Western UNIVERSITY · CANADA

GE Healthcare

Pseudo-Bound Optimization for Binary Energies

Presenter: **Meng Tang**

Joint work with

Ismail Ben Ayed

Yuri Boykov

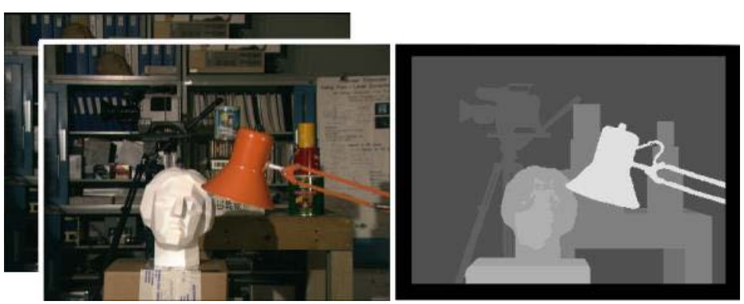
Labeling Problems in Computer Vision

Binary label

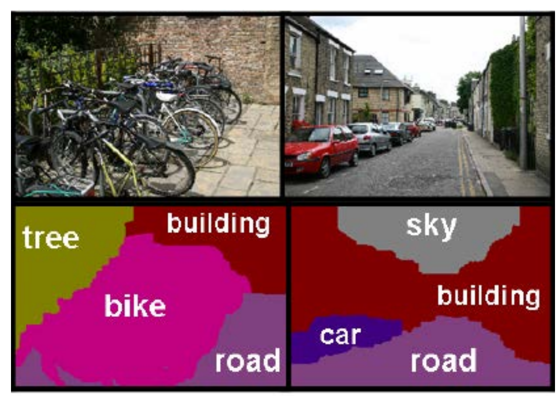
foreground selection

Multi-label

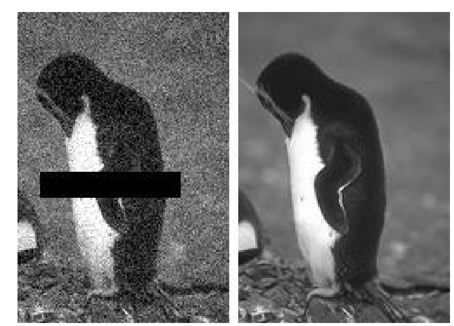
Geometric model fitting



Stereo



Semantic segmentation



Denoising inpainting

Energy Minimization for Labeling Problem

$$S^* = arg_S \min E(S)$$

s_p= 1 (FG) or 0 (BG)

foreground selection

s_p= 'sky' or 'road' or 'bike' etc.

Semantic segmentation

Basic Pairwise Energies

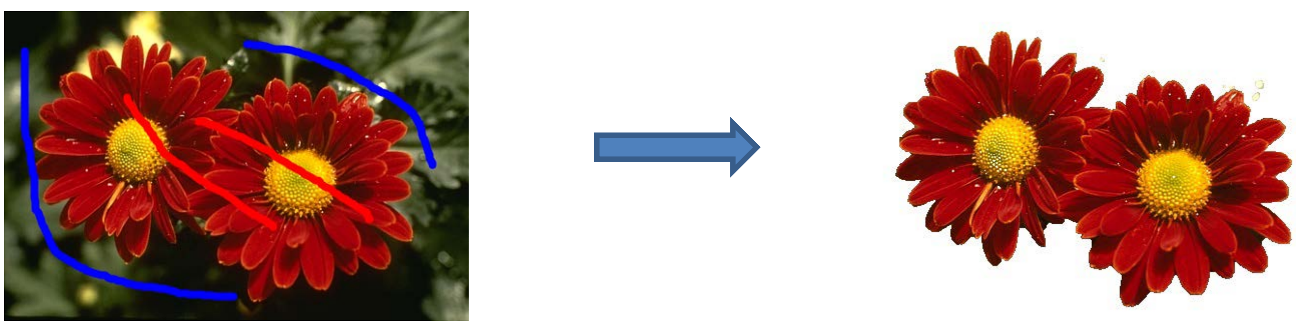
$S^* = arg_S \min E(S)$

- Common in computer vision

$$E(S) = \underbrace{\sum_{p \in \Omega} \varphi_p(s_p)}_{\text{Unary term}} + \underbrace{\sum_{pq \in N} \varphi_{pq}(s_p, s_q)}_{\text{Pairwise term}}$$

- *Submodular* case: fast global solver (Graph Cuts) e.g. Boros & Hammer. 2002

Example: interactive segmentation Boykov & Jolly. 2001



$$E(S \mid \theta_0, \theta_1) = -\sum_{p \in \Omega} \underbrace{\ln \Pr(I_p \mid \theta_{Sp})}_{\text{Unary term}} + \underbrace{\sum_{pq \in N} w_{pq} \cdot [s_p \neq s_q]}_{\text{Pairwise term}}$$

More difficult energies

Pairwise nonsubmodular energies

- ❑ Curvature regularization
- ❑ Segmentation with *repulsion*
- ❑ Binary image deconvolution
- ❑ e.t.c.

Roof duality [Boros & Hammer. 2002]
QPBO-mincut [Kolmogorov, Rother *et al*. 2007]
TRWS, SRMP [Kolmogorov *et al*. 2006, 2014]
Parallel ICM [Leordeanu *et al*. 2009]
....

High-order energies

- ❑ Entropy minimization for image segments
- ❑ Matching target distribution
- ❑ Volume constraints
- ❑ Convex shape prior

Region Competition[Zhu, Lee & Yuille. 1995]
GrabCut [Rother et al. 2004] [Vicente *et al*. 2009]
[Gould *et al*. 2011, 2012][Kohli *et al*. 2007, 2009]
[Ayed *et al*. 2010, 2013][Gorelick *et al*. 2013, 2014]
....

Our framework (Pseudo-Bound Opt.)

Pairwise nonsubmodular energies

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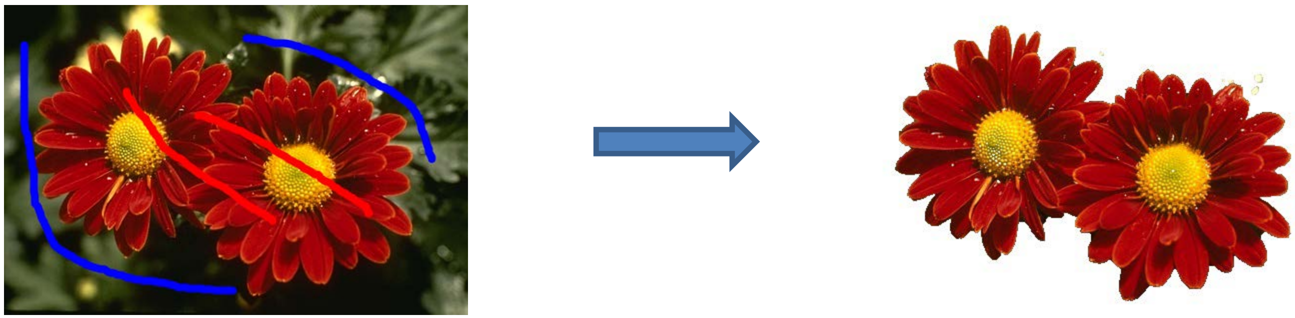
....

Example of high-order energy

- ❑ With known appearance models θ_0,θ_1.

Boykov & Jolly. 2001



$$E(S \mid \underbrace{\theta_0, \theta_1}_{\text{fixed}}) = \sum_{p \in \Omega} -\ln \Pr(I_p \mid \theta_{S_p}) + \sum_{pq \in N} w_{pq} \cdot [s_p \neq s_q]$$

- ❑ Appearance models can be optimized

GrabCut [Rother et al. 2004]

$$E(S, \underbrace{\theta_0, \theta_1}_{\text{variables}}) = \sum_{p \in \Omega} -\ln \Pr(I_p \mid \theta_{S_p}) + \sum_{pq \in N} w_{pq} \cdot [s_p \neq s_q]$$

From model fitting to **entropy** optimization:

[Delong *et al*, *IJCV* 2012] [Tang *et al*. *ICCV* 2013]

mixed optimization

$$E(S,\theta_0,\theta_1) = \sum_{p:S_p=0} -\ln \Pr(I_p|\theta_0) + \sum_{p:S_p=1} -\ln \Pr(I_p|\theta_1) + \sum_{pq\in N} w_{pq}\cdot[s_p \neq s_q]$$

$$|\bar{S}|\cdot H(\bar{S}|\theta_0) \qquad |S|\cdot H(S|\theta_1)$$

$\min\limits_{\theta_0,\theta_1}$

cross-entropy entropy

Note: $H(P|Q) \geq H(P)$ for any two distributions

entropy of intensities in $\bar{S}$ — entropy of intensities in S

binary optimization

$$E(S) = |\bar{S}|\cdot H(\bar{S}) + |S|\cdot H(S) + \sum_{pq\in N} w_{pq}\cdot[s_p \neq s_q]$$

