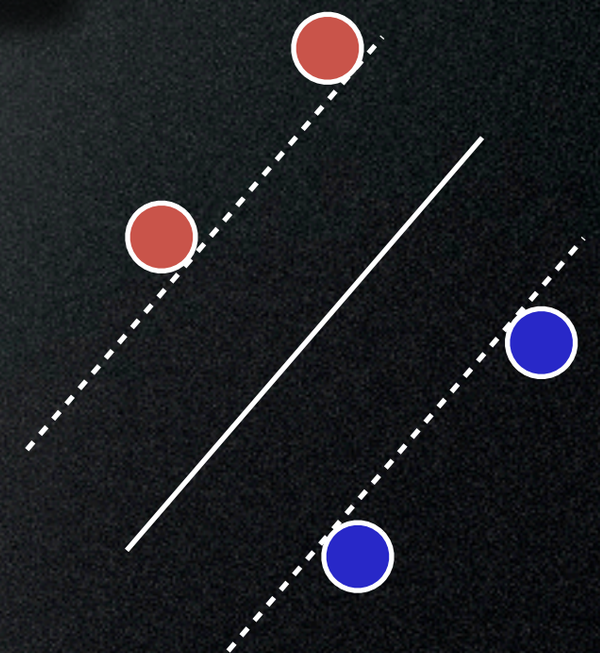
$$(\alpha, b)^\top = \mathbf{H}^{-1}(1 \cdots 1, 0)^\top$$

$$\frac{\partial(\alpha, b)}{\partial \mathbf{M}_{ij}} = -\mathbf{H}^{-1}\frac{\partial \mathbf{H}}{\partial \mathbf{M}_{ij}}(\alpha, b)^\top$$



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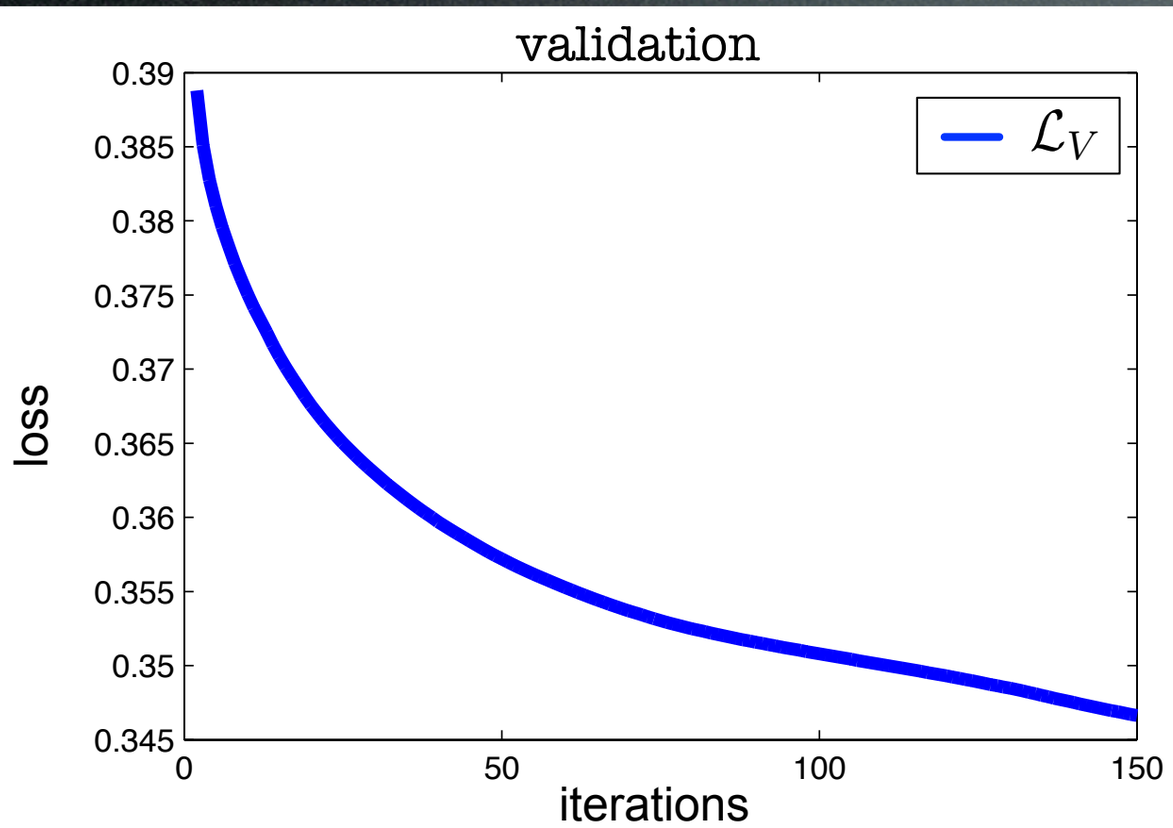
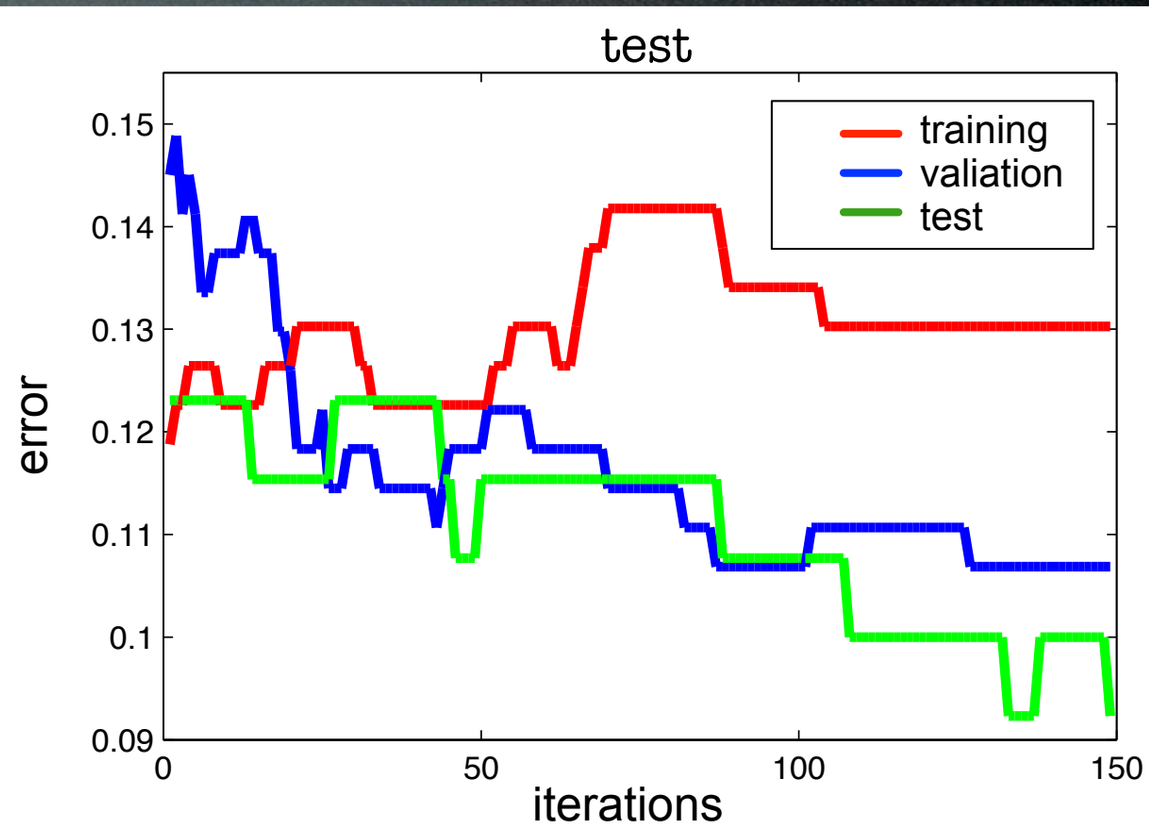
Optimization





