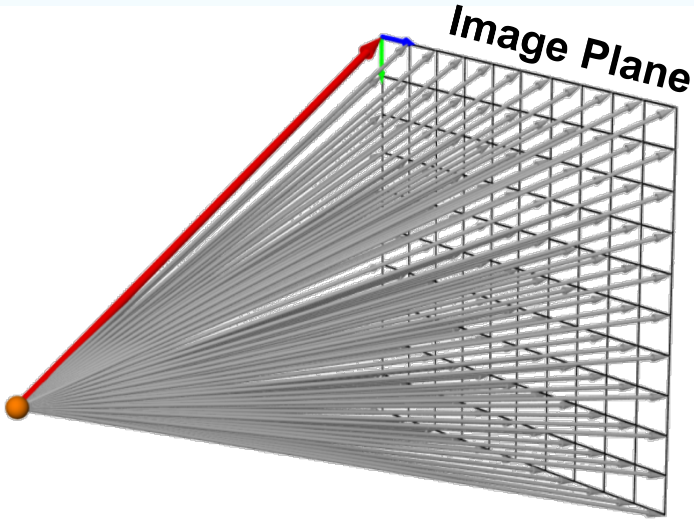
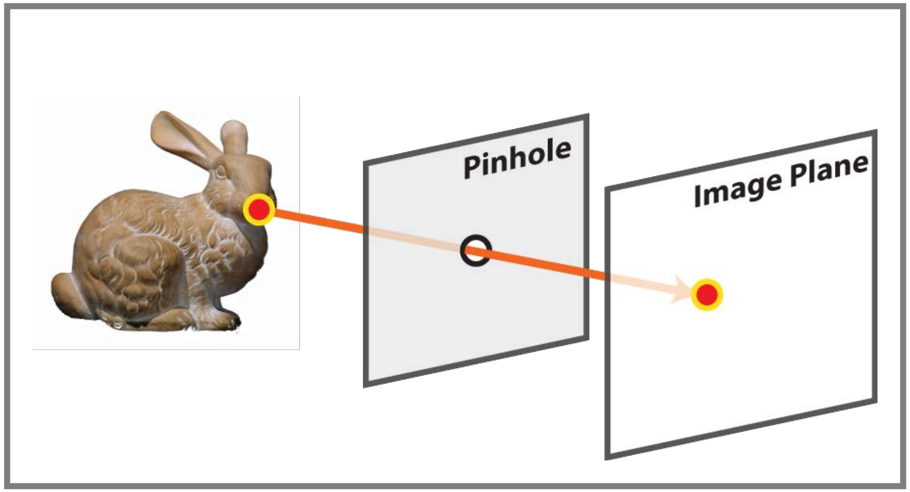
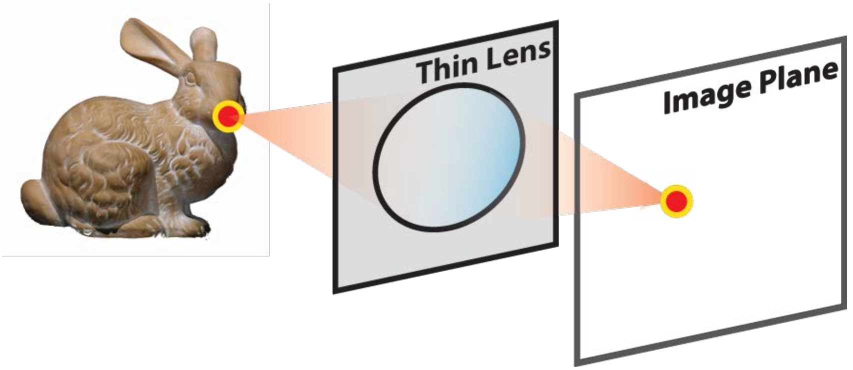
Depth-of-Field Analysis and Coded Aperture Imaging on XSlit Cameras

Jinwei Ye Yu Ji Wei Yang Jingyi Yu
University of Delaware

Pinhole Cameras

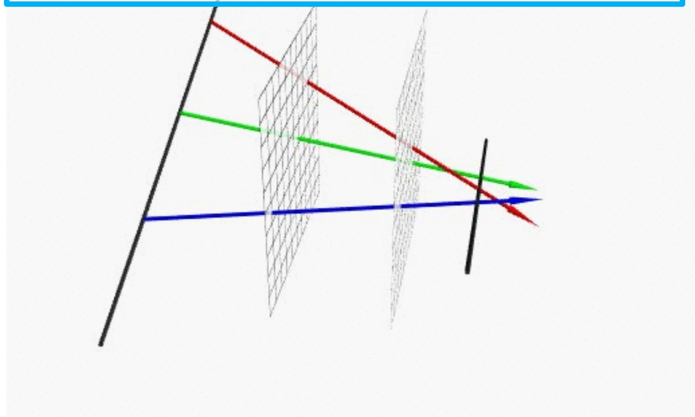


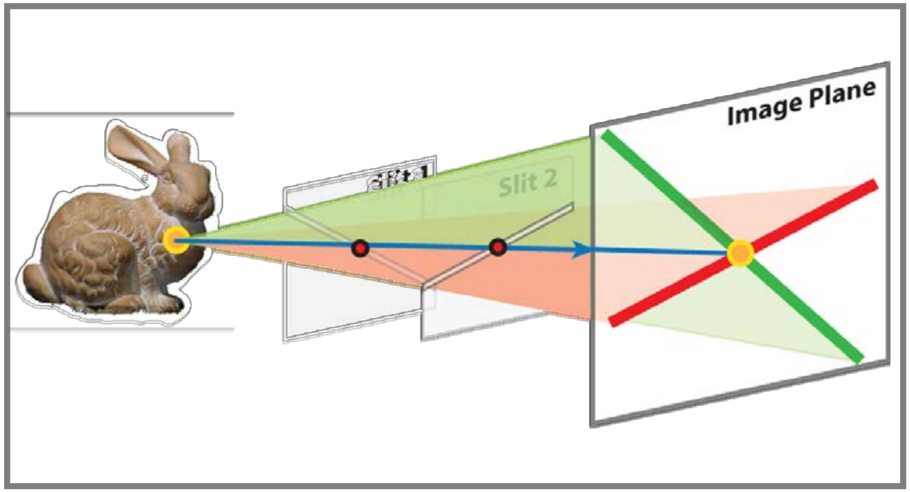


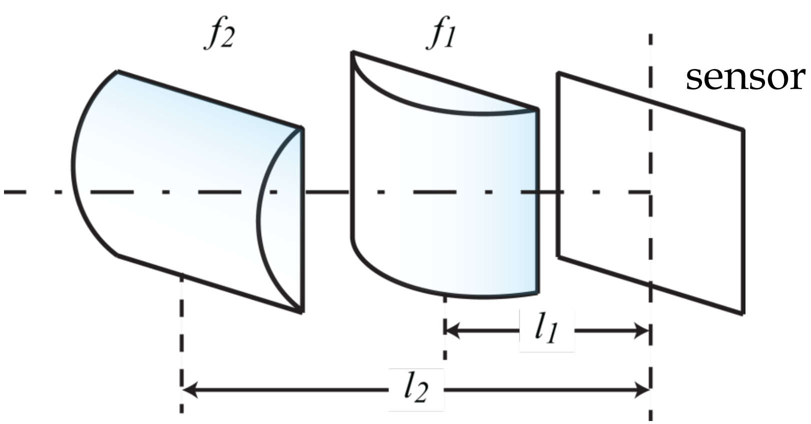


XSlit Cameras

[Pajdla '01], [Zomet '03], [Yu & McMillan'04], [Ponce '09] …

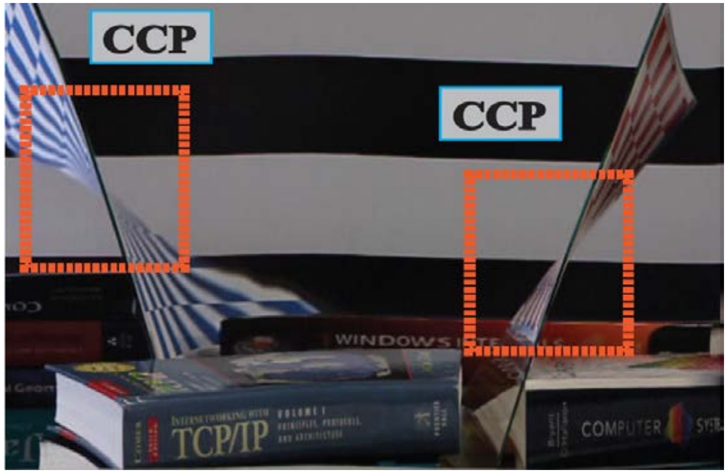




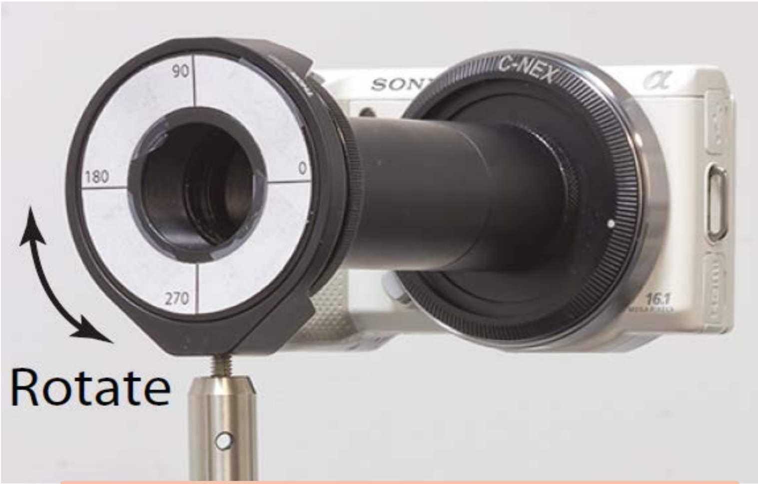


Imaging Applications Using XSlit Cameras

Panorama Stitching [Seitz ’01] [Zomet ’03] [Yu & McMillan ‘04]

