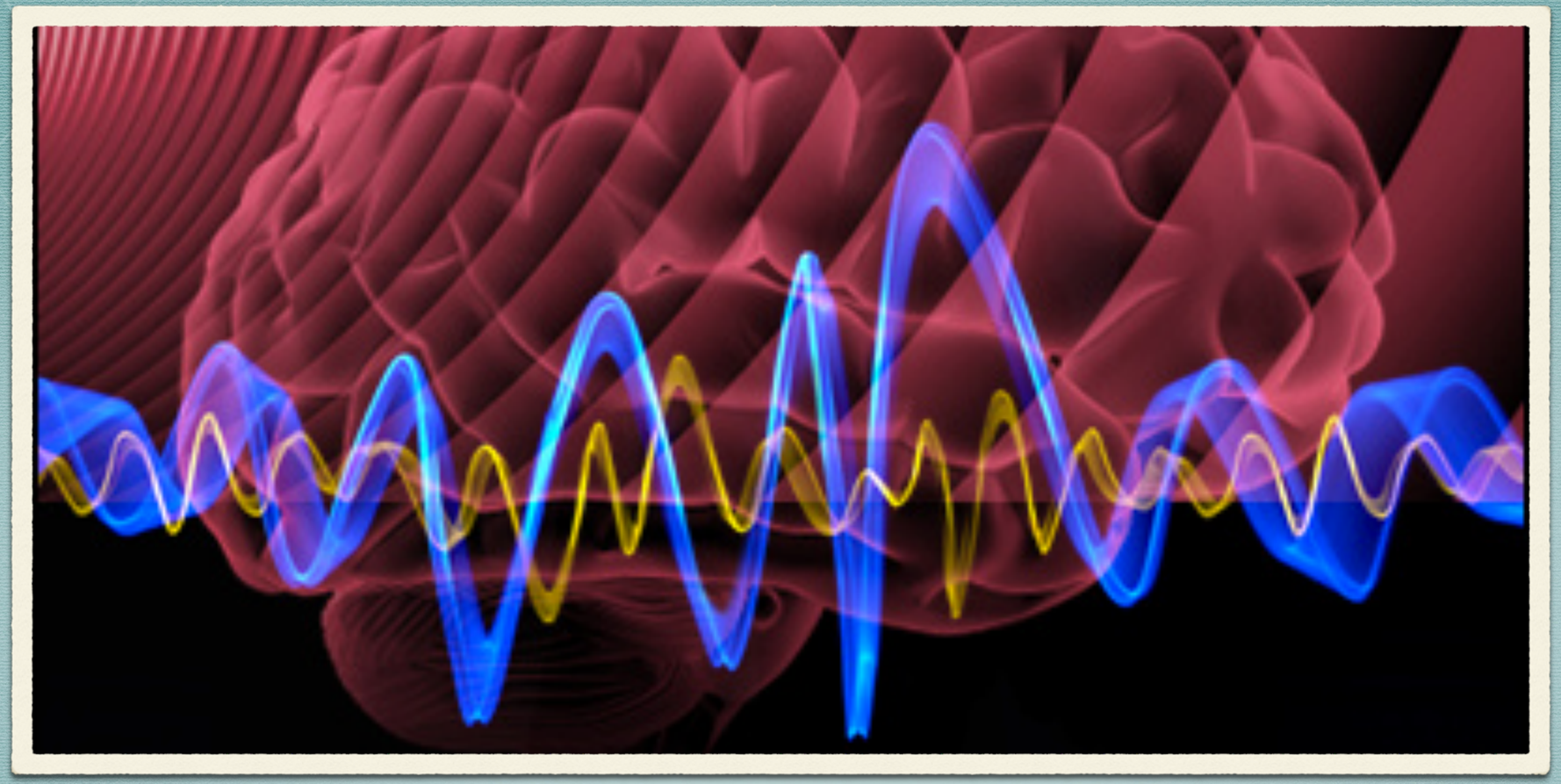
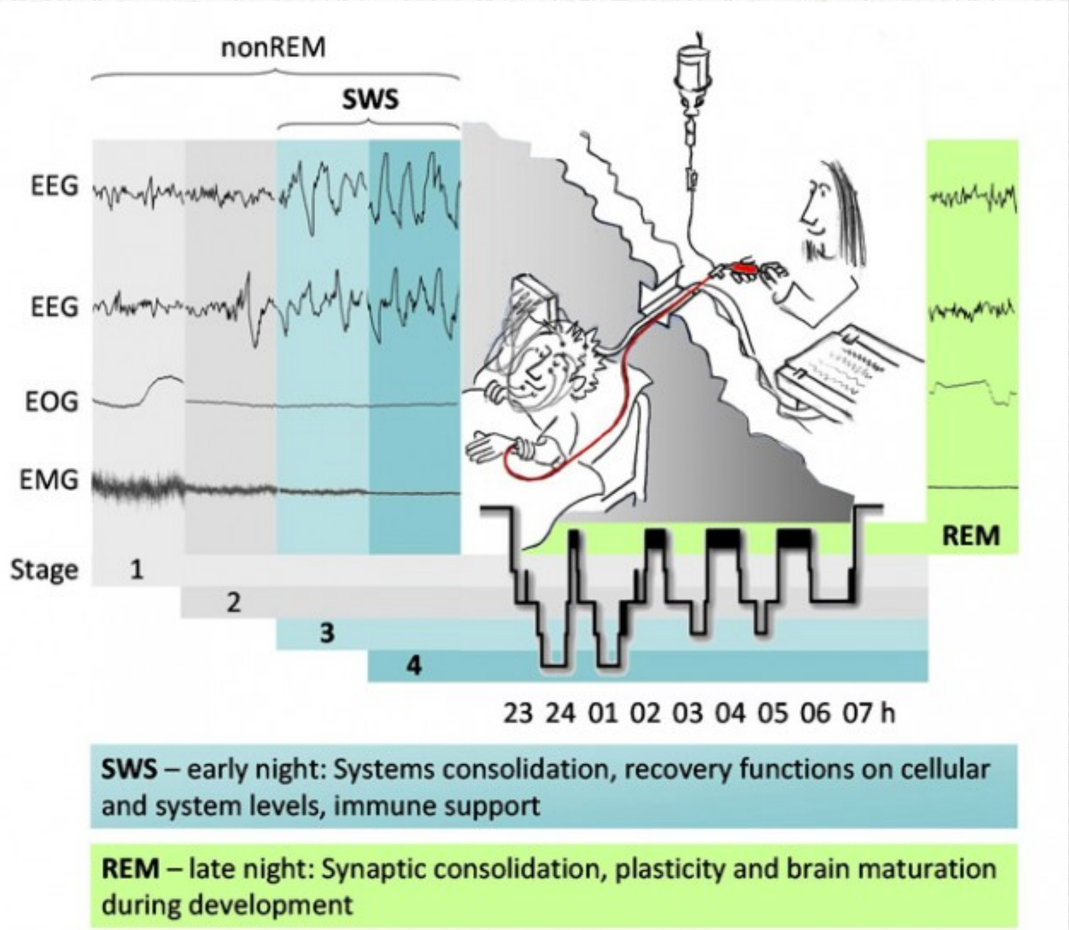
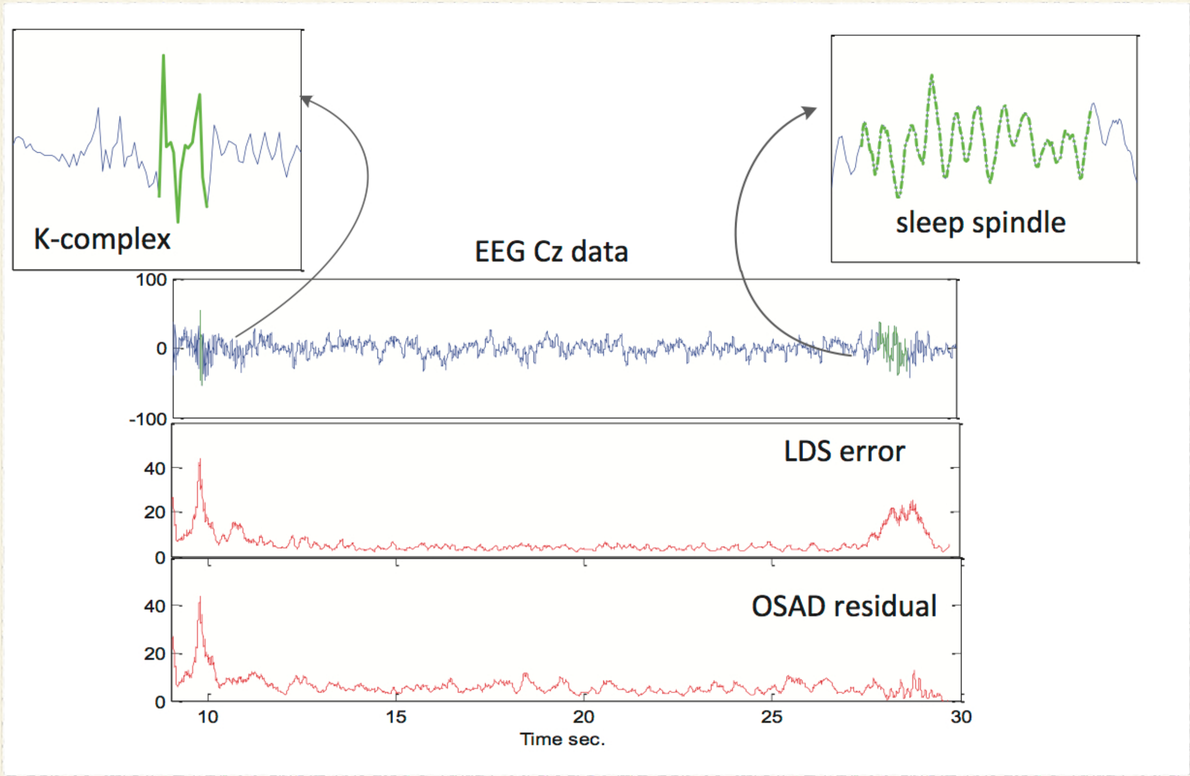


