A Theory of Multiclass Boosting

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Princeton University

Wrigley Field prepared for college **football** game

Sports

Dublin warned over ECB **liquidity**

Business

Newest **senators** Coons and Manchin sworn in

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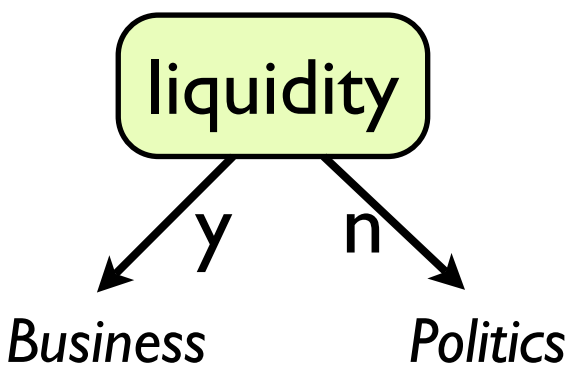

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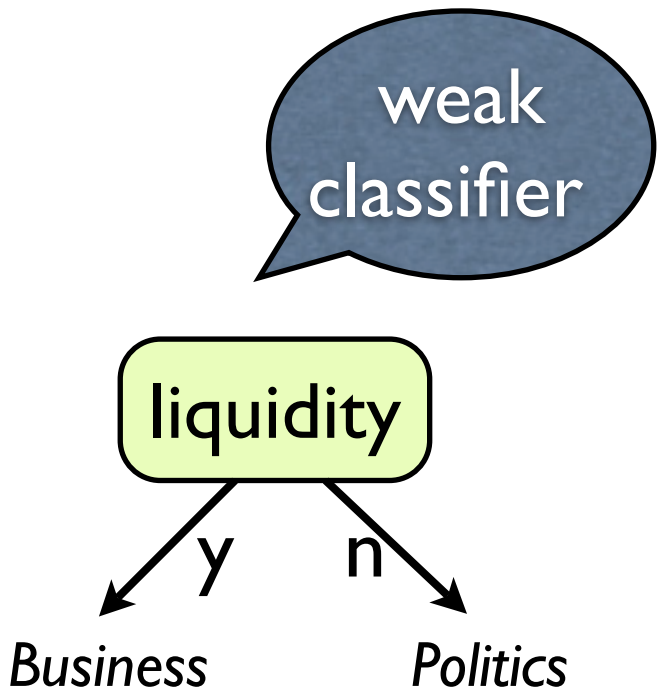

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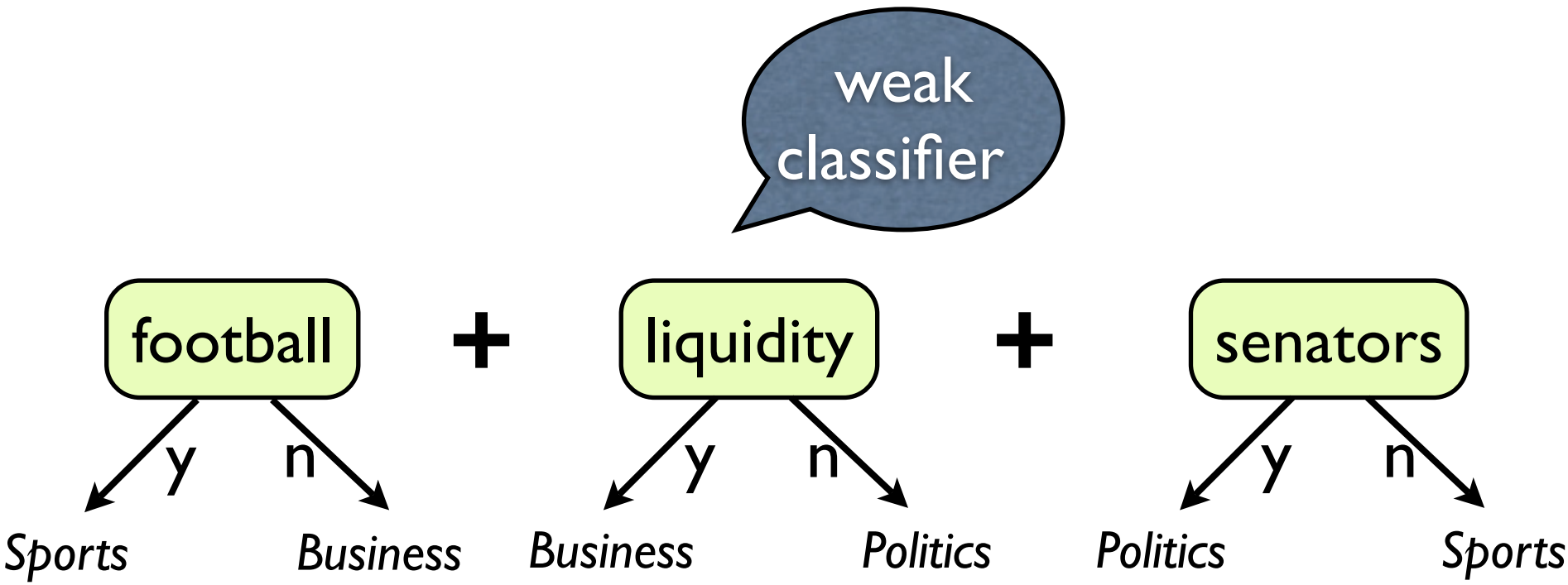

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$\mathcal{H}$ = {weak classifiers}