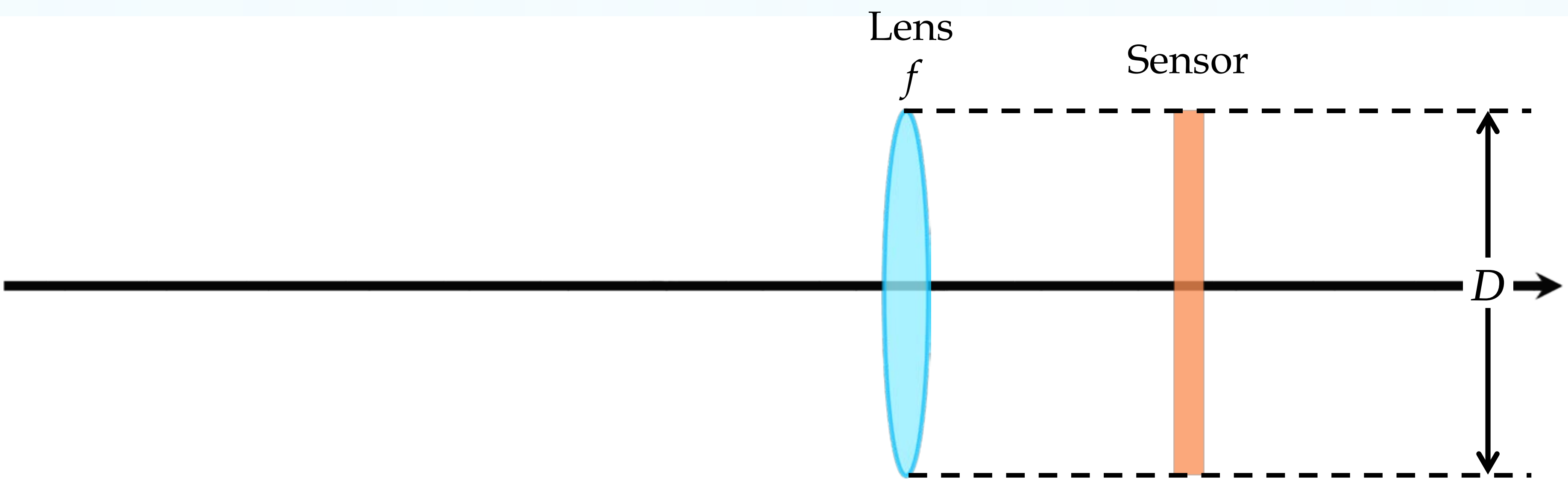


Coplanar Common Point [Ye ’13]



Rotational Stereo Mathing [Ye ‘13]

Implementing Pinhole Cameras



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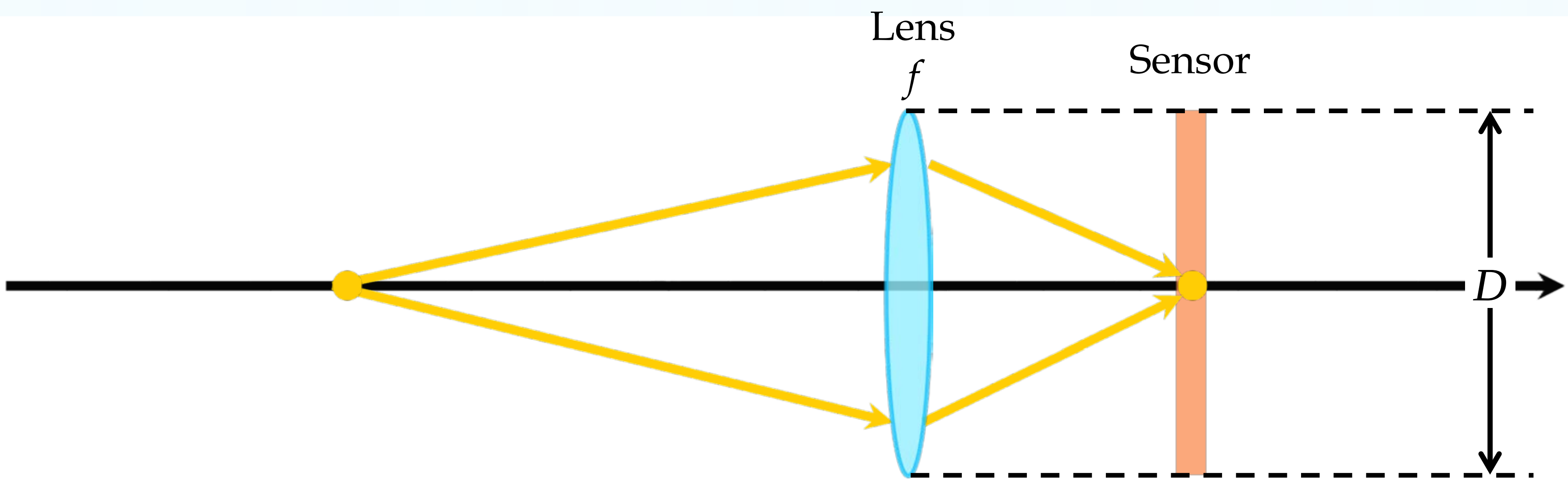

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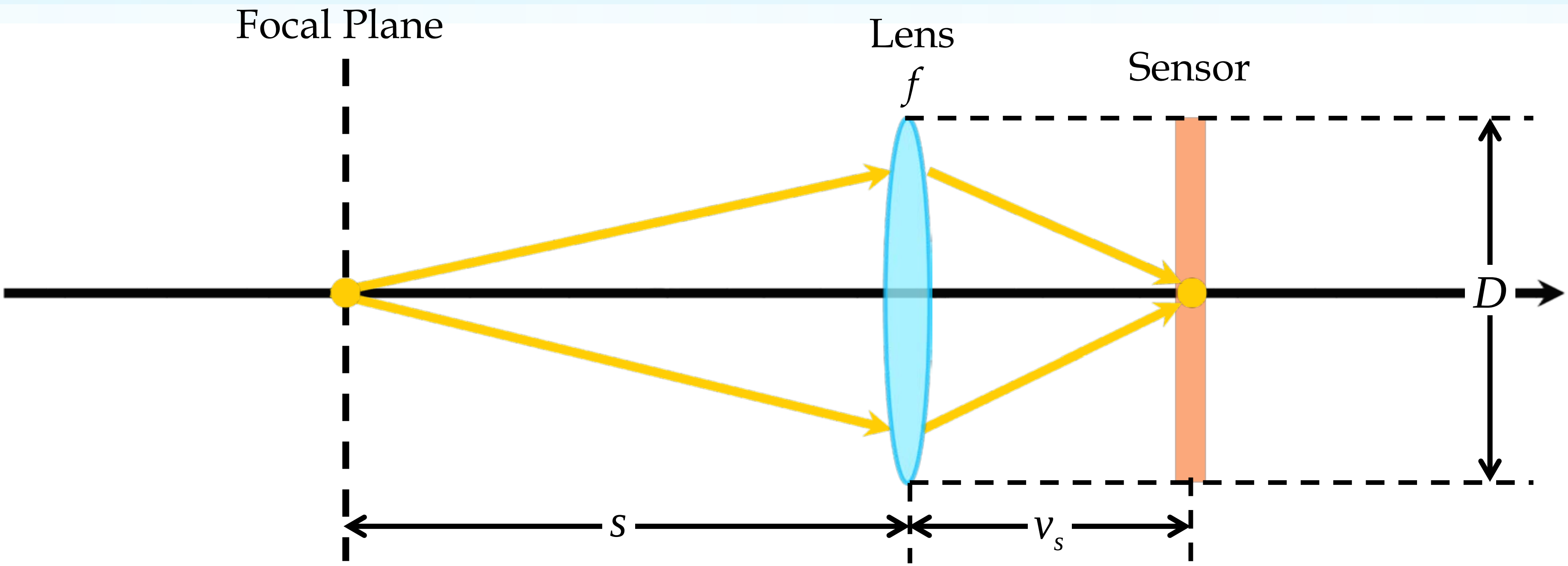
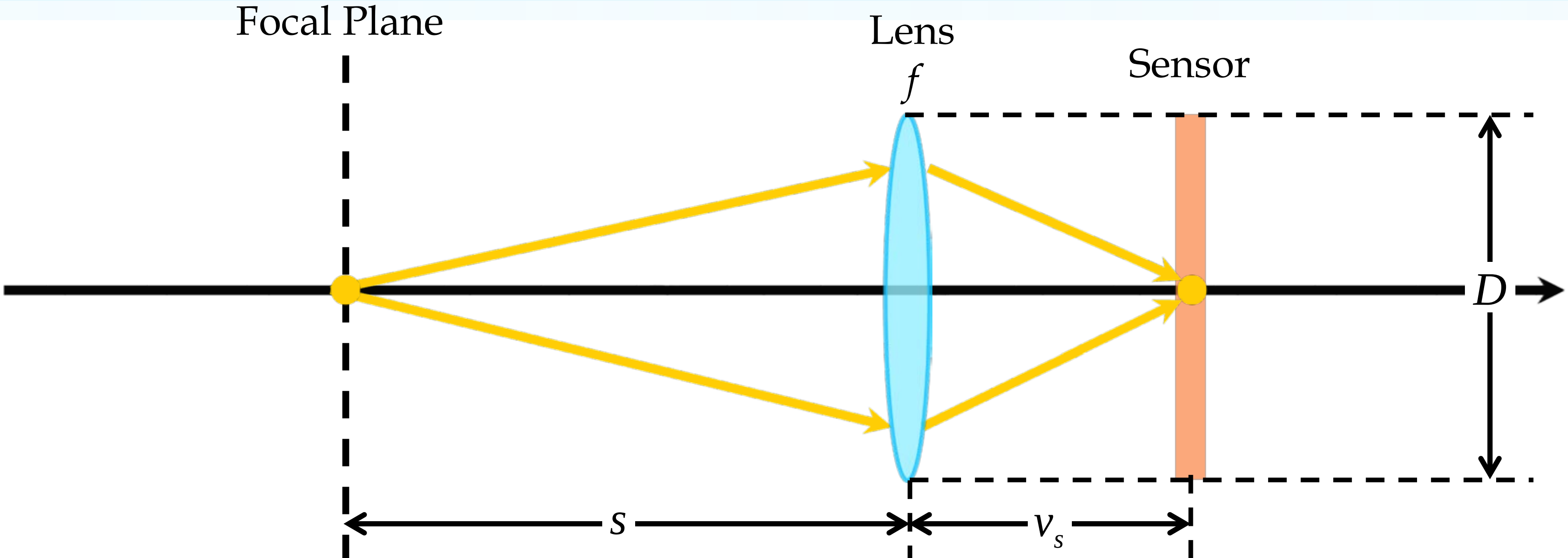