Sleep Analytics and Online Selective Anomaly Detection

Tara Babaie, Sanjay Chawla, Romesh Abey
University of Sydney

Selective Anomaly Detection ?





what we propose:

1. Introduce a new problem: Online Selective Anomaly Detection (OSAD) to address the requirement of selectively reporting sleep anomalies based on specifications by domain experts.
2. Combine techniques from data mining and control theory to solve OSAD
3. Evaluate the method on sleep EEG data

Problem: Learning

1- Given an observable time series $\{y_i\}_{i=1:N}$

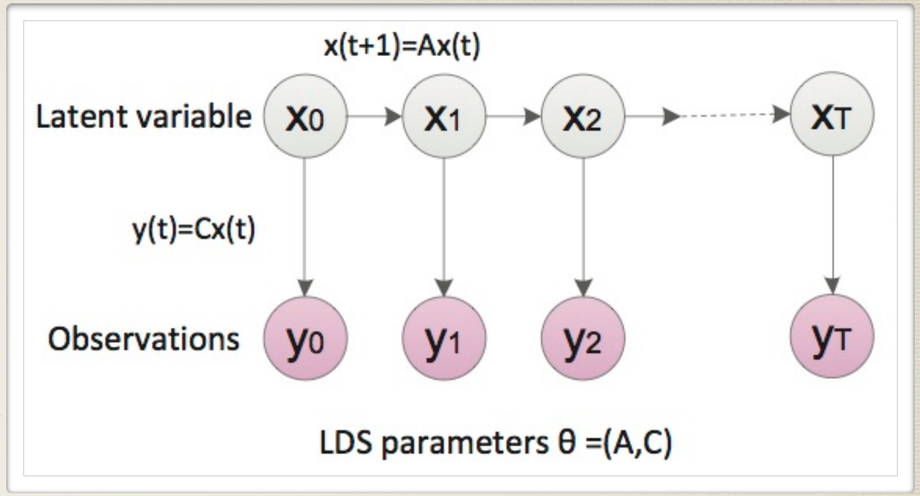
2- assuming that the observed y and the latent x are governed by a linear dynamic system (LDS),

