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Globally Optimal Inlier Set Maximization With Unknown Rotation and Focal Length

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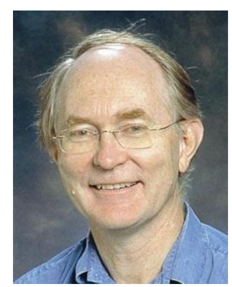
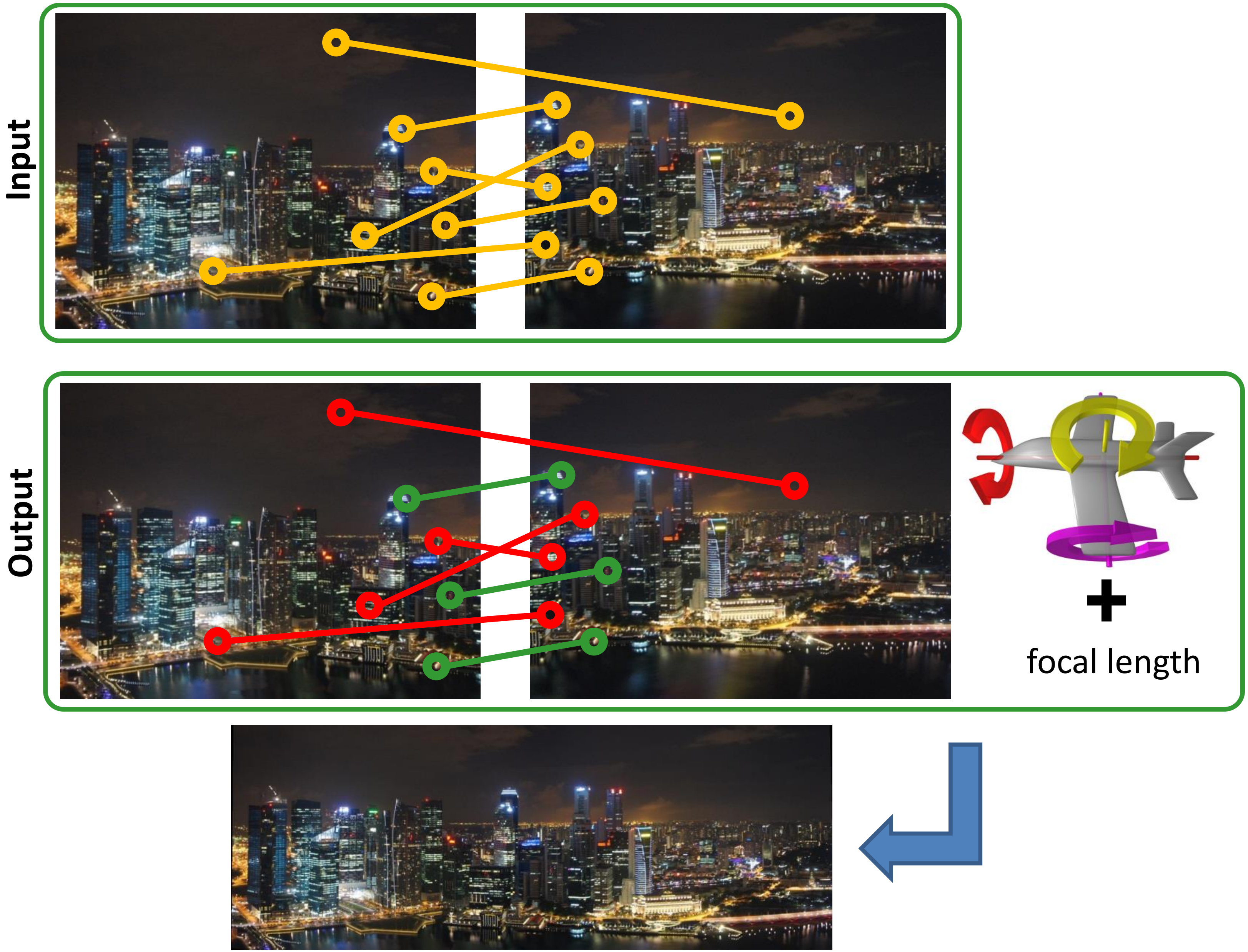


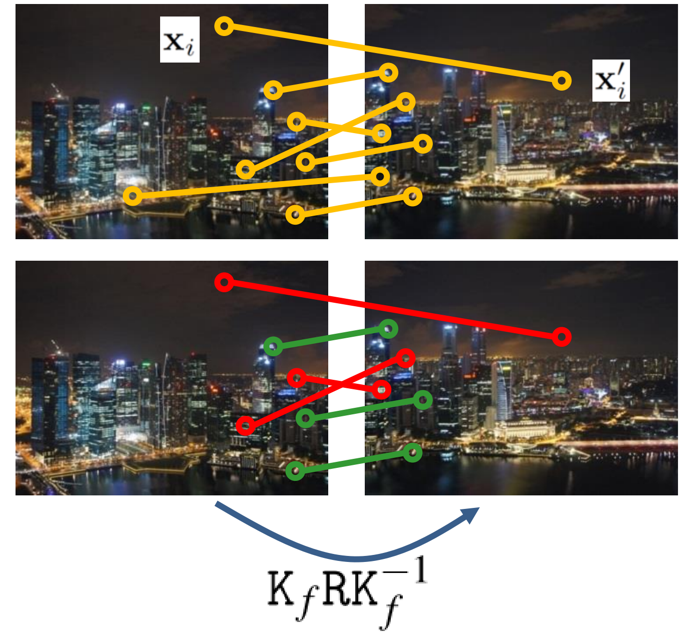
now working at Disney Research Zurich

Goal



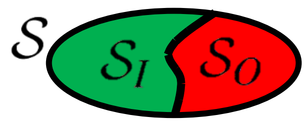
Problem statement

- Uncalibrated infinite homography
  - Unknown: rotation, focal length and inliers
  - Known: principal point, zero skew



- Mathematical formulation

$$\mathcal{S} = \{(\mathbf{x}_i, \mathbf{x}'_i), i = 1 \ldots N\}$$



$$\max_{\mathcal{S}_I, \mathrm{R}, f} \quad card(\mathcal{S}_I)$$

$$\text{s.t.} \quad d_{\mathrm{R},f}(\mathbf{x}_i, \mathbf{x}'_i) \leq \delta, \forall i \in \mathcal{S}_I \subseteq \mathcal{S}$$

$$\mathrm{R} \in SO(3)$$

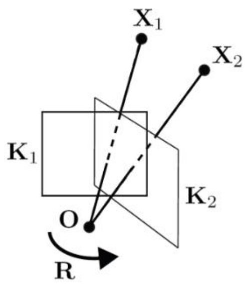
maximize the nb. of inliers

all the inliers must have a "small" reprojection error

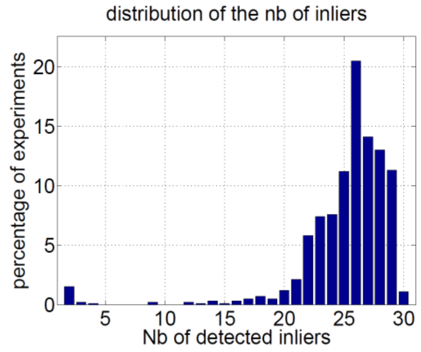
$$d_{\mathrm{R},f}(\mathbf{x}, \mathbf{x}') = \|\mathrm{K}_f \mathrm{R} \mathrm{K}_f^{-1} \mathbf{x} - \mathbf{x}'\|_2$$

A solved problem?

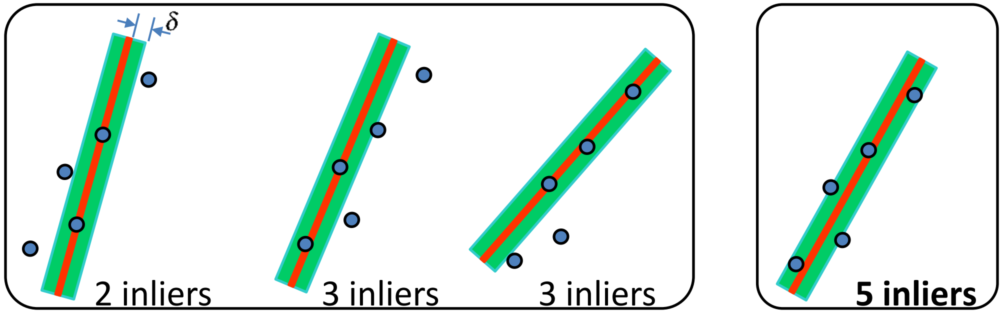
- RANSAC + 2-point algorithm [Brown et al. 2007]
  - Model: R and f



- RANSAC works great but….
- Probabilistic behavior
  - Different runs might lead to different results



- How many iterations? $N = \frac{\log(1-p)}{\log(1-(1-\varepsilon)^s)}$
- Hypothesizes only models directly supported by the samples



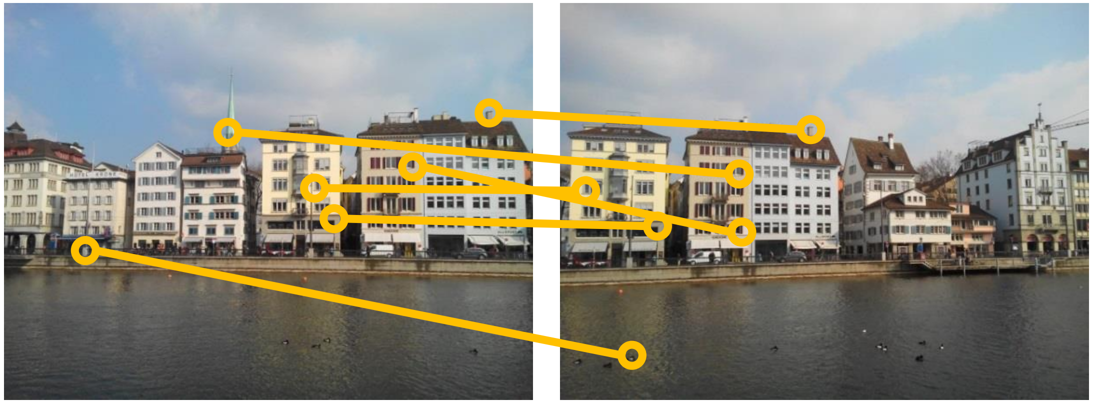
Models hypothesized from samples

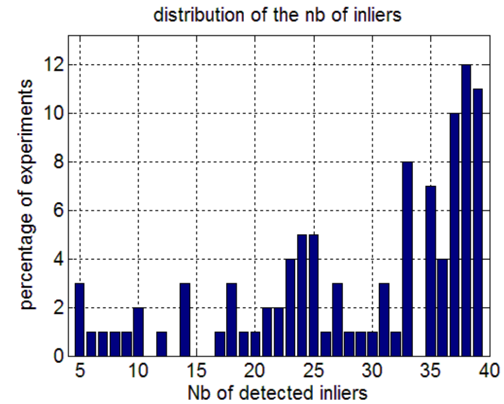
"continuous" models

No guarantee to obtain the **optimal** solution

[Brown et al 2007]: M. Brown, R. Hartley, D. Nister, *Minimal Solutions for Panoramic Stitching*, CVPR 2007

RANSAC



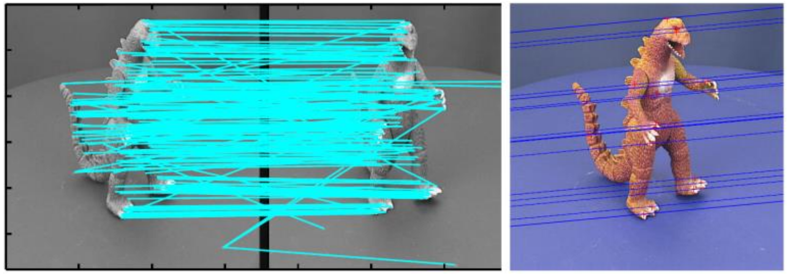


Best result of 2-pt algorithm with RANSAC +BA

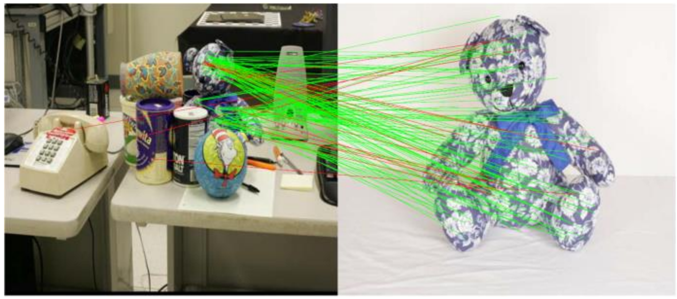
“RANSAC revisited”: how to obtain the optimal result?