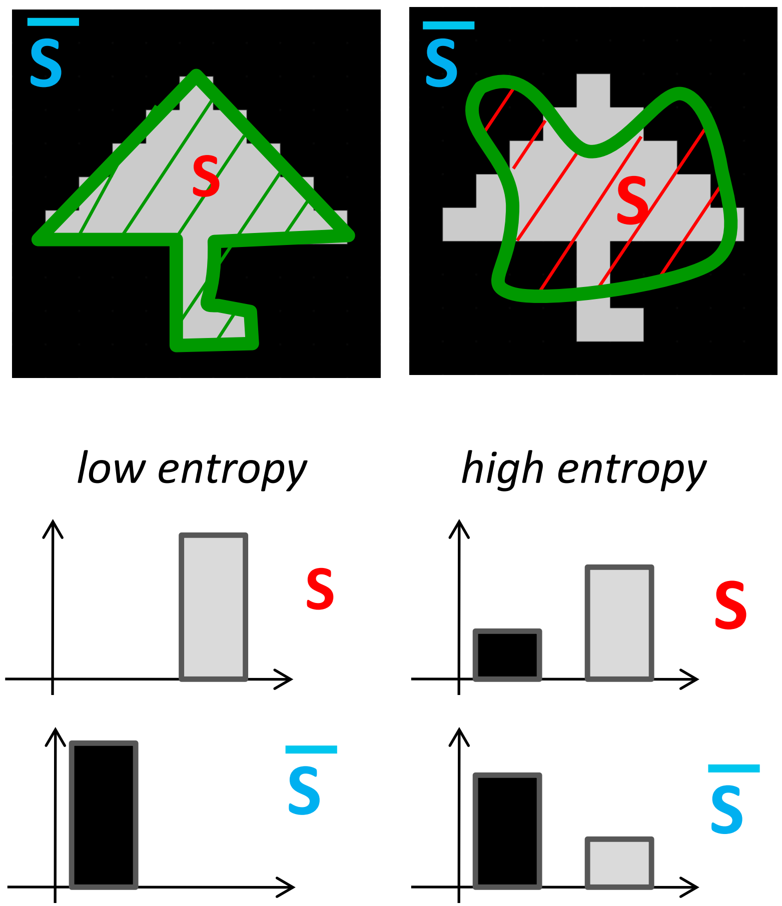
high-order energy

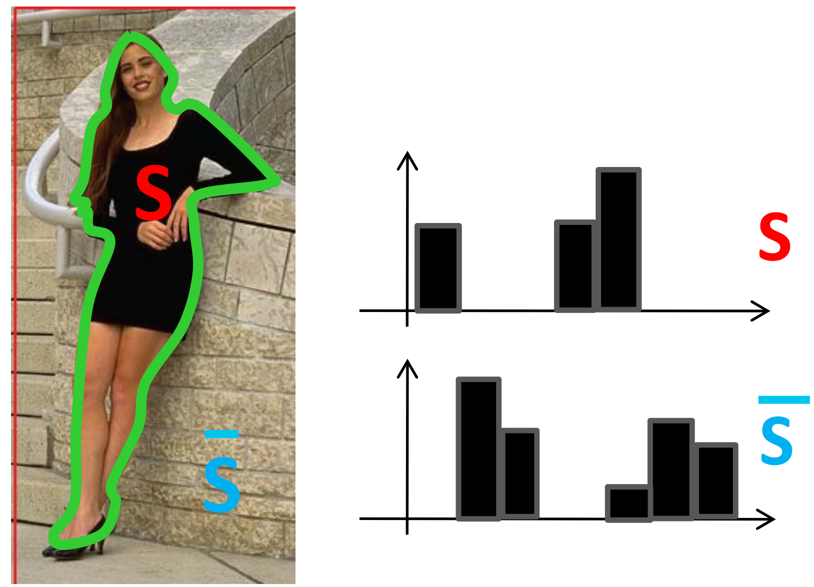
common energy for ***categorical clustering***, e.g. [Li *et al*. ICML'04]

Decision Forest Classification, e.g. [Criminisi & Shotton. 2013]

Energy example: *color entropy*

$$|\bar{S}|\cdot H(\bar{S}) + |S|\cdot H(S)$$





Pseudo-bound optimization example:

minimize our **entropy**-based energy E(S)

$$E(S) = |\bar{S}| \cdot H(\bar{S}) + |S| \cdot H(S) + \sum_{pq \in N} w_{pq} \cdot [s_p \neq s_q]$$

one standard approach:
Block-Coordinate Descent (BCD)

GrabCut [Rother et al. 2004]

$$E(S, \theta_0, \theta_1) \geq E(S)$$

mixed var. energy

our entropy energy

BCD could be seen as
bound optimization
for **entropy**

one standard approach: Block-Coordinate Descent (BCD)

GrabCut [Rother et al. 2004]

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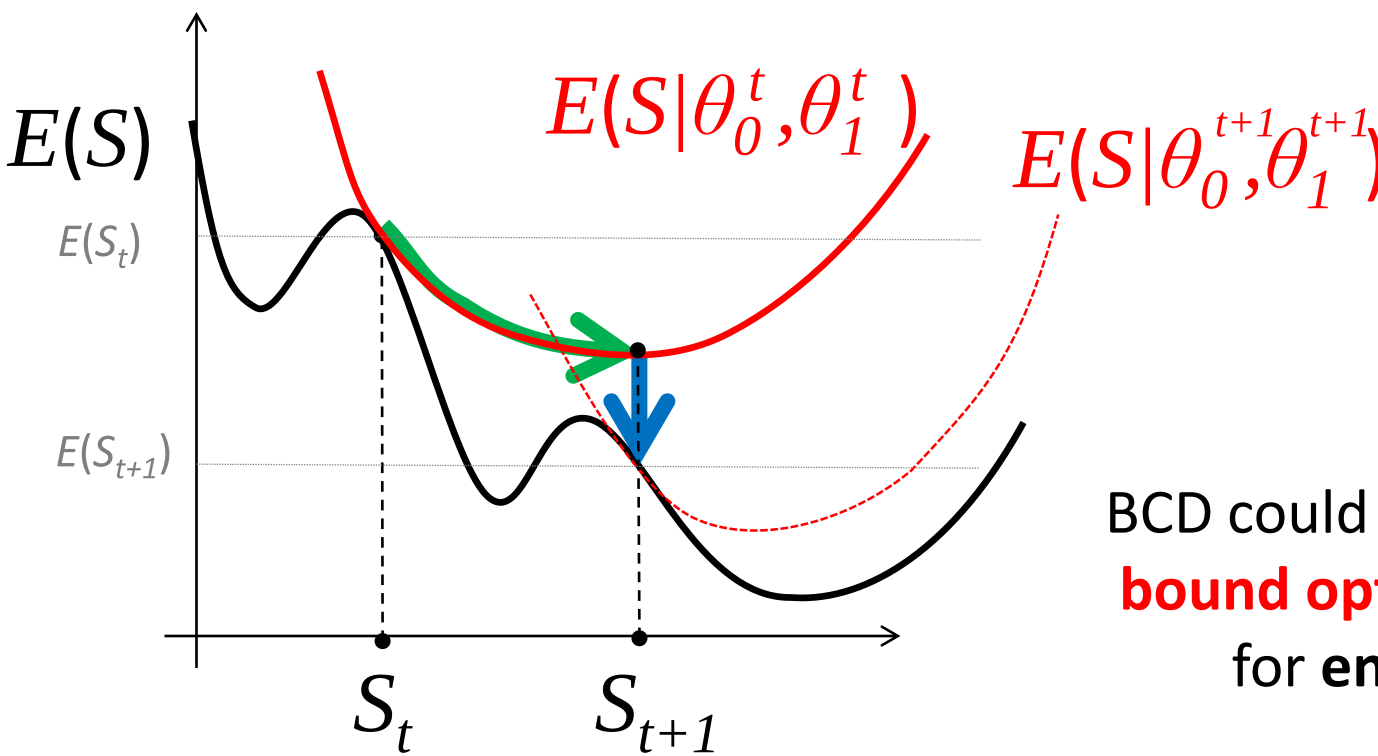
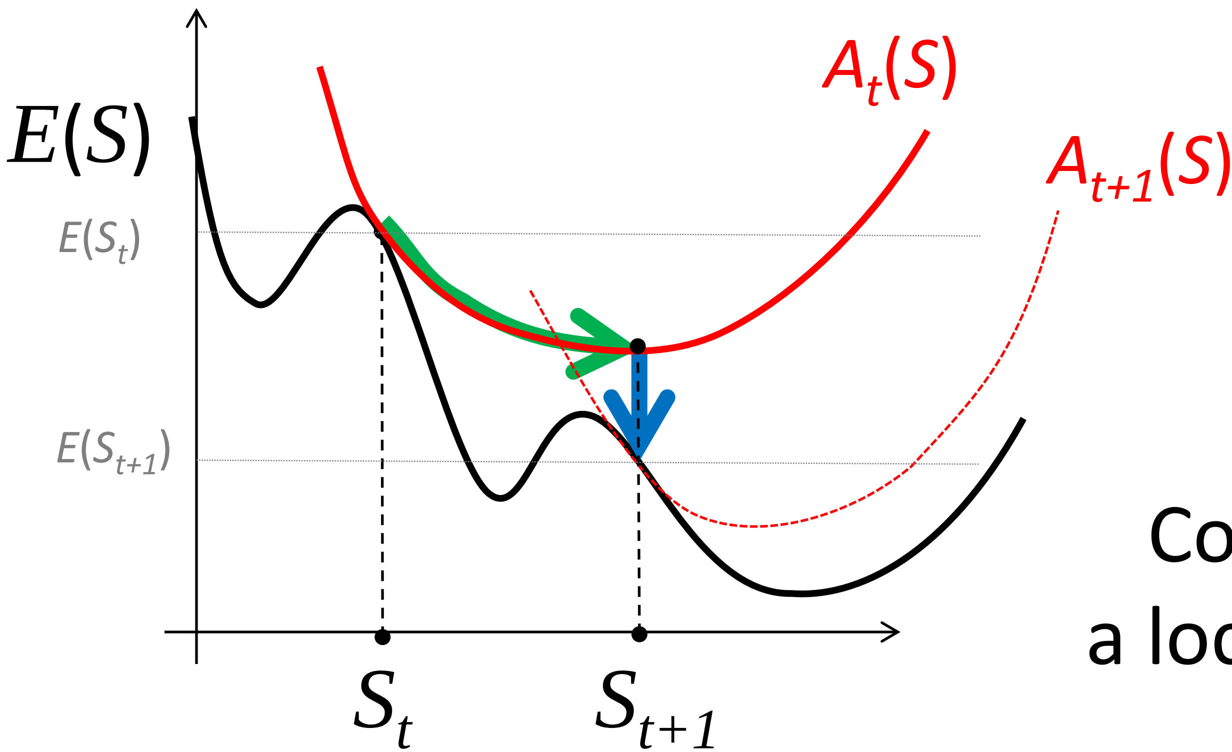
mixed var. energy — our entropy energy