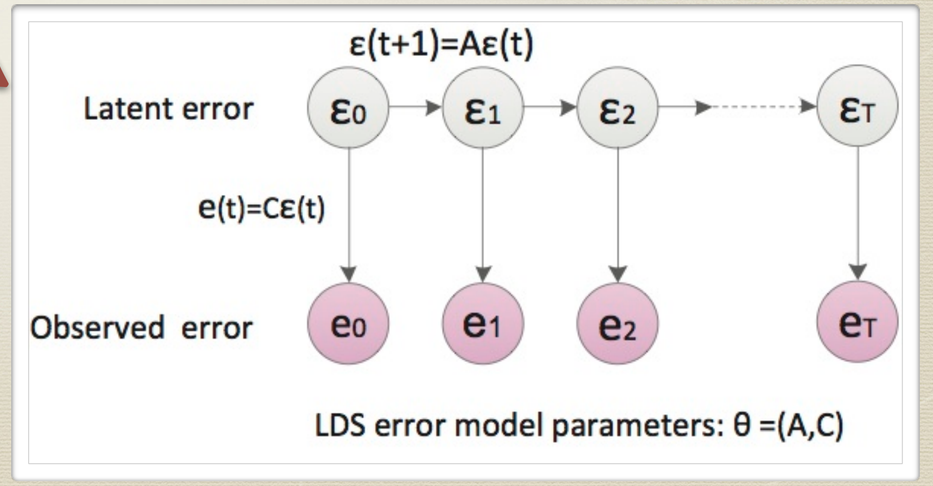
$$x(t+1) = Ax(t)$$
$$y(t) = Cx(t)$$

not_observable → $\varepsilon(t) := x(t) - \hat{x}(t)$

observable → $e(t) := y(t) - \hat{y}(t)$

linearity





- Shumway 1982 : EM algorithm as the classic solution
- Overschee 1996, Ljung 2001 : subspace method as a revolution
- Boots 2010 : re-introduced to ML community as spectral learning

Problem: Design

A **pattern P** is a user-defined matrix which operates in the latent space.

$$x(t+1) = \mathbf{A}x(t) + \mathbf{P}\xi(t)$$

$$y(t) = \mathbf{C}x(t)$$

Given an LDS, a **pattern P** in the latent space, design/control a residual r(t) such that

$$r(t) = \begin{cases} 0 & \text{if } \varepsilon(t) = P\xi(t) \\ Se(t) & \text{otherwise} \end{cases}$$

Where S is a linear transformation matrix

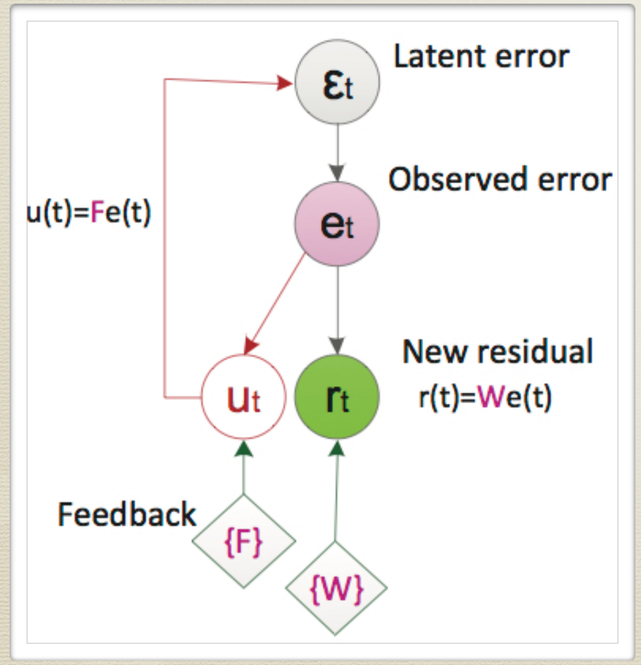
Solution: *Dynamic Residue Model* (DRM)