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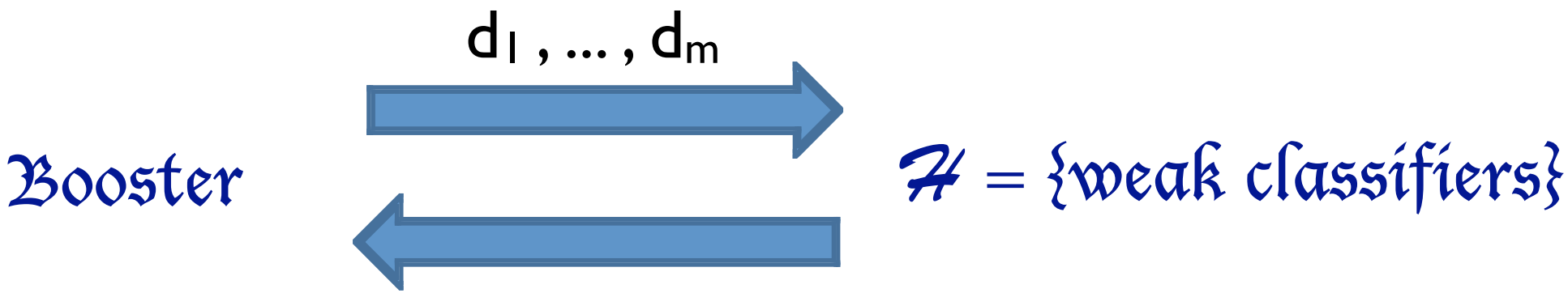

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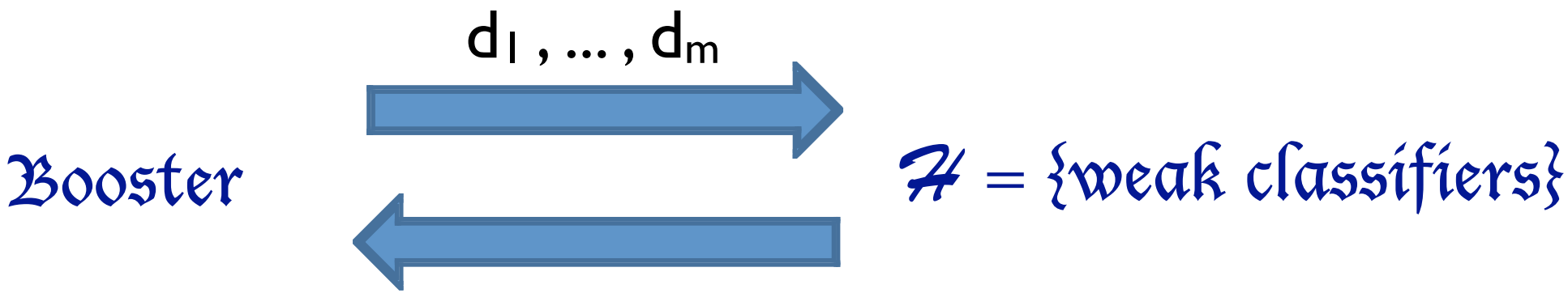

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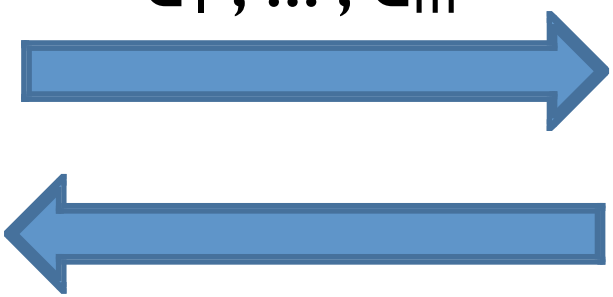

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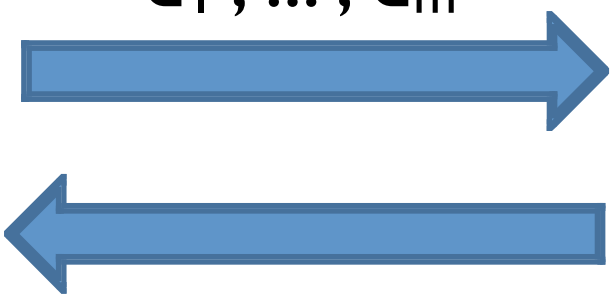

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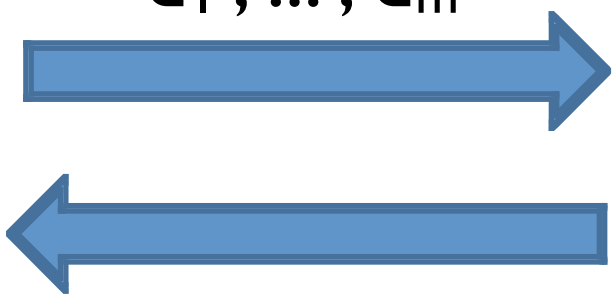

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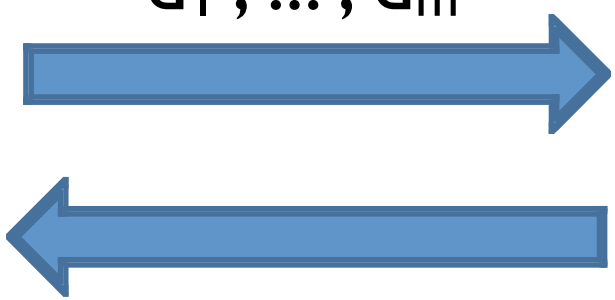

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After T rounds, $\widehat{\text{err}}$ of maj$\{h_1, \ldots, h_T\} \leq \exp(-T\gamma^2/2)$

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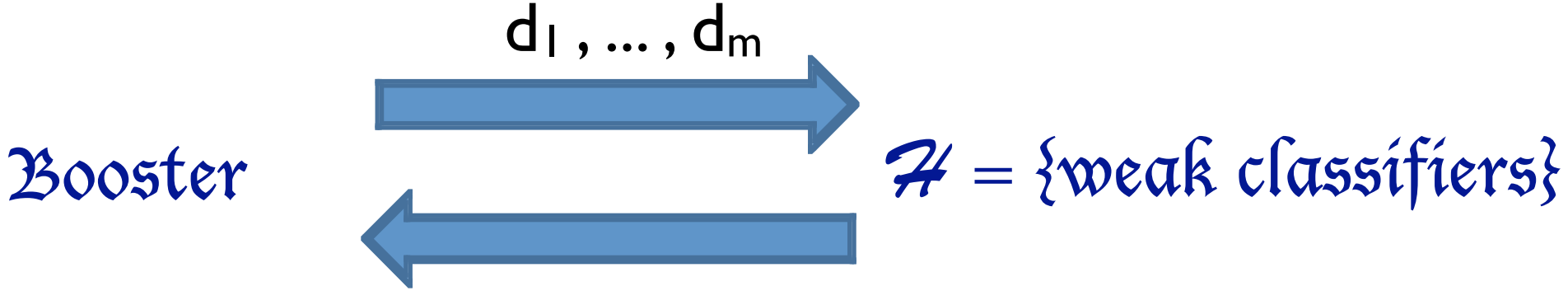

