Approximate inference in latent Gaussian-Markov models from continuous time observations

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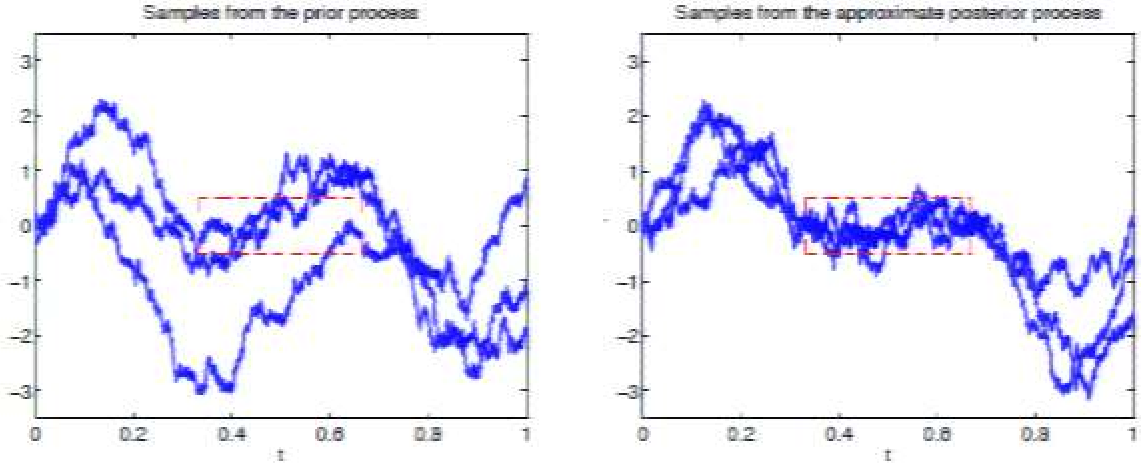


Models and inference

Bayesian model with **linear SDE prior**

$$d\boldsymbol{x}_t = (\boldsymbol{A}_t \boldsymbol{x}_t + \boldsymbol{c}_t)dt + \boldsymbol{B}_t^{1/2} d\boldsymbol{W}_t,\ t \in [0, 1]$$

and likelihood

$$p(\{\boldsymbol{y}^d_{t_i}\}_i, \{\boldsymbol{y}^c_t\} | \{\boldsymbol{x}_t\}) \propto \prod_{t_i \in T_d} p(\boldsymbol{y}^d_{t_i} | \boldsymbol{x}_{t_i}) \times \exp\left\{ - \int_0^1 dt V(t, \boldsymbol{y}^c_t, \boldsymbol{x}_t) \right\}$$