BCD could be seen as **bound optimization** for **entropy**

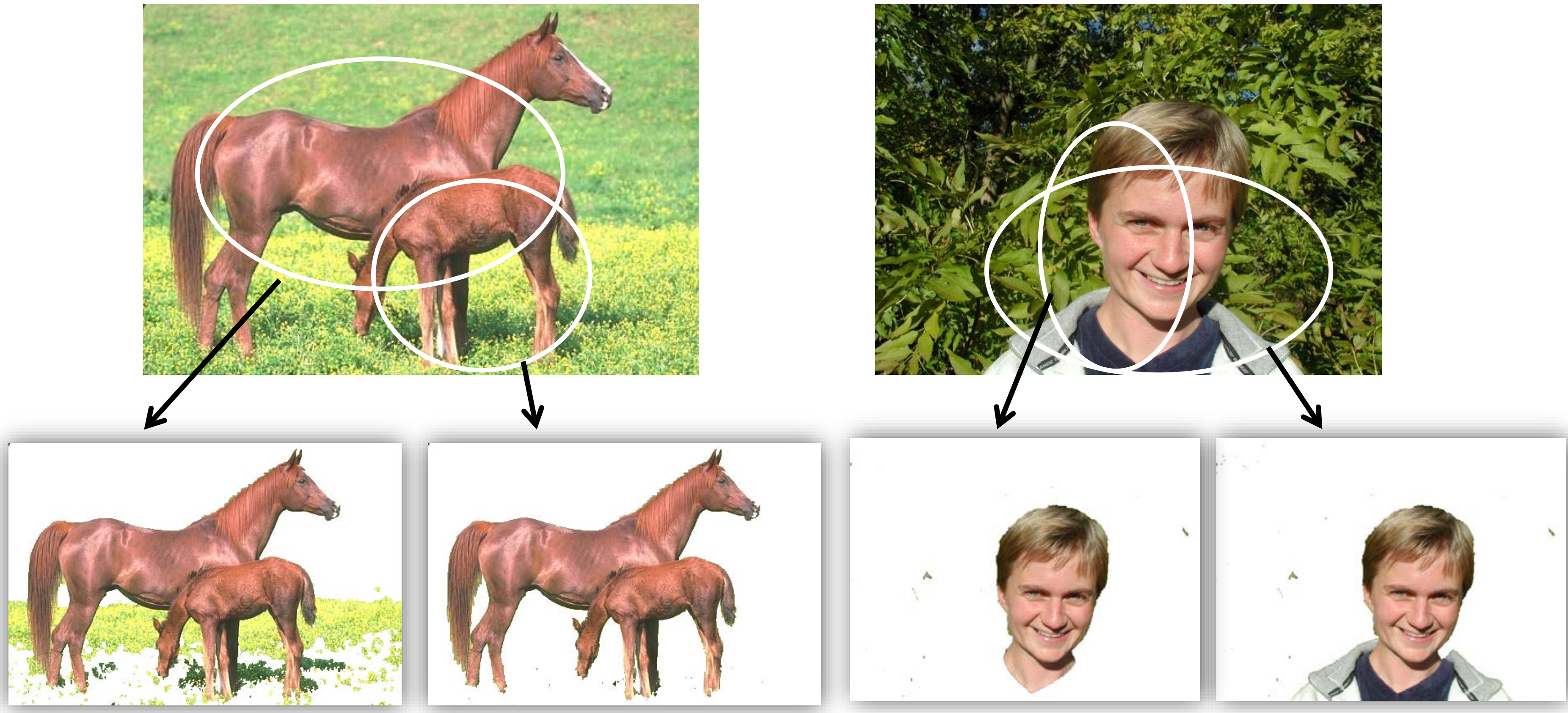
Bound optimization, in general

(Majorize-Minimize, Auxiliary Function, Surrogate Function)



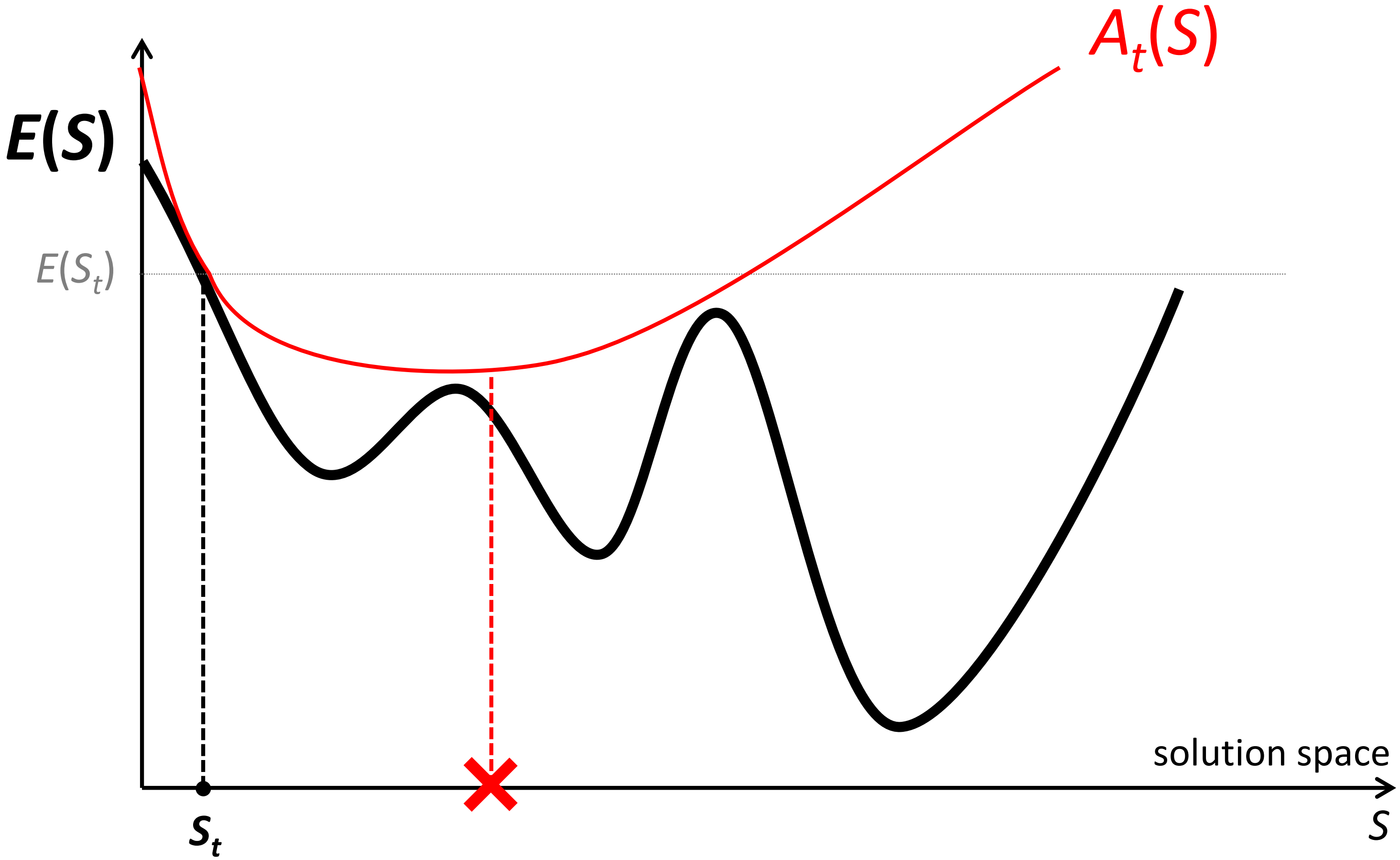
Converges to
a local minimum

Local minima examples (for GrabCut)