Related work

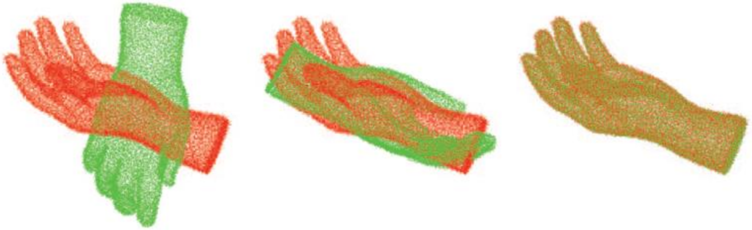
- [Li ICCV'09] [Bazin et al. TPAMI'13]
  - algebraic DLT (Direct Linear Transformation)



- [Kahl et al. IJCV'08]
  - branch-and-bound in combination with L1-norm to partially reduce the sensitivity to outliers
- [Olsson et al. CVPR'08]
  - verify whether a candidate solution is optimal or not
- [Enqvist et al. ECCV'12]
  - $O(N^{d+1})$



- [Yang et al. ICCV'13]
  - dedicated to 3D point sets (e.g. registration error in 3D space)



See ECCV'14 tutorial "Robust Optimization Techniques in Computer Vision" by Olof Enqvist, Fredrik Kahl, Richard Hartley

Related work

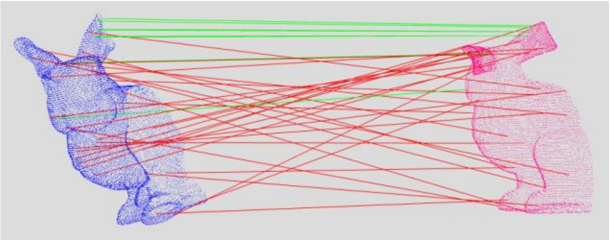
- [Bazin et al. ACCV'12]
  - maximize the inlier set under a pure rotational model

ECCV 2014

$$d_{\mathrm{R}}(\mathbf{u}_i, \mathbf{u}'_i) = \angle(\mathrm{R}\,\mathbf{u}_i, \mathbf{u}'_i) \leq \delta \quad \Rightarrow \quad d_{\mathrm{R},f}(\mathbf{x}_i, \mathbf{x}'_i) = \|\mathrm{K}_f\,\mathrm{R}\,\mathrm{K}_f^{-1}\,\mathbf{x}_i - \mathbf{x}'_i\|_2 \leq \delta$$

Fully calibrated ⇒ Unknown focal length

Angular distance ⇒ Pixel distance (image space)



- Derivation of the reprojection bounds
- Rotation parametrization that reduces the correlation between f and R

Proposed approach

$$\max_{\mathcal{S}_I,\theta} \quad card(\mathcal{S}_I)$$

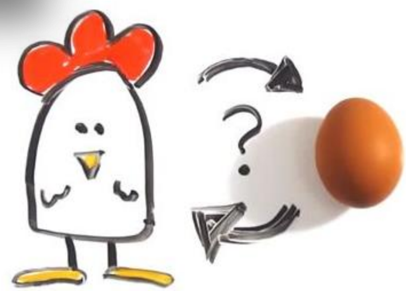
$$\text{s.t.} \quad g(x_i,\theta) \leq T, \forall i \in \mathcal{S}_I$$

maximize the nb of inliers

all the inliers must have a “small” error

If inliers are known ➔ model can be computed

If model is known ➔ inliers can be detected



Test **all** the combinations of inliers

➔ Finite (2^N) but intractable

Test **all** the models

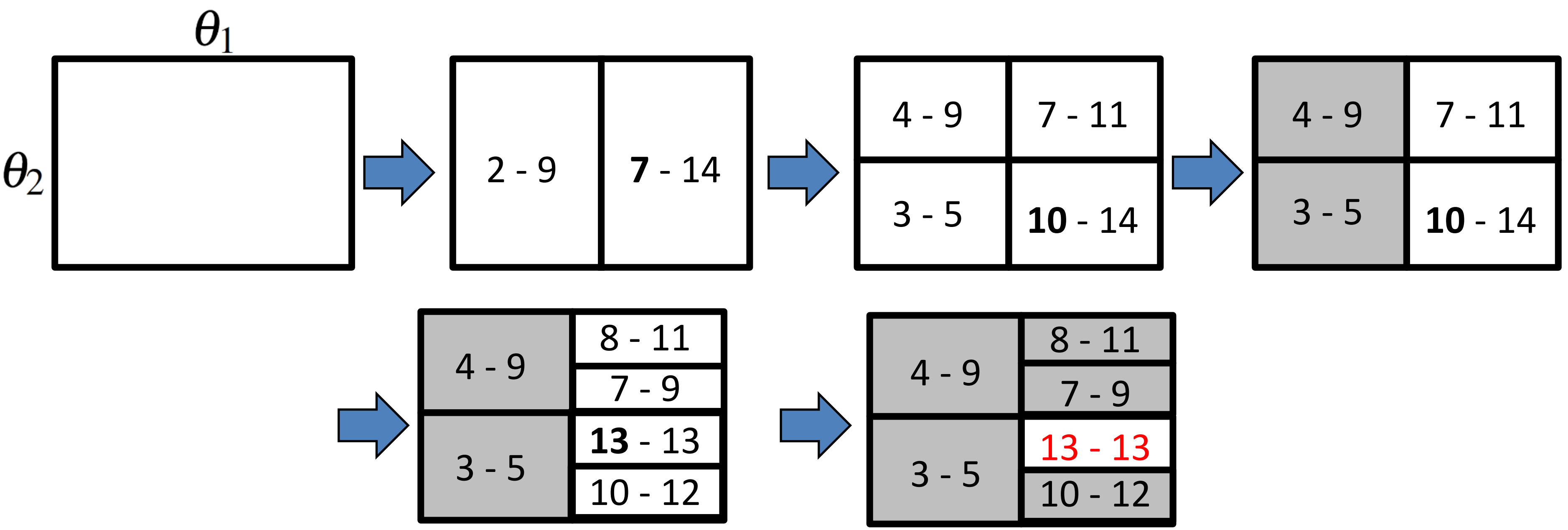
➔ Non-finite (continuous space) and intractable

A “counter-intuitive” approach:

- work on the model search space
- iteratively reduce the search space by B&B

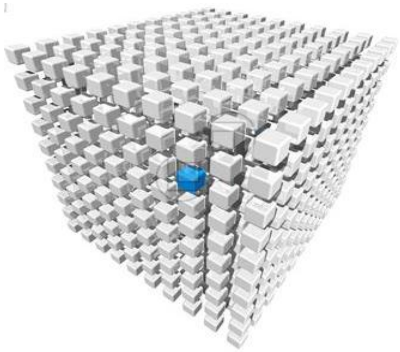
Branch-and-bound

- BnB main idea:
  - Divide the search space into smaller sub-spaces
  - For each sub-space
    - Might it contain an optimal solution?
      - Using lower and upper bounds
    - If yes, then split it
    - If no, remove it
  - Iterate on the remaining spaces

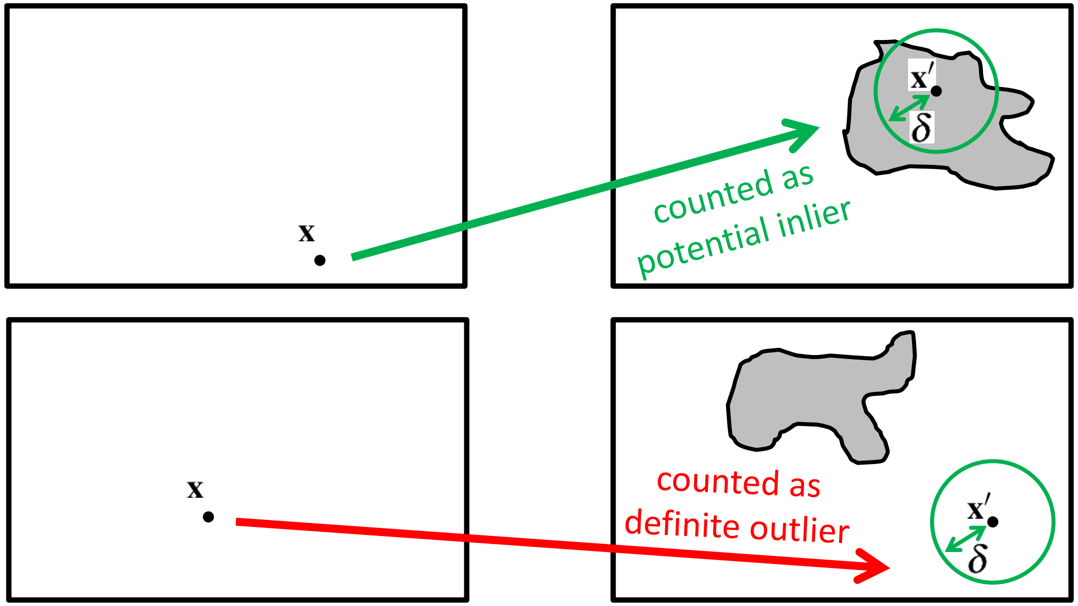


Proposed approach

- Branch and bound over R and f
  - 4D search space



- Bounds for an interval of R and f
  - Upper bound: the max nb. of inliers for this interval
  - Lower bound: the min nb. of inliers (or an evaluation) for this interval



$$\|\mathrm{K}_f \mathrm{R} \mathrm{K}_f^{-1} \mathbf{x} - \mathbf{x}'\|_2 \leq \delta$$

➔ How to get these bounds?
➔ How to get the "projection interval"?