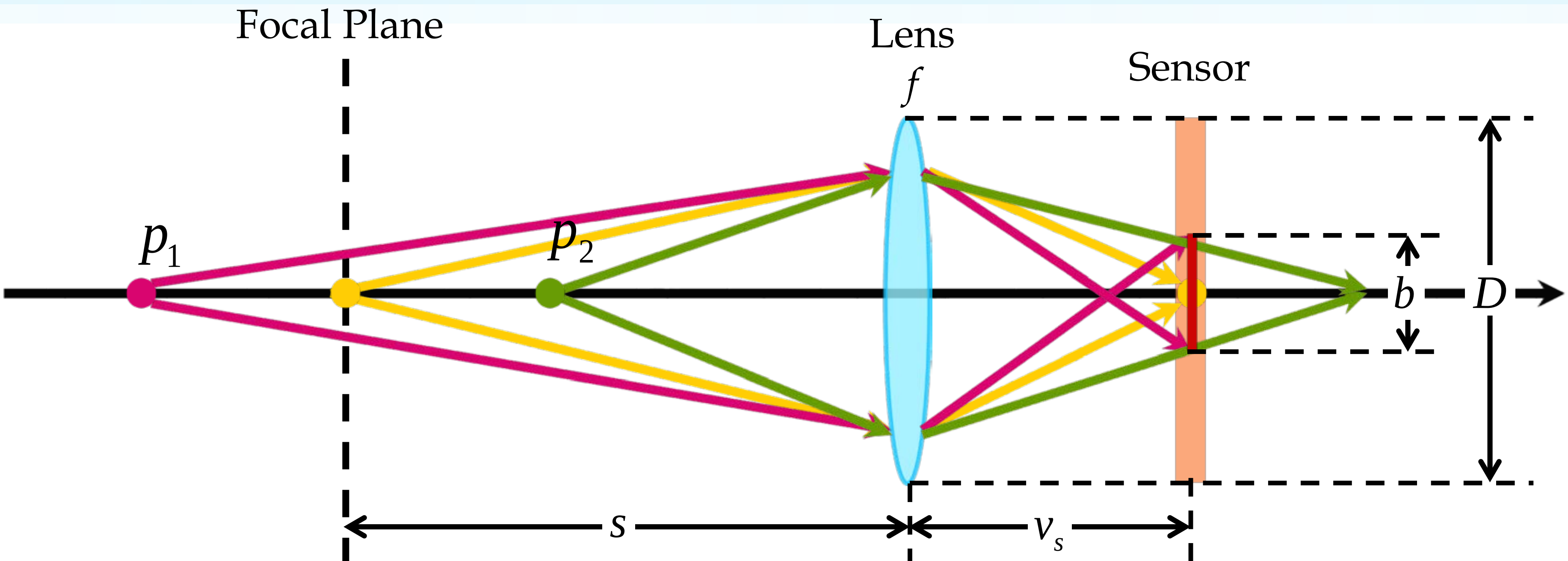
Implementing Pinhole Cameras



$$E = \frac{L\pi D^2 \cos^4 \theta}{16 {v_s}^2}$$

× **Light Efficiency**

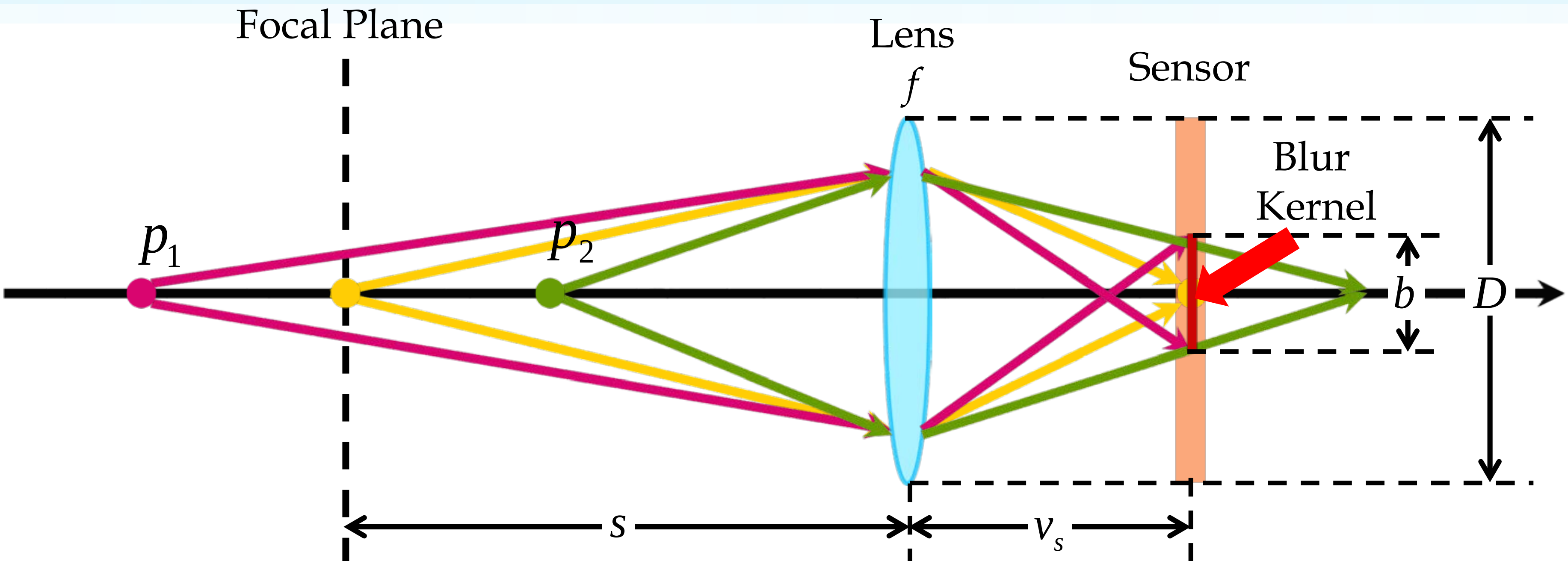
Implementing Pinhole Cameras



$$E = \frac{L\pi D^2 \cos^4\theta}{16{v_s}^2}$$

- ✗ **Light Efficiency**
- ✗ **Defocus Analysis**

Implementing Pinhole Cameras



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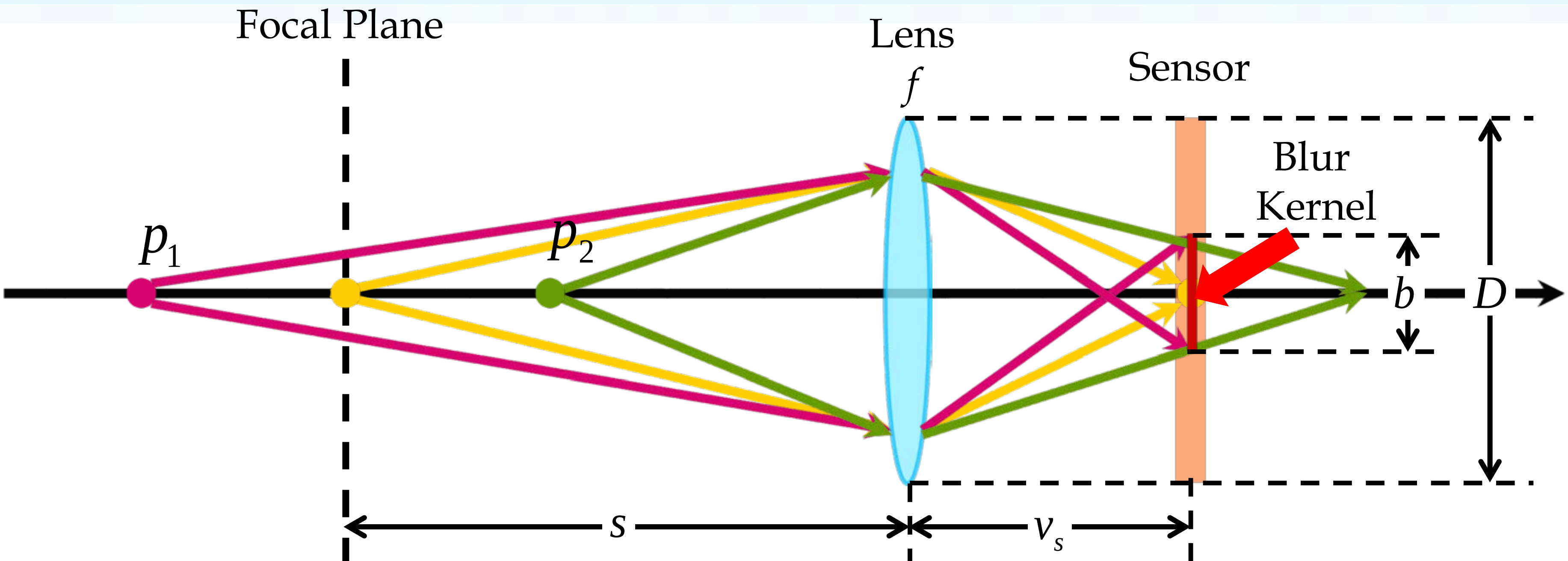
Implementing Pinhole Cameras