E=1.410×10^{6} E=1.39×10^{6} E=2.41×10^{6} E=2.37×10^{6}

This work: Pseudo-Bound Optimization



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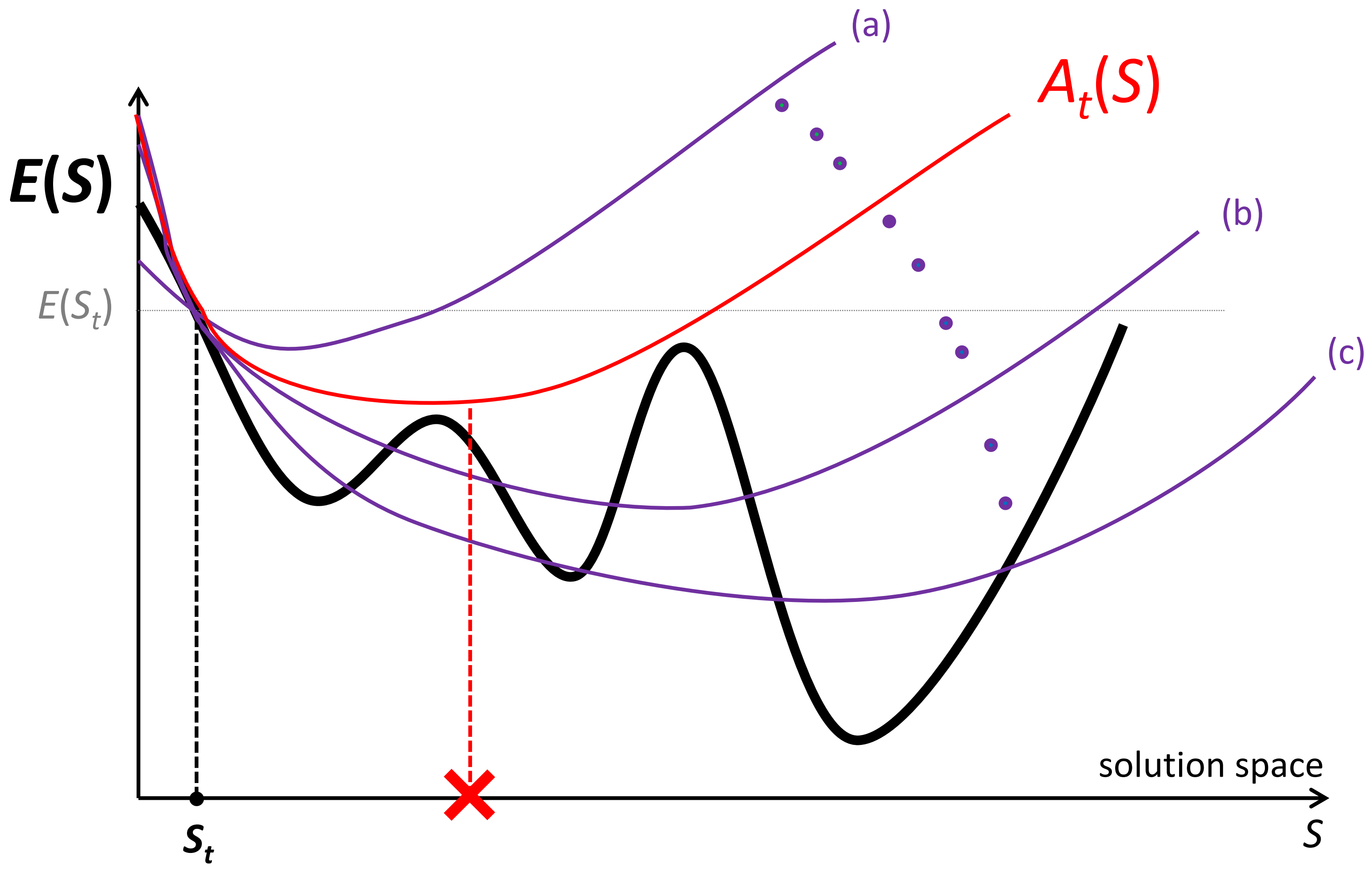

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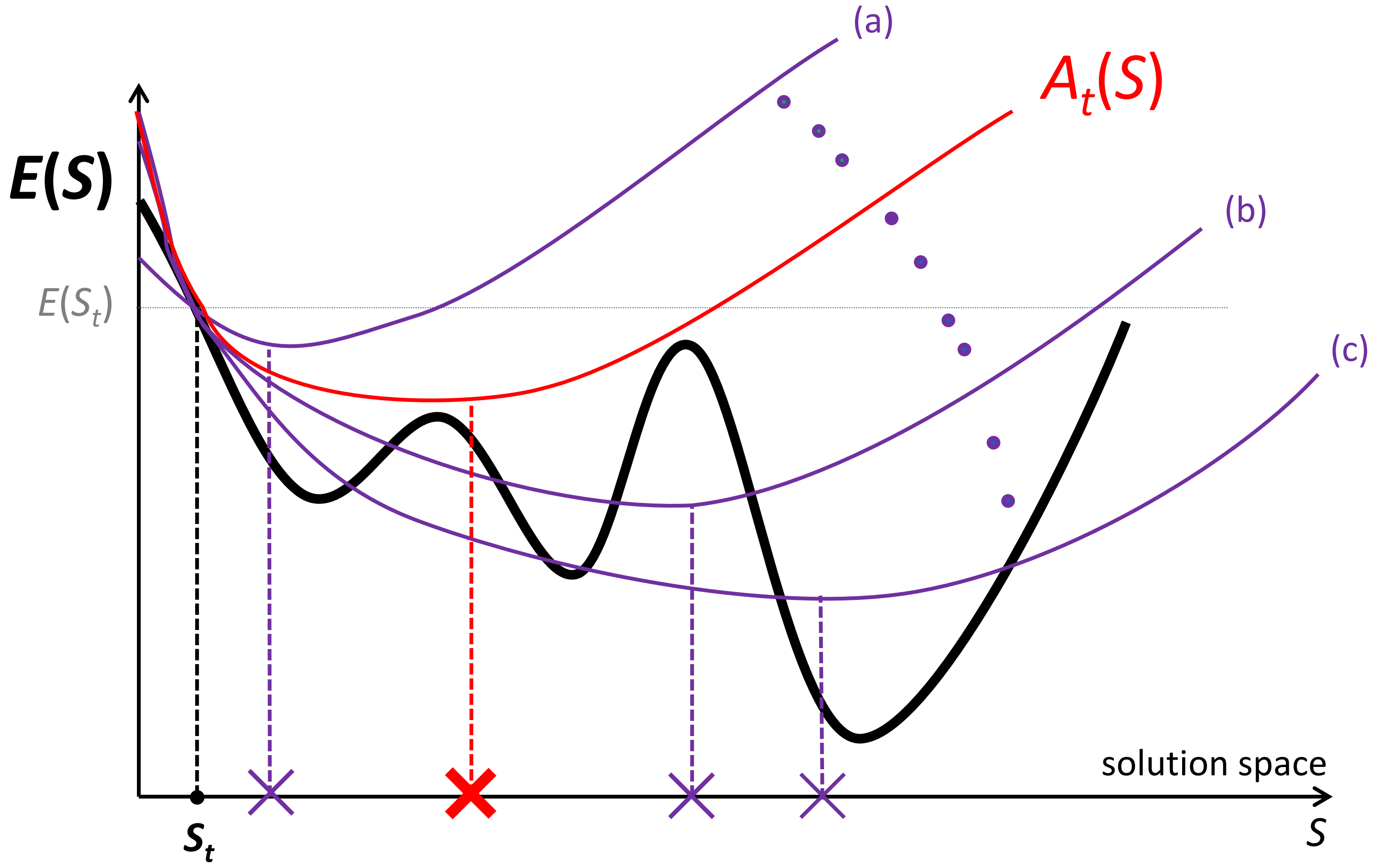

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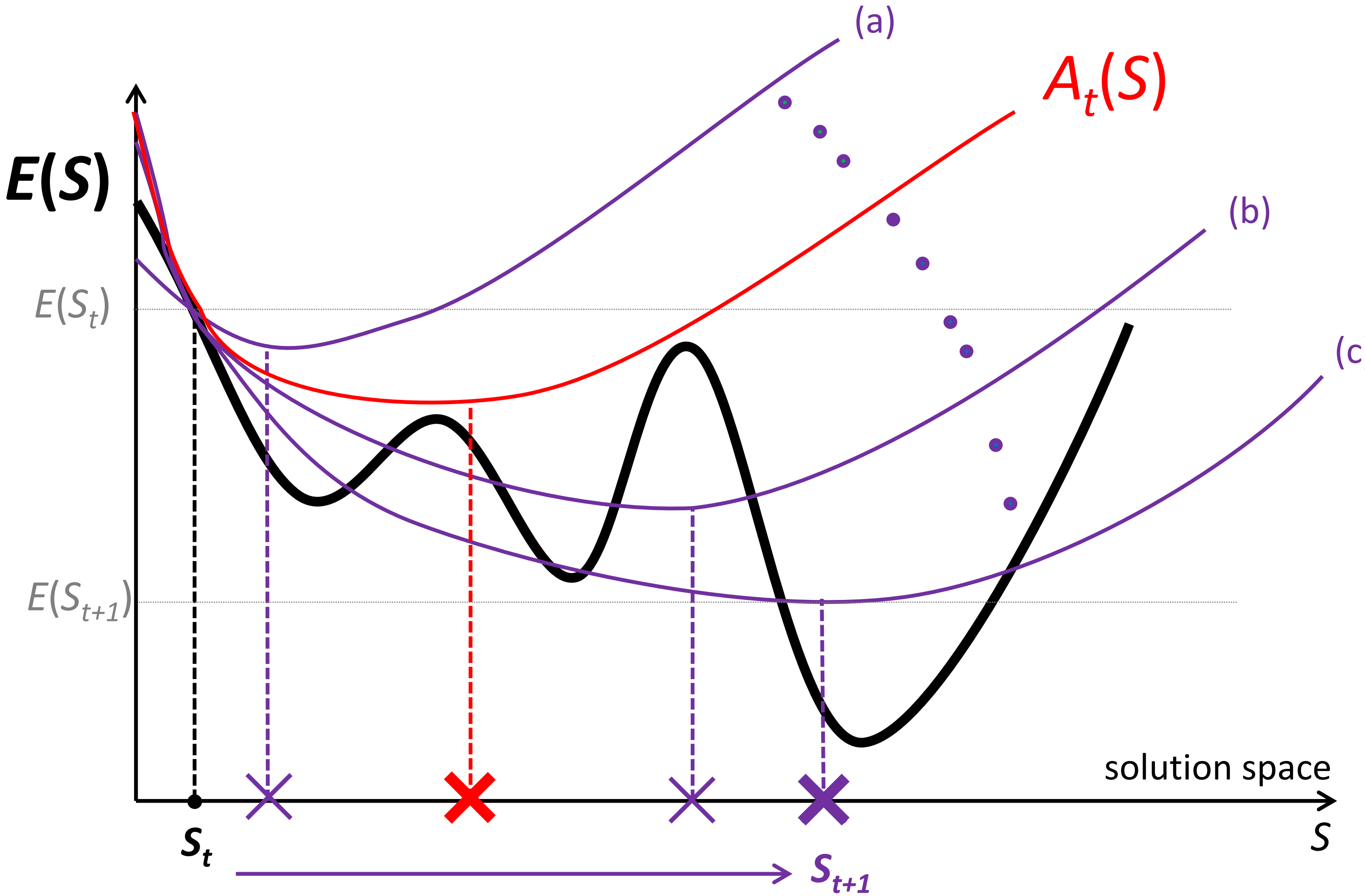