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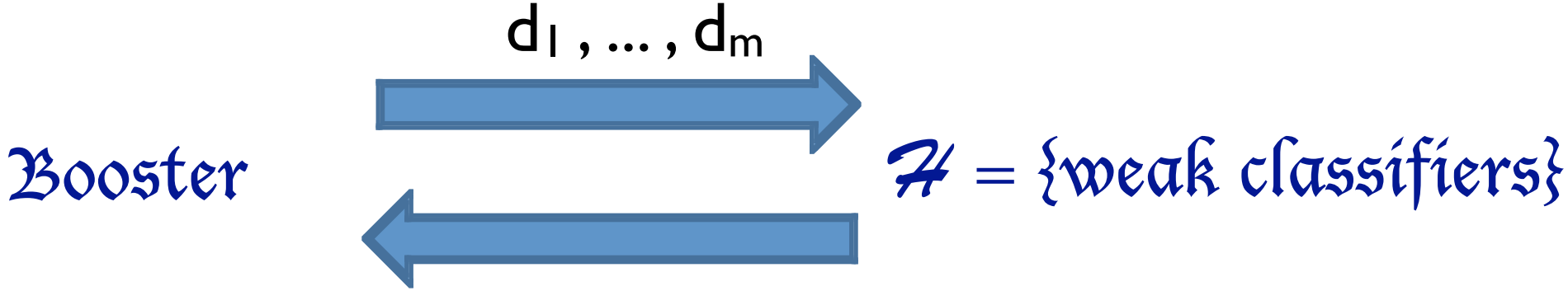

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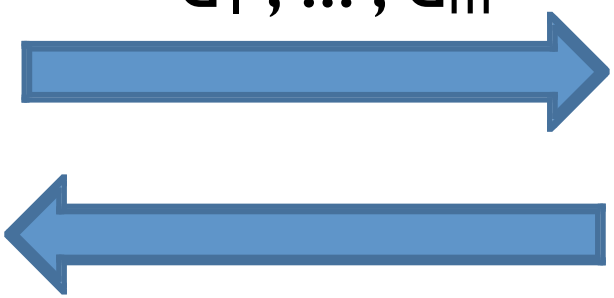

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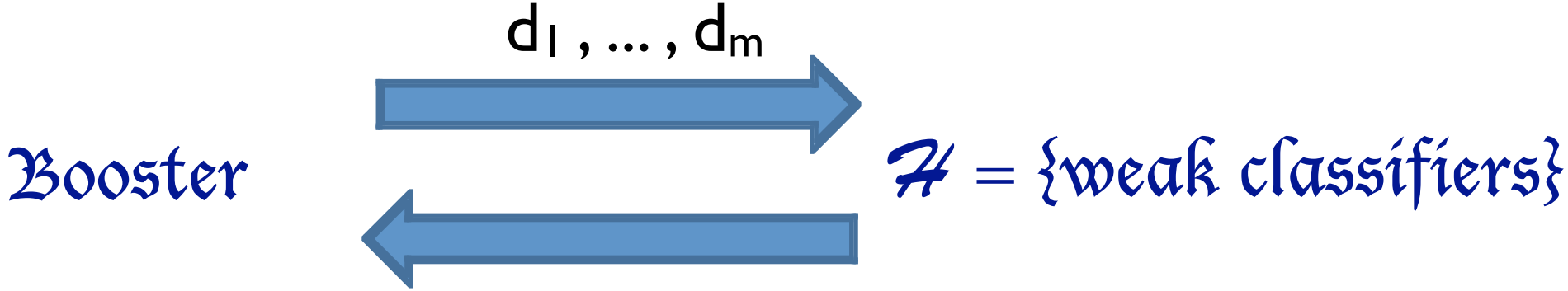

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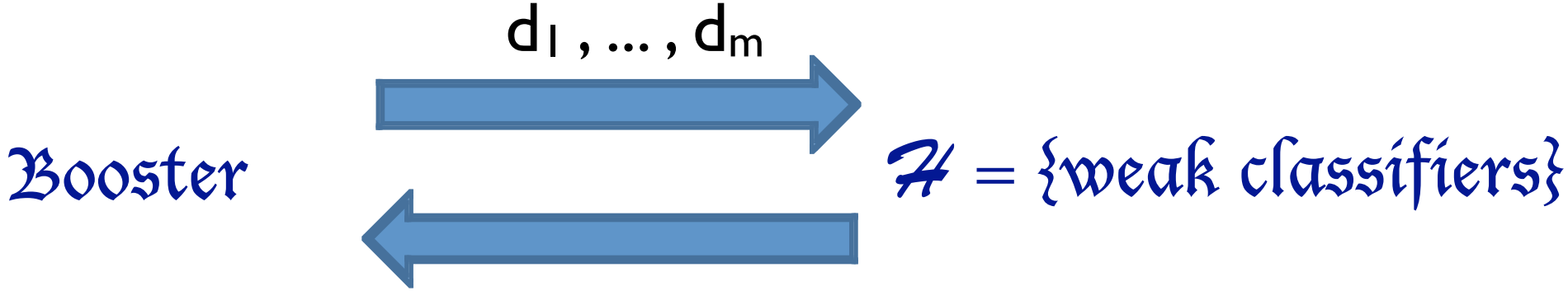