- ✗ **Light Efficiency**
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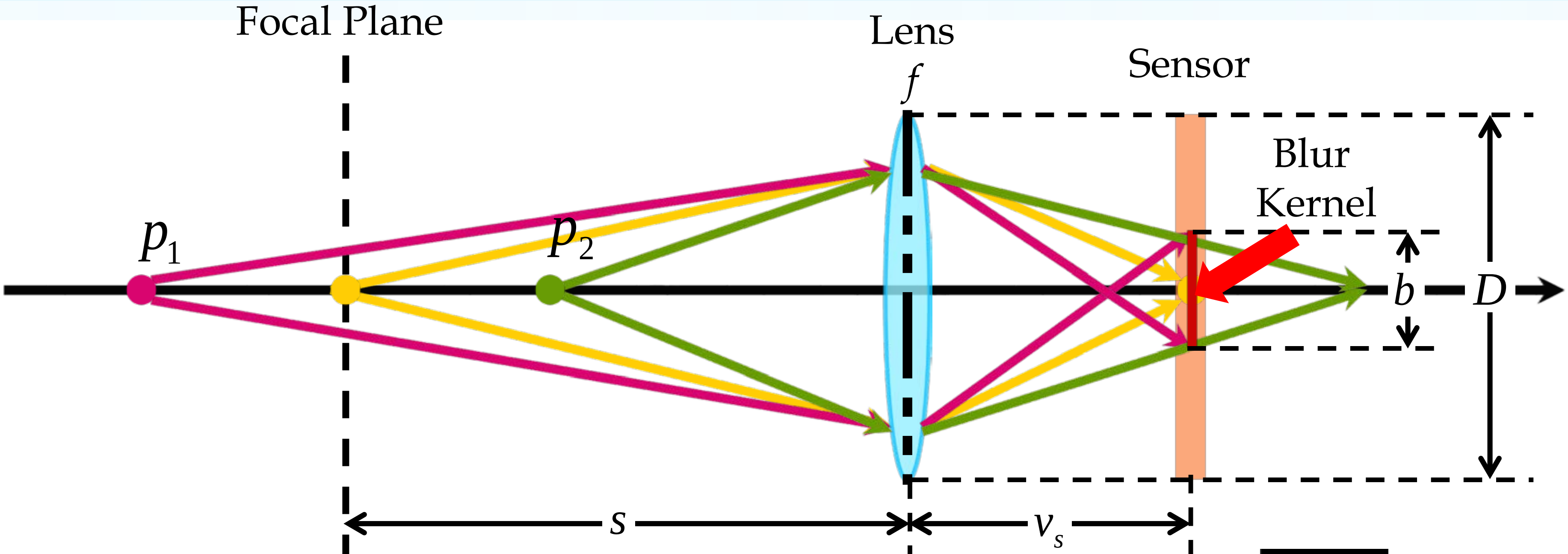
$$E = \frac{L\pi D^2 \cos^4\theta}{16{v_s}^2}$$

$$b(p) = \frac{Df\,|d(p)-s|}{d(p)(s-f)}$$

Implementing Pinhole Cameras



- ✕ **Light Efficiency**
- ✕ **Defocus Analysis**
- **Coded Aperture**

$$E = \frac{L\pi D^2 \cos^4\theta}{16{v_s}^2}$$

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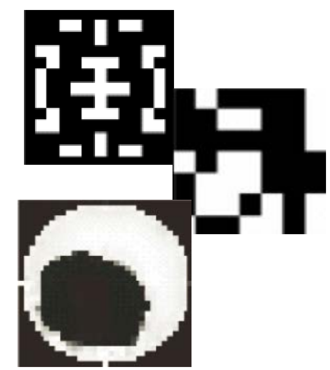
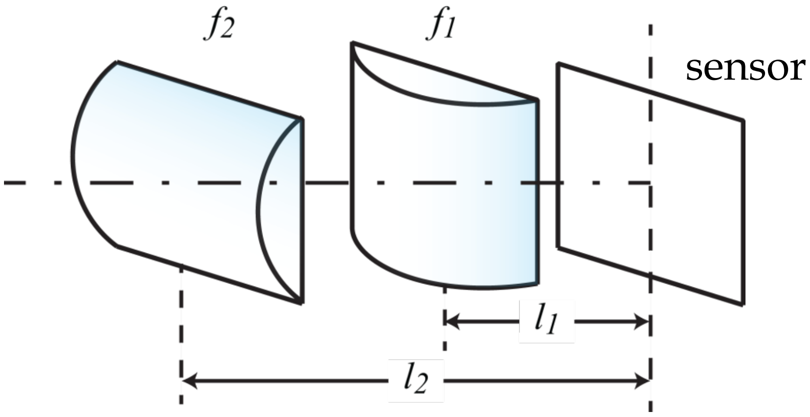
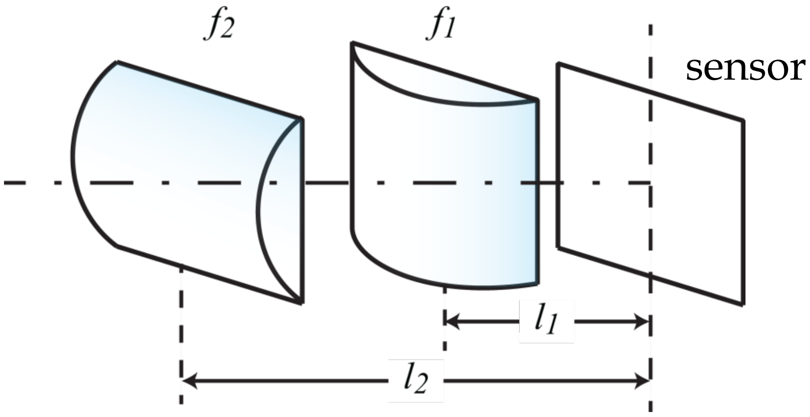


[Levin et al. '07] [Veeraraghavan et al. '07][Zhou et al. '09]…

Implementing XSlit Cameras

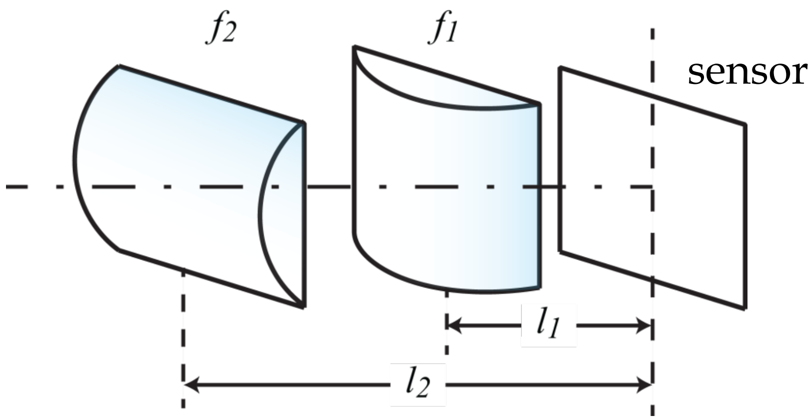


Implementing XSlit Cameras



Missing … **Defocus Analysis** | **Light Efficiency** | **Coded Aperture**

Implementing XSlit Cameras



Missing … | Defocus Analysis | Light Efficiency | Coded Aperture

XSlit vs Pinhole Lenses: Better or Worse?