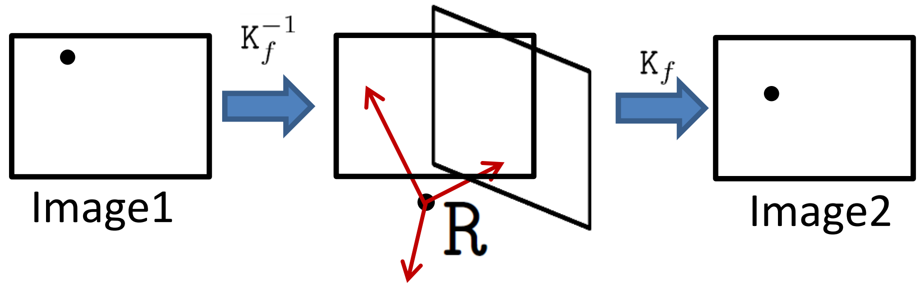
Dependency issue

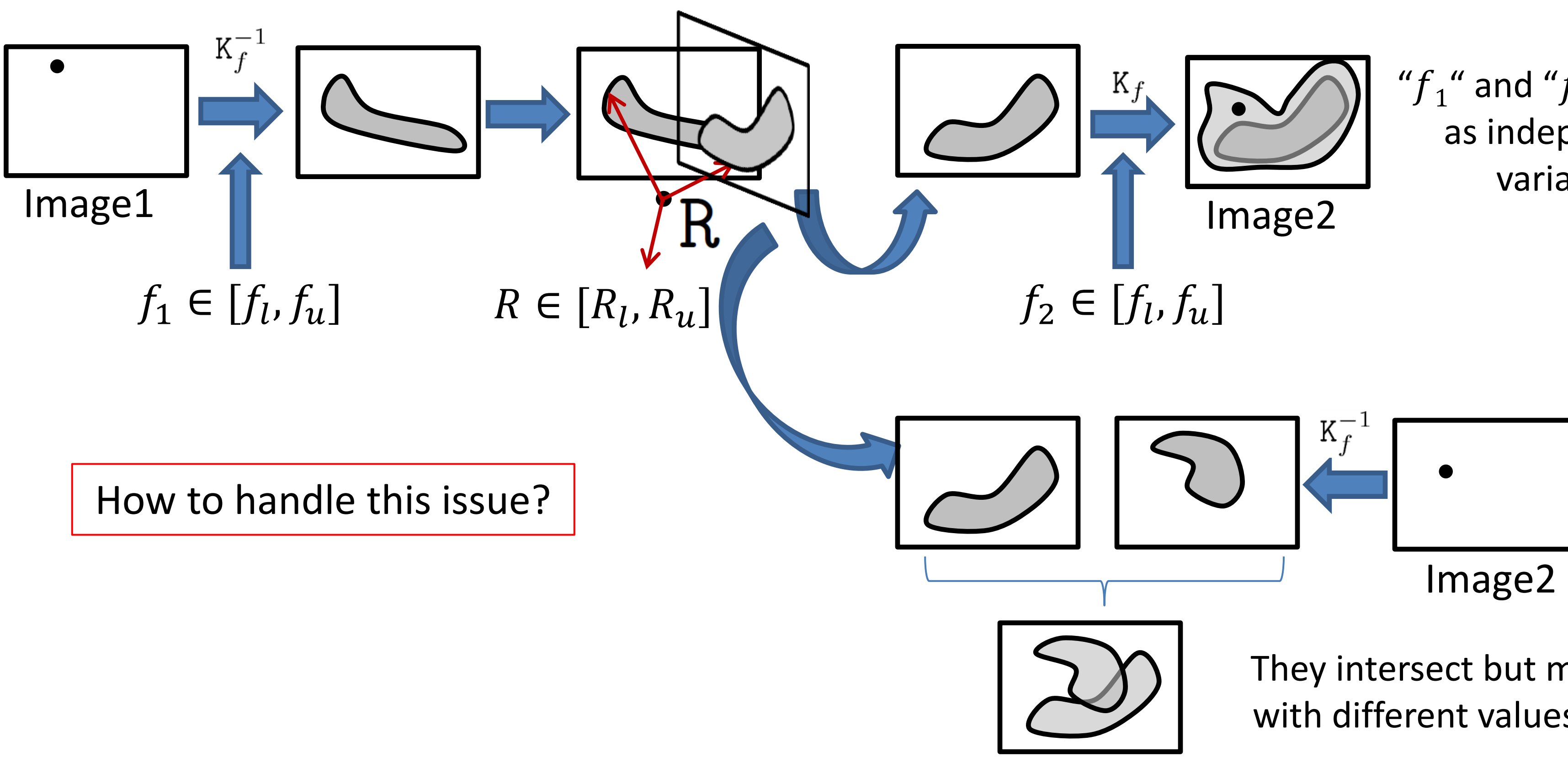
Our first attempt

$$\mathbf{x}_i' = \mathrm{T}\mathbf{x}_i = \mathrm{K}_f \, \mathrm{R} \, \mathrm{K}_f^{-1} \mathbf{x}_i$$

$f \in [f_l, f_u]$

$R \in [R_l, R_u]$





"f_1" and "f_2" treated as independent variables

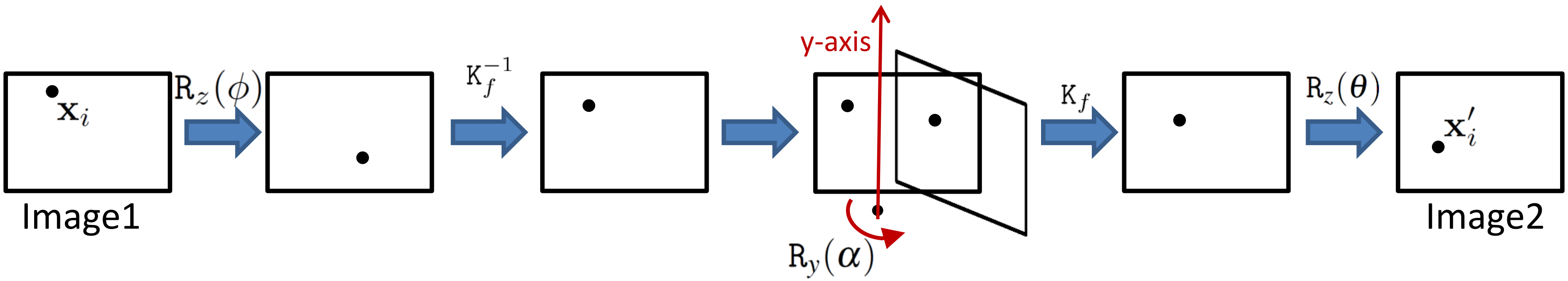
How to handle this issue?

They intersect but maybe with different values of f

Proposed approach

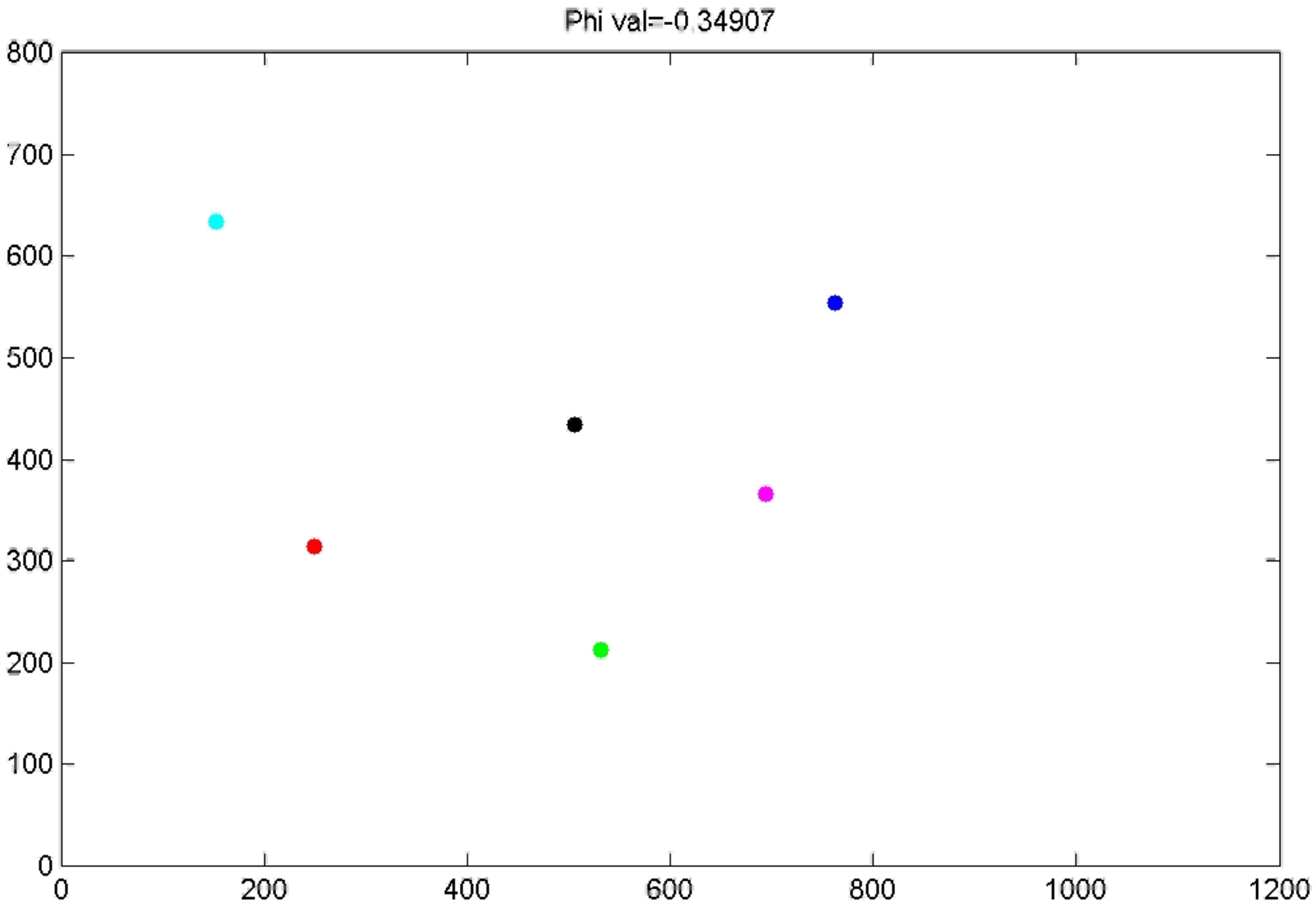
$$\mathbf{x}_i' = \mathrm{T}\mathbf{x}_i$$

$$\begin{aligned}\mathrm{T} &= \mathrm{K}_f\,\mathrm{R}\,\mathrm{K}_f^{-1}\\ &= \mathrm{K}_f\,\mathrm{R}_z\mathrm{R}_y\mathrm{R}_z\,\mathrm{K}_f^{-1}\\ &= \mathrm{R}_z\,\mathrm{K}_f\mathrm{R}_y\mathrm{K}_f^{-1}\,\mathrm{R}_z\end{aligned}$$



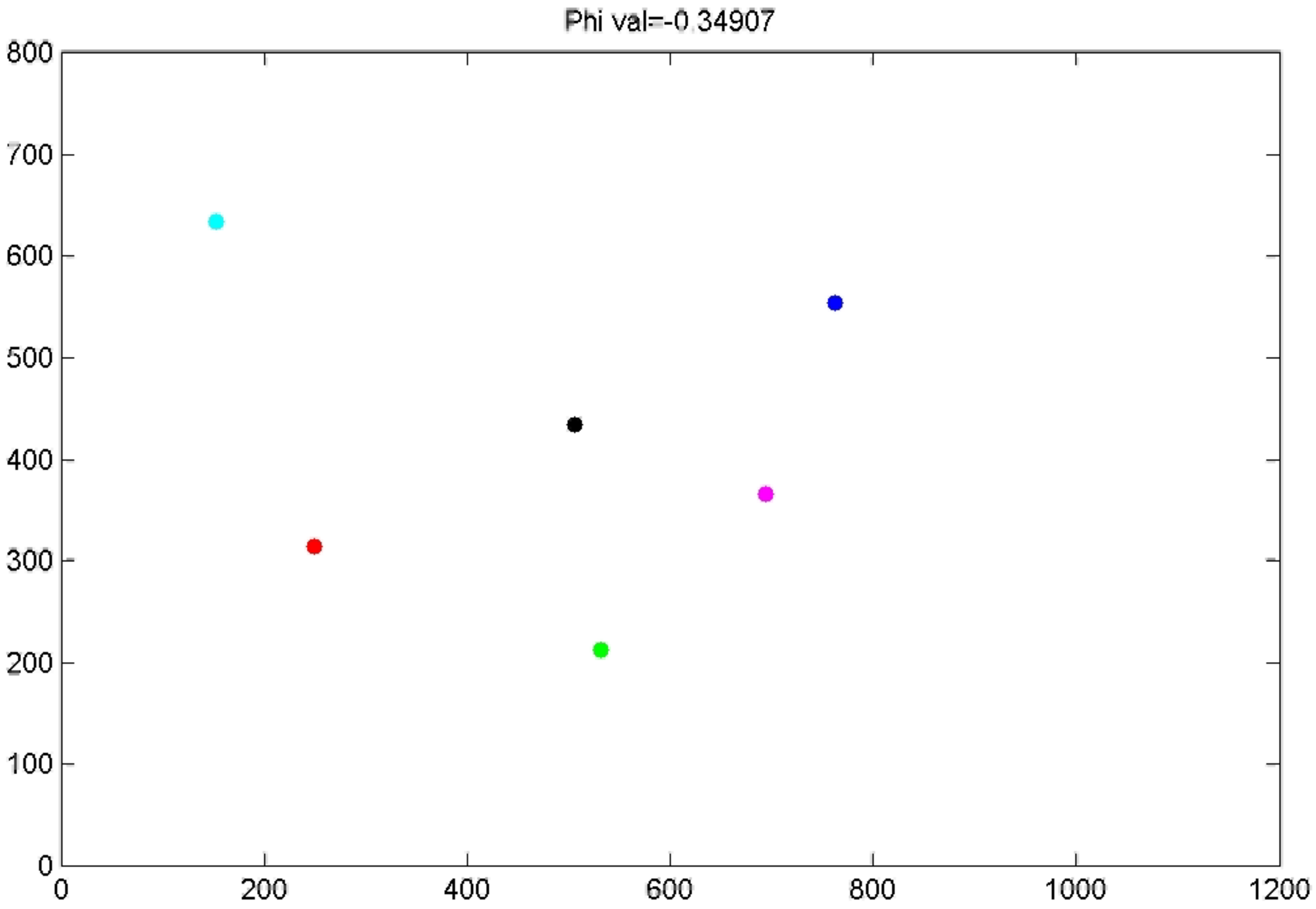
Trajectories

$$\mathbf{x}_i' = \mathrm{T}\,\mathbf{x}_i = \mathrm{R}_z(\theta)\,\mathrm{K}_f\,\mathrm{R}_y(\alpha)\,\mathrm{K}_f^{-1}\,\boxed{\mathrm{R}_z(\phi)}\,\mathbf{x}_i$$