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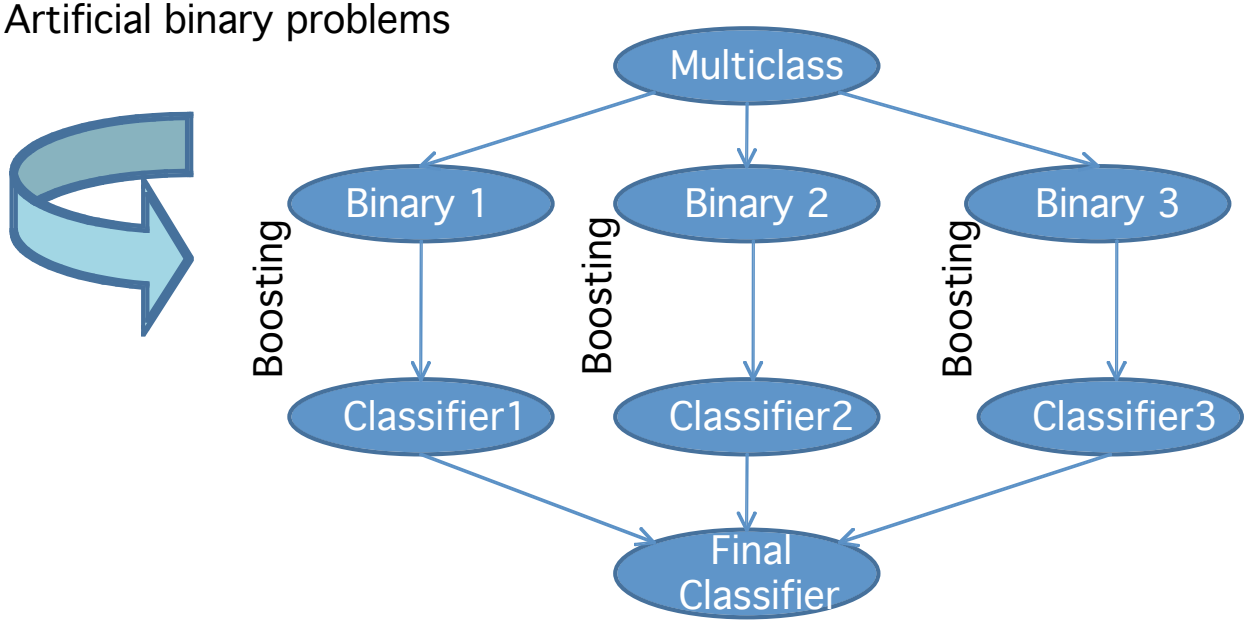

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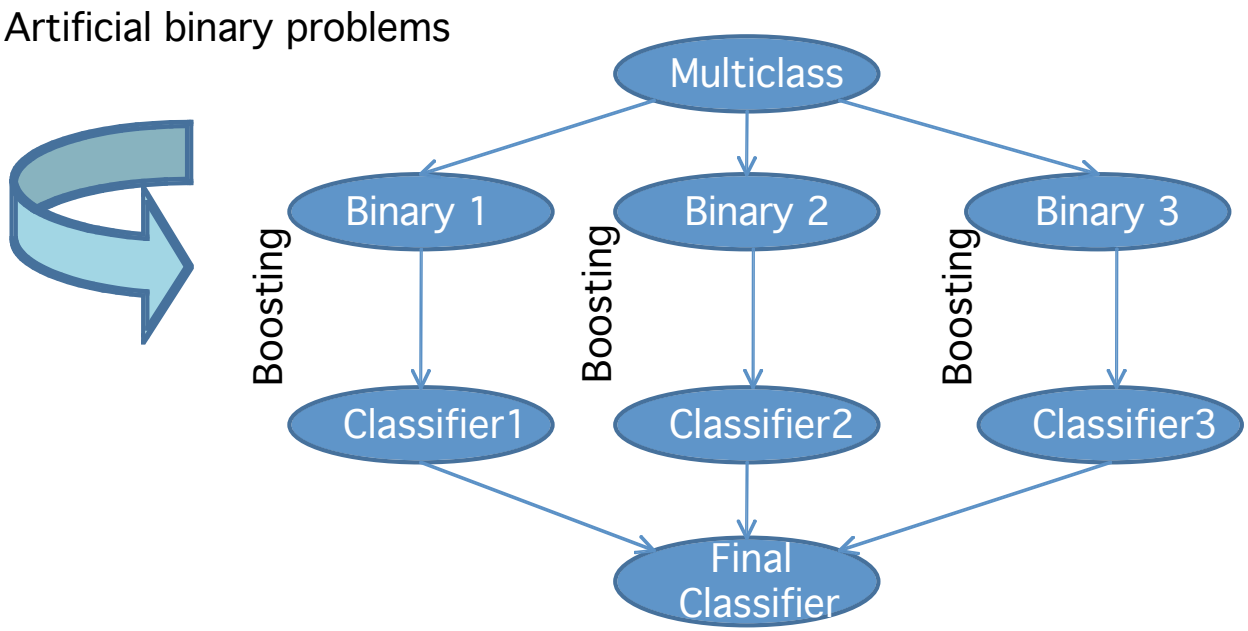

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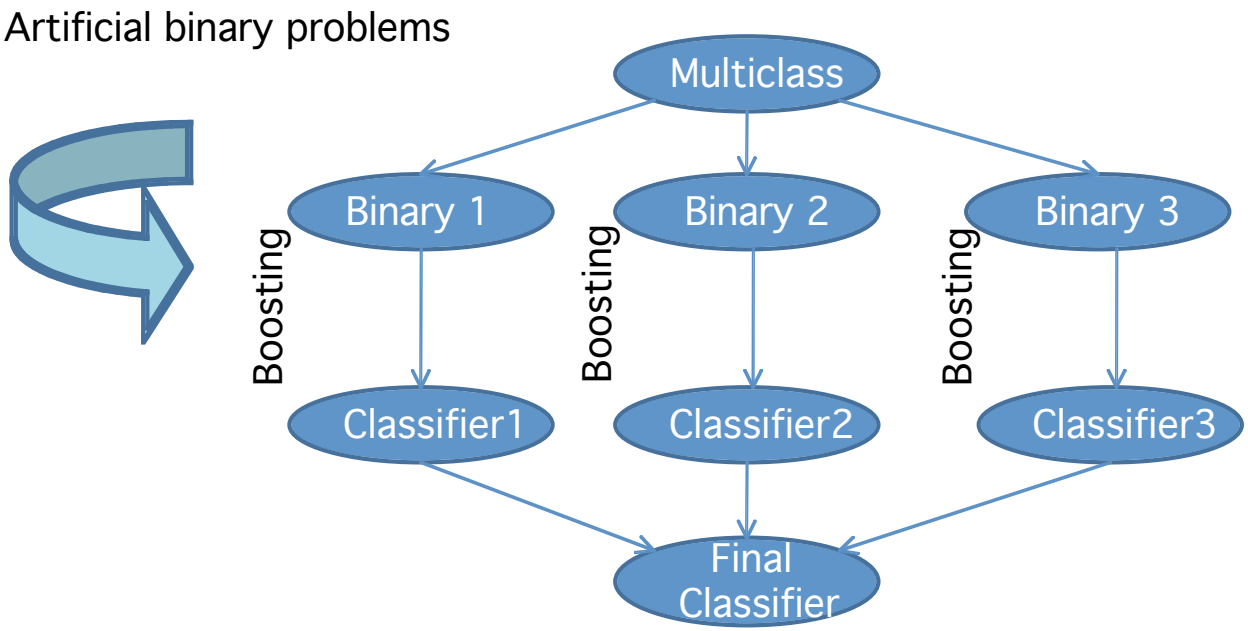