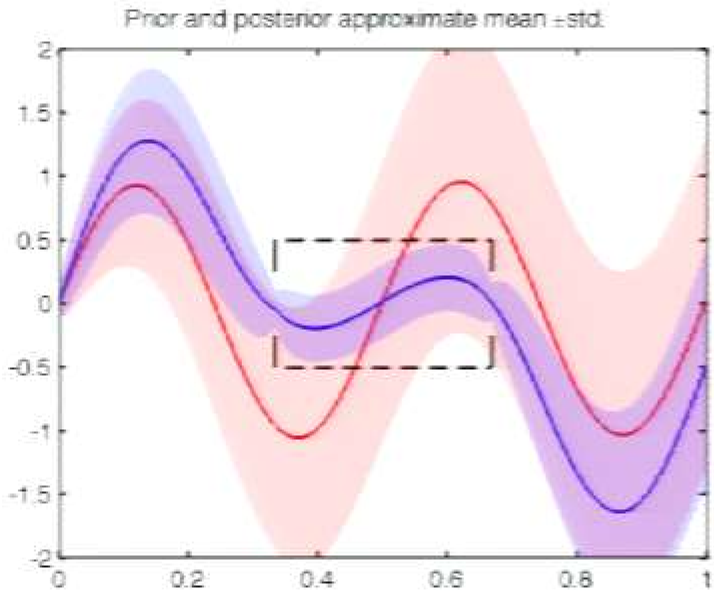
Notation

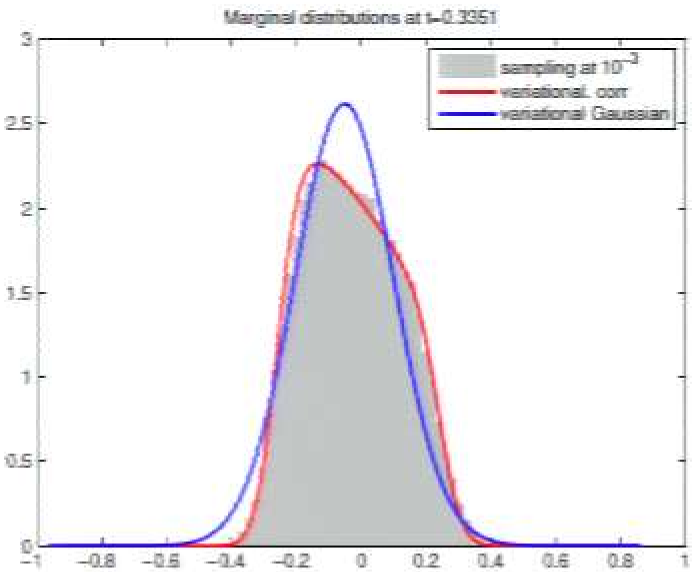
- $\{\boldsymbol{W}_t\}_t$ is the Wiener process
- $p(\boldsymbol{y}^d_{t_i} | \boldsymbol{x}_{t_i})$ stands for **discrete time observations** at times $t_i \in T_d$
- $V(t, \boldsymbol{y}^c_t, \boldsymbol{x}_t)$ is a **loss function**, coding continuous time observations/constraints

Inference

- We apply **expectation propagation** to the Euler-Maruyama discretisation
- The $\Delta t \to 0$ **limit** results in an EP/variational fixed point algorithm
- We introduce post-inference marginal **correction methods**

Inference in a (soft) box

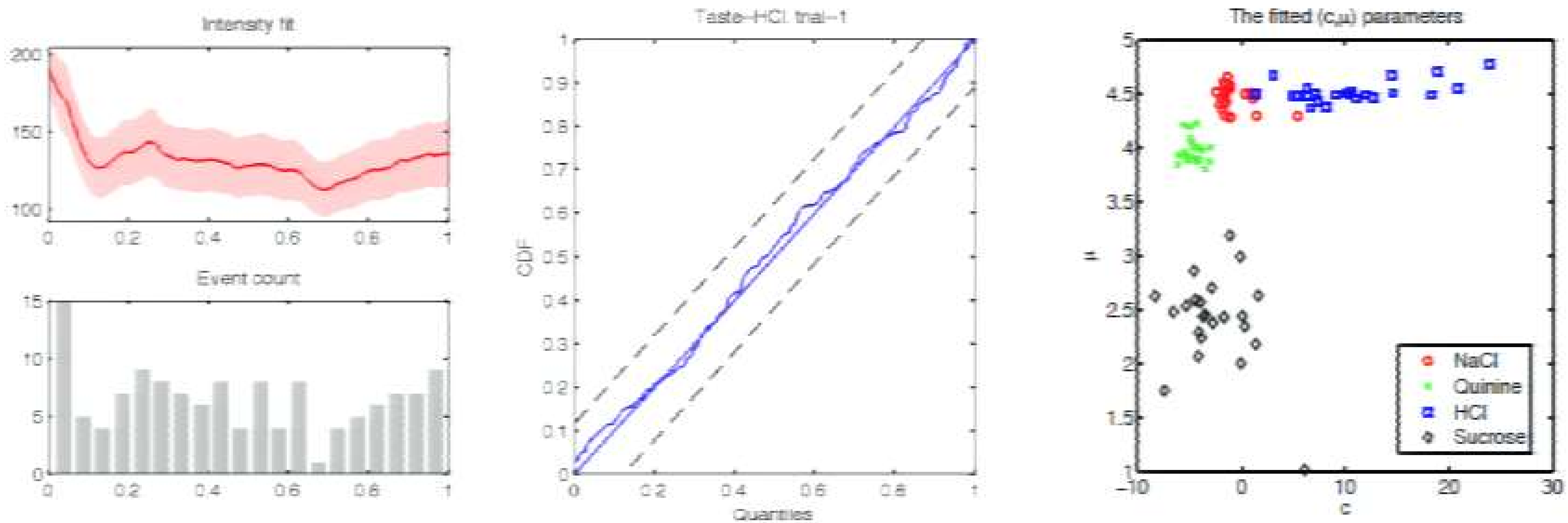




Model

- Gaussian prior with parameters $a_t = -1$, $c_t = 4\pi\cos(4\pi t)$ and $b_t = 4$
- Discrete time box likelihoods $I_{[-0.25,0.25]}(x_{1/2})$ and $I_{[-0.25,0.25]}(x_{2/3})$
- Continuous time loss function $V(t, x_t) = (2x_t)^8 I_{[1/2,2/3]}(t)$

Neural spike train data



Data and model

- Spike activity modelled using continuous time log-Gaussian Cox processes; taste receptor data from Di Lorenzo & Victor (2003)
- $p(y^d_{t_i} | x_{t_i}) \rightarrow$ point observations, $V(x_t, t) \rightarrow$ void probability
- a_t, c_t, b_t assumed constant, learned via Variational EM