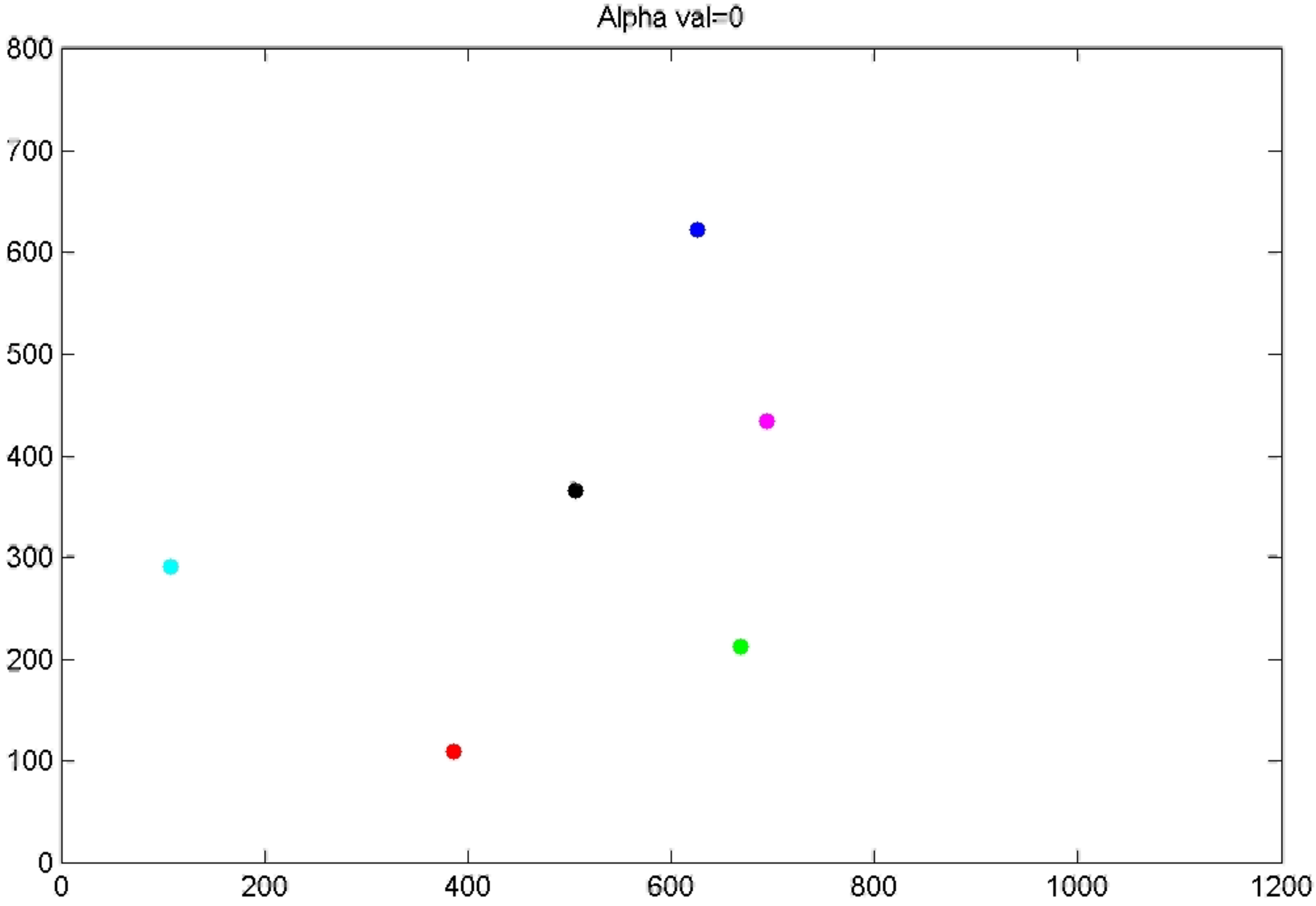
Trajectories

$$\mathbf{x}'_i = \mathrm{T}\,\mathbf{x}_i = \mathrm{R}_z(\theta)\,\mathrm{K}_f\,\mathrm{R}_y(\alpha)\,\mathrm{K}_f^{-1}\boxed{\mathrm{R}_z(\phi)}\,\mathbf{x}_i$$



Trajectories

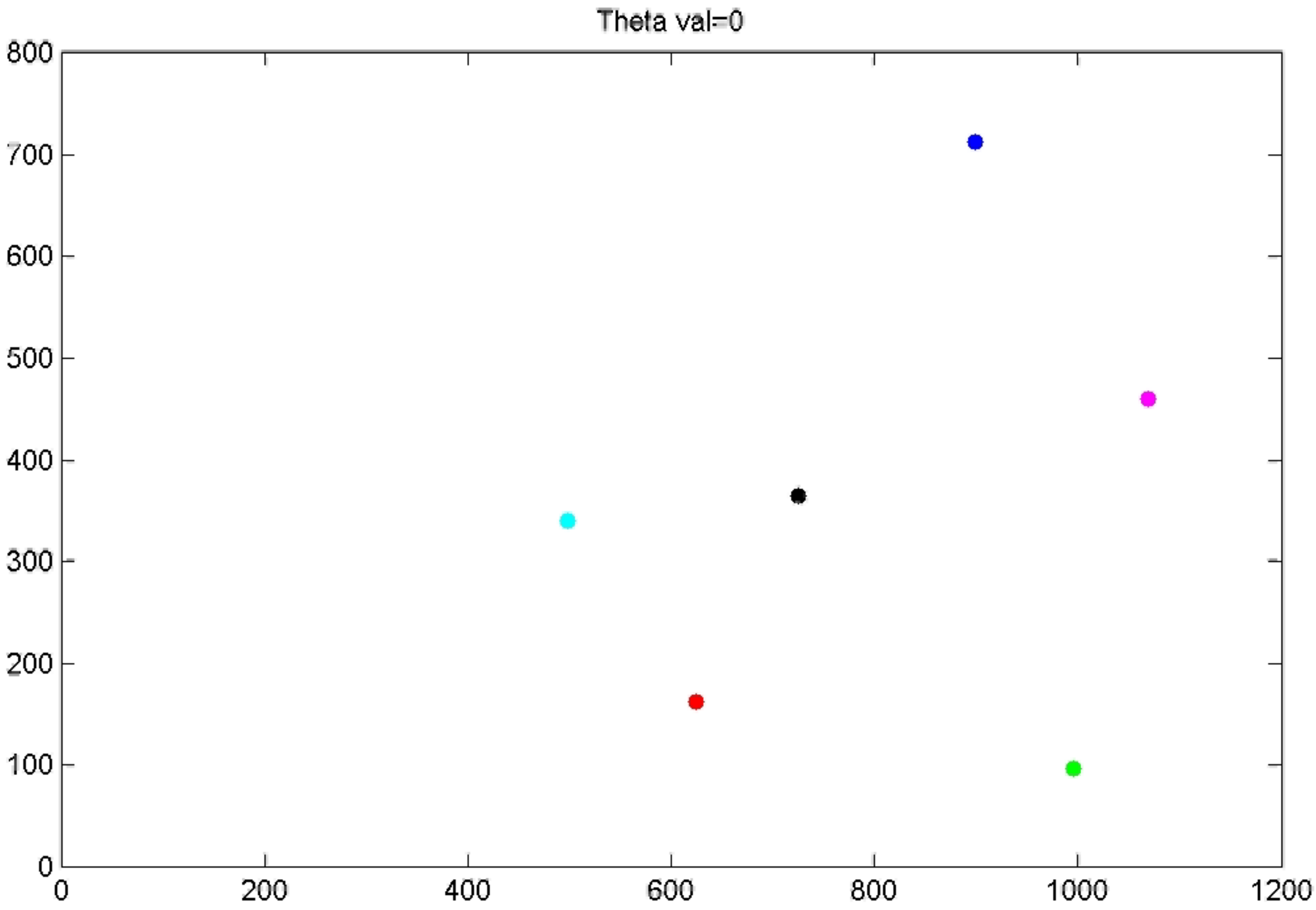
$$\mathbf{x}_i' = \mathrm{T}\,\mathbf{x}_i = \mathrm{R}_z(\theta)\,\boxed{\mathrm{K}_f\,\mathrm{R}_y(\alpha)\,\mathrm{K}_f^{-1}}\,\mathrm{R}_z(\phi)\,\mathbf{x}_i$$



with $f = 300$

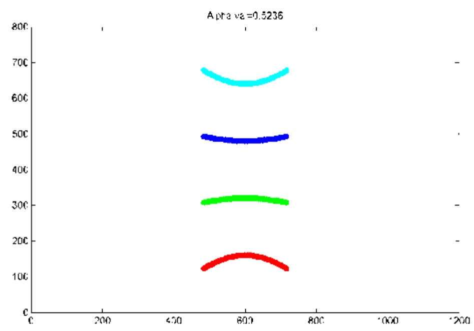
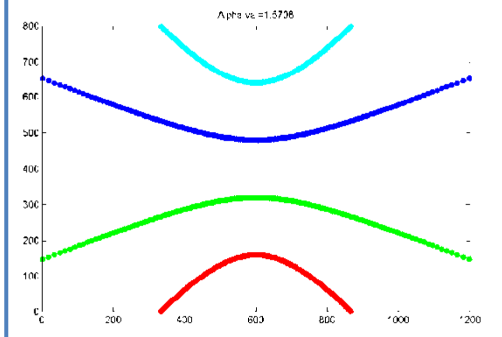
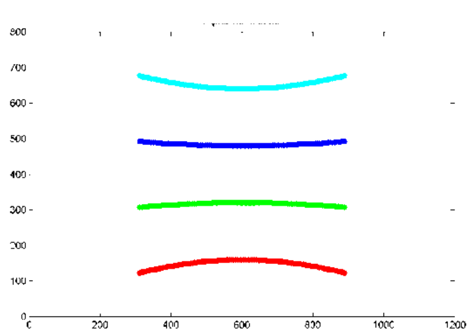
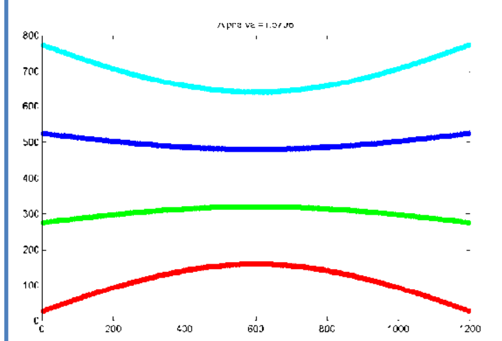
Trajectories

$$\mathbf{x}'_i = \mathrm{T}\,\mathbf{x}_i = \boxed{\mathrm{R}_z(\theta)}\,\mathrm{K}_f\,\mathrm{R}_y(\alpha)\,\mathrm{K}_f^{-1}\,\mathrm{R}_z(\phi)\,\mathbf{x}_i$$



Trajectories

$$\mathbf{x}_i' = \mathrm{T}\,\mathbf{x}_i = \mathrm{R}_z(\theta)\,\boxed{\mathrm{K}_f\,\mathrm{R}_y(\alpha)\,\mathrm{K}_f^{-1}}\,\mathrm{R}_z(\phi)\,\mathbf{x}_i$$

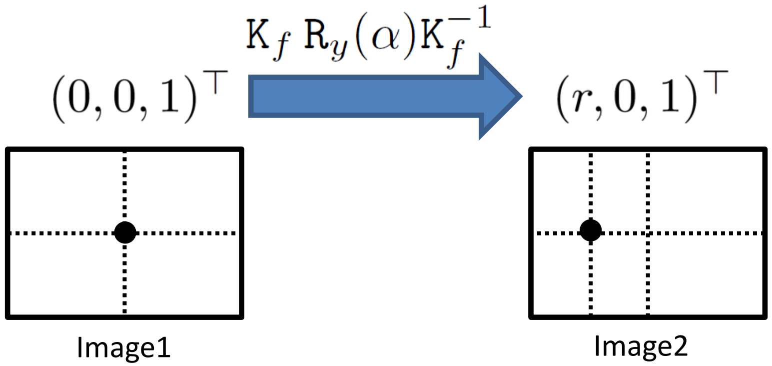
	$\alpha \in [-45°, 45°]$	$\alpha \in [-90°, 90°]$
$f = 200$		
$f = 500$		

f controls the "bending"

How much the point moves depend on both α and f
➔ Let's decouple them!