Our Contributions

- Ray Geometry Analysis
  - Ray transform operators through a single or relays of cylindrical lenses
  - Aperture operators: study light efficiency and coded aperture imaging
- XSlit coded-aperture Imaging
  - Robust devonvoluion vs. depth estimation
  - Our solution: use separate codes for individual lenses

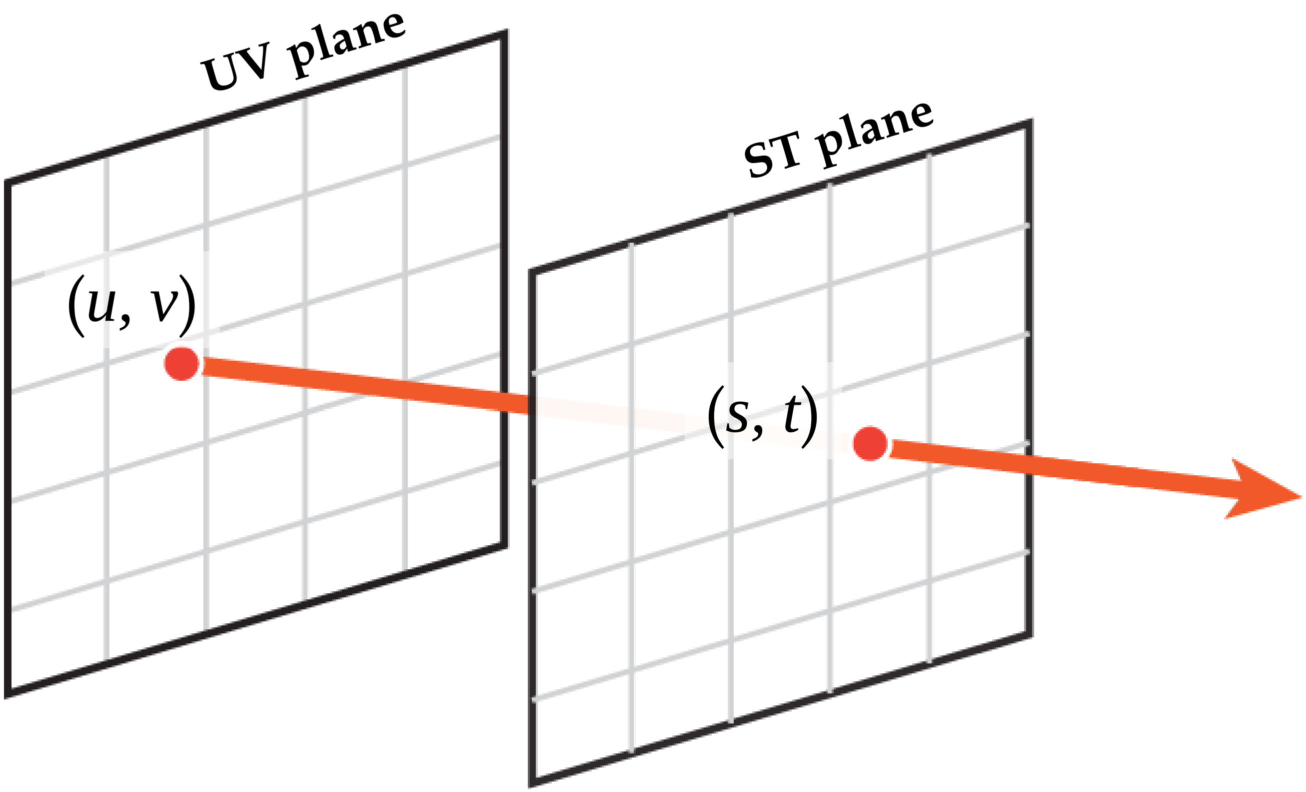
Our Tool:
Ray Geometry Analysis

- Two Plane Parameterization [Levoy and Hanranhan '96]
- 2PP Re-Parameterization [Ng '05] [Yu'05] [Ding '08]

Rays in
3D Space

Our Tool: Ray Geometry Analysis

- Two Plane Parameterization [Levoy and Hanranhan '96]
- 2PP Re-Parameterization [Ng '05] [Yu'05] [Ding '08]



Our Tool: Ray Geometry Analysis

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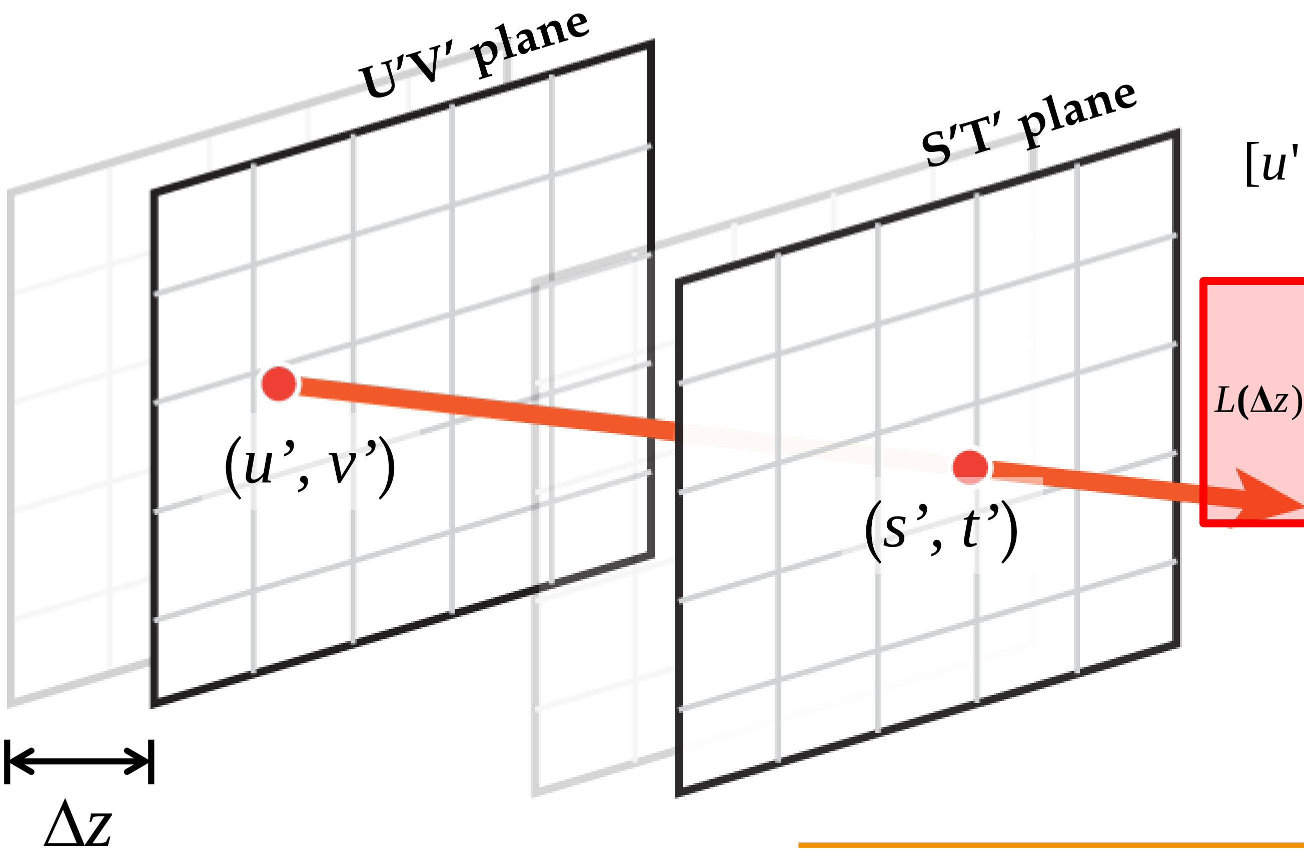
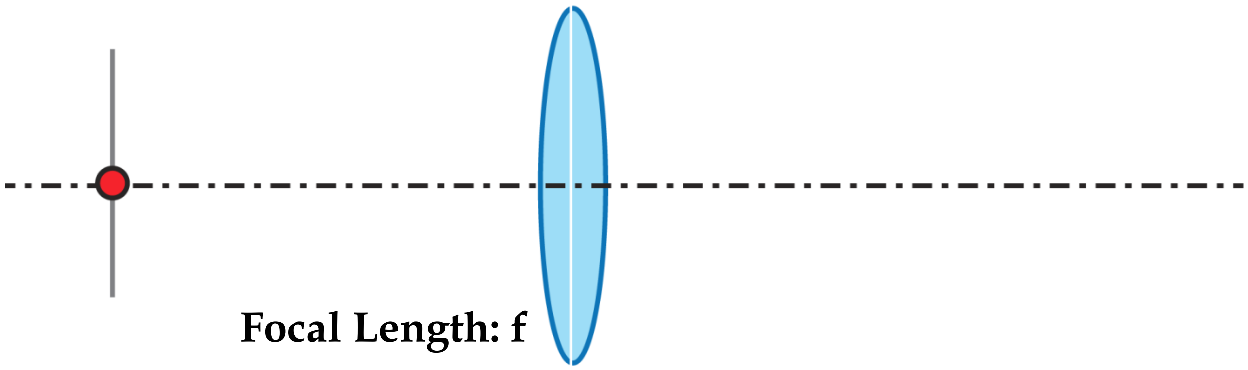