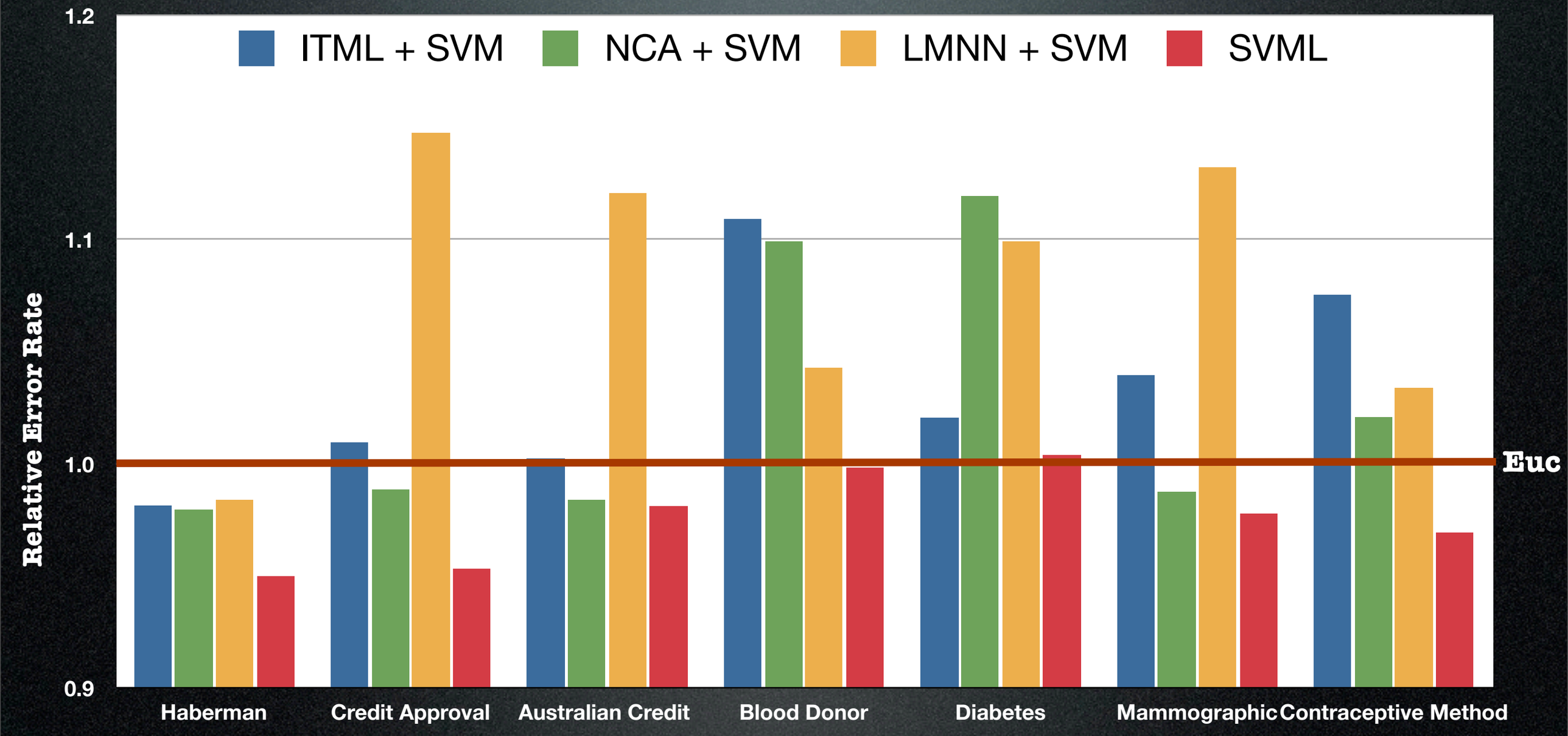
UCI credit card data set

Speed?

- Each gradient step involves solving the SVM
- But with conjugate gradient, a few steps are enough
- On average 4x faster than CV.

Results

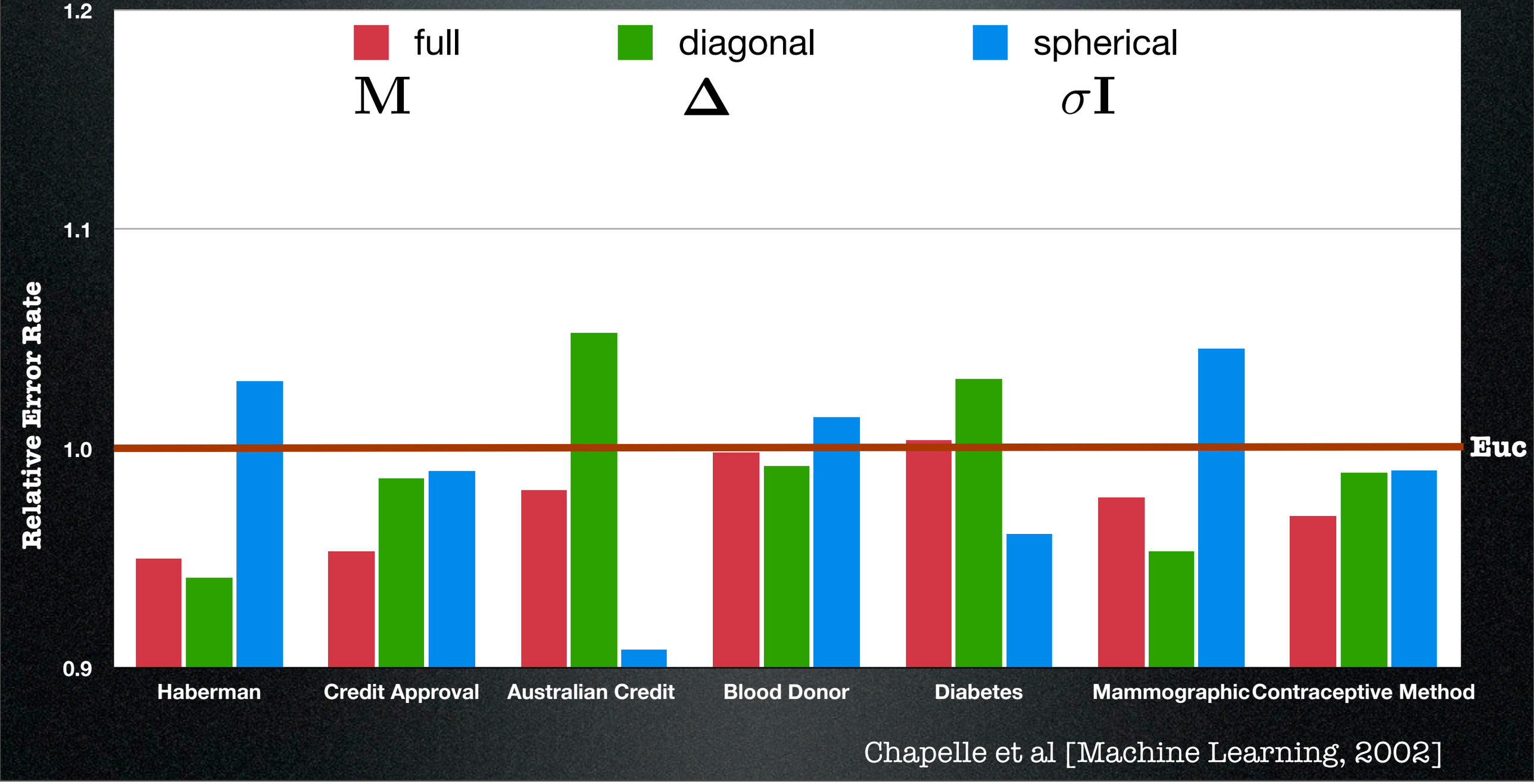
Joint Metric and SVM Learning.



