The Total Variation on Hypergraphs
Learning on Hypergraphs Revisited

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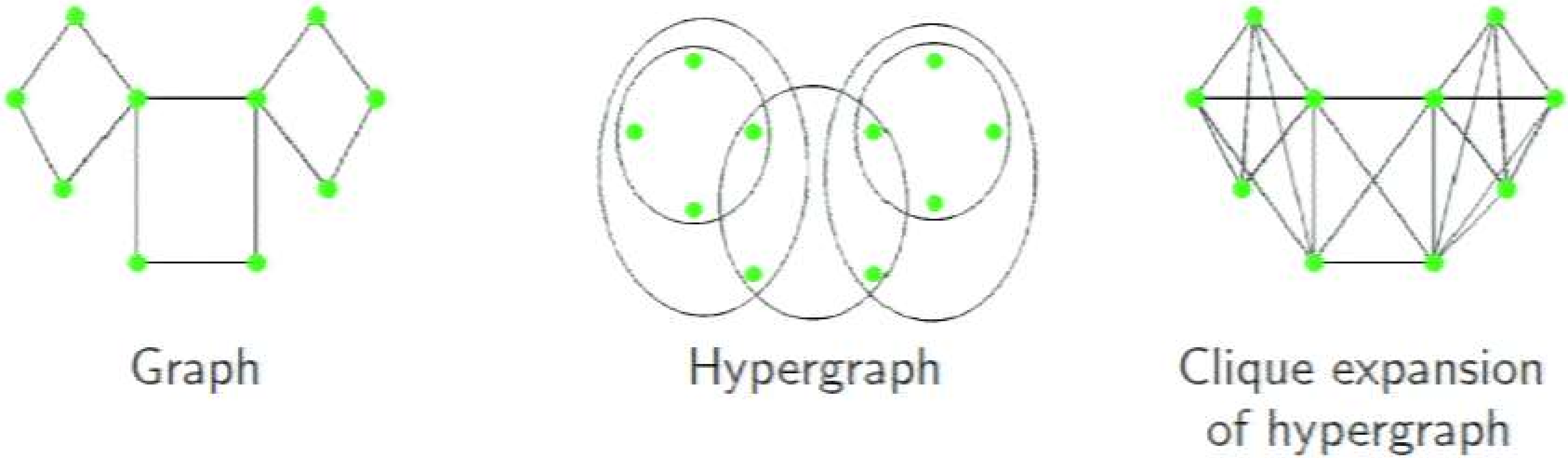
Hypergraphs vs Graphs

Modelling by hypergraphs: Motion segmentation, classification of gene expression data and Clustering categorical data



State-of-the-art: Approximation of hypergraph by standard graph (e.g., clique expansion)

- cannot exactly represent hypergraph structure
- leads to very dense graphs and hence computationally expensive

New Regularizers on Hypergraphs

	True hypergraph cut	Cut for clique expansion
$\mathrm{cut}_H(C, \overline{C})$	$\sum_{\substack{e \in E: \\ e \cap C \neq \emptyset,\, e \cap \overline{C} \neq \emptyset}} w_e$	$\sum_{\substack{e \in E: \\ e \cap C \neq \emptyset,\, e \cap \overline{C} \neq \emptyset}} \frac{w_e}{\lvert e \rvert} \lvert e \cap C \rvert \, \lvert e \cap \overline{C} \rvert$

Clique expansion is biased and cannot reproduce hypergraph cut

Our Approach: **Total variation based regularizers on hypergraphs**

$$\Omega_p(f) = \sum_{e \in E} w_e \Big(\max_{i \in e} f_i - \min_{j \in e} f_j \Big)^p$$

Exact representation of hypergraph cut is now possible:

$$\Omega_p(\mathbf{1}_C) = \mathrm{cut}_H(C, \overline{C})$$

No bias in our formulation!

Learning on Hypergraphs

- Semi-supervised learning on hypergraphs

$$\arg\min_{f \in \mathbb{R}^{|V|}} \frac{1}{2} \|f - Y\|_2^2 + \lambda \sum_{e \in E} w_e \Big(\max_{i \in e} f_i - \min_{j \in e} f_j \Big)^p$$

- Exact relaxation result for normalized hypergraph cuts

$$\min_{C \subset V} \frac{\mathrm{cut}_H(C, \overline{C})}{\hat{S}(C)} = \min_{f \in \mathbb{R}^{|V|}} \frac{\sum_{e \in E} w_e \big(\max_{i \in e} f_i - \min_{j \in e} f_j \big)}{S(f)}$$

At the poster (Sat25):

- efficient optimization of both problems + novel proximal map
- experiments showing better scalability and quality of our method over clique expansion