The Complete Rank Transform: A Tool for Accurate and Morphologically Invariant Matching of Structures

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Illumination changes in reality

source: KITTI benchmark

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- Important for optic flow
- Assumption that intensity of objects stays constant violated
- In this work: only assume invariance under monotonically increasing greyvalue rescalings (eg. additive, multiplicative)

$\rightarrow$ develop morphologically invariant descriptor

Outline

Rank Transform (RT) (Zabih and Woodfill 1994)

- Idea:

  How many pixels are smaller than me?

- Invariant under monotonically increasing transformations

4	14	83
88	25	4
3	15	65

Intensity values

	5	

Rank

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Rank

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- Idea:

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- Resulting signature:

$$s_{\mathrm{RT}} = 5$$

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Census Transform (CT) (Zabih and Woodfill 1994)

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- Census signatures carry spatial information

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Intensity values

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0		1
1	1	0

Census

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$$\boldsymbol{s}_{\mathrm{CT}} = (1, \qquad\qquad\qquad)^{\top}$$

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Census

Complete Rank Transform (CRT)

- Capture the rank of every pixel in the local patch
- Vector-valued signature in every pixel
- Invariant under monotonically increasing transformations
- Carries as much local image information as possible

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1	3	7
8	5	1
0	4	6

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Census

Outline

- Introduction
- Complete Rank Transform
- **Variational Optic Flow Model**
- Experiments
- Conclusions

Variational Optic Flow Model

- Modify model of Brox et al. (2004)
- Minimise functional for optic flow field $(u, v)^\top : \Omega \to \mathbb{R}^2$

$$E(u, v) = \int_\Omega (\, D + \alpha \, R \,) \, \mathrm{d}x \, \mathrm{d}y$$

- Data term D models constancy of Complete Rank signatures of corresponding positions

$$\left\| \boldsymbol{s}_{\mathrm{CRT}}(x + u, y + v, t + 1) - \boldsymbol{s}_{\mathrm{CRT}}(x, y, t) \right\|^2$$

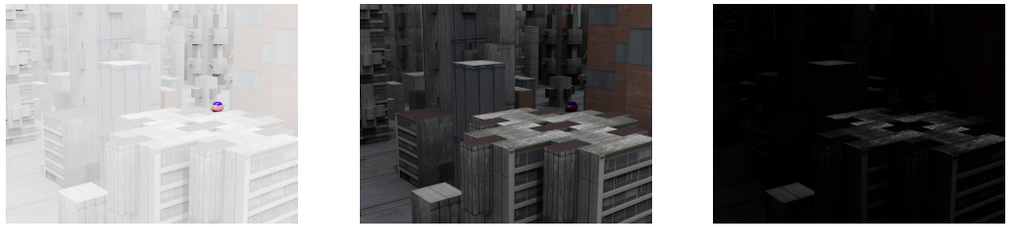
- Regularisation term R leads to piecewise smooth solution
- Both terms equipped with robust estimator functions
- Typical warping-based coarse-to-fine minimisation (Brox et al. 2004)

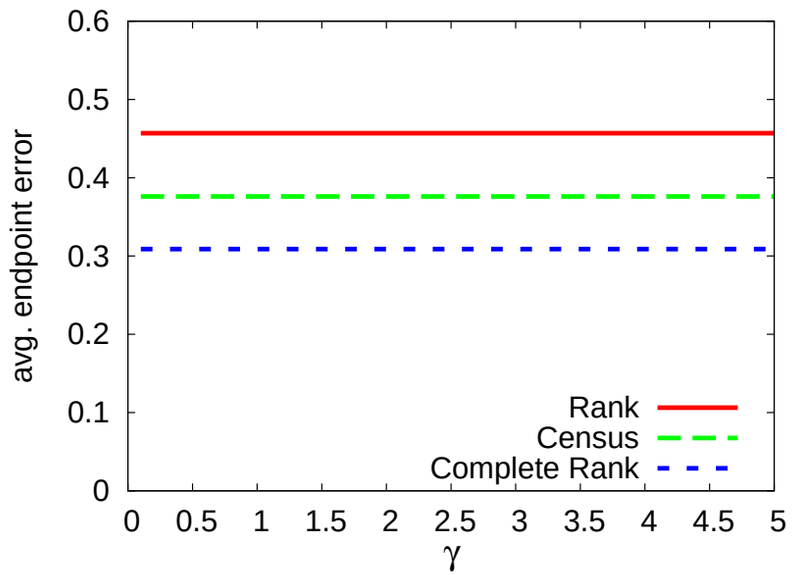
Outline

Experiments - γ Changes

- Adjust second frame with an exponential function $f_\gamma = f_{\text{max}} \cdot \left(\frac{f}{f_{\text{max}}}\right)^\gamma$