Goldberger et al [nips 2005],Weinberger et al [nips 2006],Davis et al [icml 2007]

Results

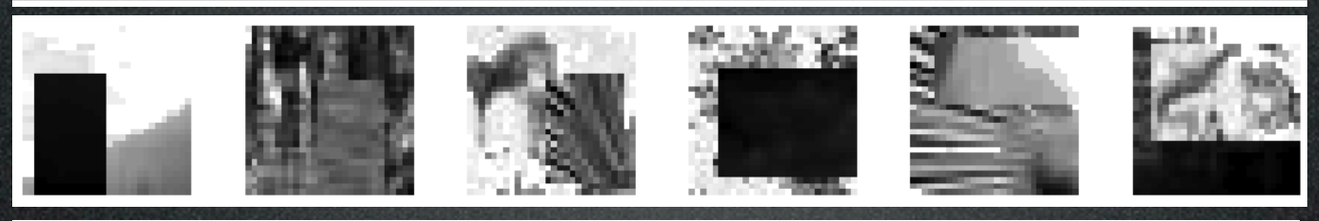
Diagonal and isotropic matrices.



Chapelle et al [Machine Learning, 2002]

Harder data sets

Deep Learning benchmarks!
(possibly designed to break SVMs)

		Train/Test
rectangles		1200/50000
rect-images		12000/50000
convex		8000/50000

[Vincent et al, 2008]

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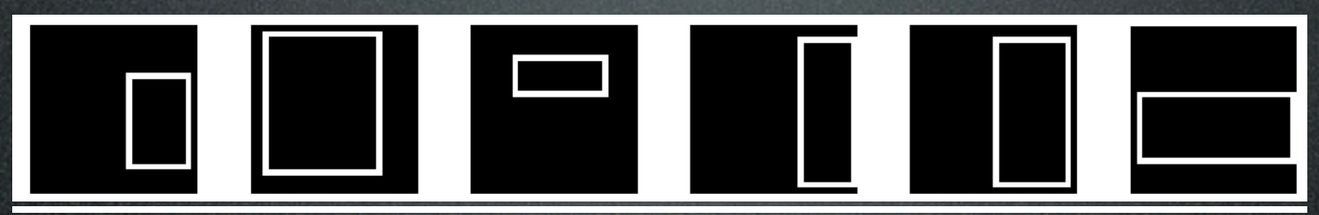
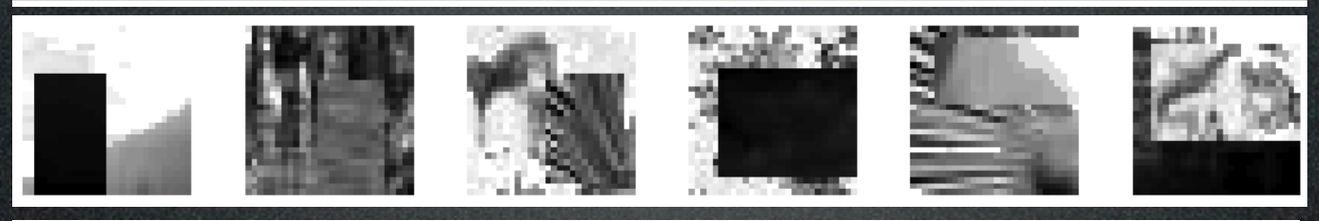
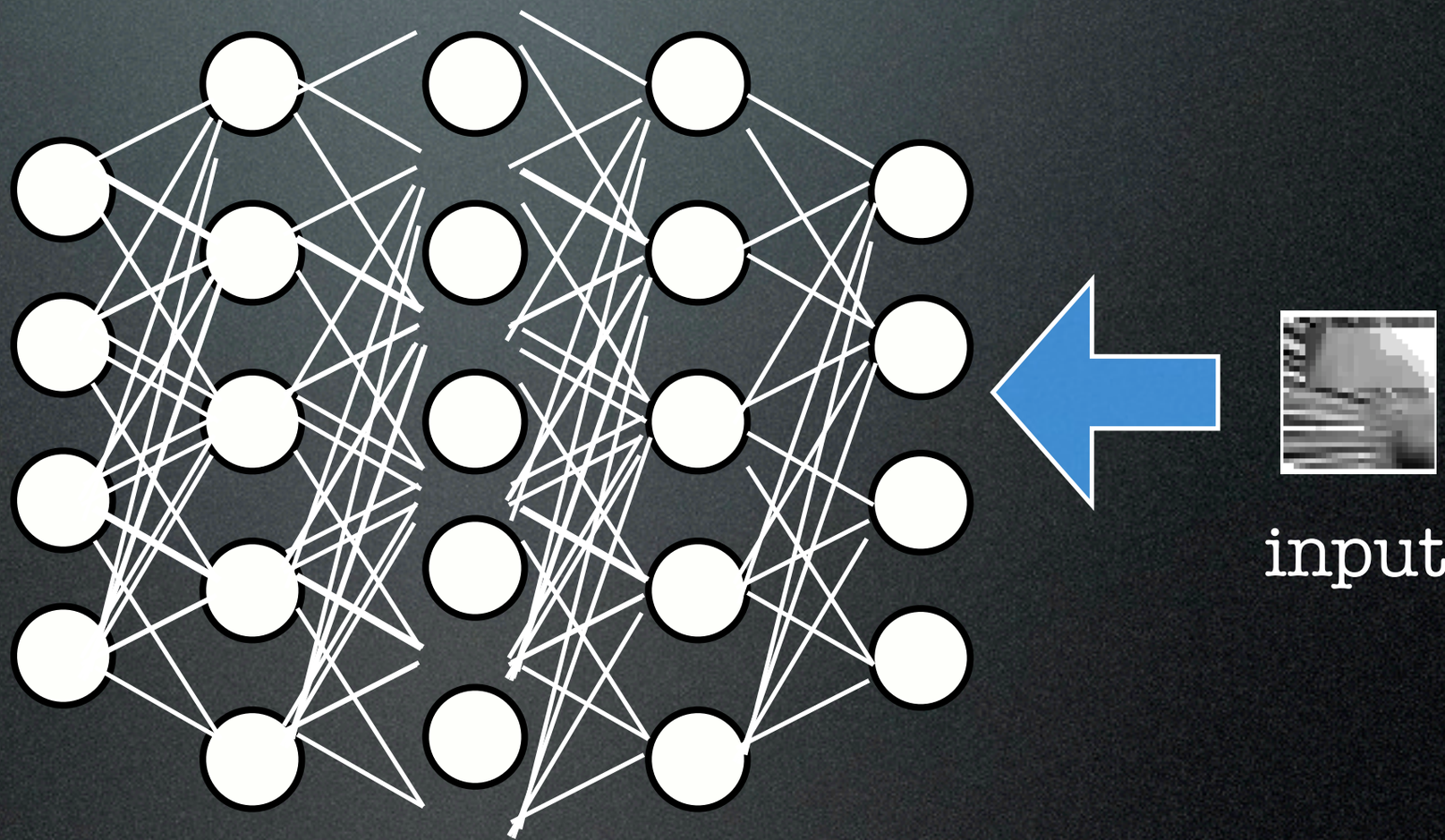
rectangles		1200/50000
rect-images		12000/50000
convex		8000/50000

But we need good features!!