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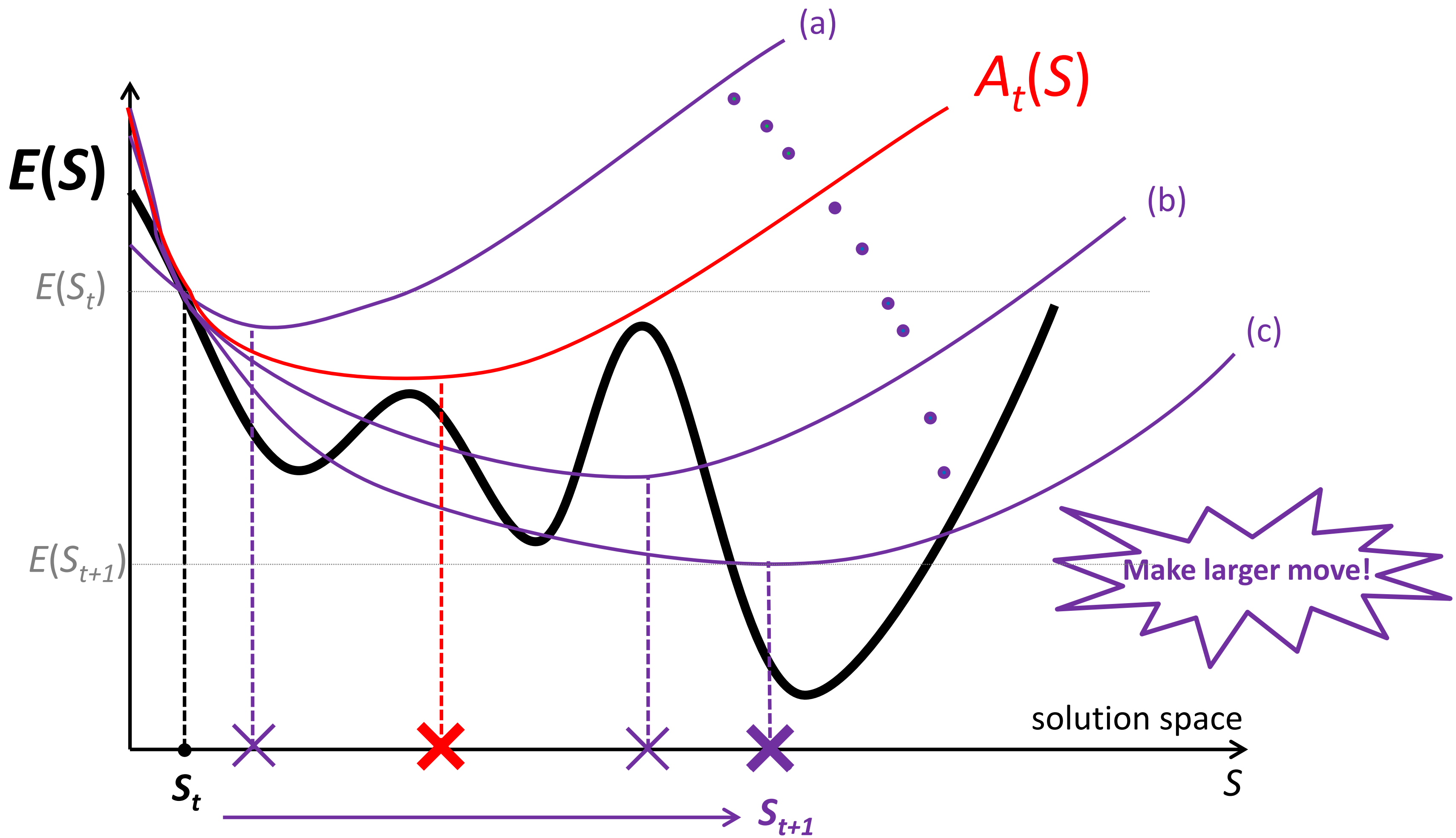
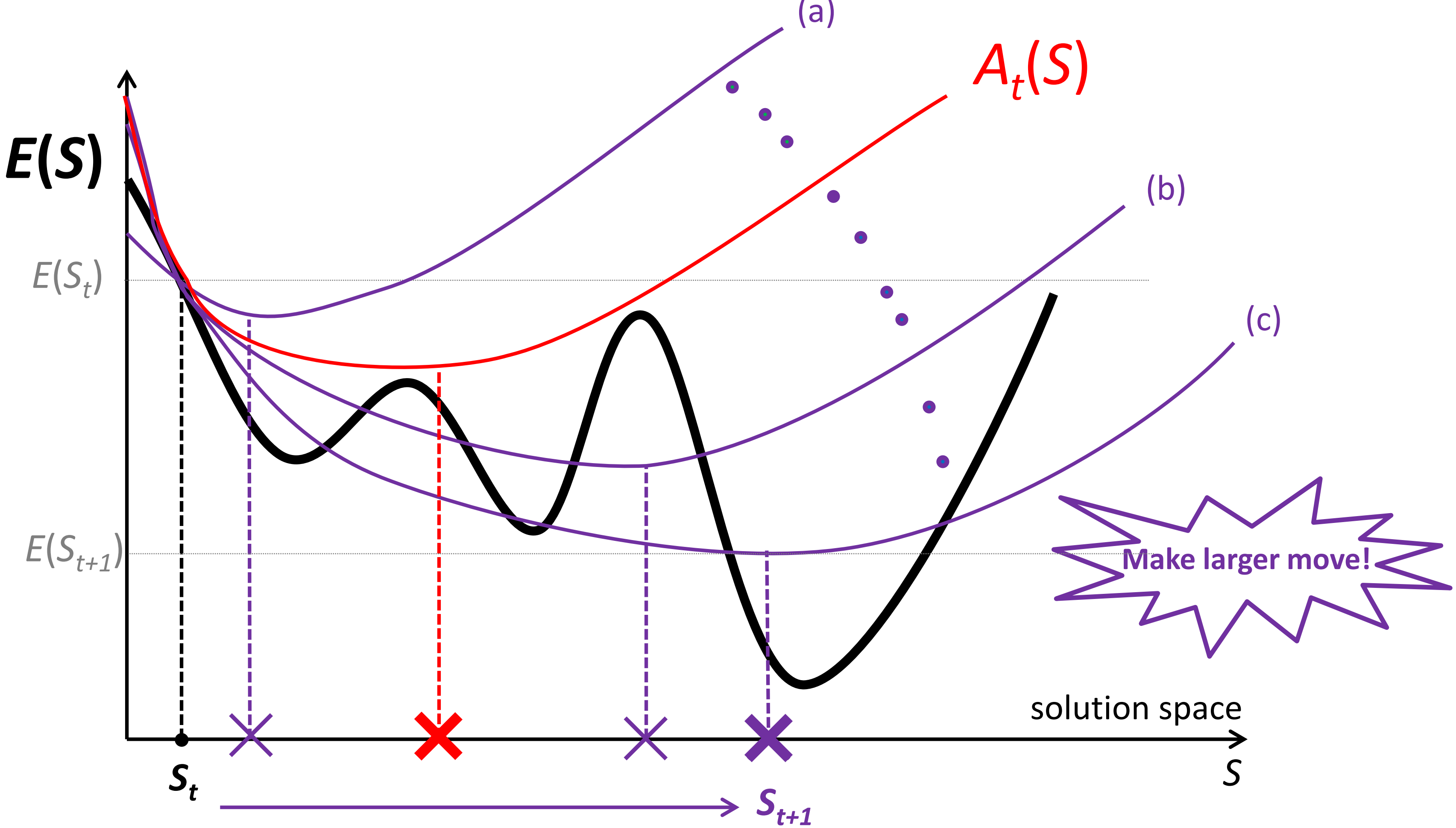


This work: Pseudo-Bound Optimization

General framework for energy-minimization



General form of Pseudo-Bounds

General form of Pseudo-Bounds

Bound

$$A_t(S)$$

Bounds

General form of Pseudo-Bounds

Bound Bounding relaxation

$$F_t(S,\lambda) = A_t(S) + \lambda\, R_t(S)$$

Parametric Pseudo-Bounds

General form of Pseudo-Bounds

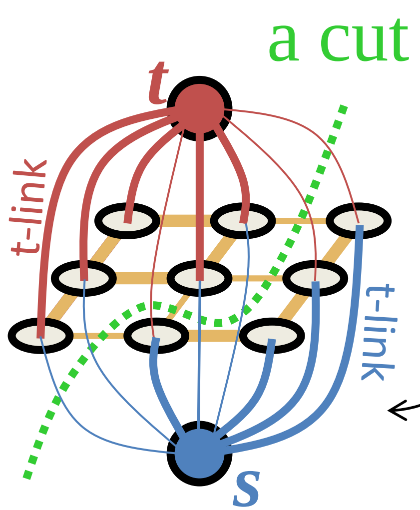
$$F_t(S,\lambda) = \underbrace{A_t(S)}_{\text{Pairwise Submodular}}^{\text{Bound}} + \lambda \underbrace{R_t(S)}_{\text{Unary}}^{\text{Bounding relaxation}}$$

Parametric Pseudo-Bounds

General form of Pseudo-Bounds

Bound — Bounding relaxation

$$F_t(S,\lambda) = \underbrace{A_t(S)}_{\textit{Pairwise Submodular}} + \lambda \underbrace{R_t(S)}_{\textit{Unary}}$$



edge capacities depend linearly on λ.