Motion model and parametrization

$$\mathrm{T} = \mathrm{R}_z(\theta)\boxed{\mathrm{K}_f \mathrm{R}_y(\alpha) \mathrm{K}_f^{-1}}\mathrm{R}_z(\phi)$$



r directly controls the "translation" (independently of f)

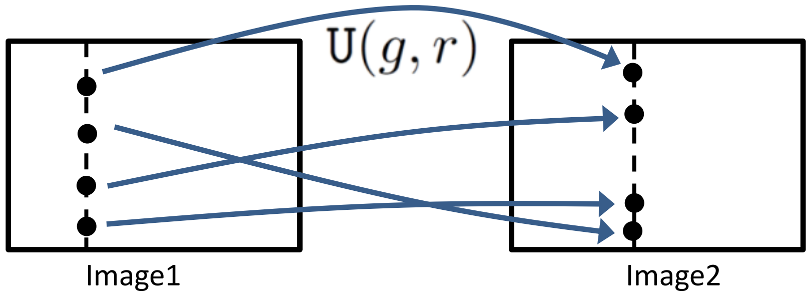
$$\Rightarrow \quad \mathrm{U}(g,r) = \mathrm{K}_f\,\mathrm{R}_r \mathrm{K}_f^{-1} = \begin{bmatrix} 1 & 0 & r \\ 0 & \sqrt{1+r^2g^2} & 0 \\ -rg^2 & 0 & 1 \end{bmatrix}$$

$$\tan(\alpha) = r/f$$
$$g = 1/f$$

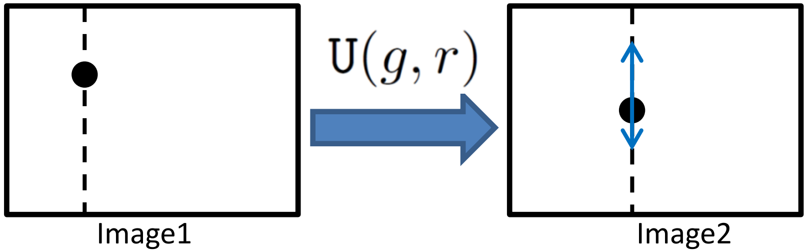
Properties of $U(g,r)$

$$(x,y) \xrightarrow{\mathtt{U}(g,r) = \mathtt{K}_f \mathtt{R}_r \mathtt{K}_f^{-1}} (x',y') = \left(\frac{x+r}{1-rxg^2}, \frac{y\sqrt{1+r^2g^2}}{1-rxg^2} \right)$$

- x' depends only on x



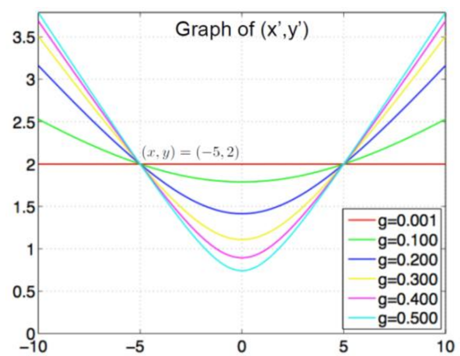
- A vertical line goes to a vertical line



as r and g vary

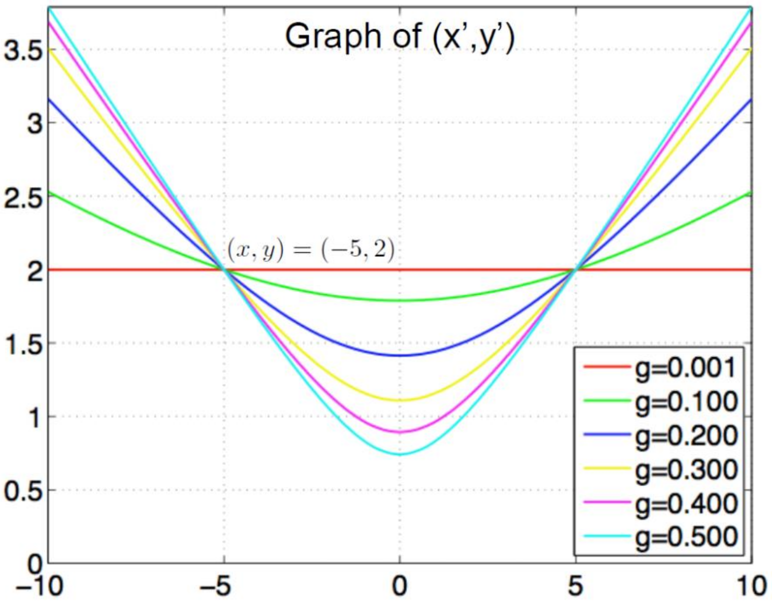
- Hyperbolic trajectory for (x', y') for a fixed $(x,y) = (x_0, y_0)$ as r varies

$$(1+g^2x_0^2)(y')^2 - (g^2y_0^2)(x')^2 = y_0^2$$



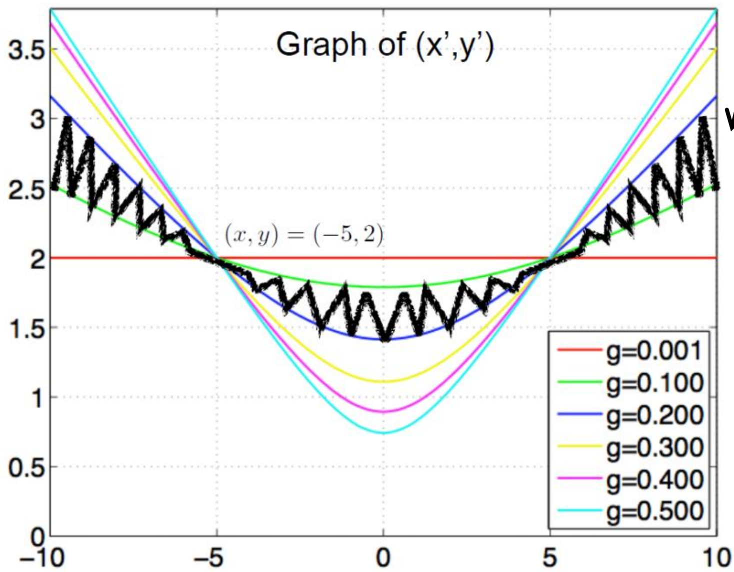
Coordinate bounds

$$(1+g^2x^2)(y')^2 - g^2y^2(x')^2 = y^2$$



Trajectories of a point transformed by $U(g,r)$ with different values of g, given the initial point $(x_0\,,y_0) = (5\,,2)$ and for any values of r.

Given $g \in [0.1, 0.2]$, y' must live here



➔ Bounds on y' for a given interval of g and any values of r

Coordinate bounds

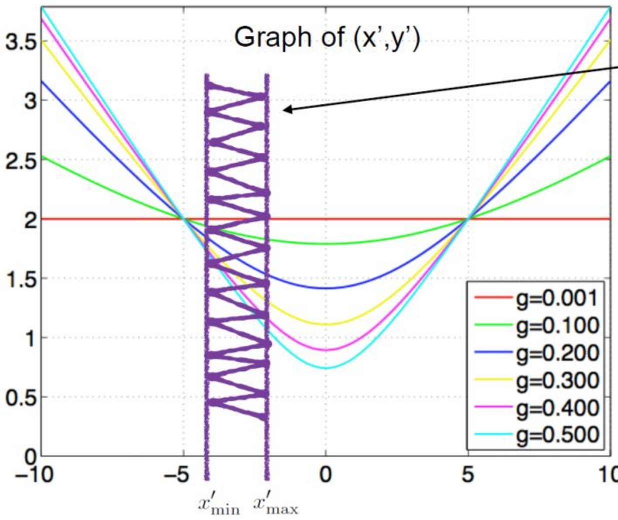
$$(x, y) \to (x', y') = \left(\frac{x + r}{1 - rxg^2}, \quad \frac{y\sqrt{1 + r^2 g^2}}{1 - rxg^2} \right)$$

$r \in [r_{min}, r_{max}]$

$$\frac{\partial x'}{\partial r} > 0 \ \forall x, r, g$$

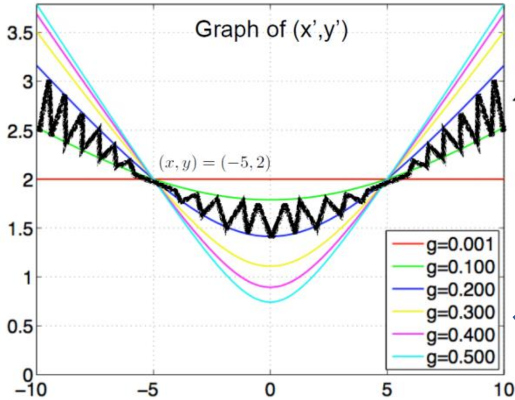
x'_{min} is achieved at r_{min}
x'_{max} is achieved at r_{max}

Given $g \in [0.1, 0.2]$, and $r \in [r_{min}, r_{max}]$, x' must live here

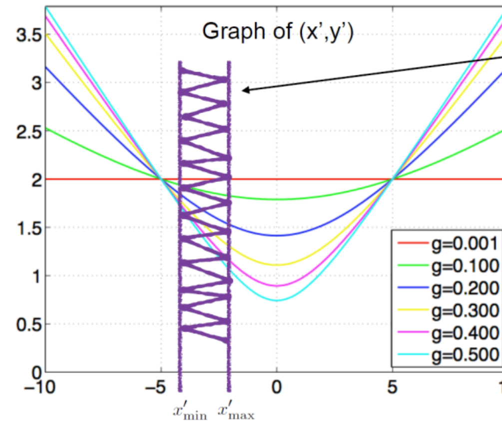


➔ Bounds on x for a given interval of r and g

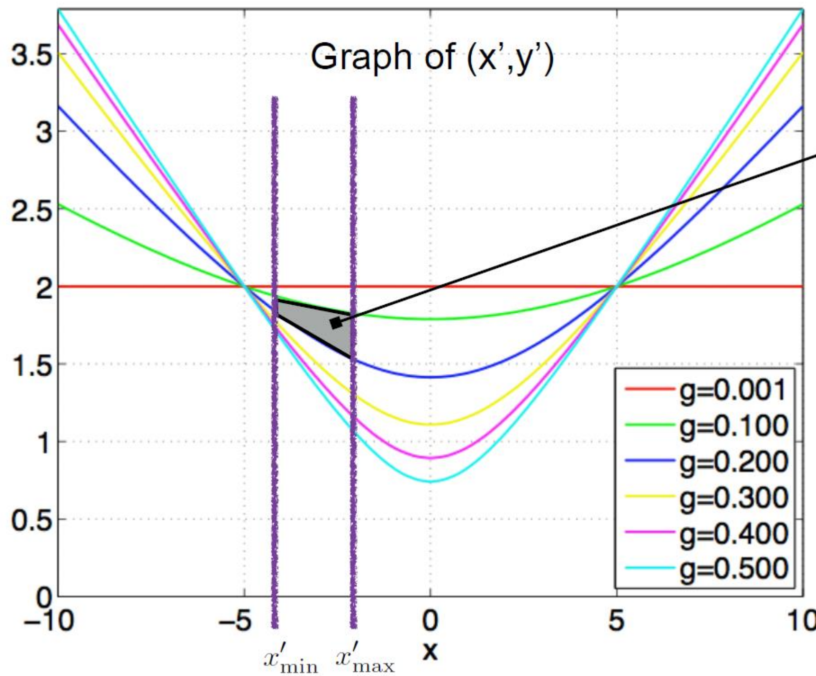
Constraining the coordinate bounds



Given $g \in [0.1, 0.2]$,
y' must live here



Given $g \in [0.1, 0.2]$,
and $r \in [r_{min}, r_{max}]$,
x' must live here



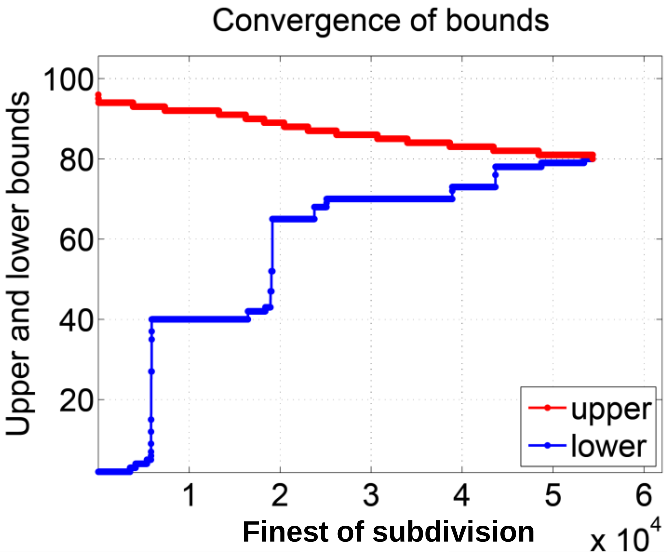
Given $g \in [0.1, 0.2]$, and
$r \in [r_{min}, r_{max}]$,
(x', y') must live here