[Vincent et al, 2008]

Feature Generator: Denoising Autoencoder



[Vincent et al, 2008]

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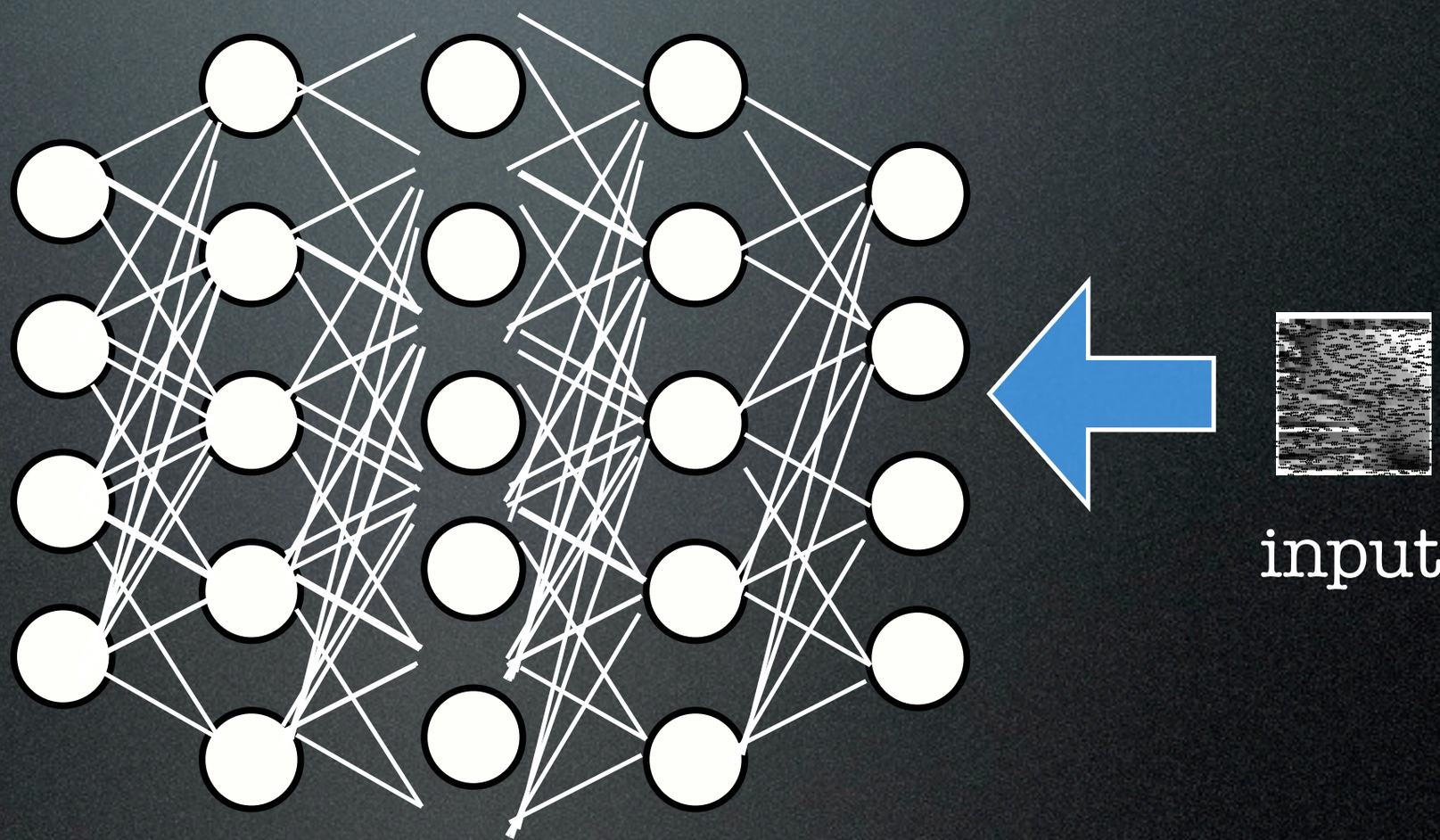
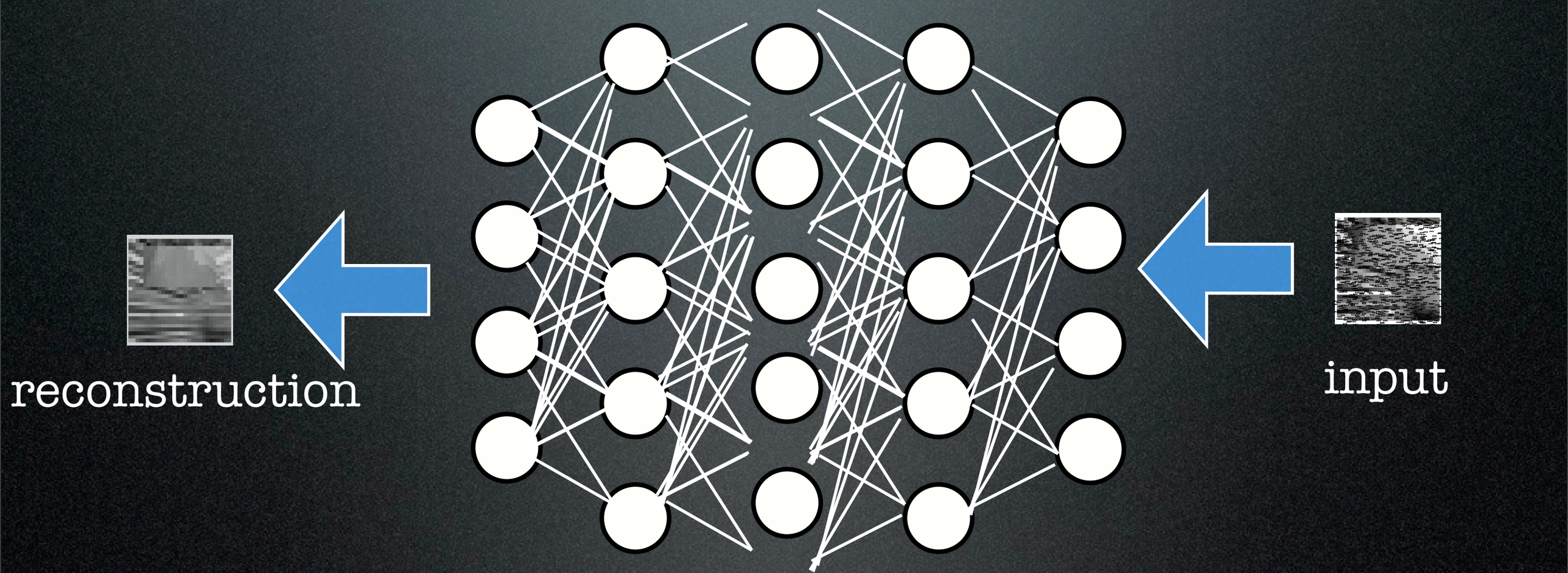


[Vincent et al, 2008]

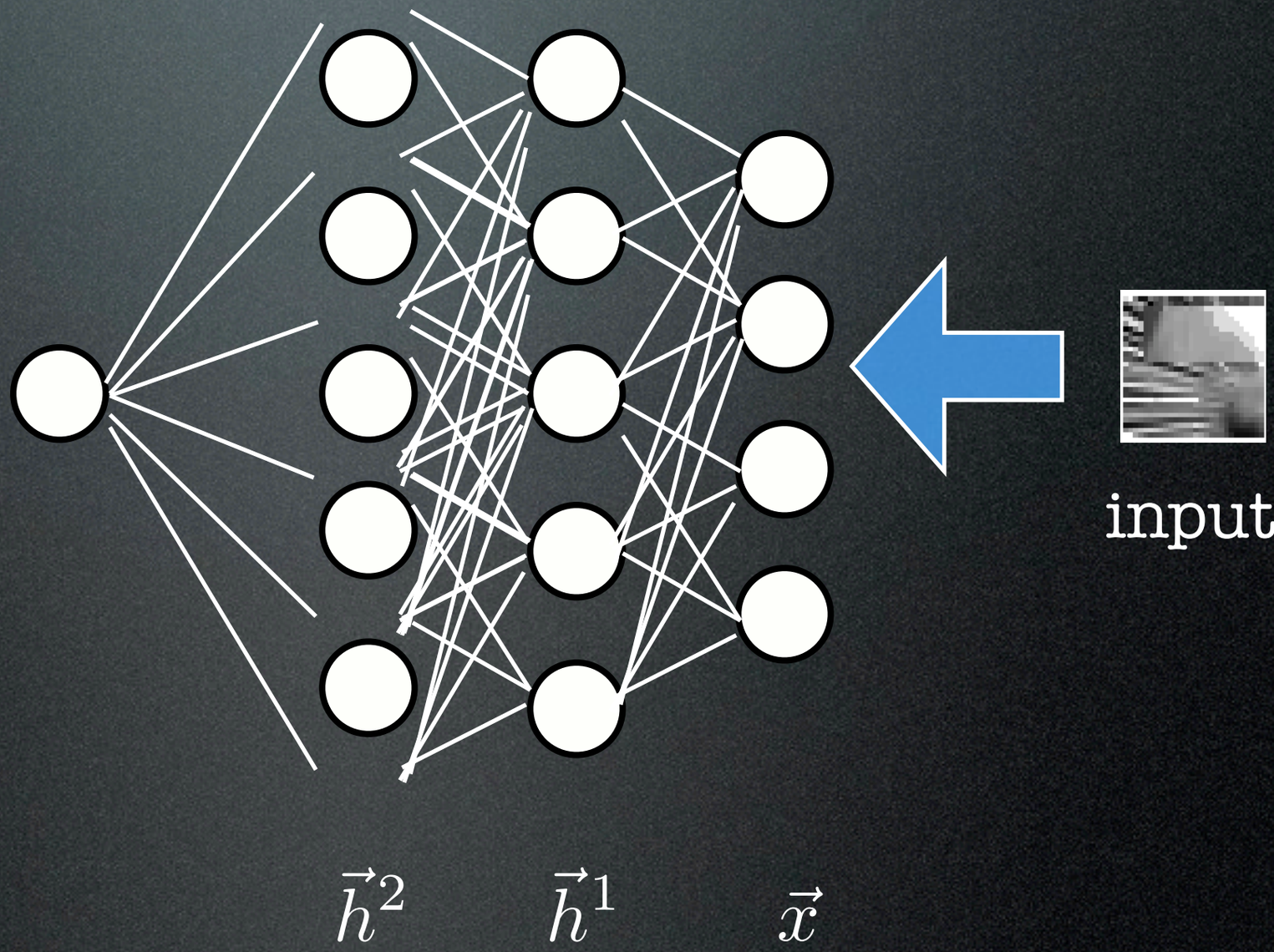
Feature Generator: Denoising Autoencoder