Parametric Max-flow

Gallo et al. 1989

Hochbaum et al. 2010

Kolmogorov et al. 2007

Parametric Pseudo-Bounds

General form of Pseudo-Bounds

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Parametric Pseudo- Bounds

General form of Pseudo-Bounds

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$$S^{\lambda} = \min_S F_t(S,\lambda)$$

Parametric Pseudo-Bounds

General form of Pseudo-Bounds

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$$S^\lambda = \min_s F_t(S,\lambda) \xrightarrow{\lambda \in [-\infty, +\infty]} \text{Parametric Maxflow}$$

Parametric Pseudo-Bounds Cuts

General form of Pseudo-Bounds

Bound — Bounding relaxation

$$F_t(S,\lambda) = A_t(S) + \lambda\, R_t(S)$$

Pairwise Submodular — *Unary*

$\lambda \in [-\infty, +\infty]$

$$S^{\lambda} = \min_s F_t(S,\lambda)$$ → Parametric Maxflow

$$S_{t+1} = \operatorname{argmin}_{S^{\lambda}} E(S^{\lambda})$$ → Update

Parametric Pseudo-Bounds Cuts (pPBC)

General form of Pseudo-Bounds

Bound — Bounding relaxation

$$F_t(S,\lambda) = \underbrace{A_t(S)}_{\text{Pairwise Submodular}} + \lambda \underbrace{R_t(S)}_{\text{Unary}}$$

Iterate

$$\begin{cases} S^\lambda = \min_S F_t(S,\lambda) \\ S_{t+1} = \arg\min_{S^\lambda} E(S^\lambda) \end{cases}$$

$\lambda \in [-\infty, +\infty]$ → Parametric Maxflow

→ Update

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Parametric Pseudo-Bounds Cuts (pPBC)

Specific pseudo-bounds Example 1

$$F_t(S,\lambda) = \underbrace{A_t(S)}_{\text{Pairwise Submodular}} + \lambda \underbrace{R_t(S)}_{\text{Unary}}$$

How to choose auxiliary and bound relaxation functions?

$$E(S) = |\bar{S}| \cdot H(\bar{S}) + |S| \cdot H(S) + \sum_{pq \in N} w_{pq} \cdot [s_p \neq s_q]$$

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$$F_t(S,\lambda) = E(S|\theta_0^t, \theta_1^t) + \lambda(|S| - |S_t|)$$

Specific pseudo-bounds Example 2

$$F_t(S,\lambda) = \underbrace{A_t(S)}_{\text{Pairwise Submodular}} + \lambda \underbrace{R_t(S)}_{\text{Unary}}$$

How to choose auxiliary and bound relaxation functions?

Specific pseudo-bounds Example 2

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How to choose auxiliary and bound relaxation functions?

High-order example:

Soft volume constriants

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High-order example:

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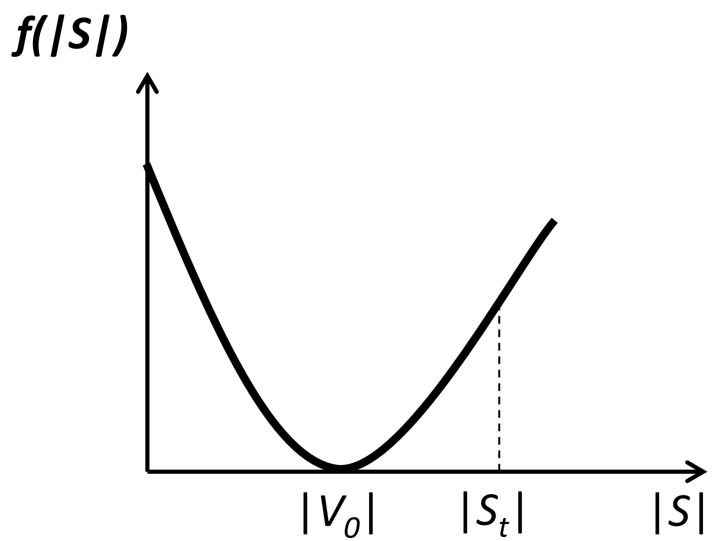

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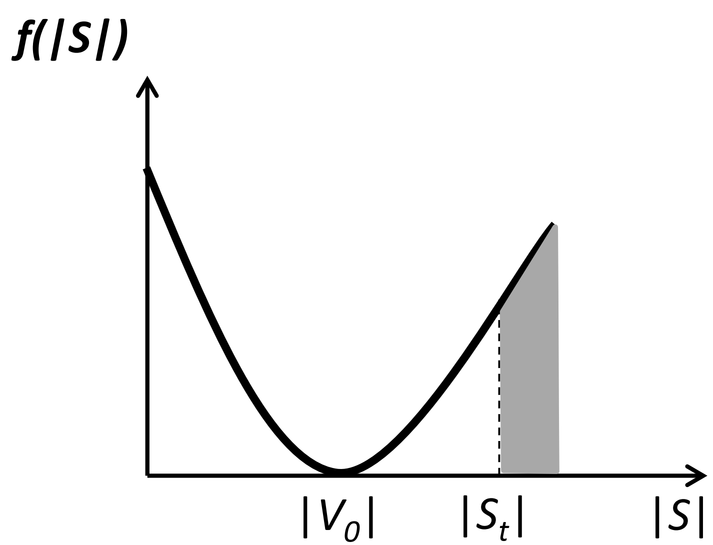

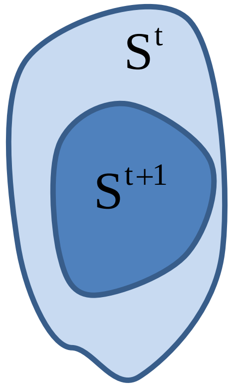

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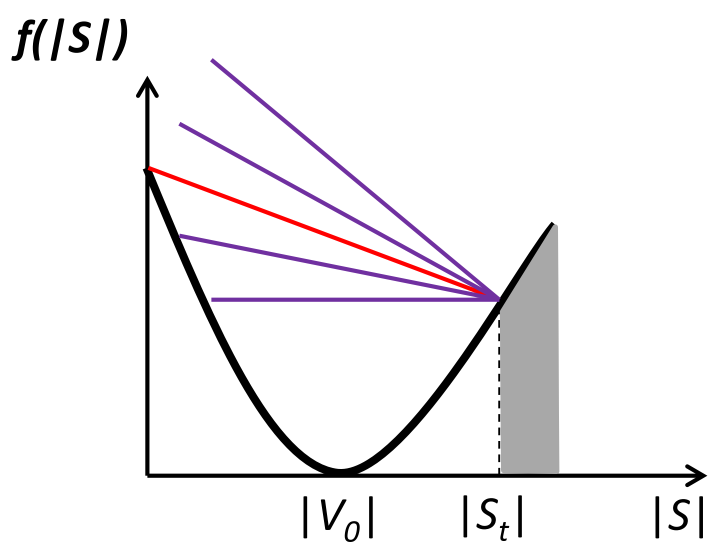
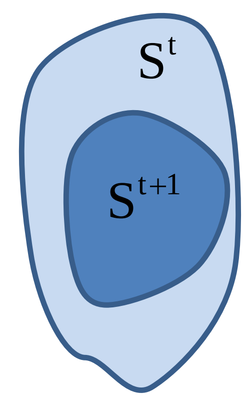




Specific pseudo-bounds Example 3

$$\underbrace{F_t(S,\lambda)}\; = \underbrace{A_t(S)}_{\textit{Pairwise Submodular}} + \lambda \underbrace{R_t(S)}_{\textit{Unary}}$$

How to choose auxiliary and bound relaxation functions?

Specific pseudo-bounds Example 3

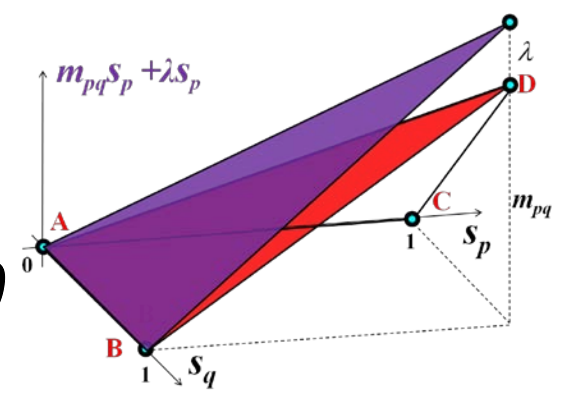
$$F_t(S,\lambda) = \underbrace{A_t(S)}_{\textit{Pairwise Submodular}} + \lambda \underbrace{R_t(S)}_{\textit{Unary}}$$

How to choose auxiliary and bound relaxation functions?