$$r(t) := \mathbf{W}e(t)$$

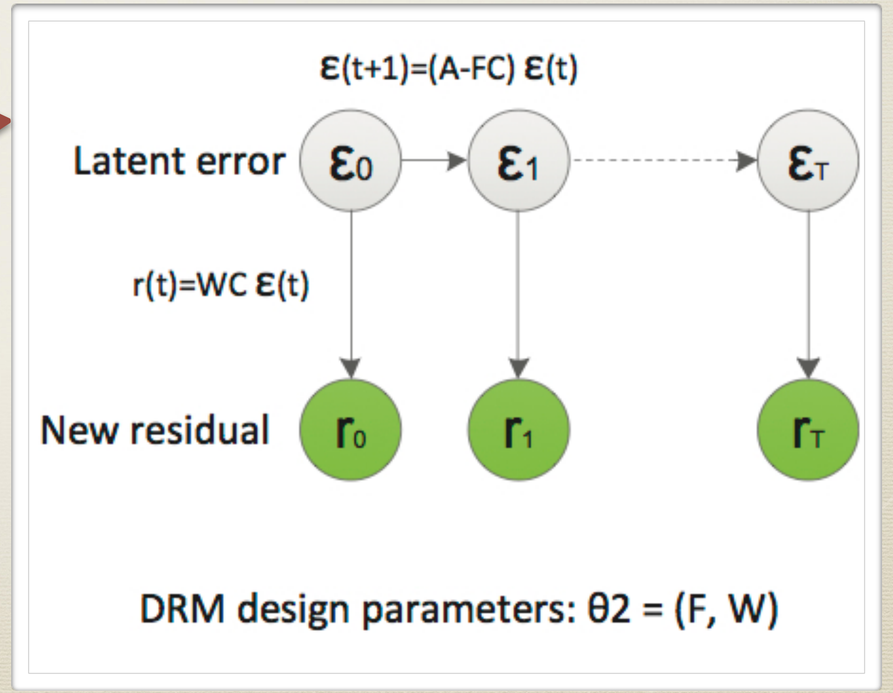
$$u(t) := \mathbf{F}e(t)$$

$$\varepsilon(t+1) = \mathbf{A_f}\,\varepsilon(t)$$

$$e(t) = \mathbf{C_f}\,e(t)$$



DRM



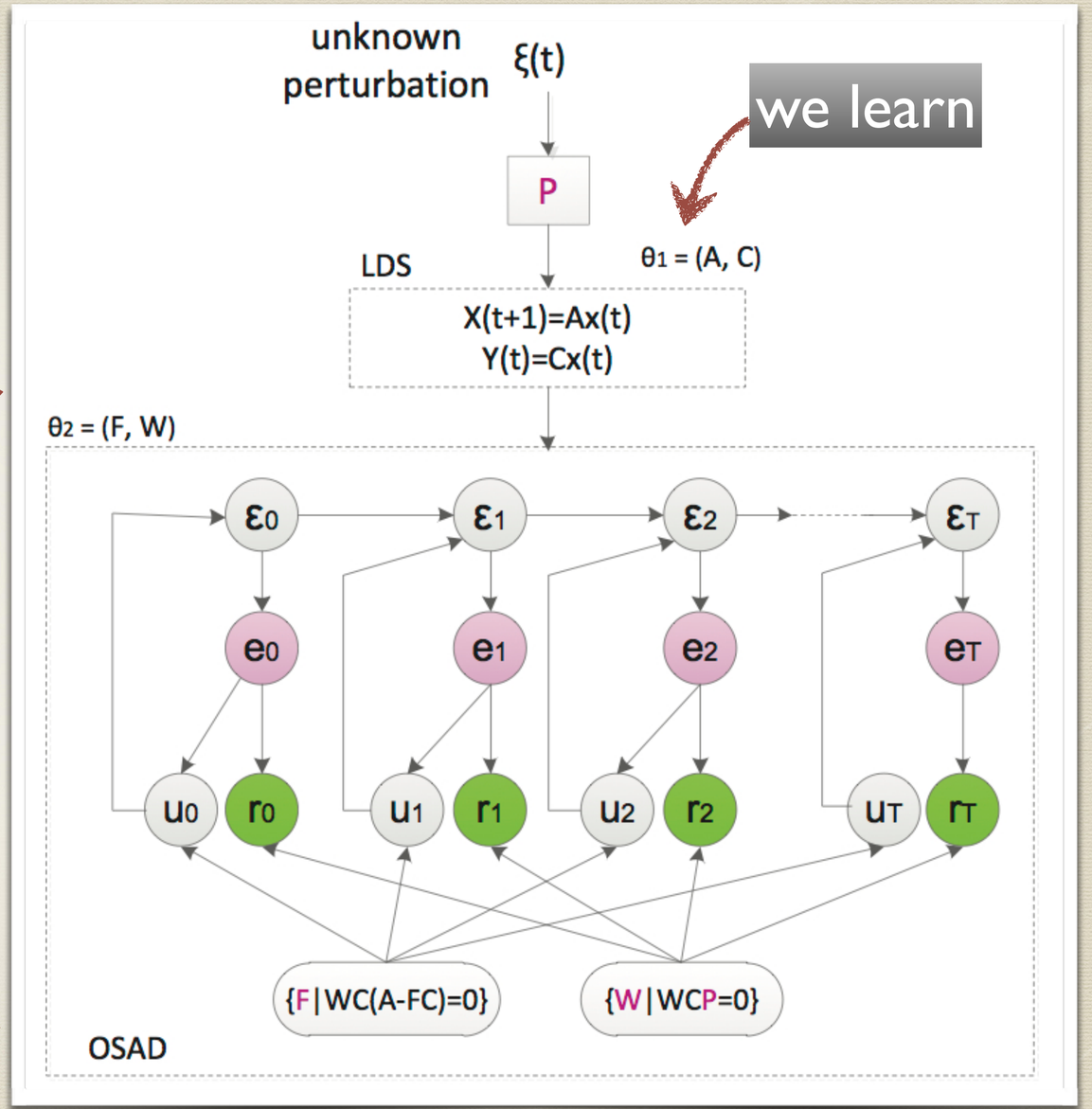
1- Using parameter **F** a virtual input u(t) is generated to feed the error back to the latent space.

2- The error e(t) is then calibrated by **W** to generate a new residual space r(t).

we control

$$\varepsilon(t+1) = \mathbf{A_f}\,\varepsilon(t)$$
$$r(t) = \mathbf{C_f}\,e(t)$$

$$r(t) = \begin{cases} 0 & \text{if } \varepsilon(t) = P\xi(t) \\ Se(t) & \text{otherwise} \end{cases}$$



Z-transform as a design approach

$$\varepsilon(t+1) = \mathbf{A_f}\,\varepsilon(t)$$
$$r(t) = \mathbf{C_f}\,e(t)$$

$\xrightarrow{\text{Z-transform}}$

$$\begin{cases} z\varepsilon(z) = A_f\varepsilon(z) + P\xi(z) \\ R(z) = C_f\varepsilon(z) \end{cases}$$

$\xrightarrow{\text{soln}}$

$$\begin{cases} \varepsilon(z) = (zI - A_f)^{-1}P\xi(z) \\ R(z) = [C_f(zI - A_f)^{-1}P]\xi(z) \end{cases}$$

$\longrightarrow$

$$\boxed{G_\xi(z) \doteq C_f(zI - A_f)^{-1}P}$$

$\downarrow$

$$R(z) = G_\xi(z)\xi(z)$$

Theorem. For a DRM, a sufficient condition for $\mathbf{G}_\xi(z) = 0$ is

$$\mathbf{C}_f\mathbf{P}=0 \quad \text{and} \quad \{\mathbf{C}_f\mathbf{A}_f=0 \ \text{ or } \ \mathbf{A}_f\mathbf{P}=0\}$$

$\mathbf{C}_f\mathbf{P} = \mathbf{WCP} = 0 \Rightarrow$ find $\mathbf{W}$: $\mathbf{W} \perp \mathbf{CP}$

linear algebraic problem

for $\mathbf{C}_f\ \mathbf{A}_f = 0$, it is sufficient to design a matrix $\mathbf{A}_f$ such that its left eigenvectors corresponding to the zero eigenvalue are orthogonal to $\mathbf{P}$. $\Rightarrow$ **find F : WC(A-FC)=0**

for $\mathbf{A}_f\ \mathbf{P} = 0$, it is sufficient to design a matrix $\mathbf{A}_f$, such that the right eigenvectors corresponding to the zero eigenvalues are the columns of $\mathbf{P}$. $\Rightarrow$ **find F : (A-FC)P=0**