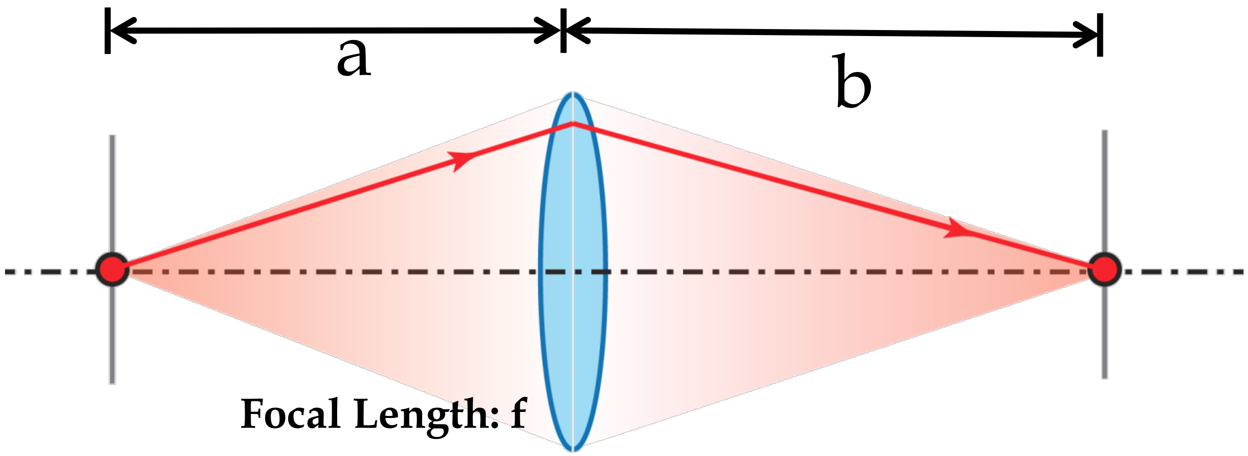
$$[u',v',s',t']^{T} = L[u,v,s,t]^{T}$$

$$L(\Delta z) = \begin{bmatrix} 1-\Delta z & 0 & \Delta z & 0 \\ 0 & 1-\Delta z & 0 & \Delta z \\ -\Delta z & 0 & 1+\Delta z & 0 \\ 0 & -\Delta z & 0 & 1+\Delta z \end{bmatrix}$$

Thin-Lens Ray Transform



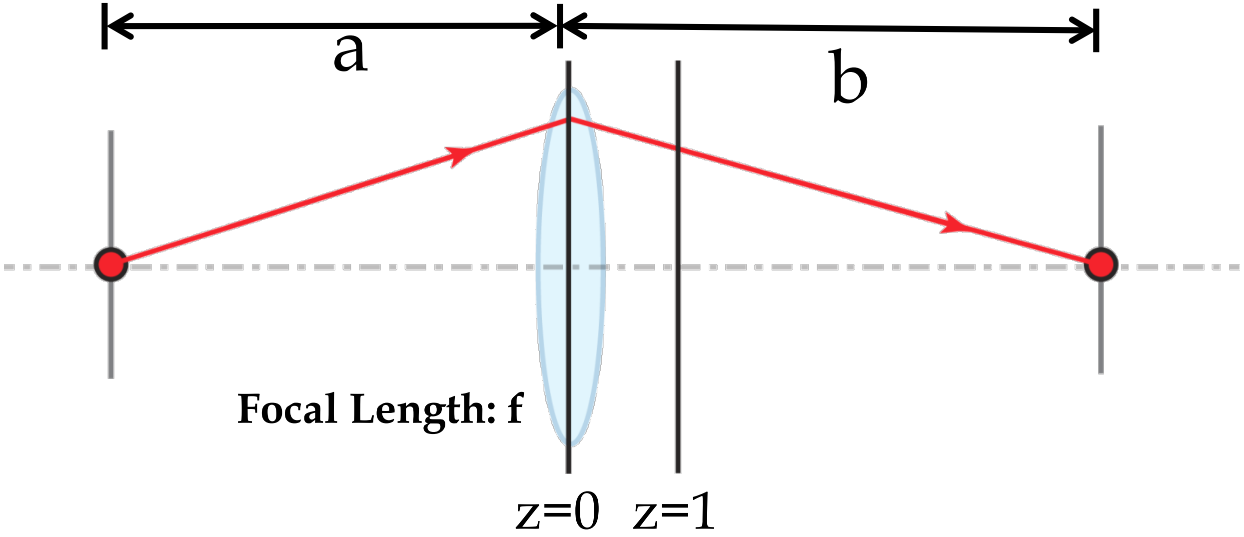
Thin-Lens Ray Transform



The Thin Lens Eq:

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

Thin-Lens Ray Transform



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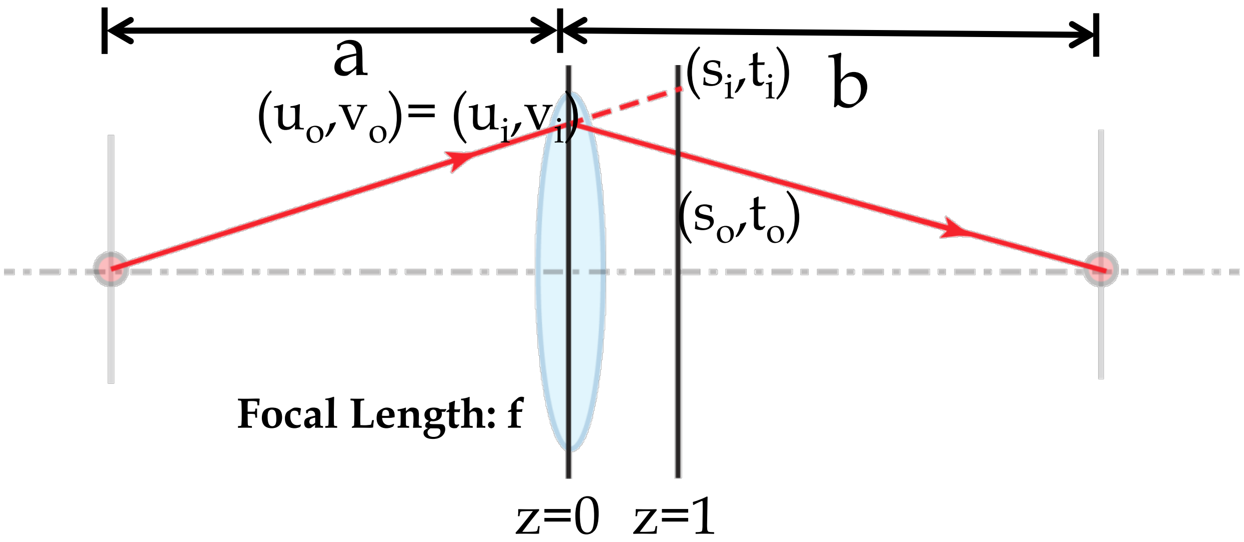

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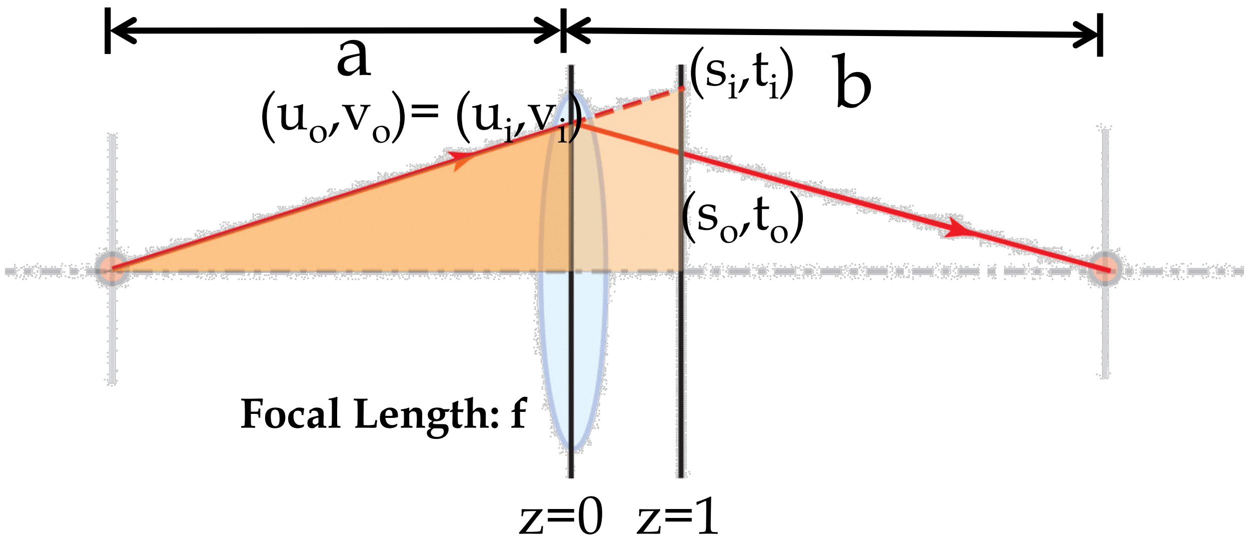
Thin-Lens Ray Transform



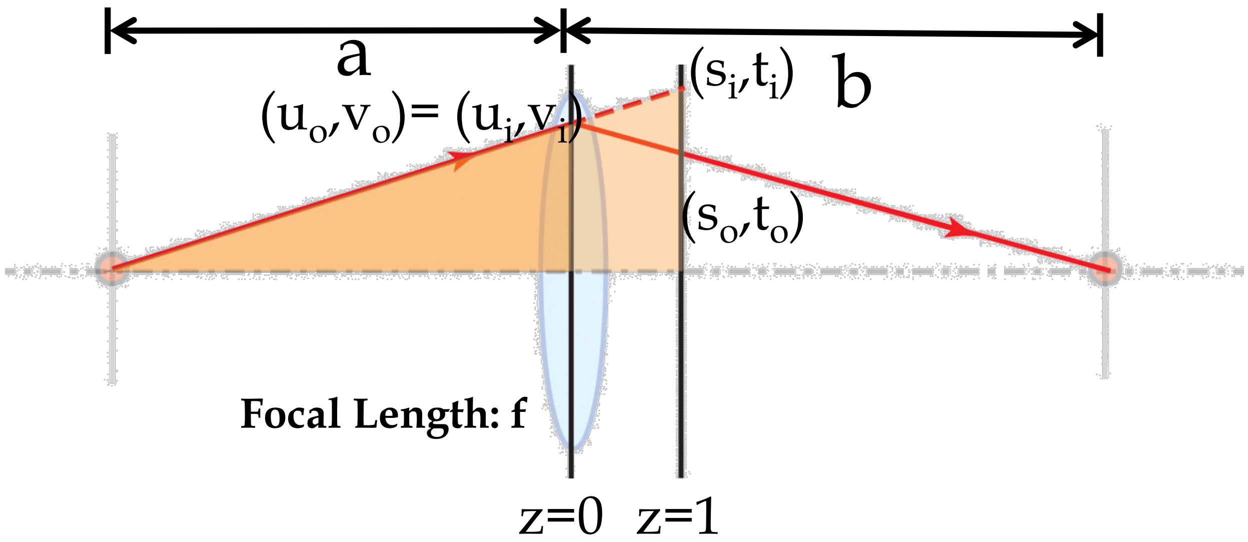
The Thin Lens Eq:

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

$$\frac{s_i}{u_o} = \frac{(a+1)}{a} = 1 + \frac{1}{a}$$

$$\frac{s_o}{u_o} = \frac{(b-1)}{b} = 1 - \frac{1}{b}$$

Thin-Lens Ray Transform



The Thin Lens Eq:

$$\frac{1}{a}+\frac{1}{b}=\frac{1}{f}$$

$$\frac{s_i}{u_o}=\frac{(a+1)}{a}=1+\frac{1}{a}$$

$$\frac{s_o}{u_o}=\frac{(b-1)}{b}=1-\frac{1}{b}$$

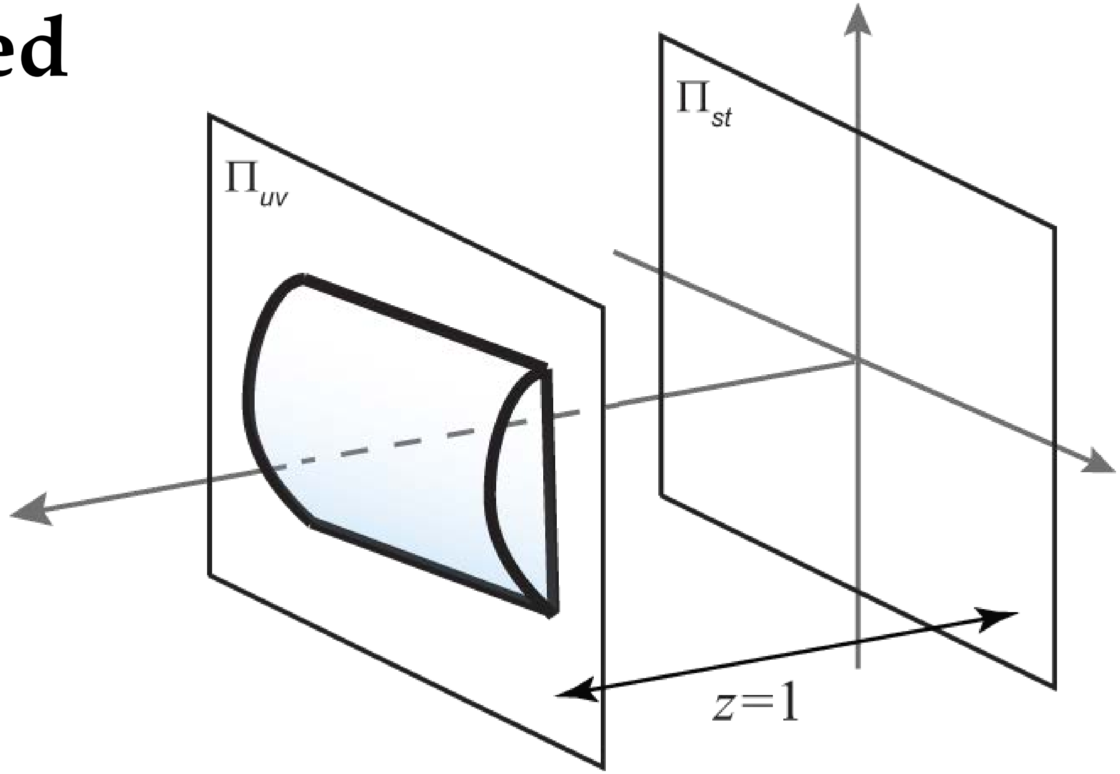
$$[u_o, v_o, s_o, t_o]^T = \mathbf{C}_{thin}[u_i, v_i, s_i, t_i]^T$$

C_{thin}: Thin-Lens Operator

$$\mathbf{C}_{thin} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1/f & 0 & 1 & 0 \\ 0 & -1/f & 0 & 1 \end{bmatrix}$$

Cylindrical Lens Operator

- X-Axis Aligned



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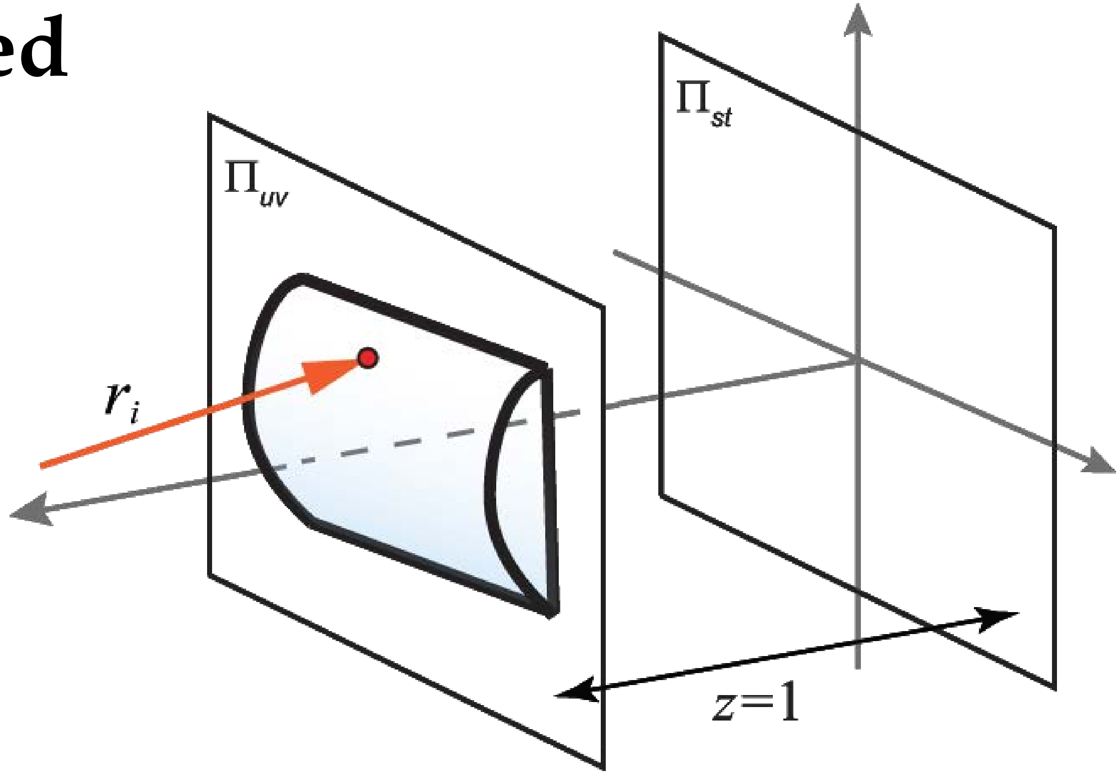