Pairwise example:

Nonsubmodluar pairwise

$$m_{pq}s_p s_q,\ for\ m_{pq}>0$$

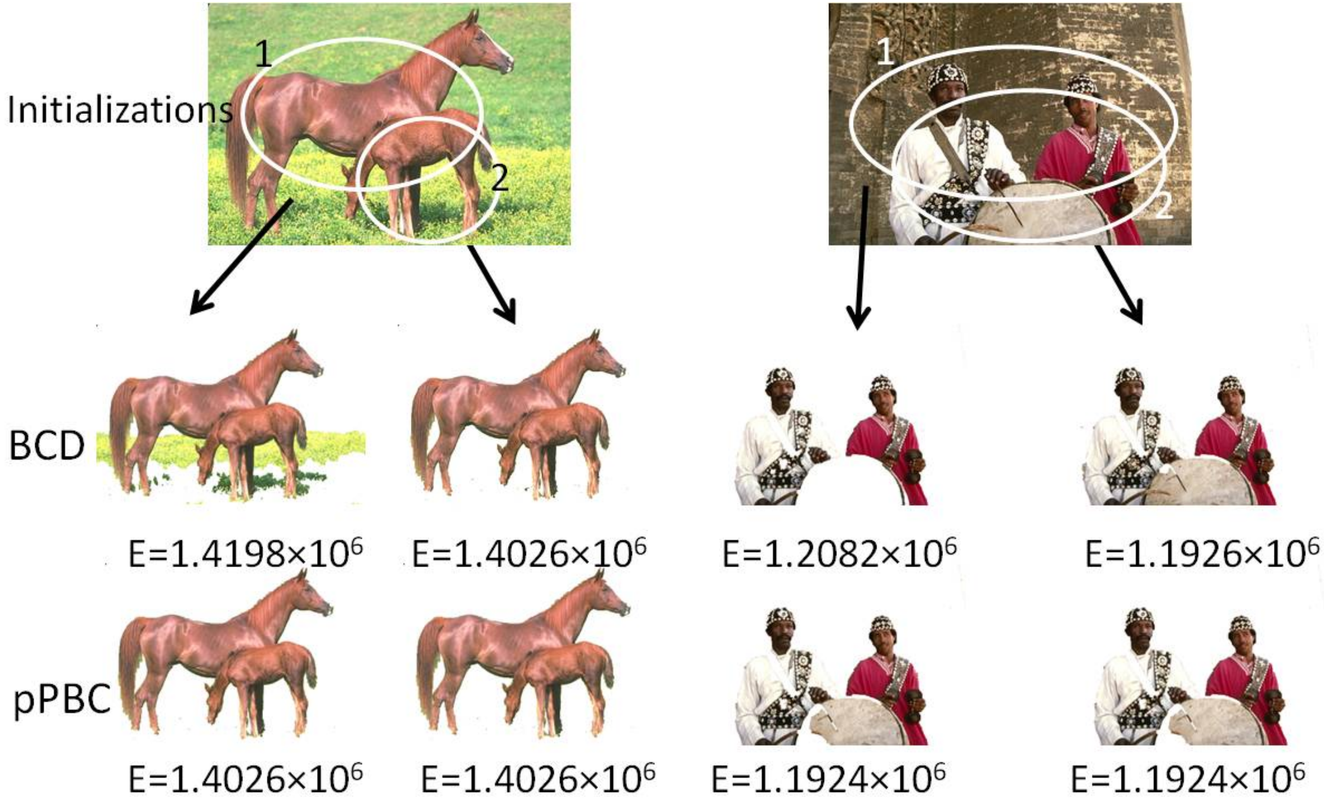


Gorelick et al. in *CVPR* 2014

EXPERIMENTAL RESULTS

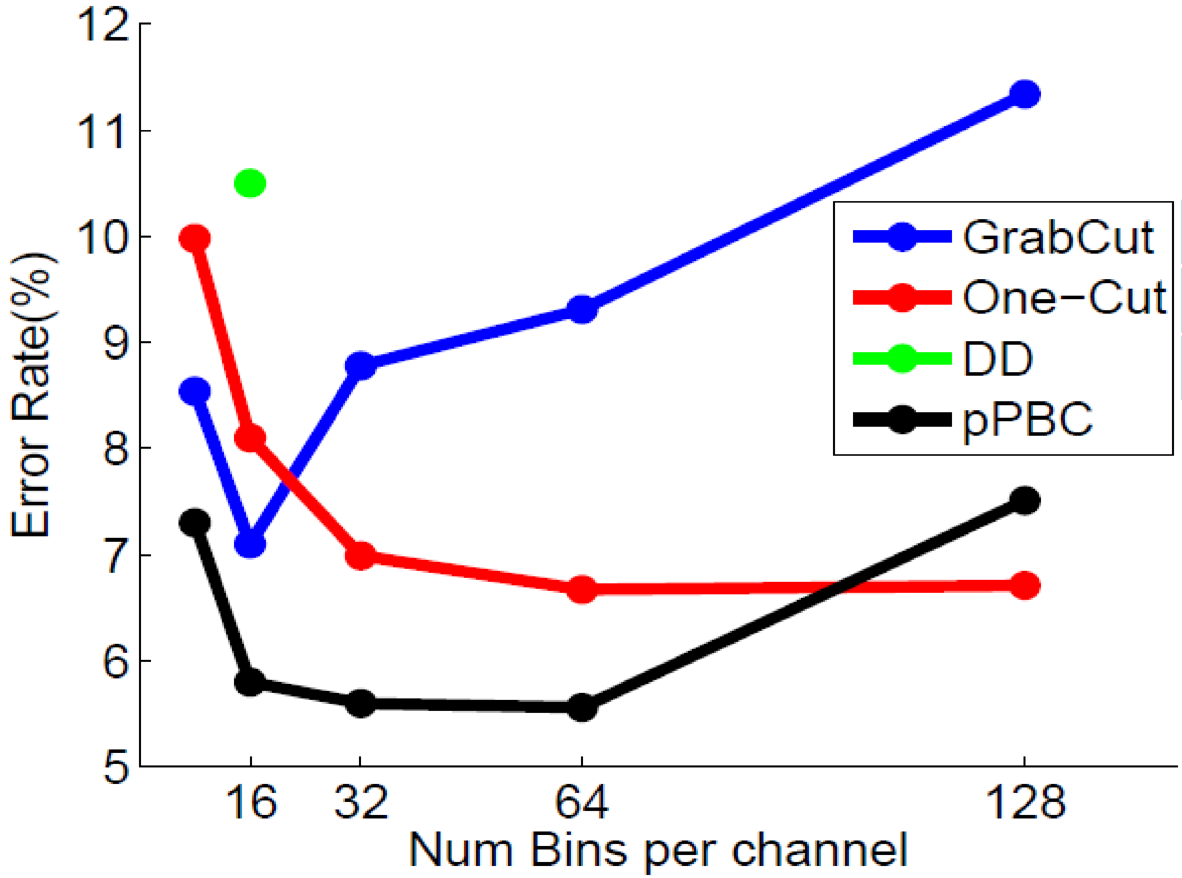
Experiment results (high-order)

Interactive segmentation (entropy minimization)



Experiment results (high-order)

Interactive segmentation (GrabCut database)



Rother et al. 2004

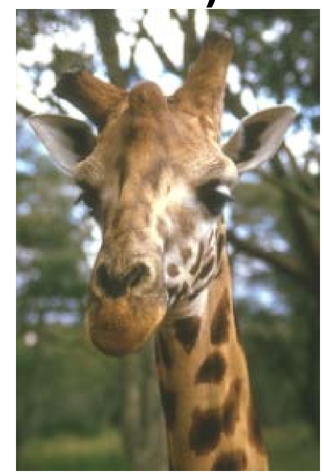
Tang et al. 2013

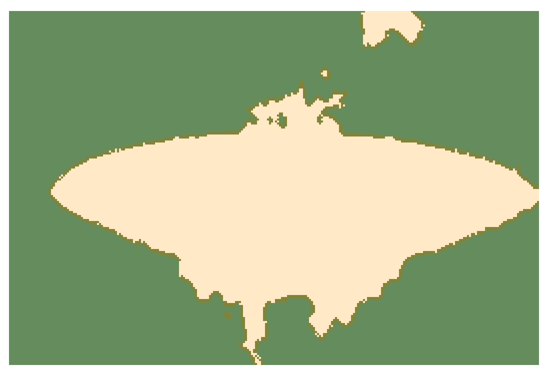
Vicente et al. 2009

Experiment results (high-order)