➔ Bounds on x' and y'
for a given interval of r and g

➔ If (x', y') lies in this area
then potential inlier
else definite outlier

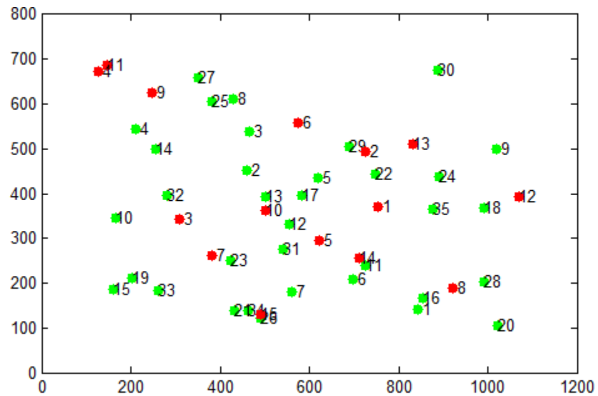
In practice

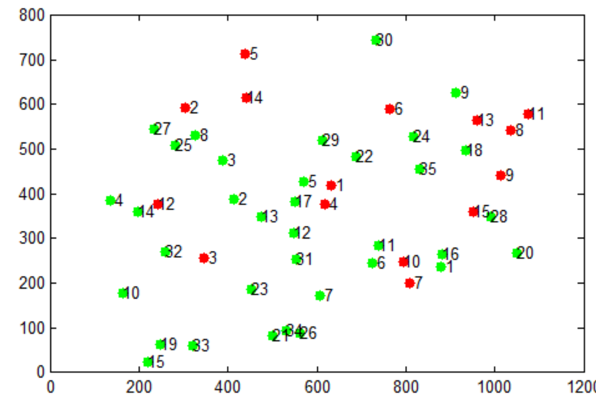
- Branch and bound on ϕ, θ, r and g
  - Divided into sub-cubes
  - Upper bound: nb. inliers inside the reprojection area
  - Lower bound: nb. inliers obtained at the “cube center”
  - Converges to the optimal solution
    - Bounds get tighter along the BnB subdivisions

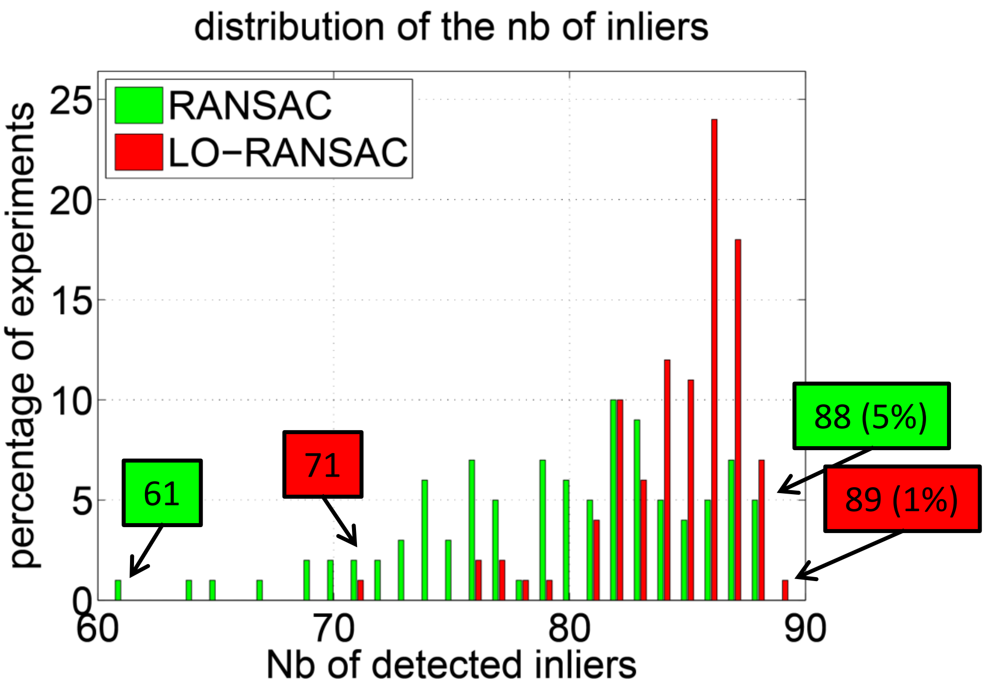


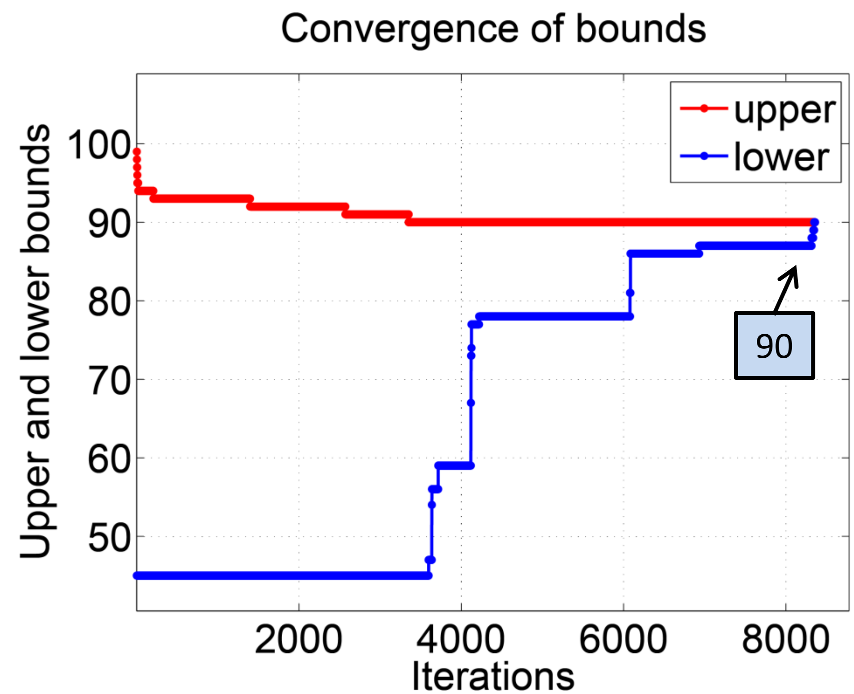
Results

100 points,
10% outliers



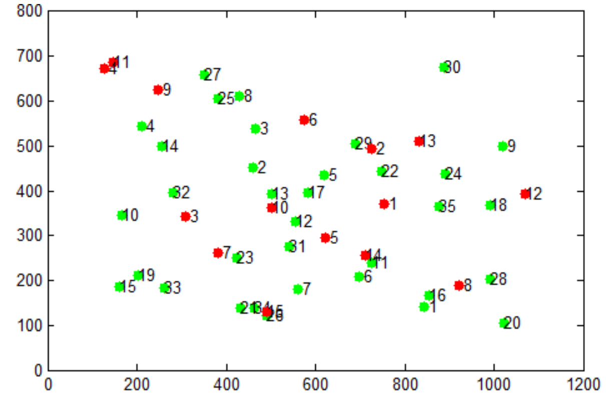






"Locally optimized RANSAC", Ondřej Chum, Jiří Matas, Josef Kittler, Pattern Recognition, 2003

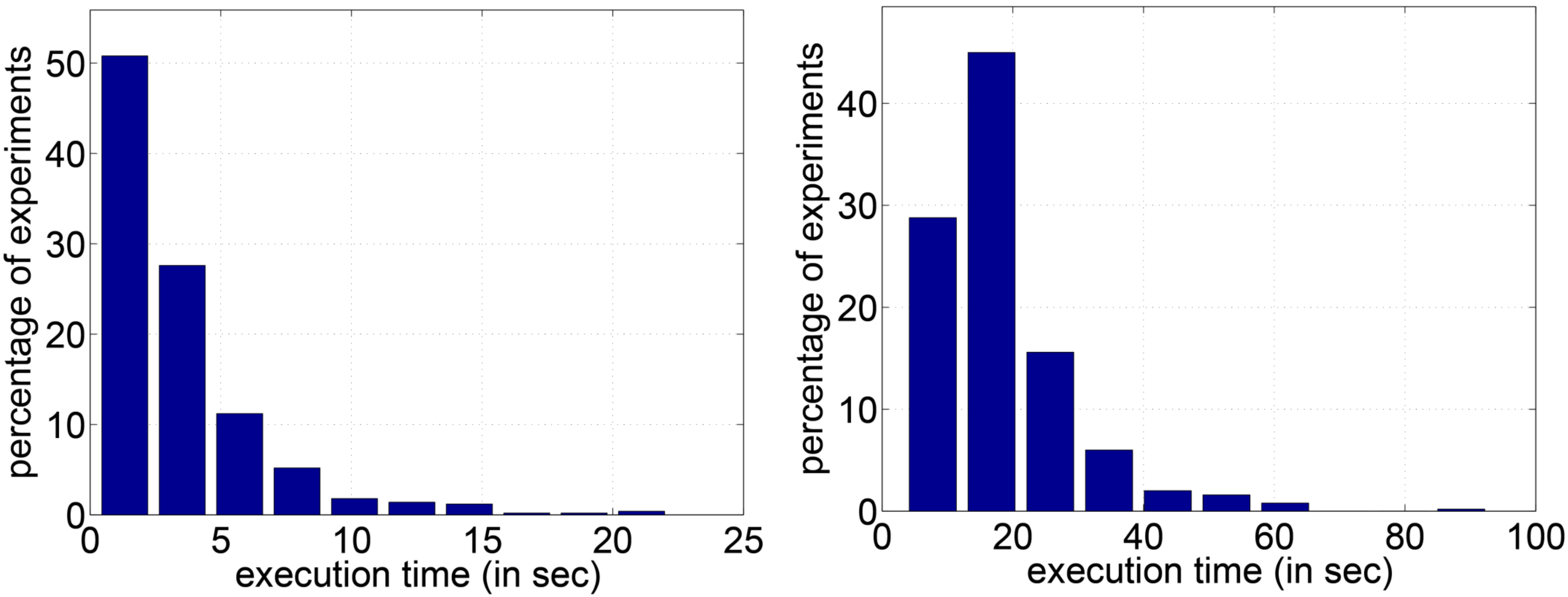
Results



- More than 100,000 experiments
  - Nb. of data points, noise, outlier ratio, etc
- Our results were always equal to or higher than all the runs of RANSACs
- Exhaustive search
  - Every combination of 2 points