$\gamma = 0.1$ $\gamma = 1$ $\gamma = 3$





→ Unconditional morphological invariance
→ Outperform Rank and Census

Experiments - KITTI Vision Benchmark

- GCPR Special Session Imagery (all scenes with lighting changes)

KITTI image sequence:	#11	#15	#44	#74	average
Zimmer et al. 2011	37.3	32.3	23.2	62.9	38.9
Bruhn/Weickert 2005	33.9	47.7	32.4	71.4	46.7
Census Transform	36.5	28.6	28.5	63.8	39.4
Complete Rank Transform	**29.8**	**22.8**	**22.6**	**61.5**	**34.2**

[%] bad pixel error measure bp3

→ CRT outperforms established methods in difficult lighting conditions

- Whole KITTI benchmark (195 testing image sequences)
  - Our method ranks 10th of 33
  - Top 4 methods use stereo information
  - Census-based method by Ranftl et al. (2012) ranks 14th

→ CRT carries enough image information for real-world applications

Experiments - Middlebury Benchmark

- Middlebury Training Set

	rw	dimetr.	grove2	grove3	hydr.	urban2	urban3	yos	avg
RT	.111	.092	.191	.764	.191	.457	1.03	.211	.381
CT	.102	.090	.169	.646	**.147**	.378	.819	.169	.316
CRT	**.100**	**.076**	**.154**	**.585**	.158	**.324**	**.529**	**.150**	**.260**

[pixel] average endpoint error

→ CRT clearly preferable over RT and CT

- Middlebury Benchmark (July 31st 2013)

Method	Average rank
Zimmer et al. (2009)	40.5
ours	45.8
Brox et al. (2004)	52.1

→ even without illumination changes, CRT gives acceptable quality

Conclusions

- Morphological invariance handles illumination changes
- Problem: Invariance discards information
- Our solution: CRT that carries as much local image information as possible
- Intentionally kept variational optic flow model simple
- Our proposed CRT clearly preferable over Census and Rank transforms

