

Speeding up Permutation Testing in Neuroimaging

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http://pages.cs.wisc.edu/~vamsi/pt_fast.html

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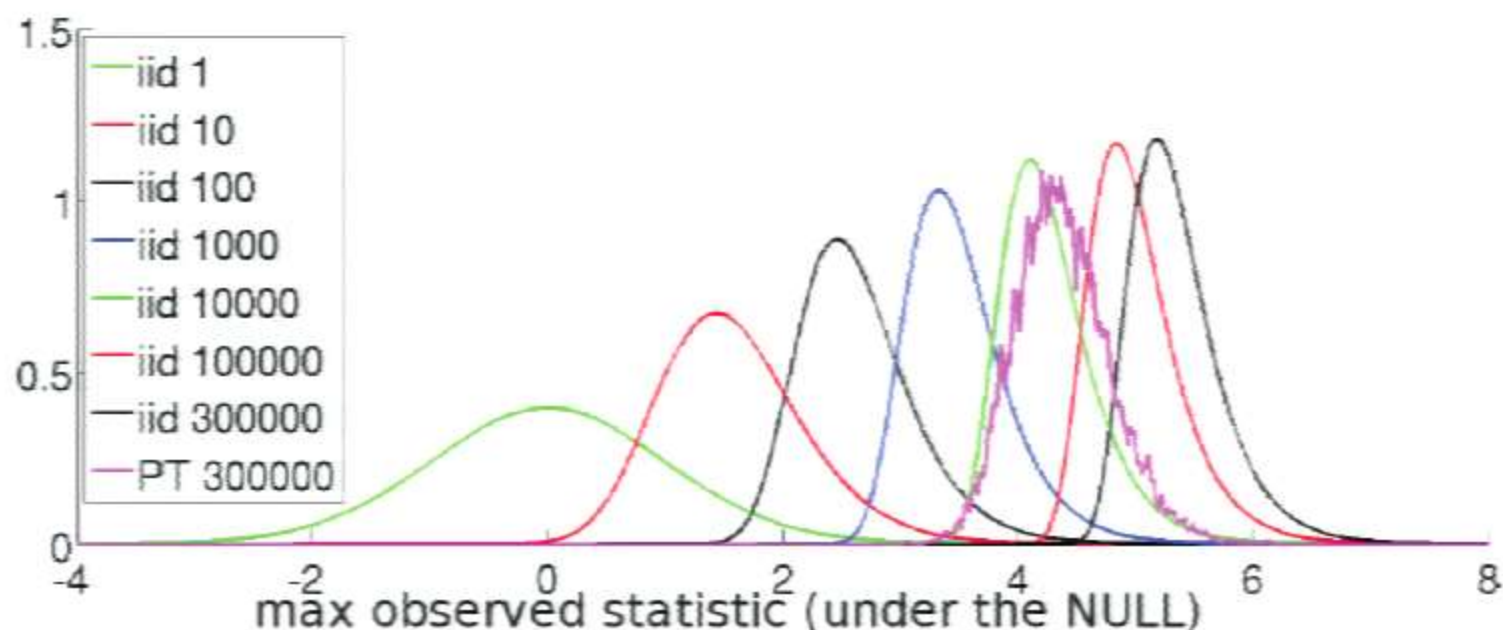


Permutation Testing

Setting

- ▶ **High-dimensional** measurements;
- ▶ **Highly correlated** covariates;
- ▶ Comparing **distinct** phenotype populations, statistically.

Under the **Global (Joint) Null Hypothesis**, the **max** observed test statistic is distributed as a function of the # of covariates:



Permutation Testing is an unbiased way of estimating this distribution from the sampled data.



Modelling Assumptions and Approach

Low-rank matrix completion

$$P = UW + S; \quad P, UW, S \in \mathbb{R}^{v \times t}$$

$$S_{i,j} \sim \mathcal{N}(0, \sigma^2).$$

- P : Permutation test matrix; v : voxels; t : tests
 UW : Low-rank component;
 $U \in \mathbb{R}^{v \times r}$, $W \in \mathbb{R}^{r \times t}$; **r is small**
 S : Approx. iid Normal residual

Optimization

$$\min_{\tilde{P}, U, W} \|\mathbf{P}_\Omega - \tilde{\mathbf{P}}_\Omega\|_F^2 \quad \text{s.t. } \tilde{\mathbf{P}} = UW; \quad U \text{ is column-wise orthogonal}$$

