Exact and Stable Recovery of Pairwise Interaction Tensor

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a scalable algorithm for recovering pairwise interaction tensors from limited observations

Recovery of Pairwise Interaction Tensor

Object	Decomposition	Recovery
rank-*k* matrix $\mathbf{M} \in \mathrm{R}^{n_1 \times n_2}$	$M_{ij} = \langle u_i, v_j \rangle$	[Candes et al. 2009] guaranteed recovery of $\mathbf{M}$ from $O(nk \log^2(n))$ observations
rank-*k* tensor $\mathbf{T} \in \mathrm{R}^{n_1 \times n_2 \times n_3}$	$T_{ijk} = \langle u_i, v_j, w_k \rangle$	computing the rank is **NP-hard** best rank-1 approximation is **NP-hard**
pairwise interaction tensor $\mathbf{T} \in \mathrm{R}^{n_1 \times n_2 \times n_3}$	$T_{ijk} = \langle u_i^{(a)}, v_j^{(a)} \rangle + \langle v_j^{(b)}, w_k^{(b)} \rangle + \langle w_k^{(c)}, u_i^{(c)} \rangle$	**this paper**: guaranteed recovery of $\mathbf{T}$ from $O(nk \log^2(n))$ observations.

- Pairwise interaction tensor is a special case of tensor.
  - Good performance in various applications
    - Tag recommendation [Rendle et al. 2009]
    - Sequential data analysis [Rendle et al. 2010]
  - Existing recovery algorithms are local optimization heuristics.

Key ideas and results

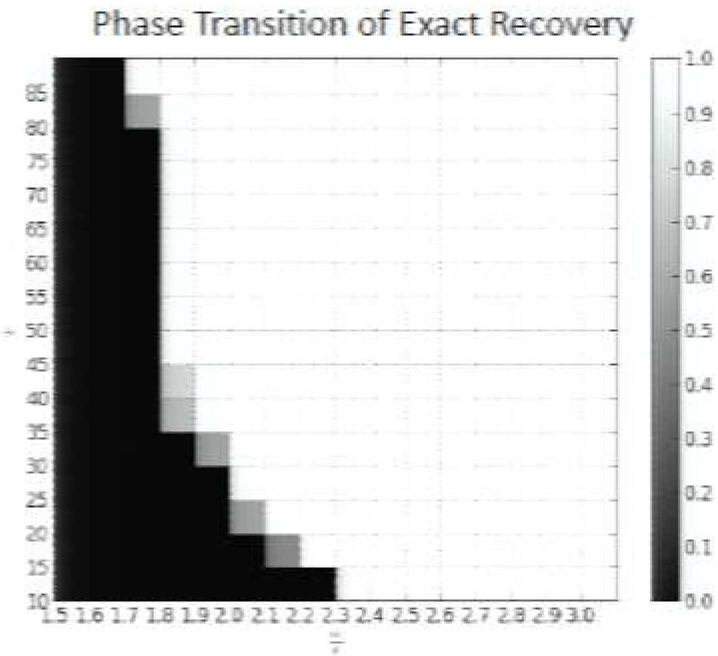
- Reduce to a **matrix completion** problem.
- Formulate a **constrained trace norm minimization** objective.
- Optimize using **SVT**.

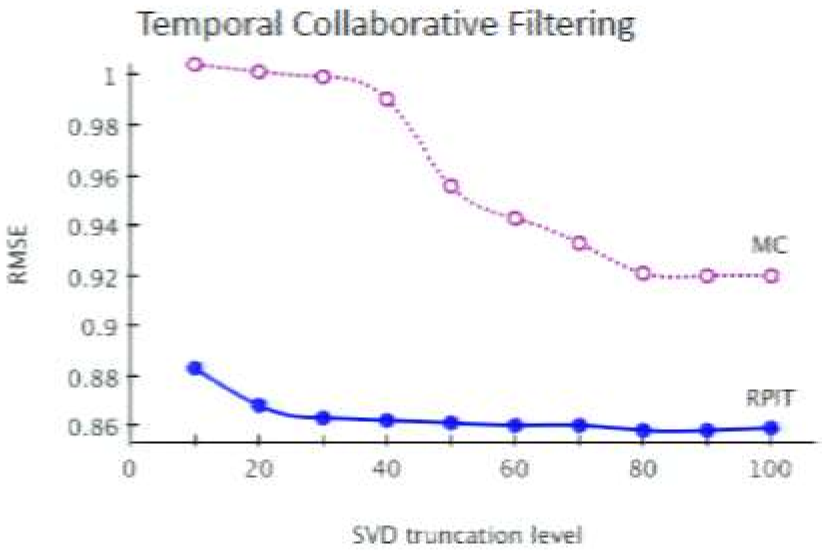
$$\min_{(X,Y,Z)\in S} \sqrt{n_3}\|X\|_* + \sqrt{n_1}\|Y\|_* + \sqrt{n_3}\|Z\|_*$$
$$\text{s.t.} \quad X_{ij} + Y_{jk} + Z_{ki} = T_{ijk}, \forall ijk \in \Omega$$

- S is a constraint for ensuring **uniqueness** of recovery.
- Ω is the set of observations
- (formulation of exact recovery)

Theoretical results

- Exact observations.
  - Recovery is exact from $O(nk\log^2(n))$ observations
- Noisy observations
  - Provable approximation guarantees.





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