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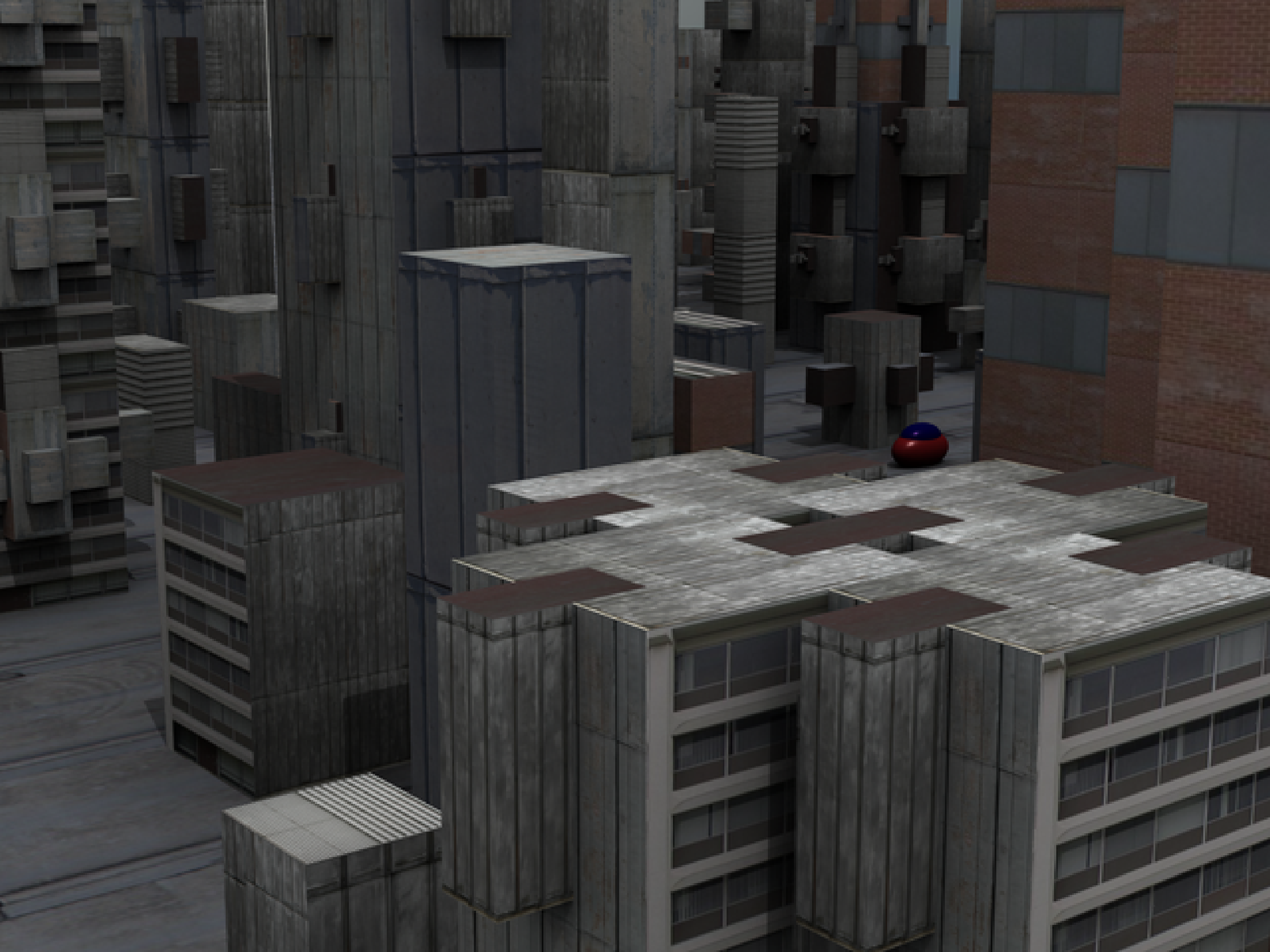
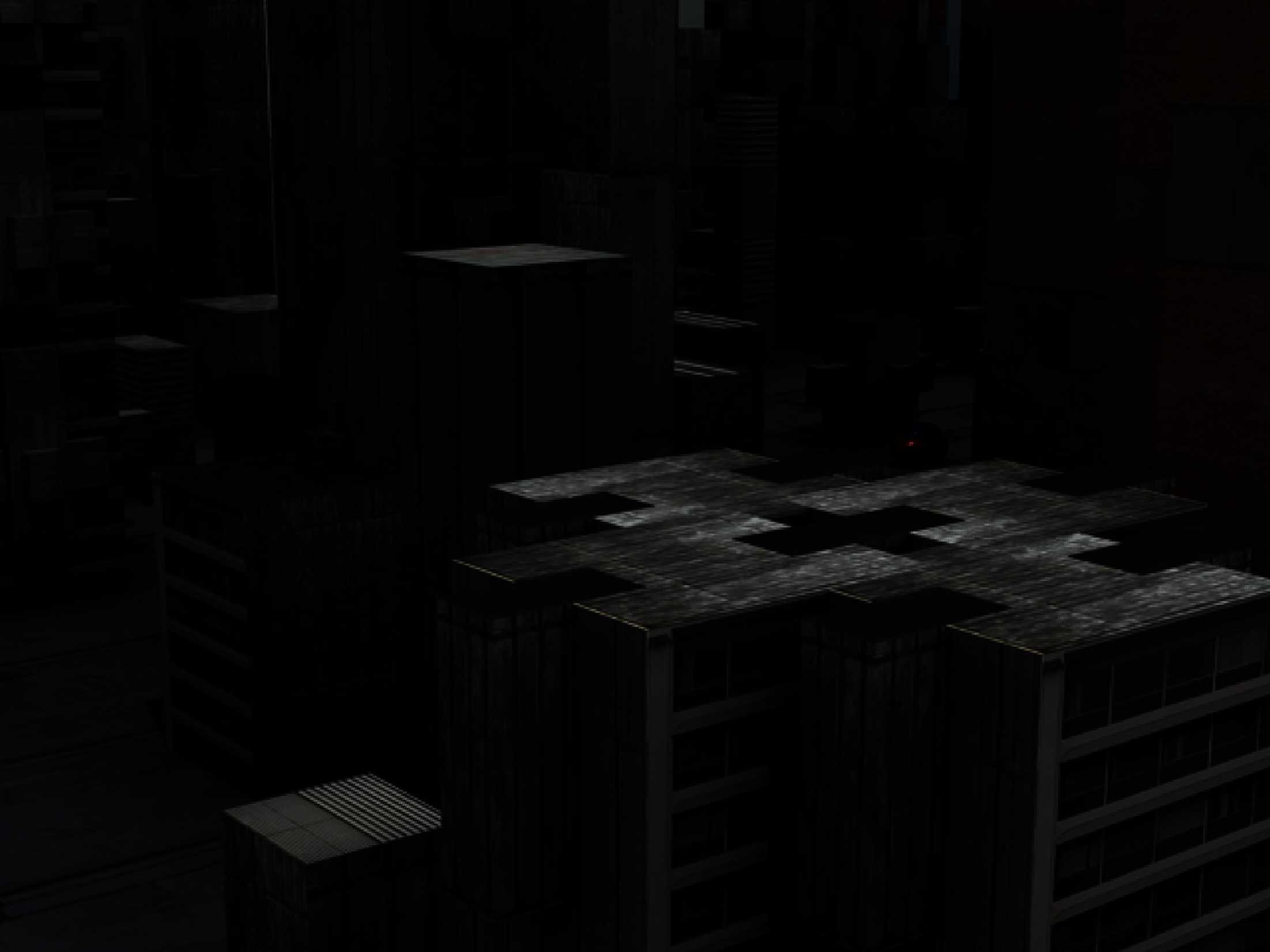
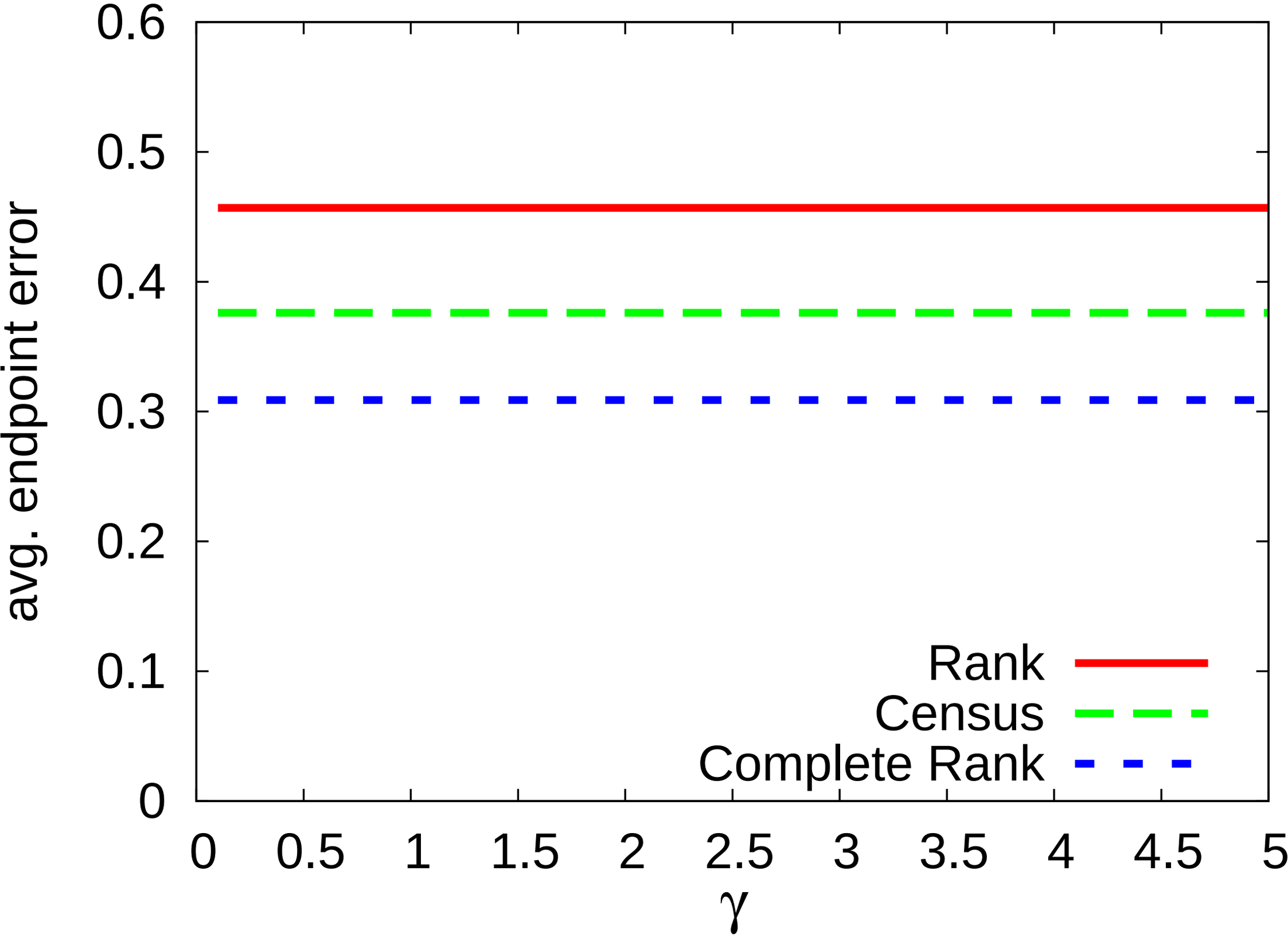
Thank You !






0.6
0.5
0.4
0.3
0.2
0.1
0
avg. endpoint error
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
γ
Rank
Census
Complete Rank