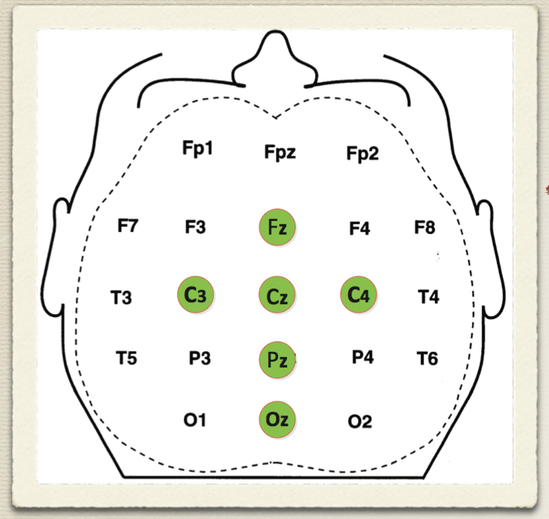
eigen-space assignment problem

Eigen-Structure Assignment (EA) vs SVD

SVD: Given: Matrix X $\longrightarrow$ singular-values λ_i, singular-vectors u_i

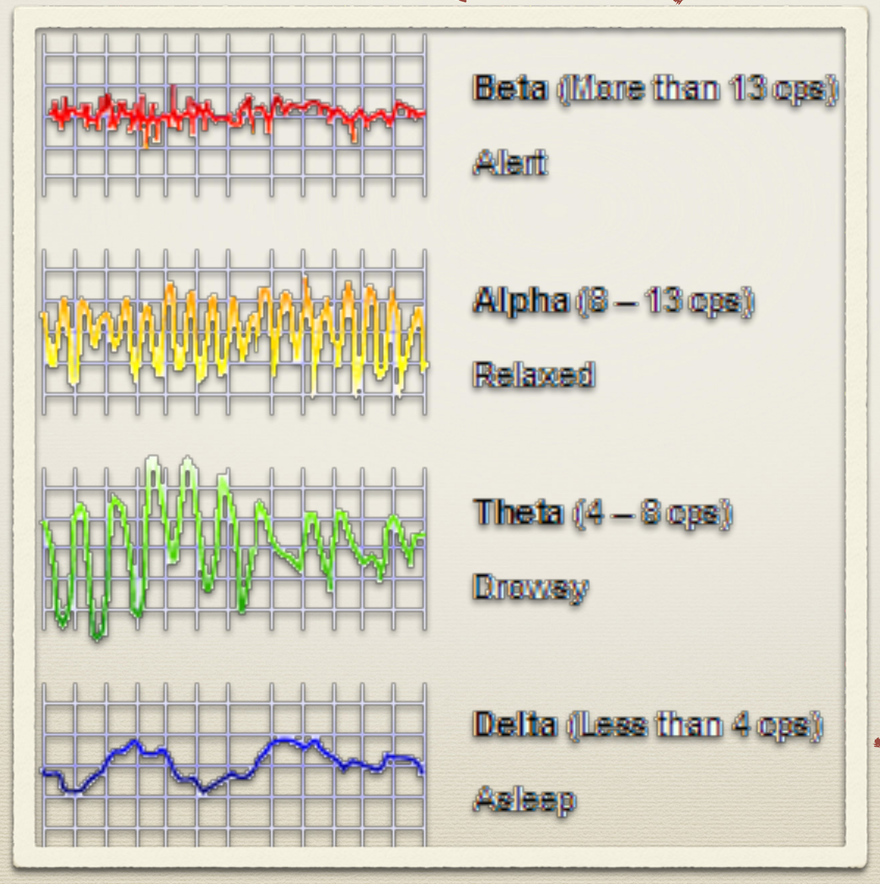
EA: Given: eigen-values λ_j, eigen-vectors u_j $\longrightarrow$ Matrix X

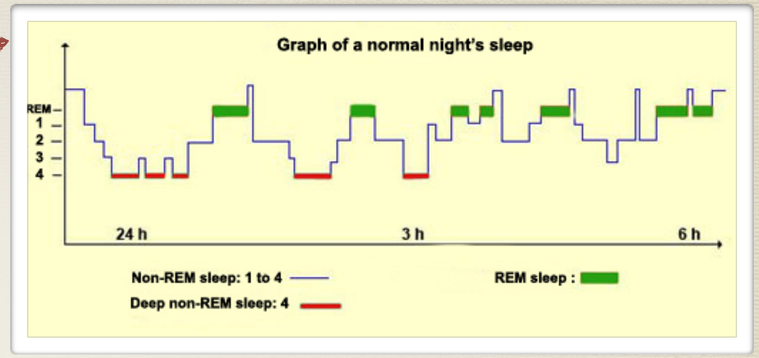
Experiments: Sleep Staging and EEG

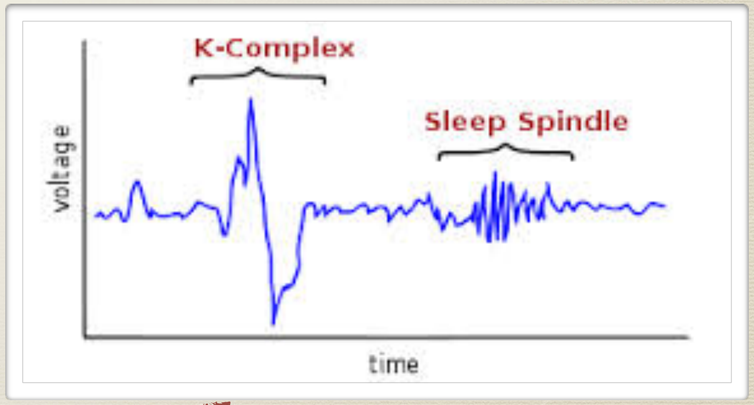


data set : EEG time series from four health controls (age 25-36).

Recordings: at the Woolcock Institute of Medical Research, at Sydney University, using 6 EEG channels with a sampling rate of 200 Hz.