[Vincent et al, 2008]

Fine-Tuning

(with backprop)



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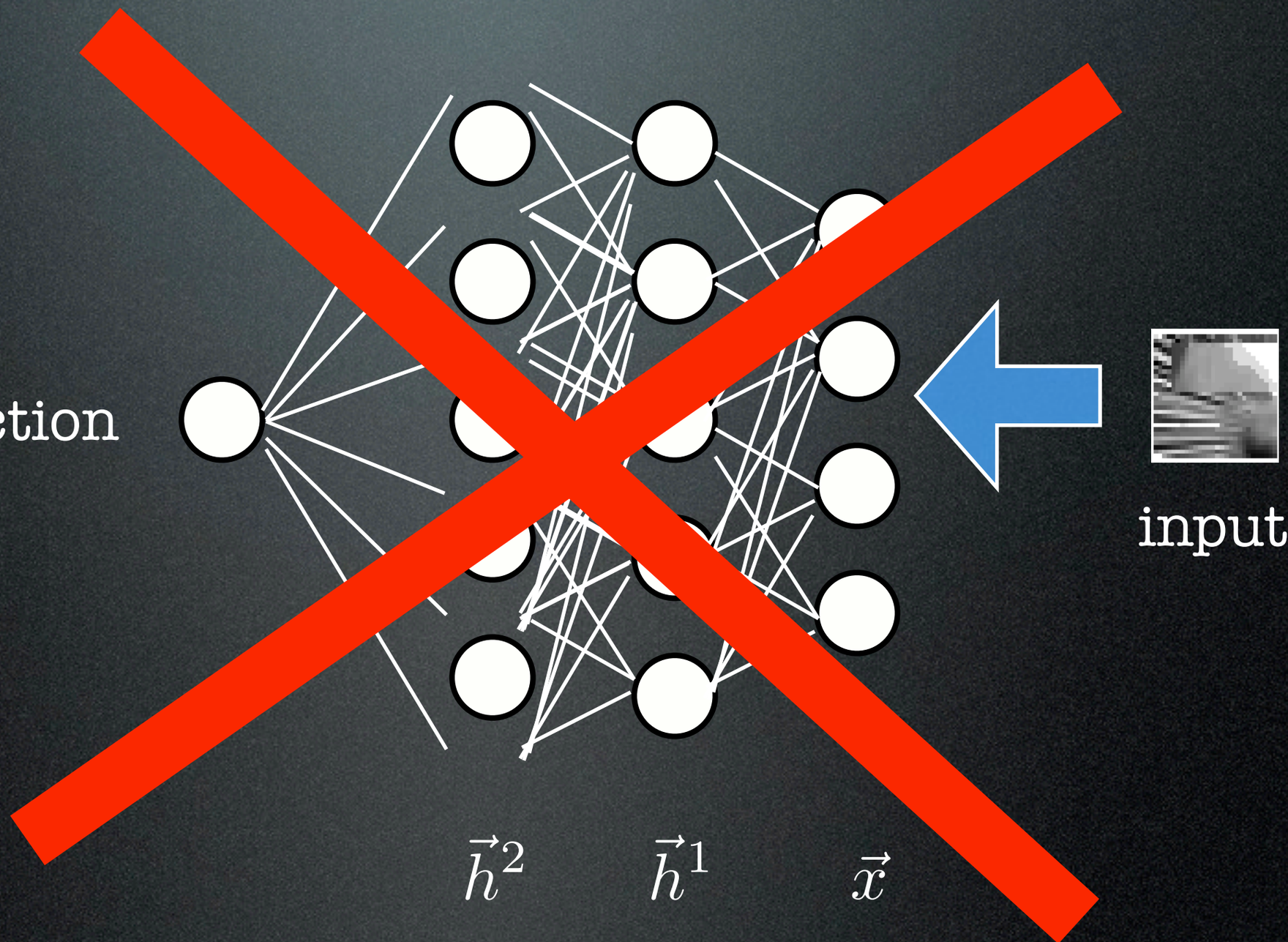
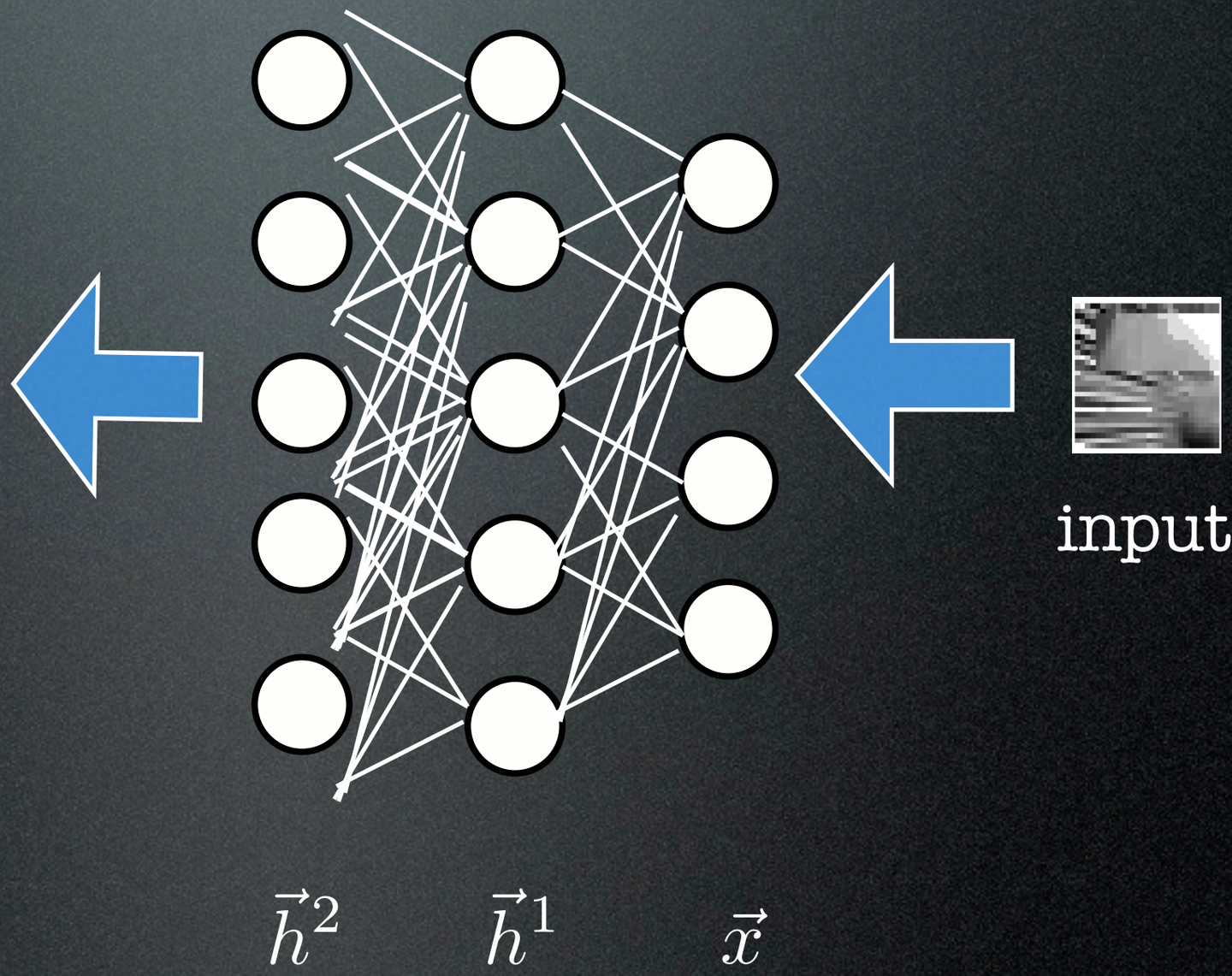