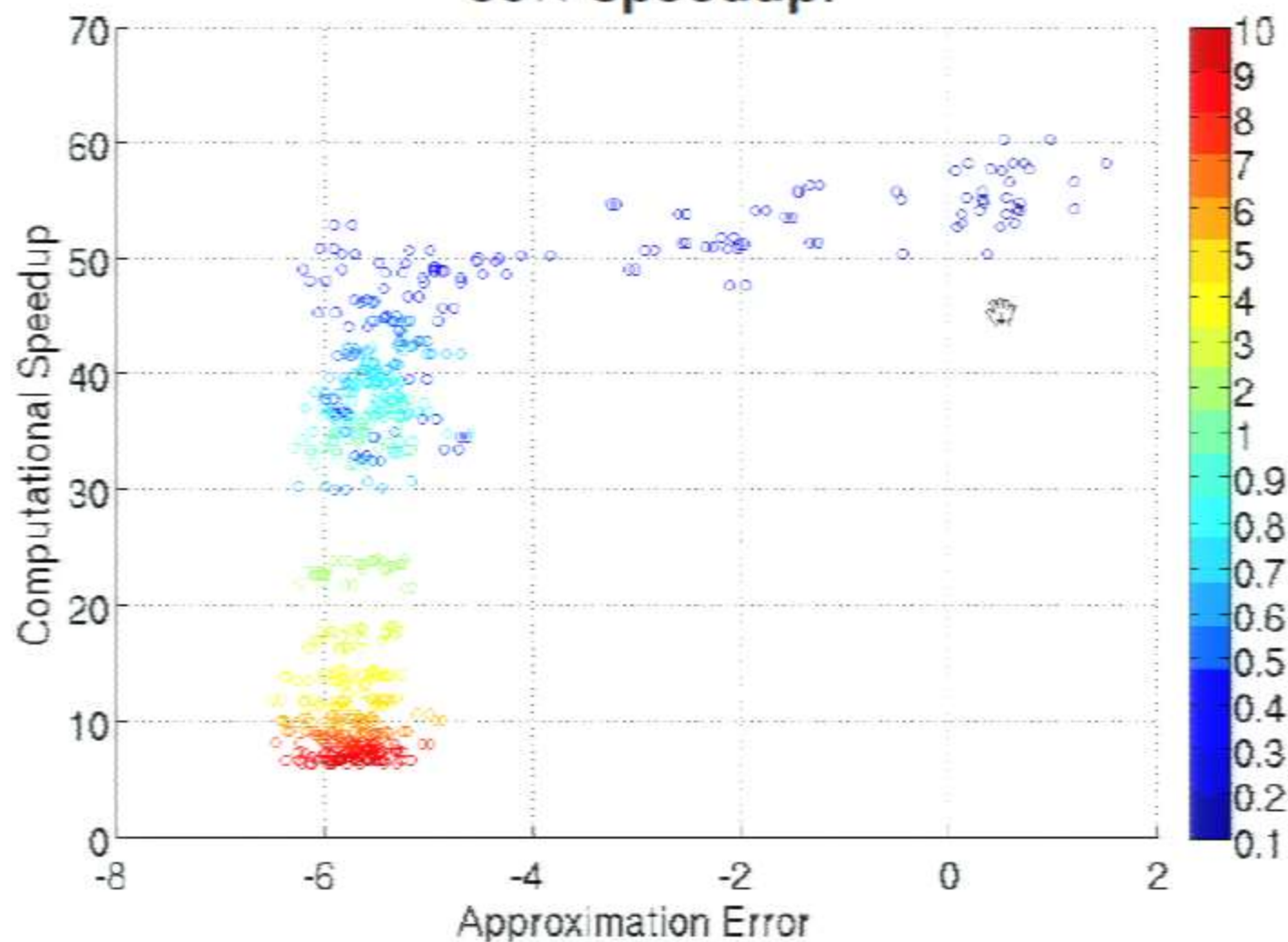
Theoretical guarantees

- ▶ Under realistic assumptions, we can model PP^T as a **low-rank perturbation** of a Wishart matrix, SS^T .
- ▶ The desired sample Null max distribution can be recovered with **bounded error**.



Trade-off

Thresholds can be recovered with **high fidelity** with a **50 \times speedup**.



Look for us at poster **Sun34**, and on the web at http://pages.cs.wisc.edu/~vamsi/pt_fast.html