Detection of Sleep Spindle and K-Complex

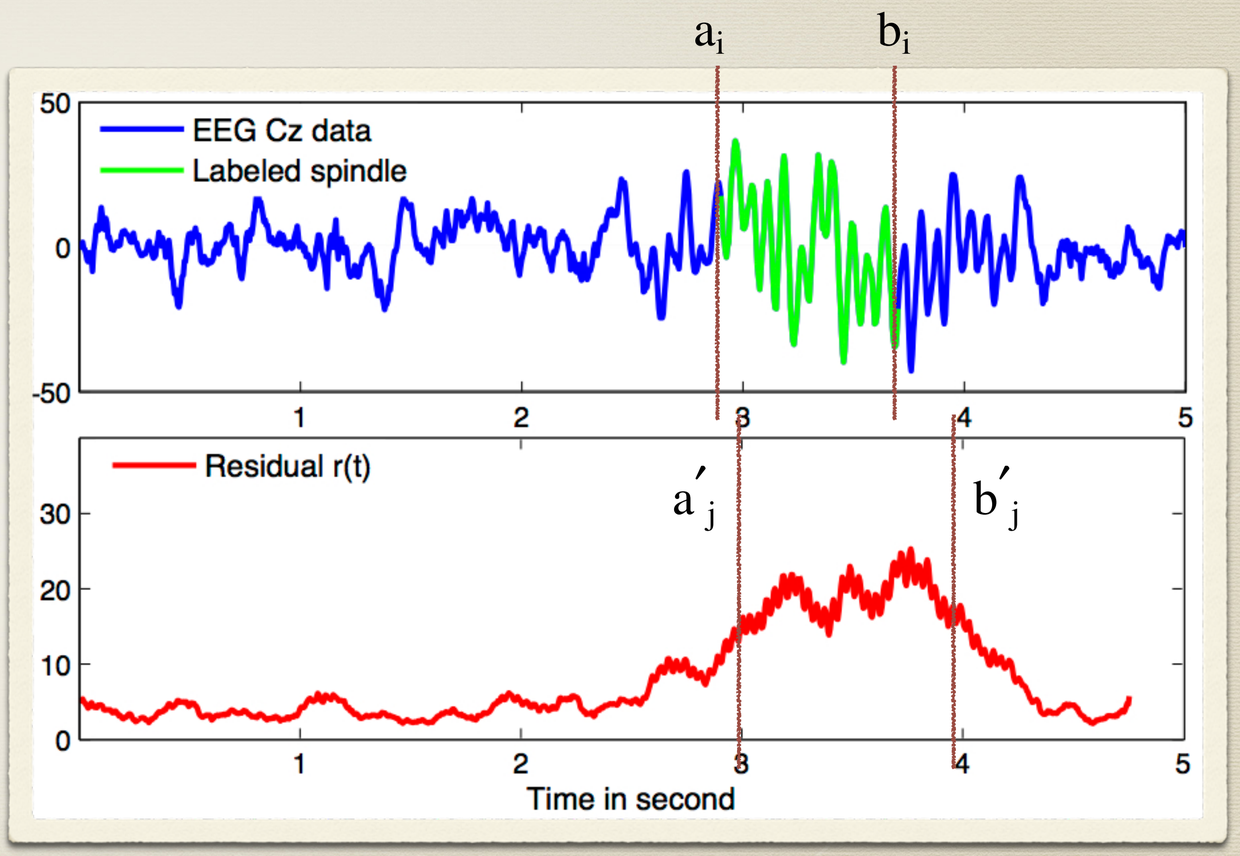
For a given subject,

Let $\{[a_i, b_i]\}_{i=1:n}$: the intervals of the labeled spindles

Let $\{[a'_j, b'_j]\}_{j=1:m}$: the intervals of predicted spindles

$$\texttt{precision} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}|[a_i, b_i] \cap [a'_j, b'_j]|}{\sum_{j=1}^{m}|[a'_j, b'_j]|}$$

$$\texttt{recall} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}|[a_i, b_i] \cap [a'_j, b'_j]|}{\sum_{j=1}^{m}|[a_j, b_j]|}$$



	No. of Labeled Anomalies		No. of Detected Anomalies	
	Spindle	K-Complex	Spindle	K-Complex
subject 1	170	277	164	251
subject 2	6	13	6	11
subject 3	23	38	21	37
subject 4	141	205	132	186

LDS tends to under-predict the number of anomalies

	Total time of K-Complex		Performance	
	Labeled in sec	Detected in sec	Precision	Recall
subject 1	198.23	216.35	90.45%	93.43%
subject 2	11.48	11.25	92.76%	94.12%
subject 3	21.39	24.56	91.01%	92.06%
subject 4	147.68	160.49	91.28%	93.73%

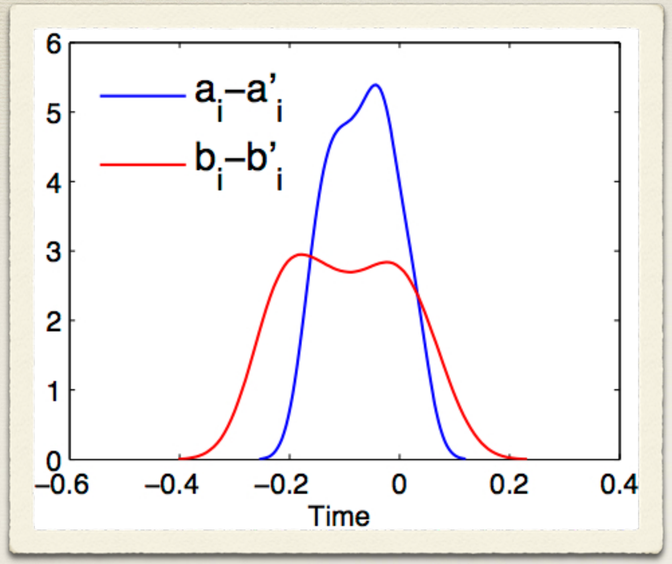
Sleep Spindles: tends to over-predict the length of interval