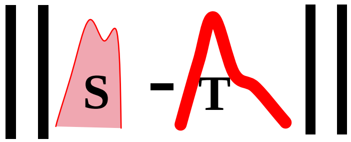
Unsupervised binary segmentation

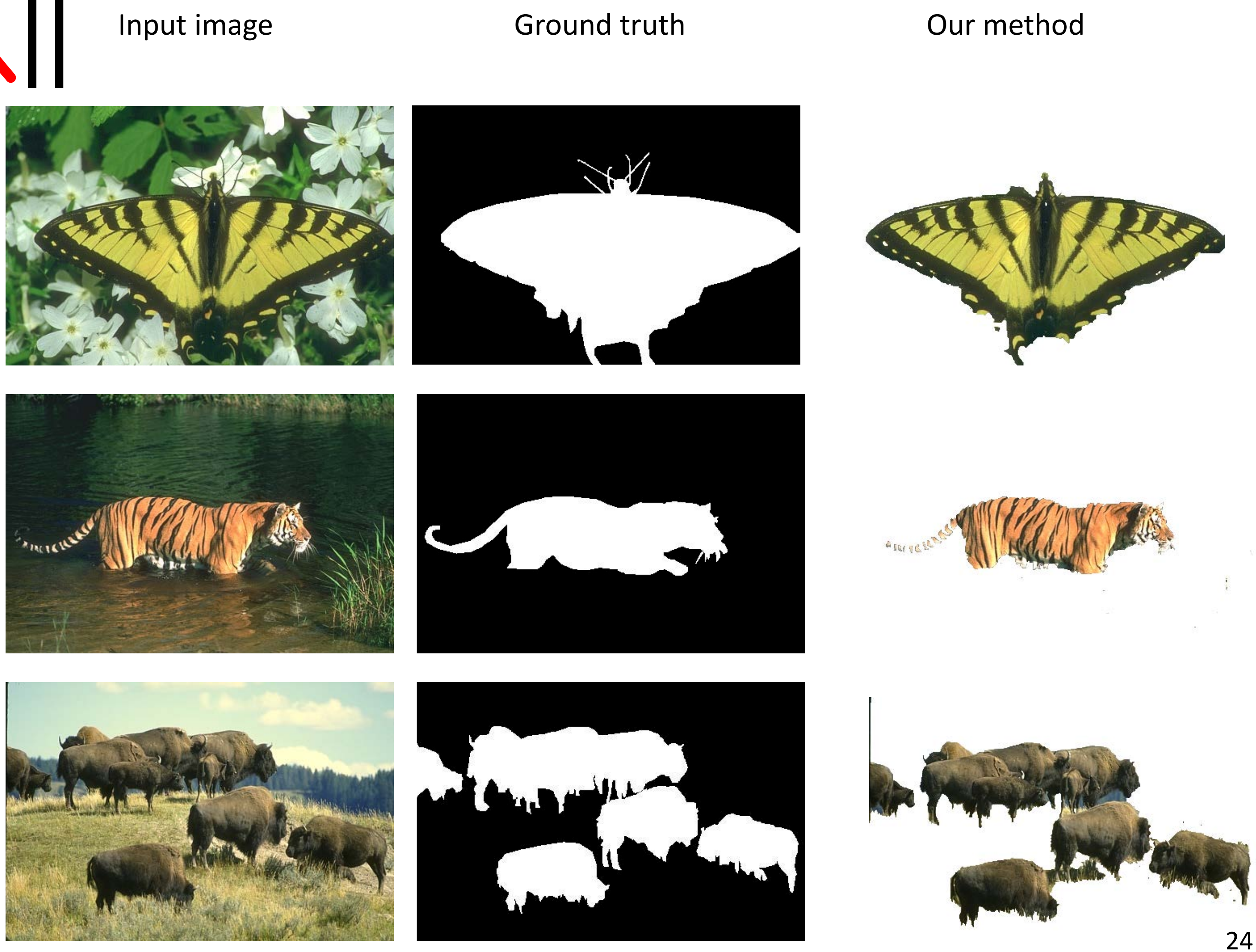
– without prior (bounding box, appearance etc.)





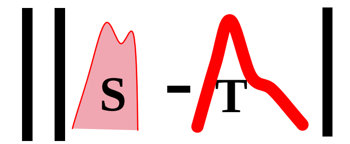
Matching appearance distribution

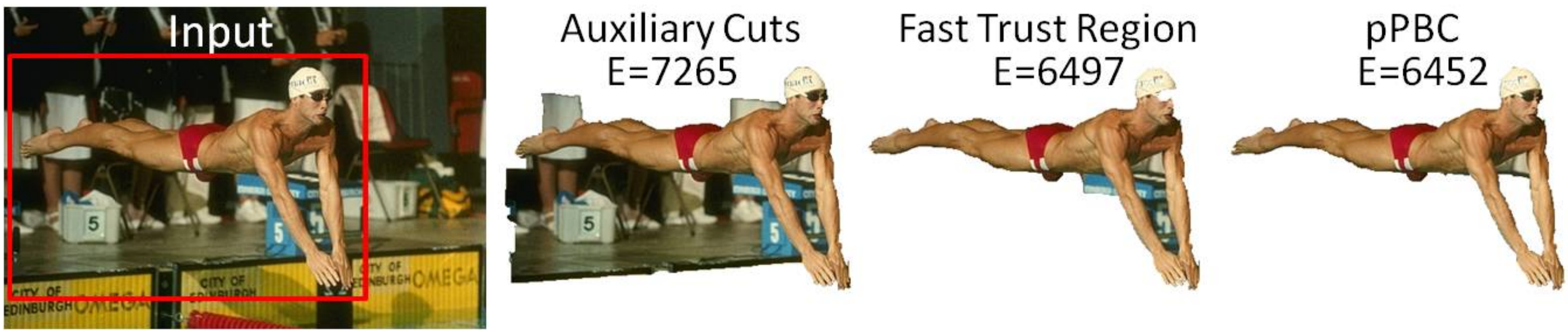




Experiment results (high-order)

Matching color distribution

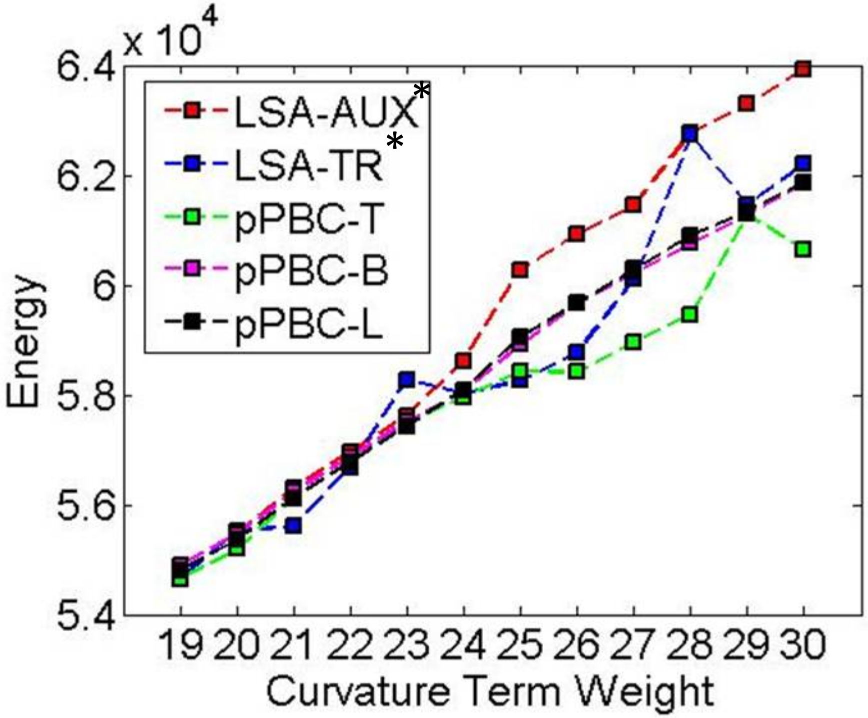


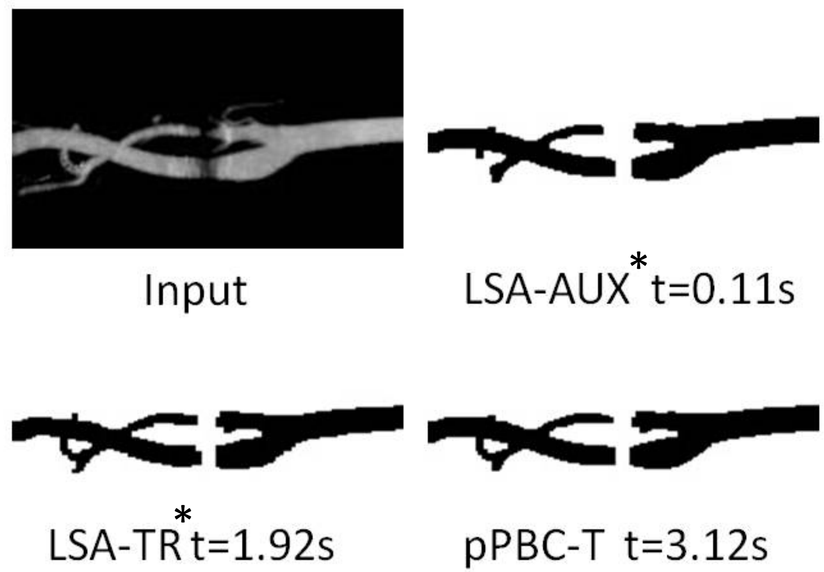


		KL divergence			Bhattacharyya distance		
	Method	Mean energy	Mean error	Time	Mean energy	Mean error	Time
Ayed et al. 2013	Auxiliary Cuts	6189	16.54%	**1.8s**	-12402	24.1%	**1.7s**
	pPBC($\lambda \leq 0$)	6150	14.88%	N/A	-12451	23.7%	N/A
Gorelick et al. 2013	FTR	5868	7.70%	4.40s	-14499	3.2%	2.71s
	pPBC($\lambda \in [-\infty, +\infty]$)	**5849**	**3.63%**	2.98s	**-14504**	**2.9%**	1.99s

Experiment results (pairwise)

Segmentation with curvature regularization





*Submodularization for Binary Pairwise Energies, Gorelick et al. in *CVPR* 2014

Conclusion

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- ❑ Several options of pseudo-bounds
- ❑ Achieve state-of-the-art for many binary energy minimization problems

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