Features for SVMs



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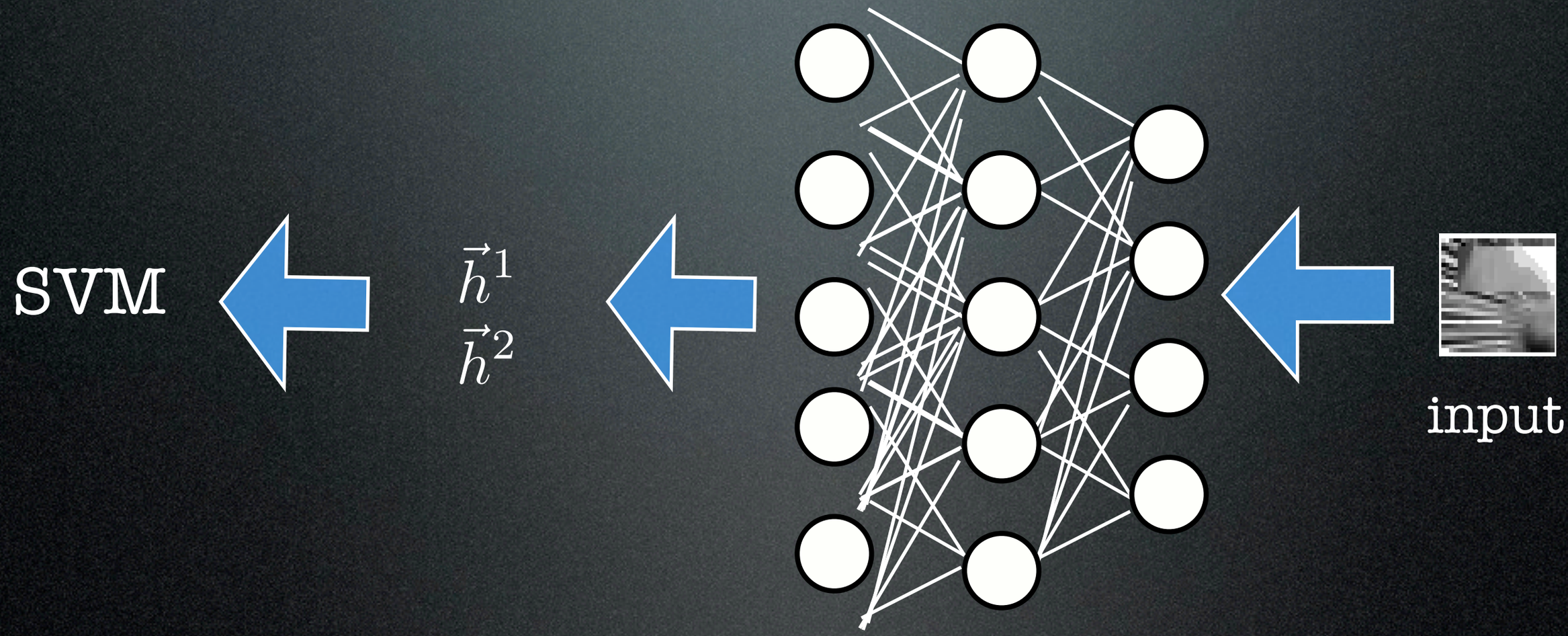

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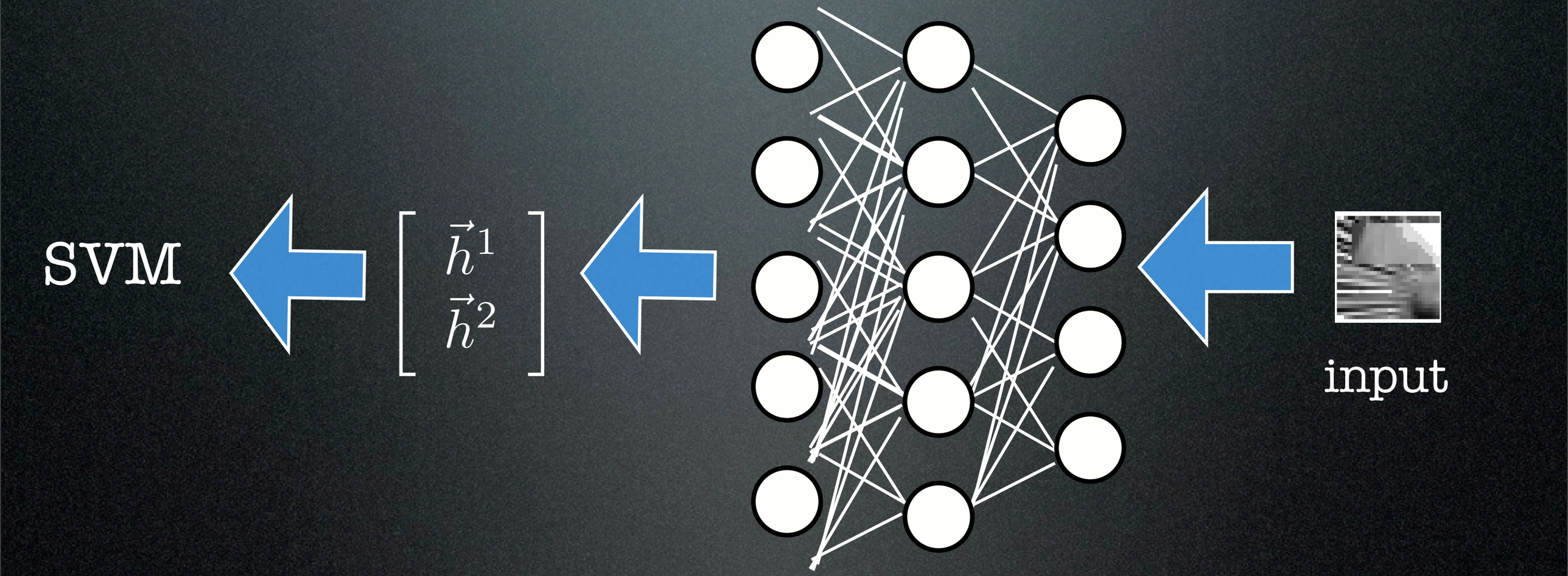

Deep Learning 101

$$\begin{bmatrix} \vec{h}^1 \\ \vec{h}^2 \end{bmatrix} \Rightarrow \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} \begin{bmatrix} \vec{h}^1 \\ \vec{h}^2 \end{bmatrix}$$

- Hidden layers are incompatible.
- Need to scale them appropriately.
- We use a diagonal matrix.