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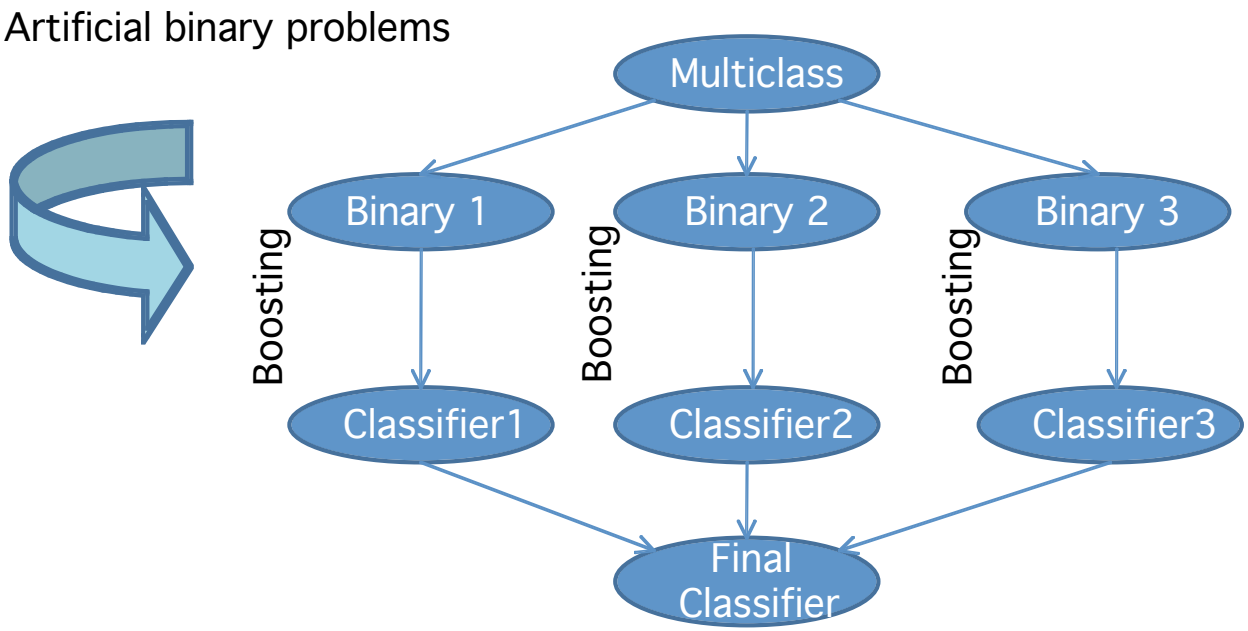

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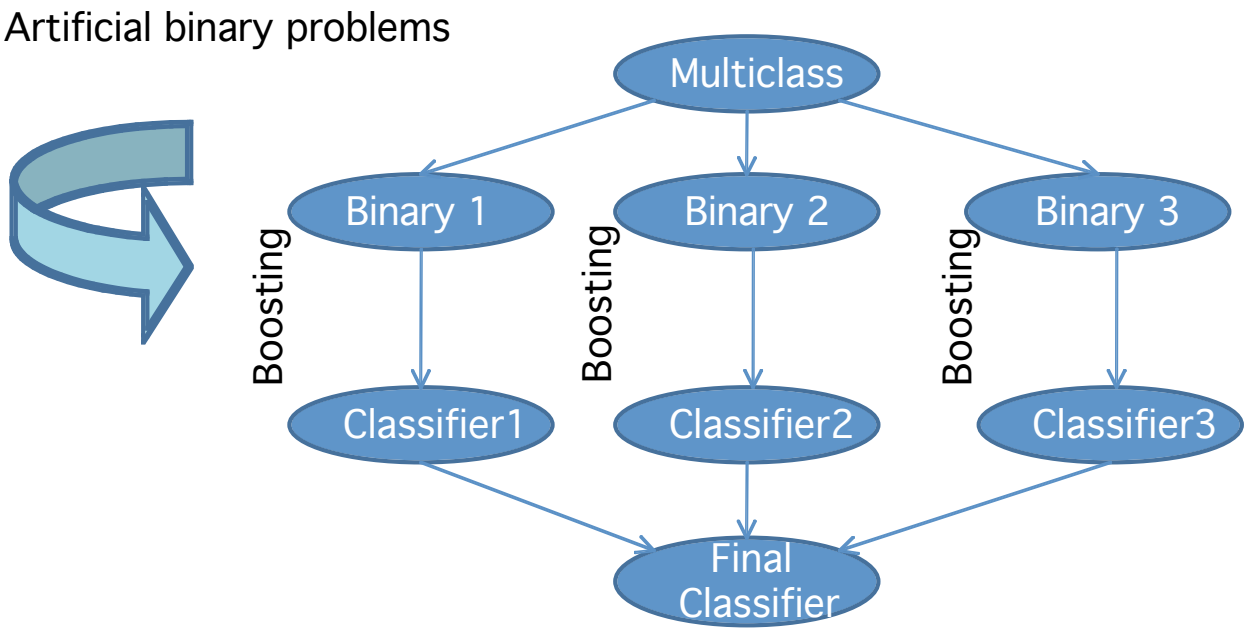