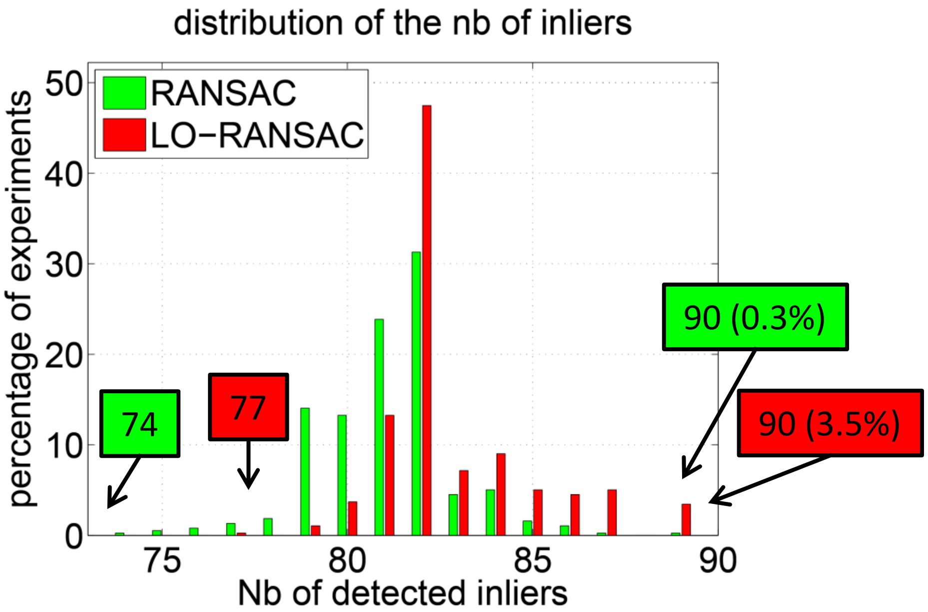
RANSAC ≤ Exhaustive search ≤ Our

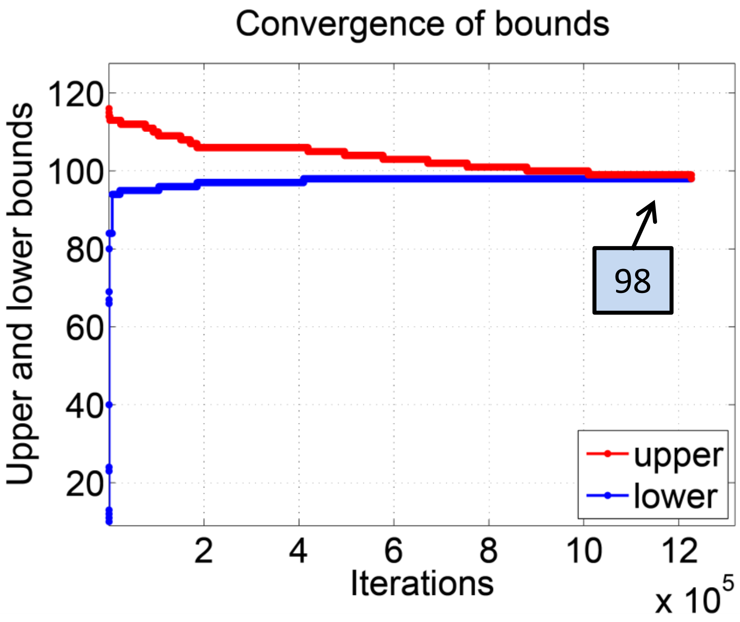
Results

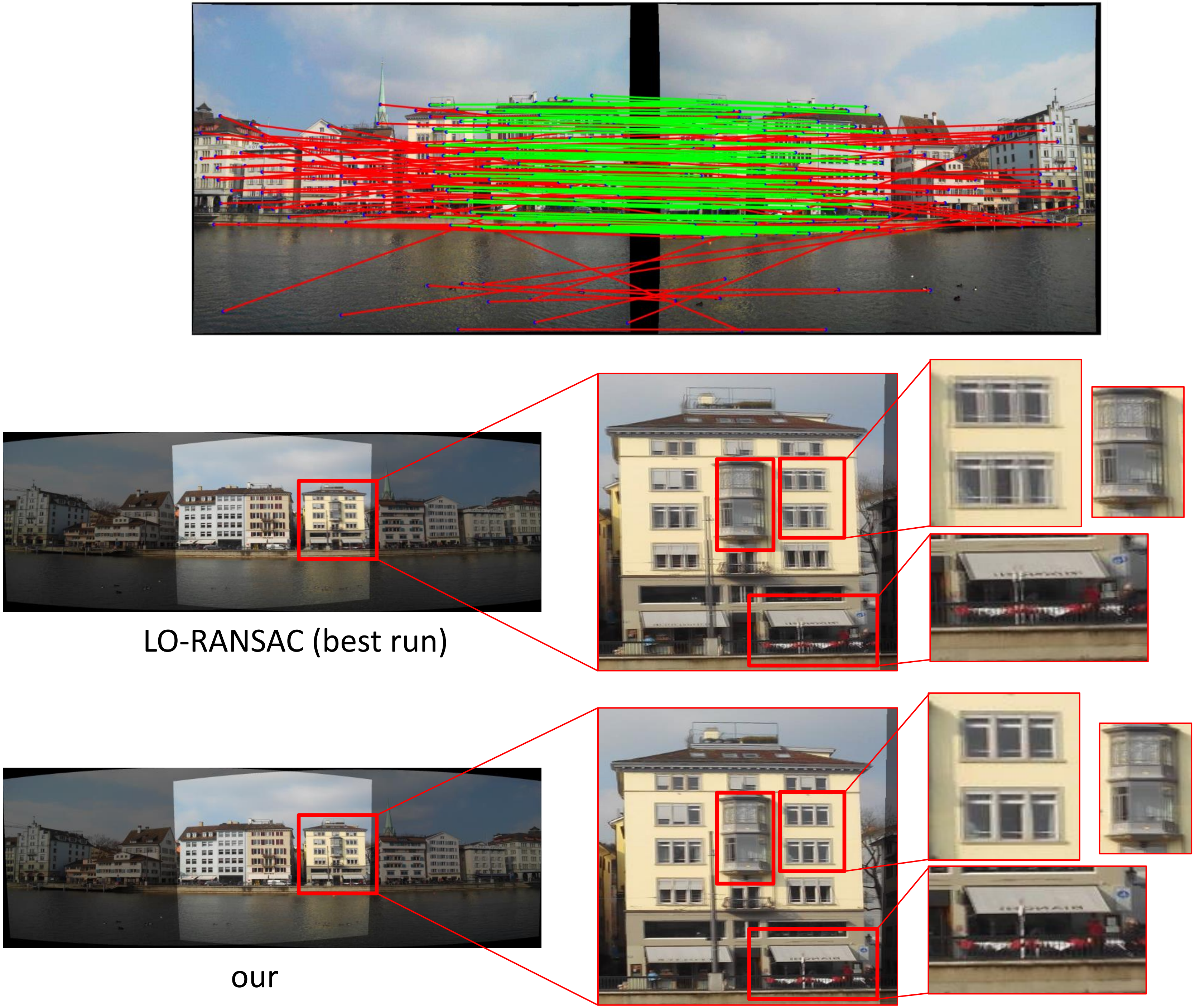


Distribution of the execution time of our approach for $N = 30$ (left) and $N = 300$ data points (right) with 70% of outliers.

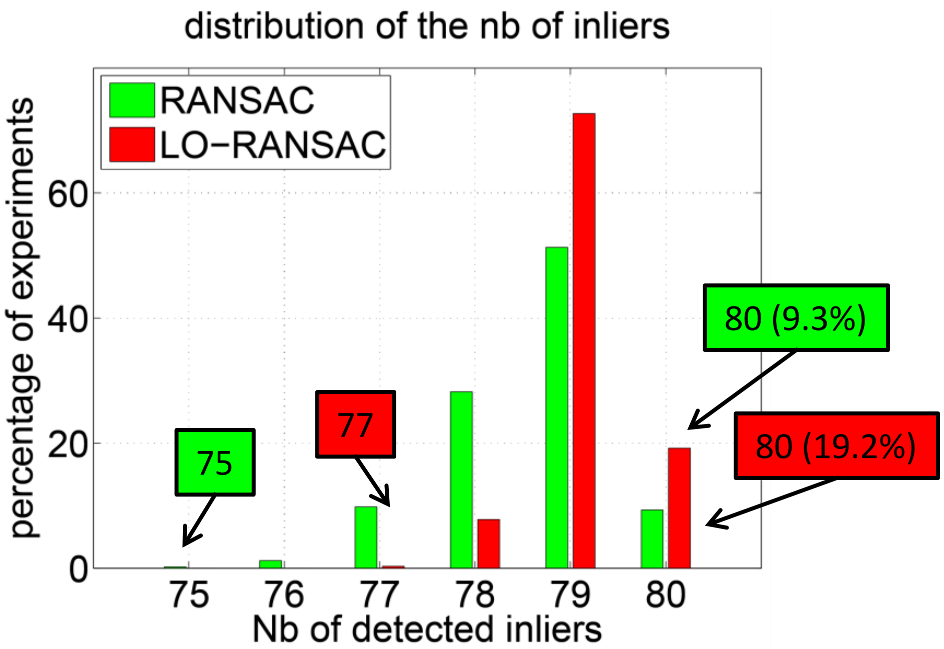
Results

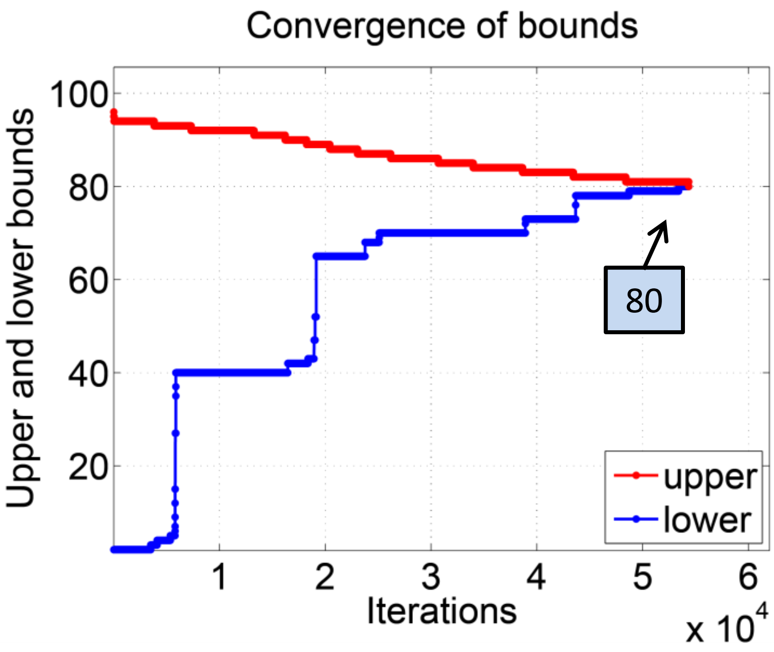

distribution of the nb of inliers
RANSAC
LO−RANSAC
percentage of experiments
0
10
20
30
40
50
75
80
85
90
Nb of detected inliers
74
77
90 (0.3%)
90 (3.5%)

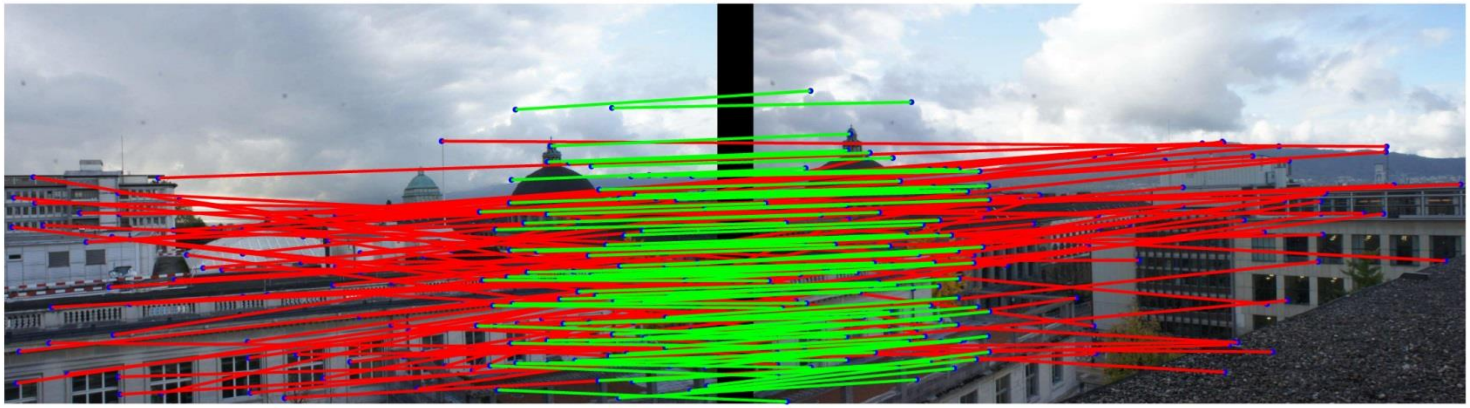

Convergence of bounds
Upper and lower bounds
20
40
60
80
100
120
2
4
6
8
10
12
Iterations
x 10^5
98
upper
lower