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Deep Learning 101

$$k(\vec{x}_i, \vec{x}_j) = \exp\left(-\sum_{t=0}^{l} \frac{\|\vec{h}_i^t - \vec{h}_j^t\|_2^2}{\sigma_t^2}\right)$$

- Hidden layers are incompatible.
- Need to scale them appropriately.
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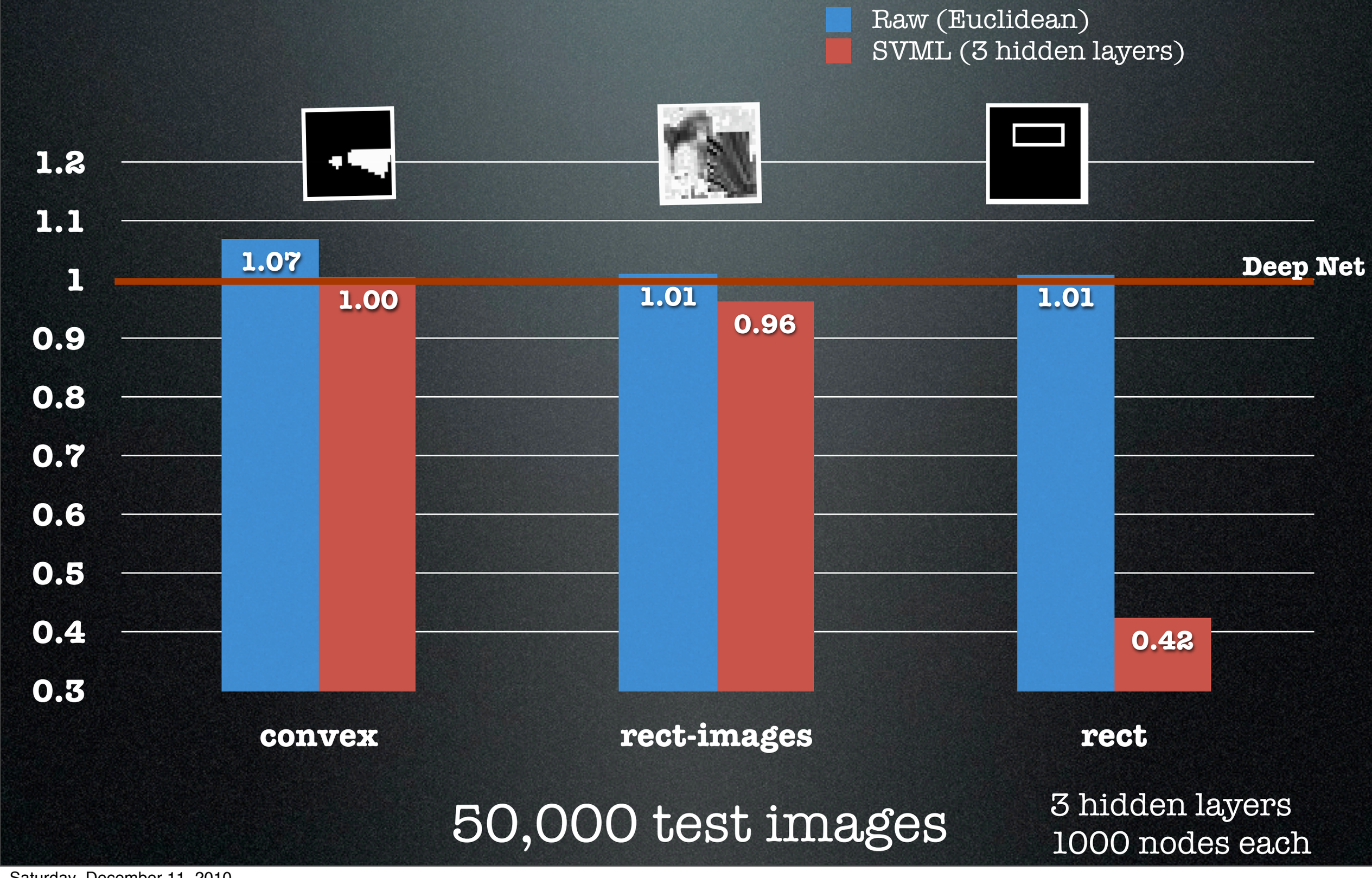
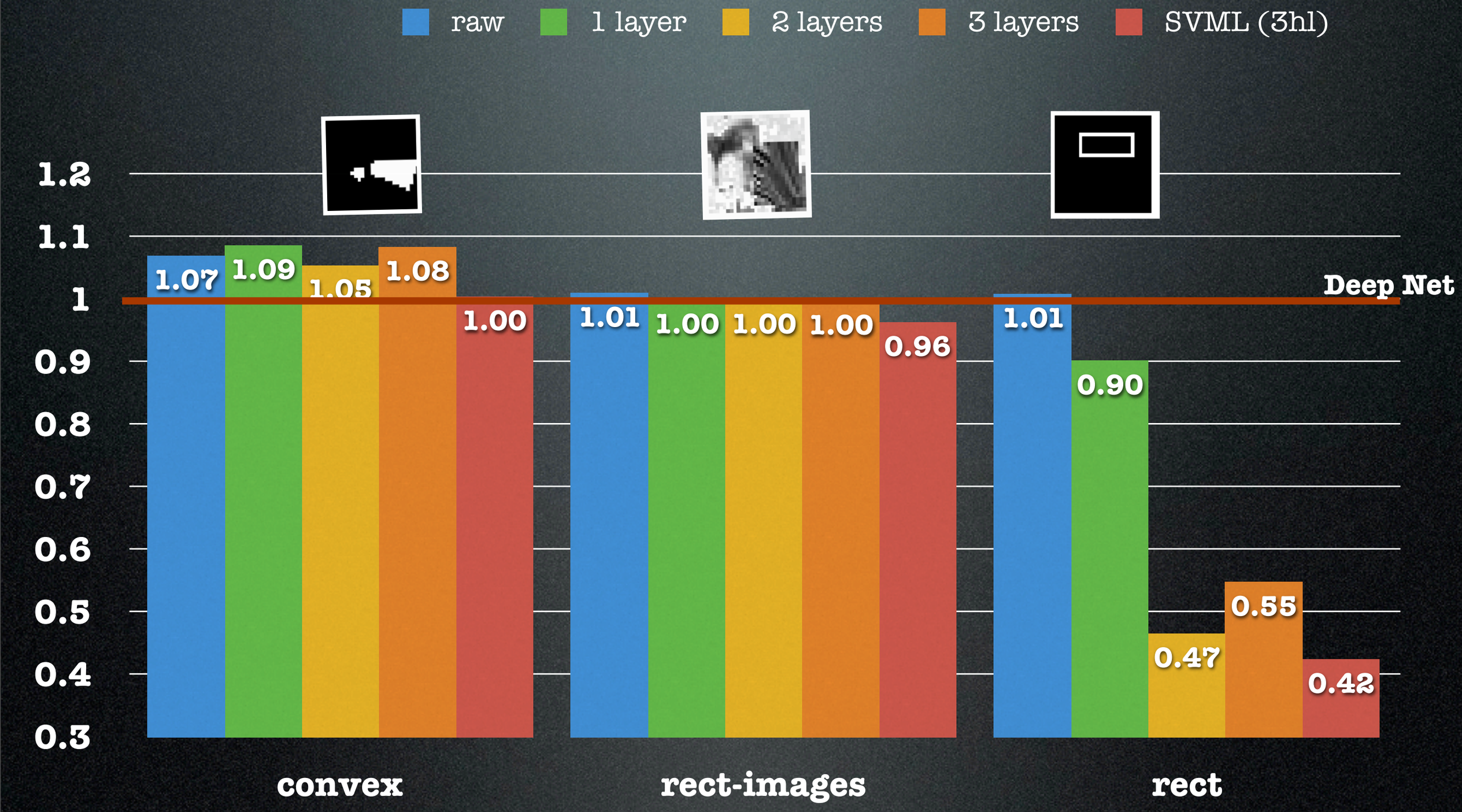
Results