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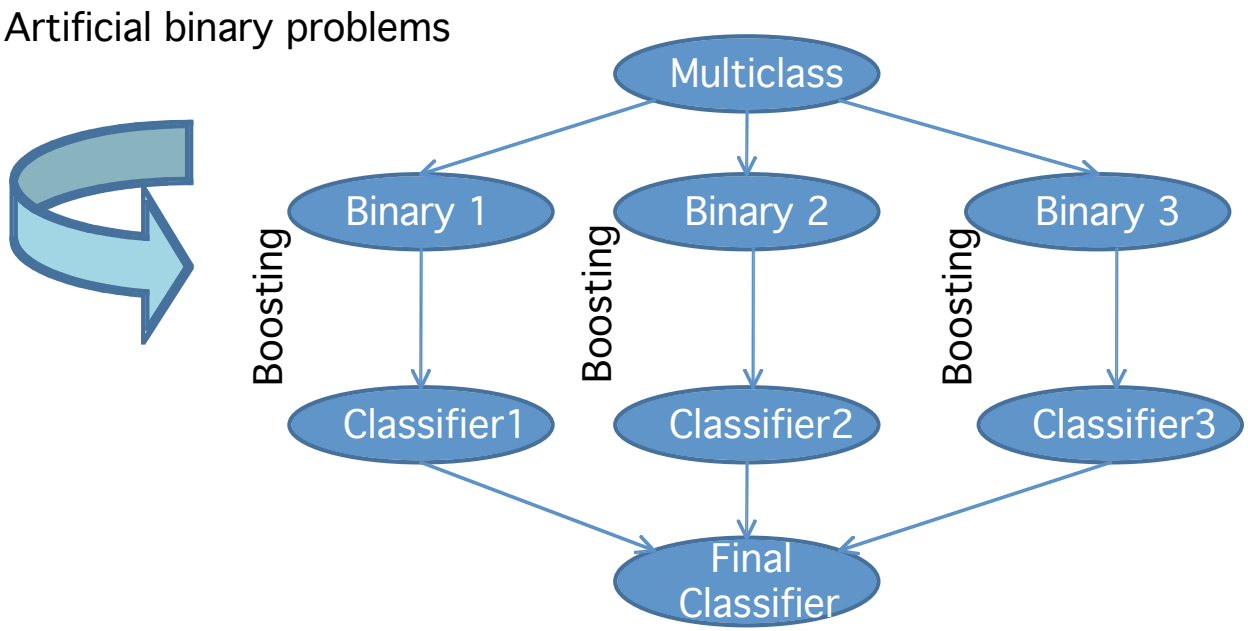

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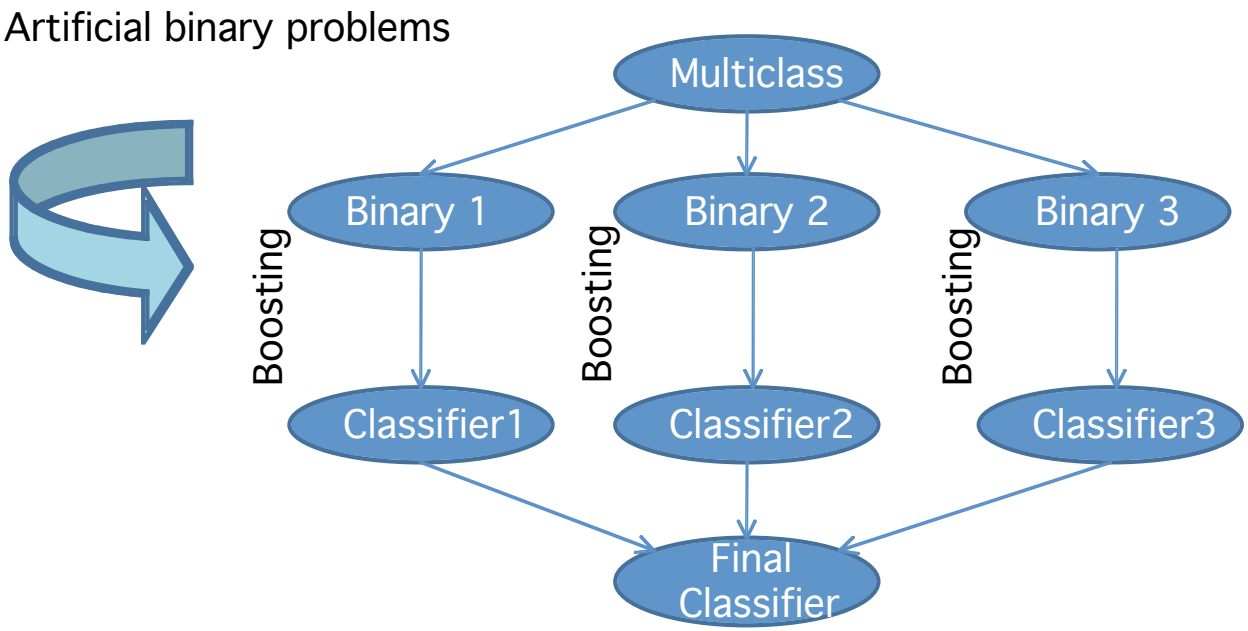

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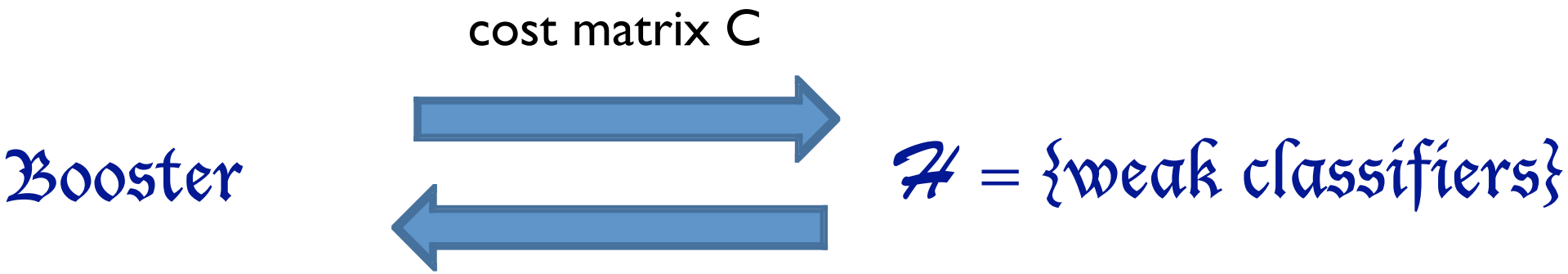