LO-RANSAC (best run)
our

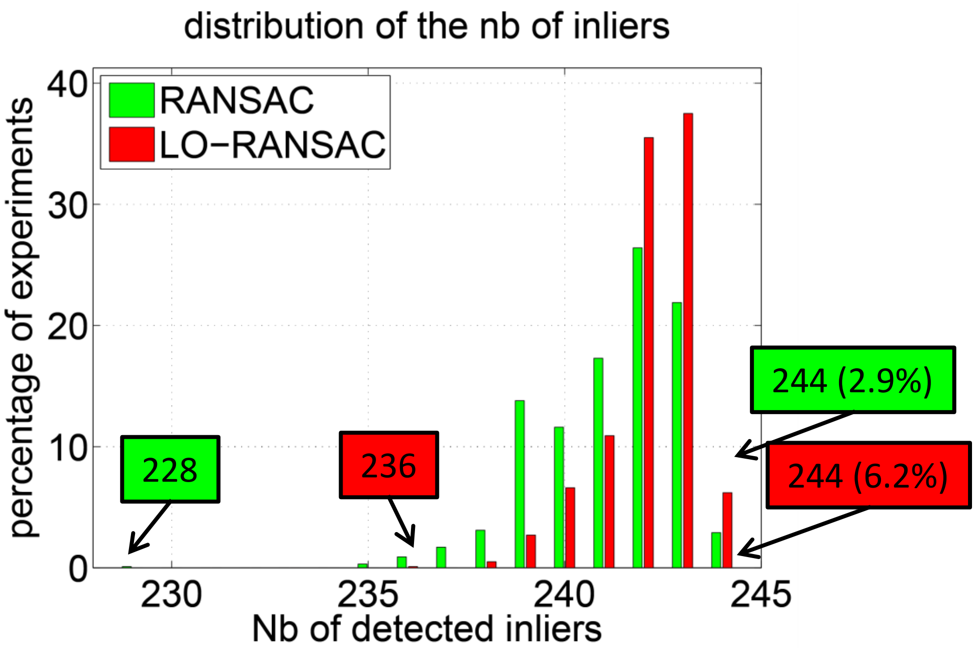
Results

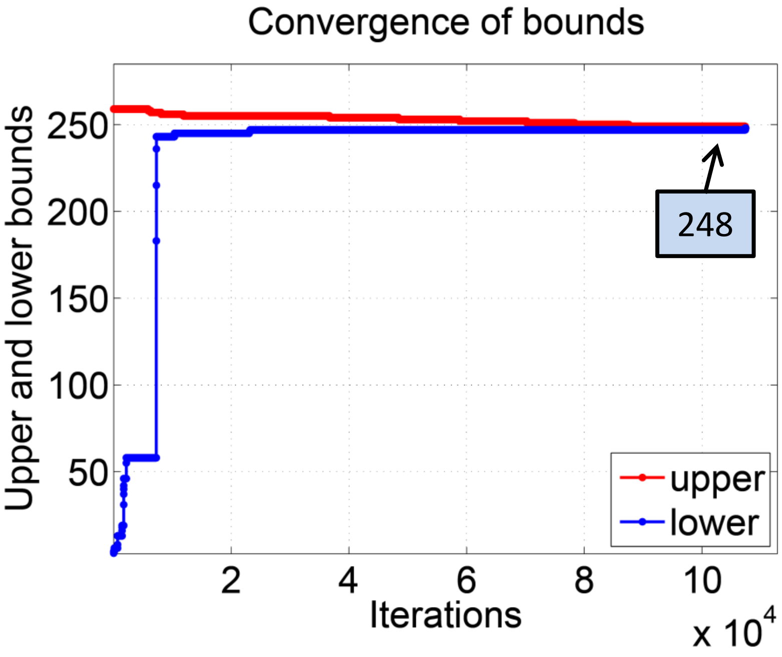

distribution of the nb of inliers
RANSAC
LO−RANSAC
percentage of experiments
0
20
40
60
75
76
77
78
79
80
Nb of detected inliers
75
77
80 (9.3%)
80 (19.2%)

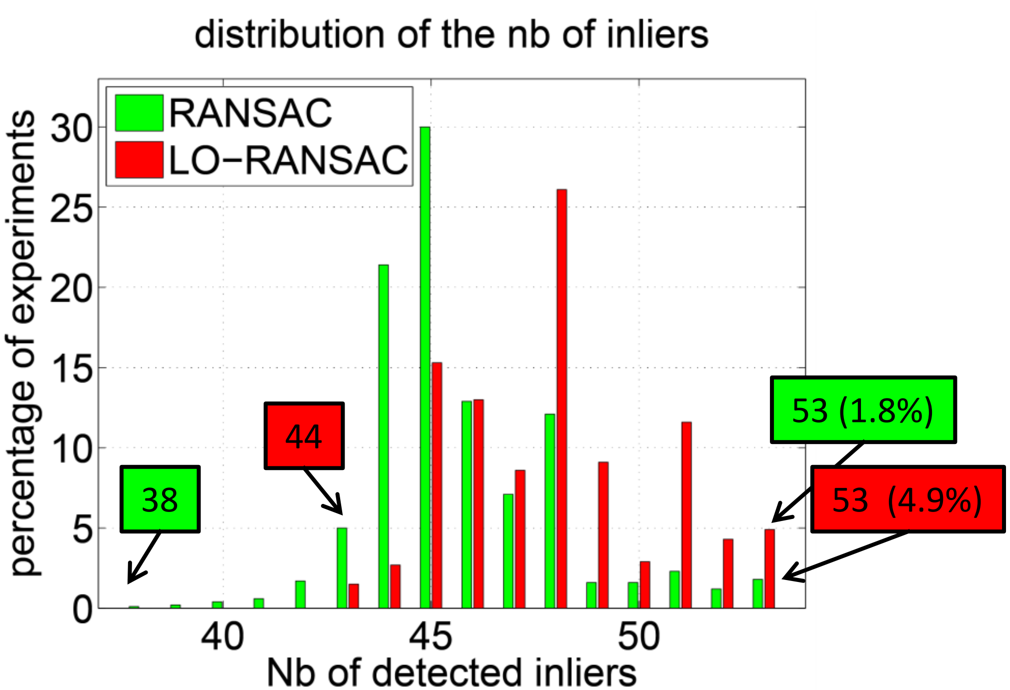

Convergence of bounds
Upper and lower bounds
20
40
60
80
100
1
2
3
4
5
6
Iterations
x 10^4
80
upper
lower

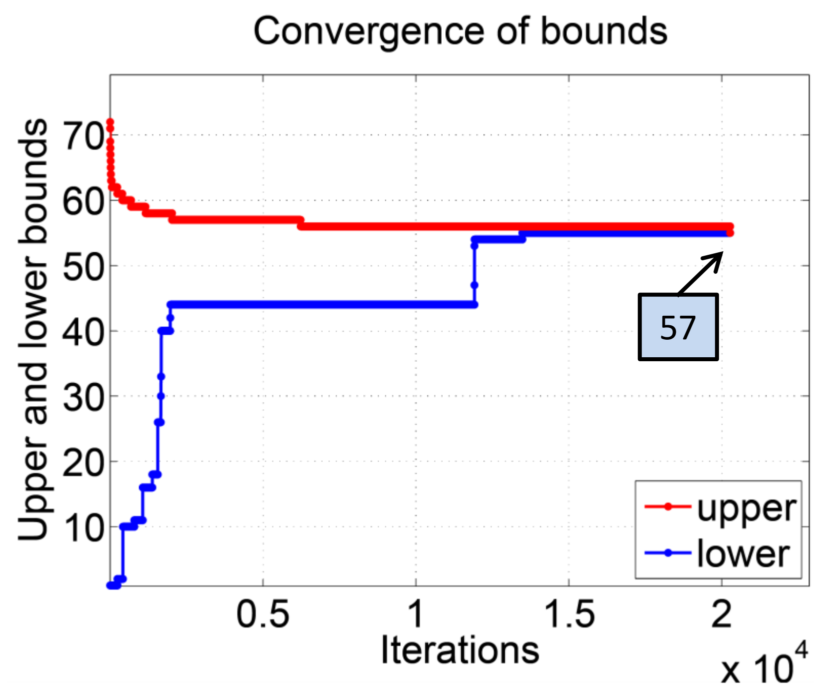


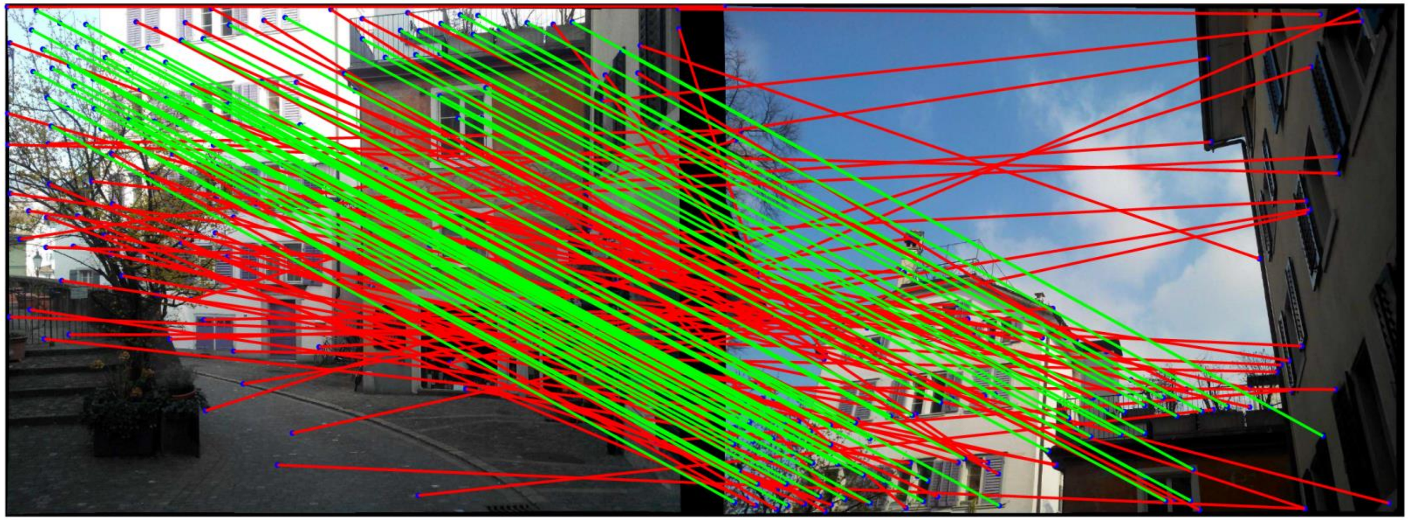
Results


distribution of the nb of inliers
RANSAC
LO−RANSAC
percentage of experiments
0
10
20
30
40
230
235
240
245
Nb of detected inliers
228
236
244 (2.9%)
244 (6.2%)

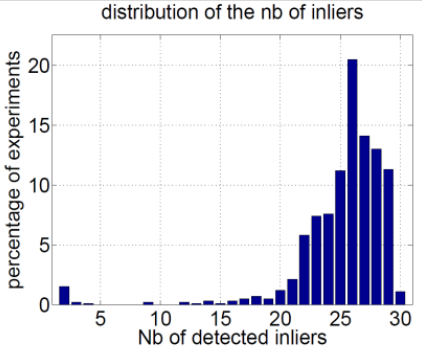

Convergence of bounds
Upper and lower bounds
50
100
150
200
250
2
4
6
8
10
Iterations
x 10^4
248
upper
lower


distribution of the nb of inliers
RANSAC
LO−RANSAC
percentage of experiments
Nb of detected inliers
0
5
10
15
20
25
30
40
45
50
38
44
53 (1.8%)
53 (4.9%)


Convergence of bounds
Upper and lower bounds
Iterations
x 10^4
0.5
1
1.5
2
10
20
30
40
50
60
70
upper
lower
57



Conclusion



- RANSAC works great…
  - But probabilistic behavior, not optimal, etc
- Maximize the number of inlier correspondences w.r.t. rotation and focal length in a **globally optimal** way
  - "globally optimal RANSAC" for uncalibrated infinite homography
  - For benchmark
  - For offline, post-processing/validation, and where quality/safety must be guaranteed