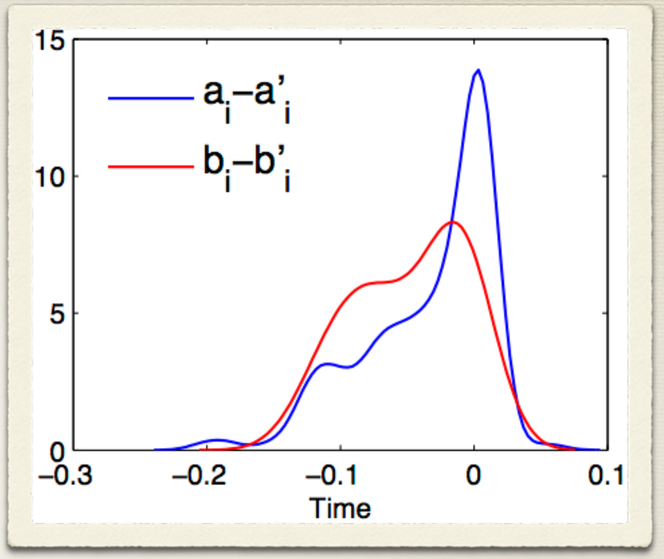
	Total time of spindles		Performance	
	Labeled in sec	Detected in sec	Precision	Recall
subject 1	129.8	168.74	71.24%	95.53%
subject 2	3.45	3.55	97.18%	97.38%
subject 3	15.15	16.23	83.88%	95.66%
subject 4	93.5	103.2	79.15%	95.42%

K-Complex: tends to over-predict the length of interval

On-Line Analysis

subject 1





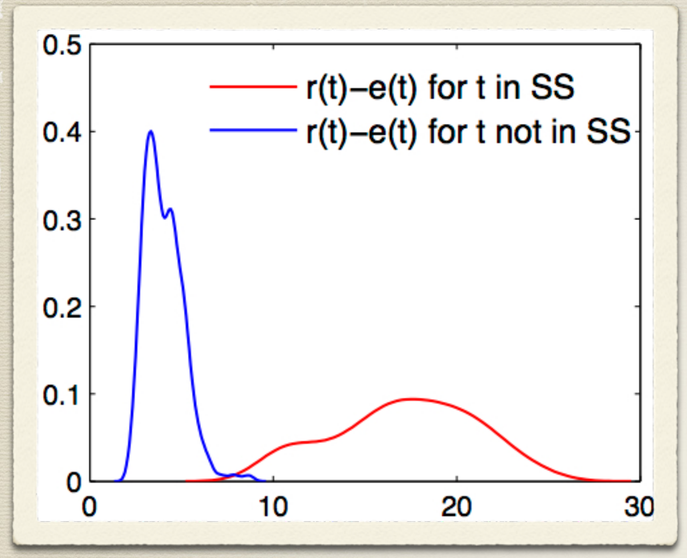
subject 4

1- OSAD provides near real time detection.

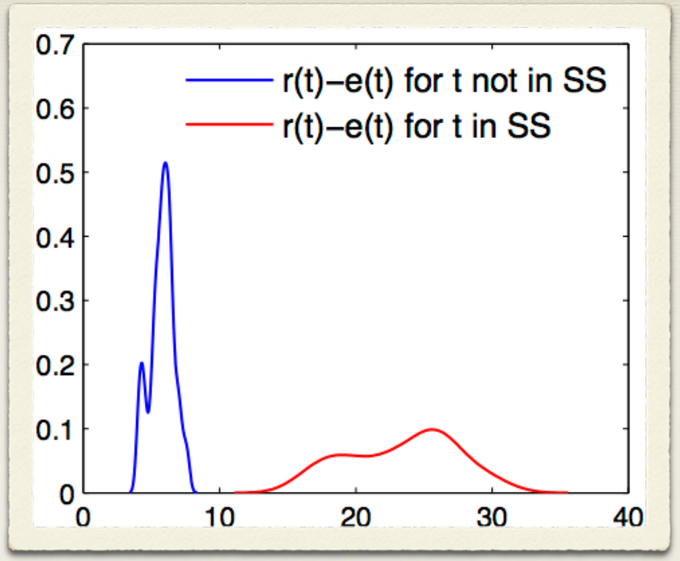
2- There is a tendency to start early and end early.

3- There is subject to subject difference.

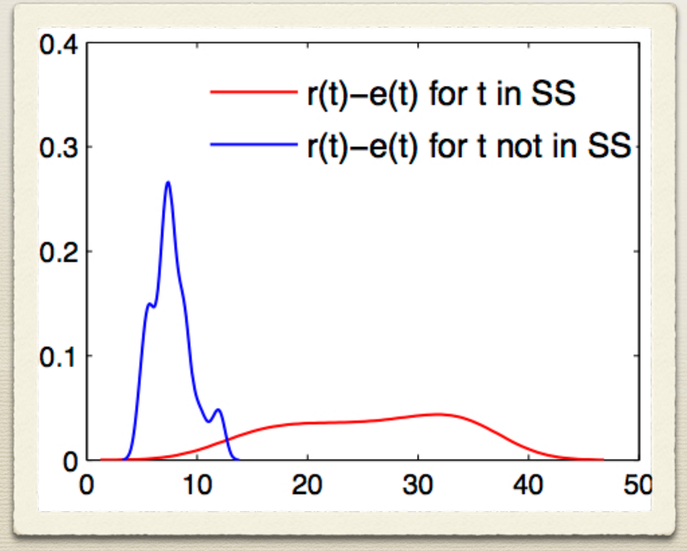
Performance of Designed Residual for Sleep Spindle (SS)



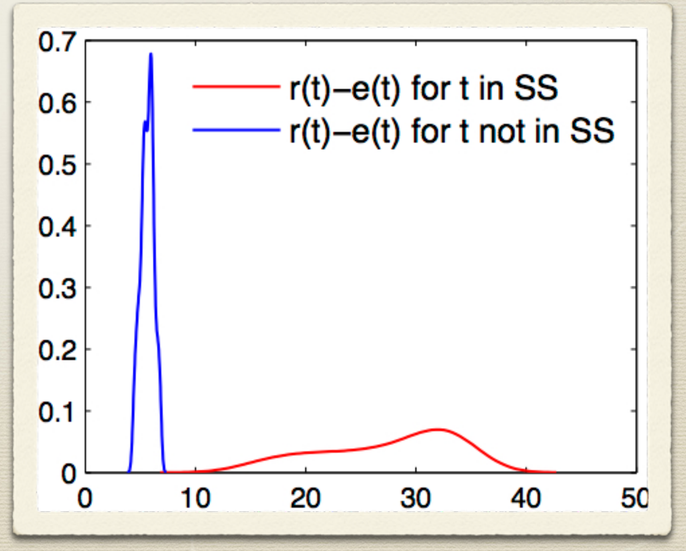
subject 1



subject 3



subject 2



subject 4

Conclusion

1. Linear Dynamical Systems (LDS) show high accuracy in modelling multidimensional EEG sleep data.

2. The advantage of a model-based approach is the capability to control the system, the residual system in our case.

3. OSAD problem is general and can be applied in many other situations.

Questions?