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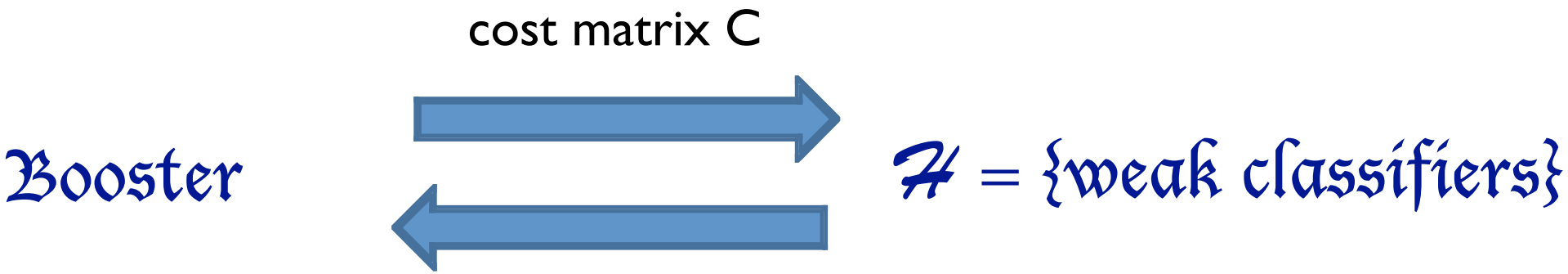

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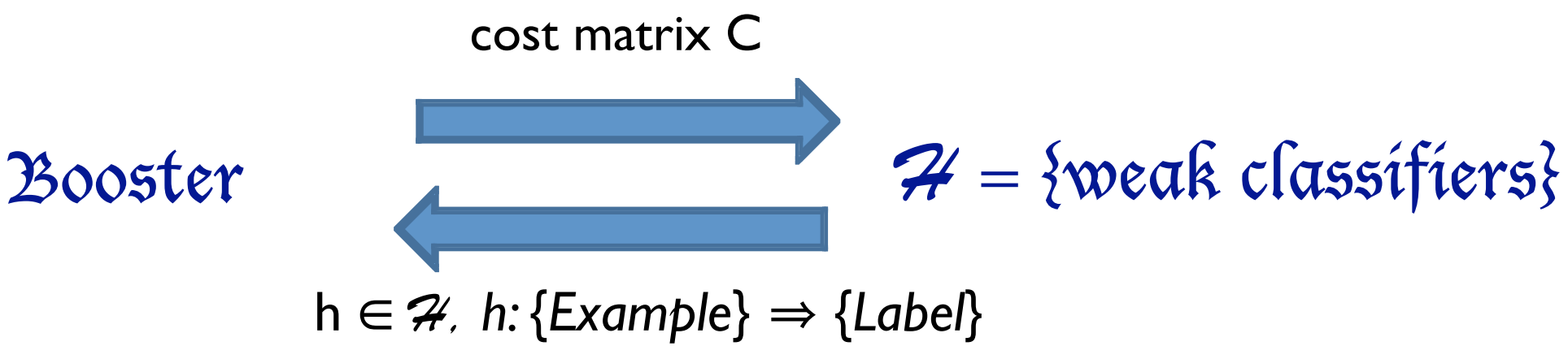
$$B(i, \ell) = \begin{cases} \frac{1}{2} + \gamma & \text{if } \ell \text{ correct} \\ \frac{1}{2} - \gamma & \text{if } \ell \text{ wrong} \end{cases}$$



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  - Not optimal, but still provably very efficient

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Weight

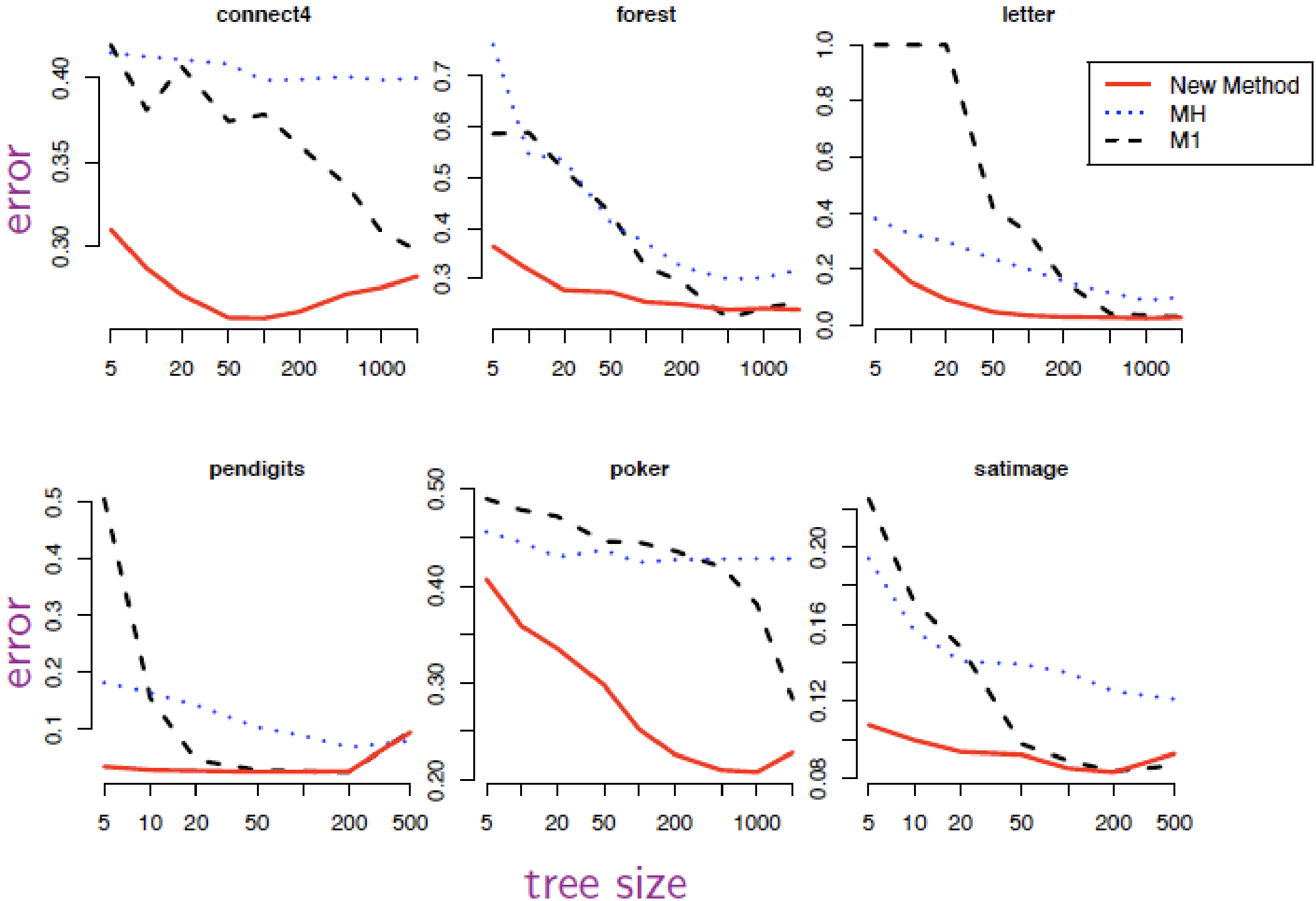
$$\alpha_t = \ln\left\{\frac{1+\delta_t}{1-\delta_t}\right\}$$