50,000 test images

3 hidden layers
1000 nodes each

Results



50,000 test images

3 hidden layers
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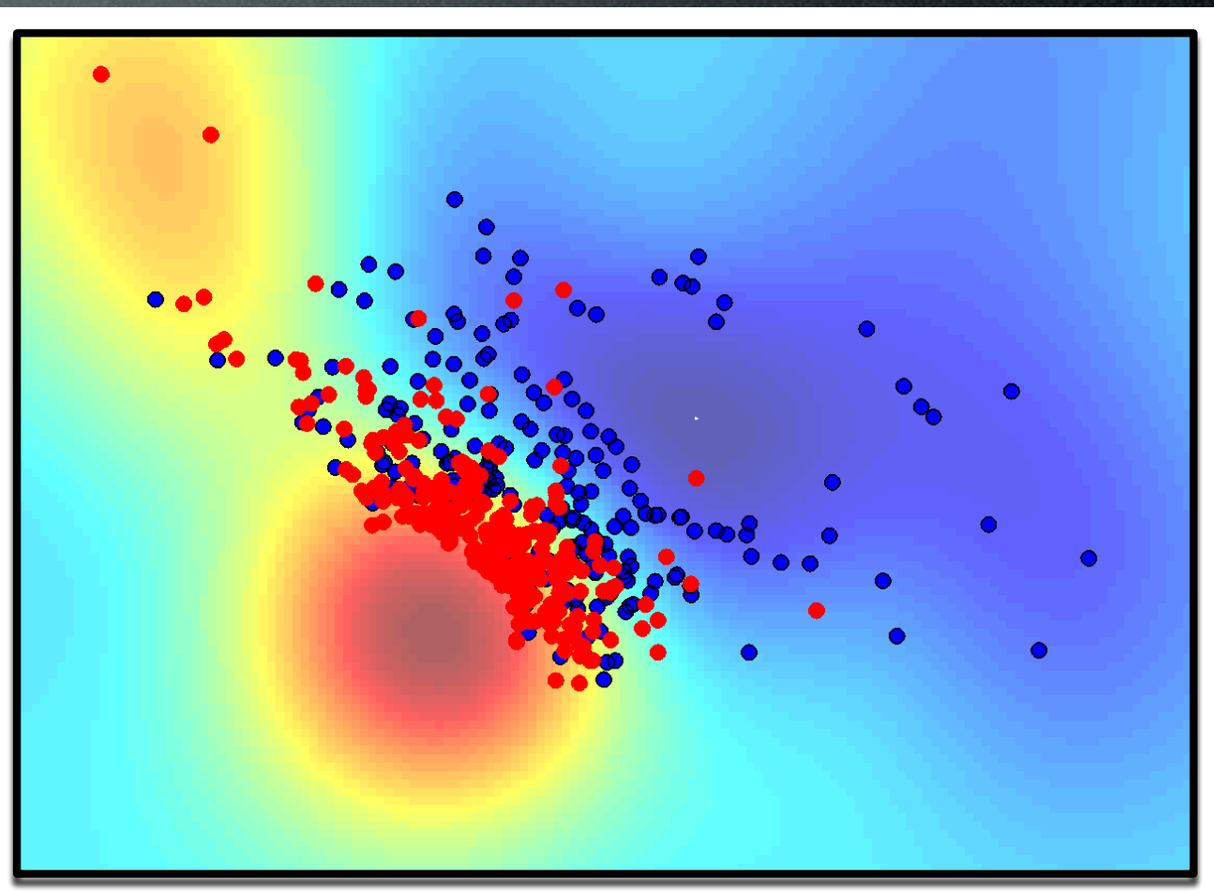
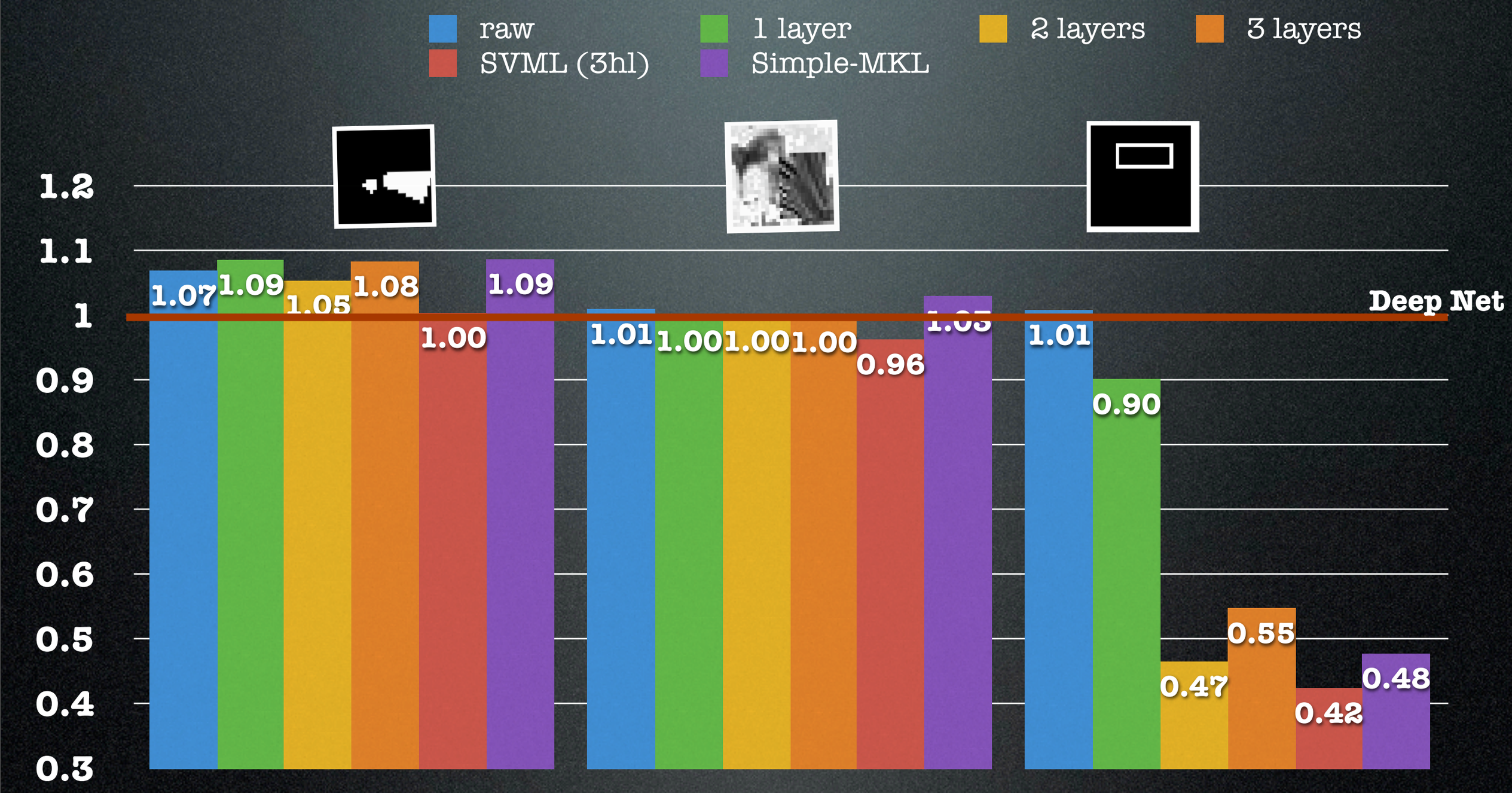
Conclusion

- For Metric Learning to work, it must be integrated into the SVM optimization
- SVML maybe a better alternative to cross validation
- Helps for challenging problems, if features come from incompatible sources
- Denoising auto-encoders are great feature generators for SVMs



SVML-2d

Results



50,000 test images