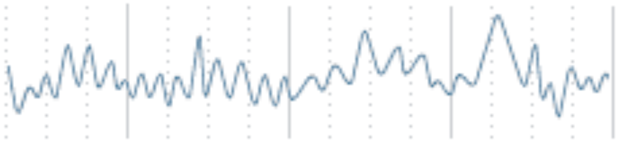
“Spare Slides”

Four Categories of Brain Wave Patterns

Beta (14-30 Hz)

Concentration, arousal, alertness, cognition

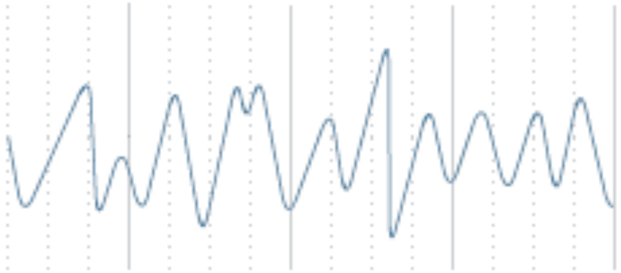
Higher levels associated with anxiety, disease, feelings of sparation, fight or flight



Alpha (8-13.9 Hz)

Relaxation, superlearning, relaxed focus, light trance, increased serontonin production

Pre-sleep, pre-waking drowsiness, mediation, beginning of access to unconscious mind

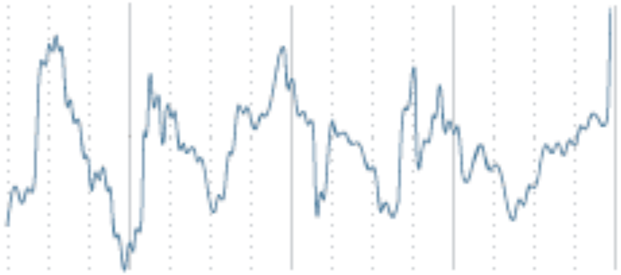


Theta (4-7.9 Hz)

Dreaming sleep (REM sleep)
Increased production of catecholamines (vital for learning and memory), increased creativity

Integrative, emotional experiences, potential change in behavior, increased retention of learned material

Hypnagogic imagery, trance, deep mediation, access to unconscious mind

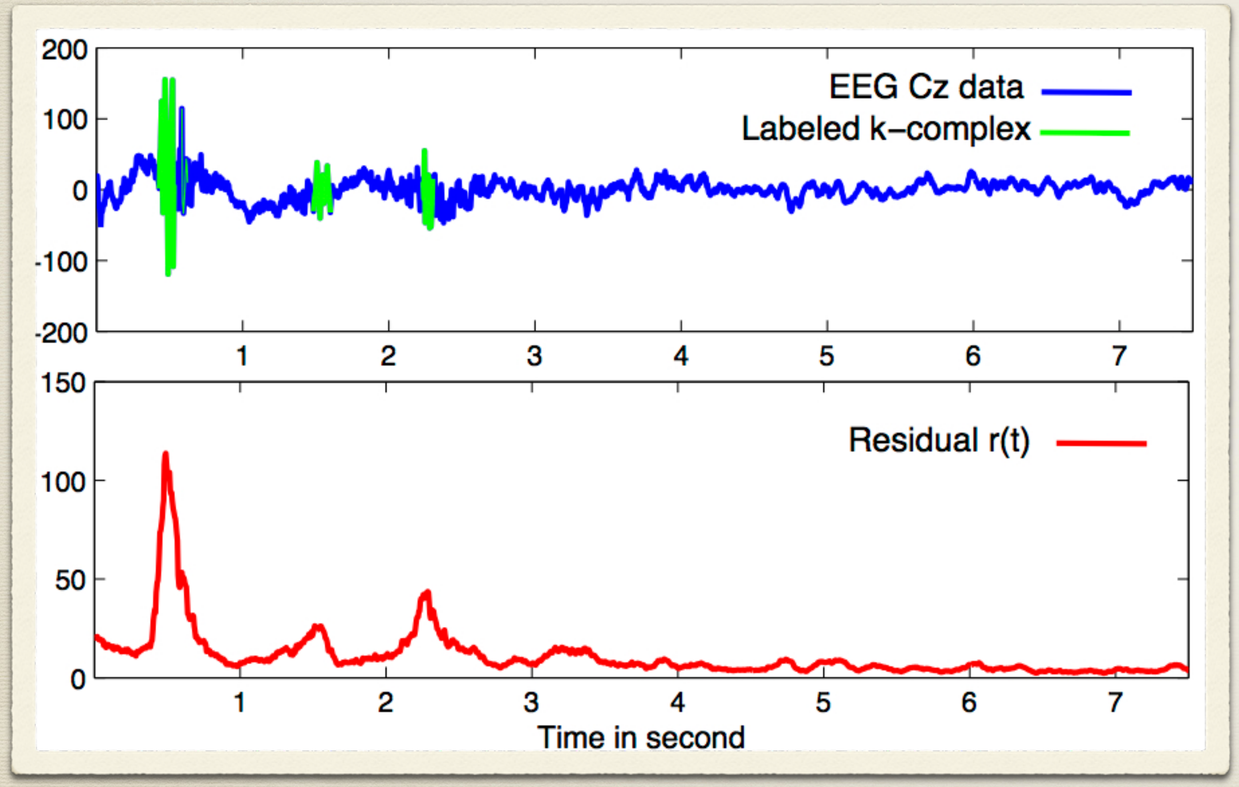


Delta (.1-3.9 Hz)

Dreamless sleep
Human growth hormone released

Deep, trance-like, non-physical state, loss of body awareness

Access to unconscious and "collective unconscious" mind, greatest "push" to brain when induced with Holosync®


200
100
0
-100
-200
EEG Cz data
Labeled k−complex
1
2
3
4
5
6
7
150
100
50
0
Residual r(t)
Time in second