Cost Matrix

$$C_{t+1}(i,l) = \begin{cases} e^{f_t(i,l)-f_t(i,y_i)} & \text{if } l \neq y_i \\ -\sum_{l' \neq y_i} e^{f_t(i,l')-f_t(i,y_i)} & \text{if } l = y_i \end{cases}$$

Experiments

- Ran adaptive algorithm using minimal WLC
- Compared with AdaBoost.M1, AdaBoost.MH
- Tested on benchmark datasets
- Weak classifiers: bounded size decision trees


connect4
forest
letter
pendigits
poker
satimage
error
error
tree size
New Method
MH
M1
0.30
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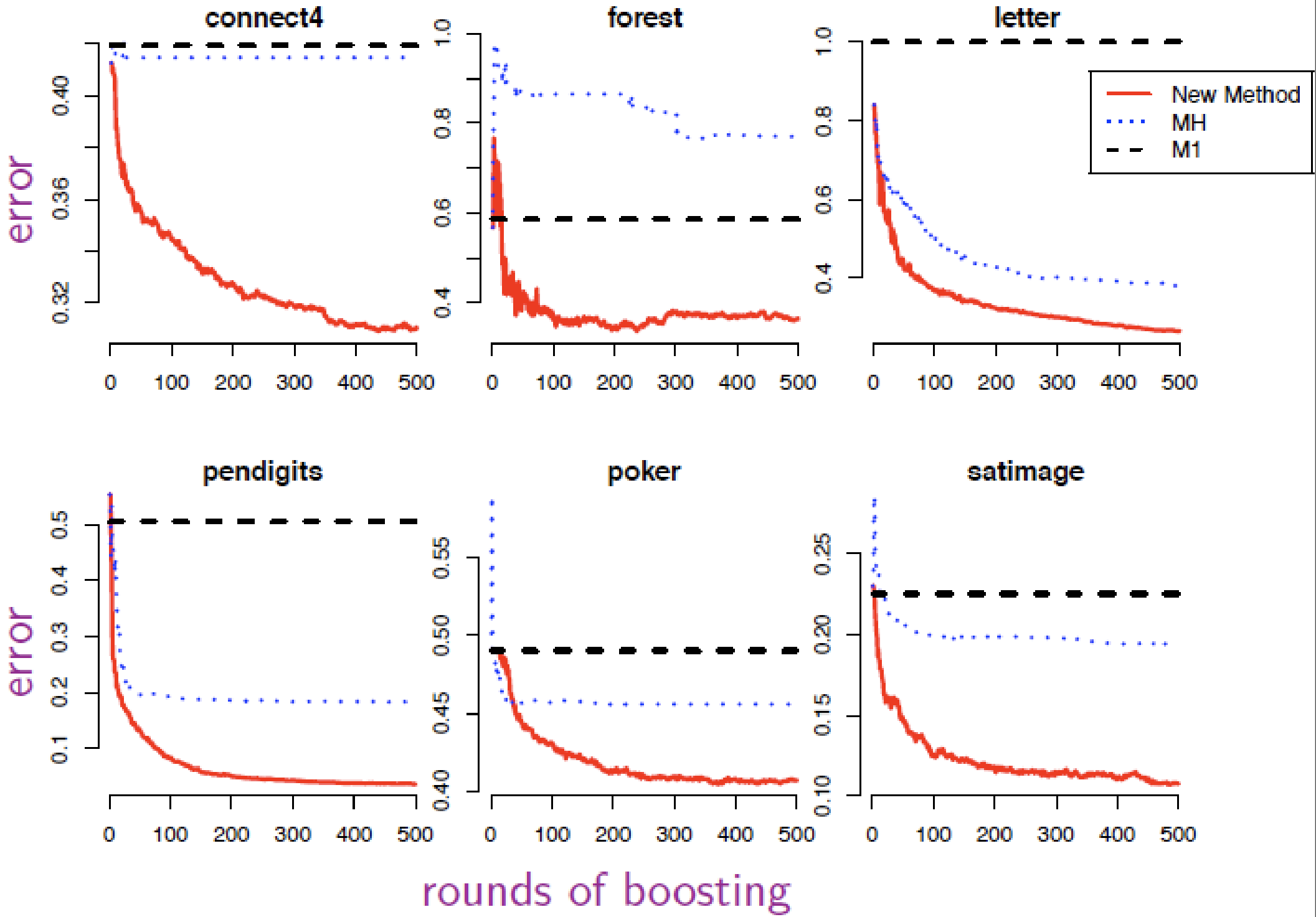
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0.10
0.15
0.20
0.25
0
100
200
300
400
500

Future Work

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- What happens with multi-label / confidence rated weak classifiers?

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- Extensions to ranking.

Thank you