Cylindrical Lens Operator

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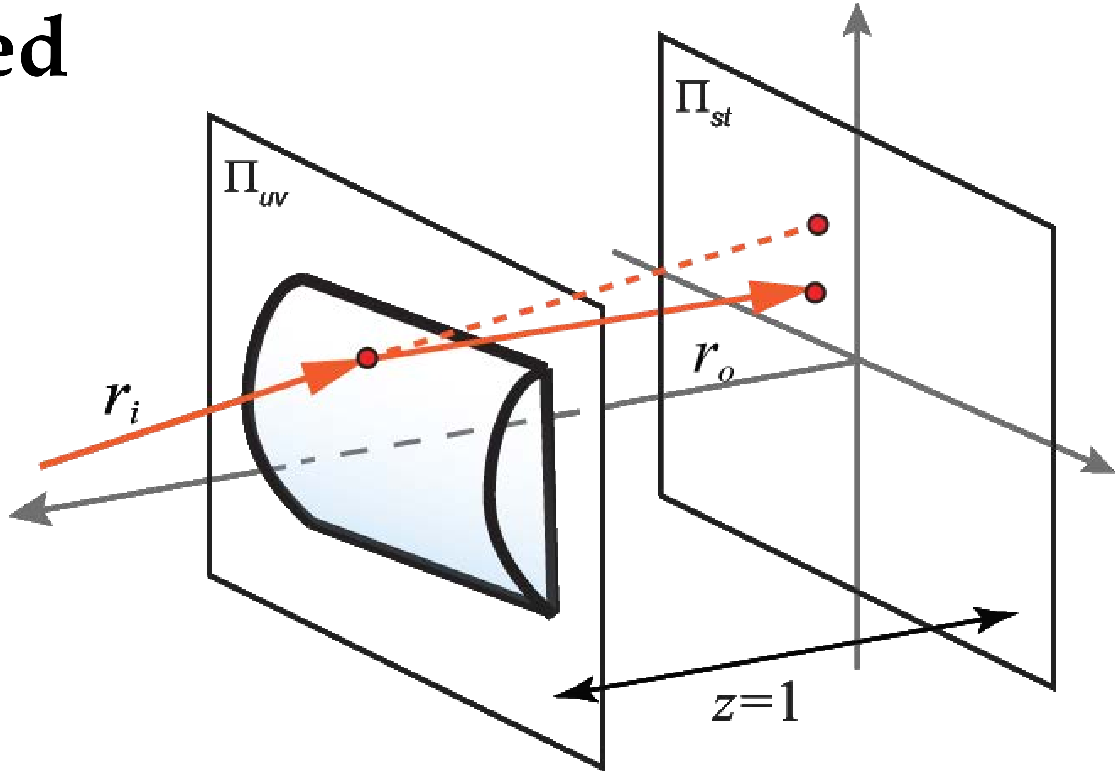
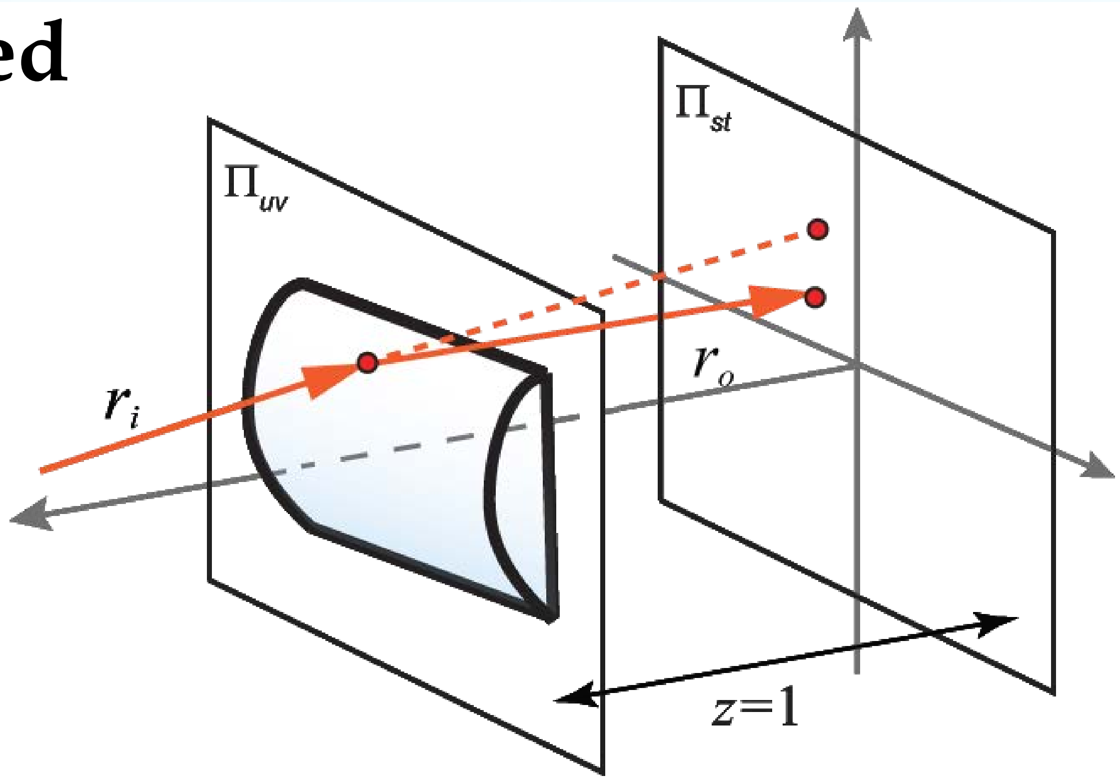


Cylindrical Lens Operator

- X-Axis Aligned

$f_{horizontal} \Rightarrow \infty$

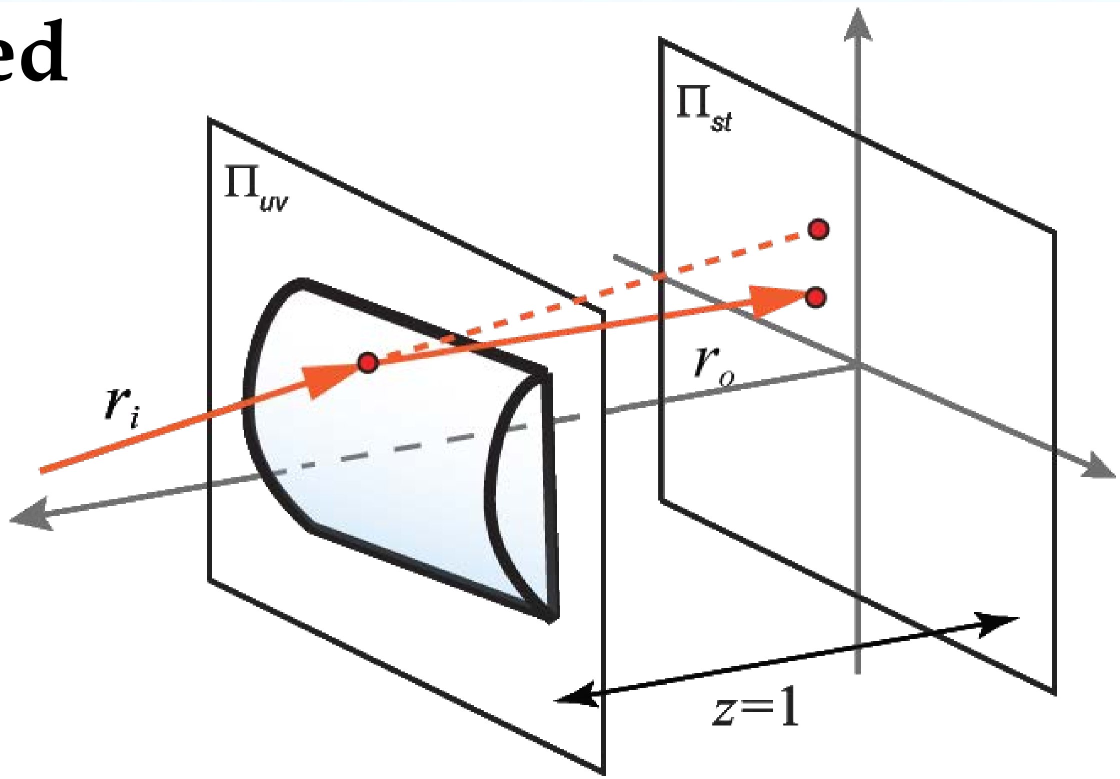


Cylindrical Lens Operator

- **X-Axis Aligned**

$\mathbf{f}_{\mathbf{horizontal}} \Rightarrow \infty$

$$[u_o, v_o, s_o, t_o]^T = \mathbf{C}[u_i, v_i, s_i, t_i]^T$$



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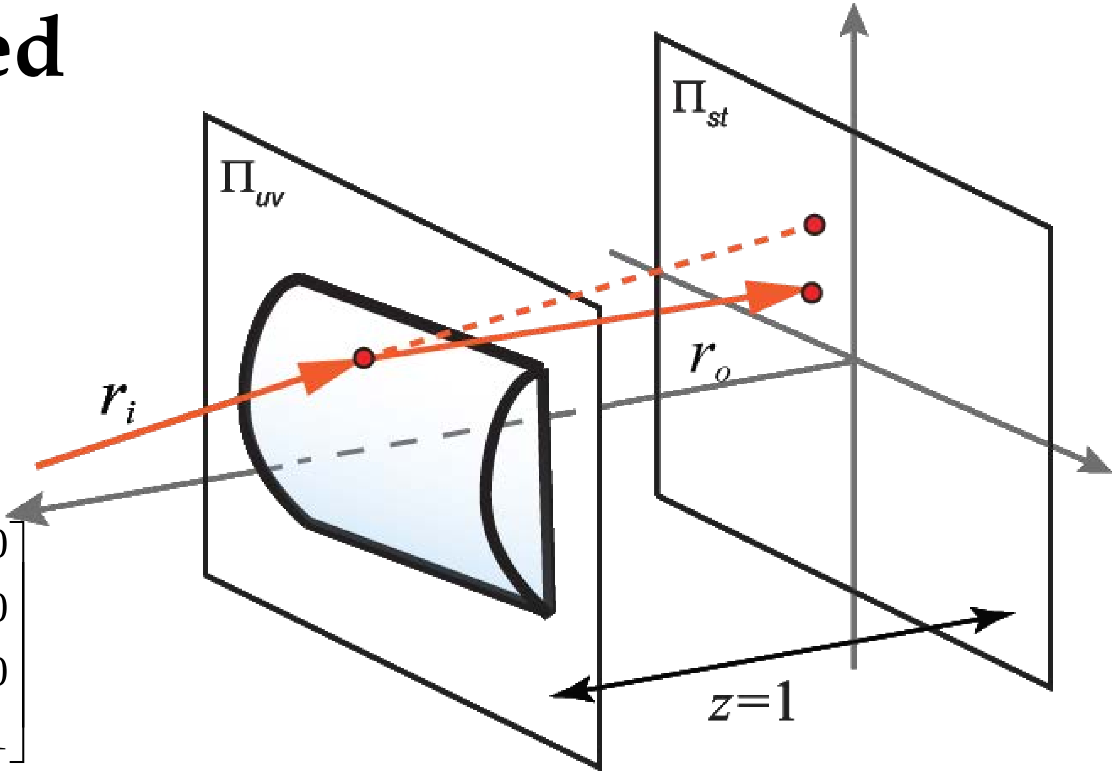
$\mathbf{f}_{\mathbf{horizontal}} \Rightarrow \infty$

$$[u_o, v_o, s_o, t_o]^T = \mathbf{C}[u_i, v_i, s_i, t_i]^T$$

$$horizontal: \mathbf{C}_h(f) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1/f & 0 & 1 \end{bmatrix}$$



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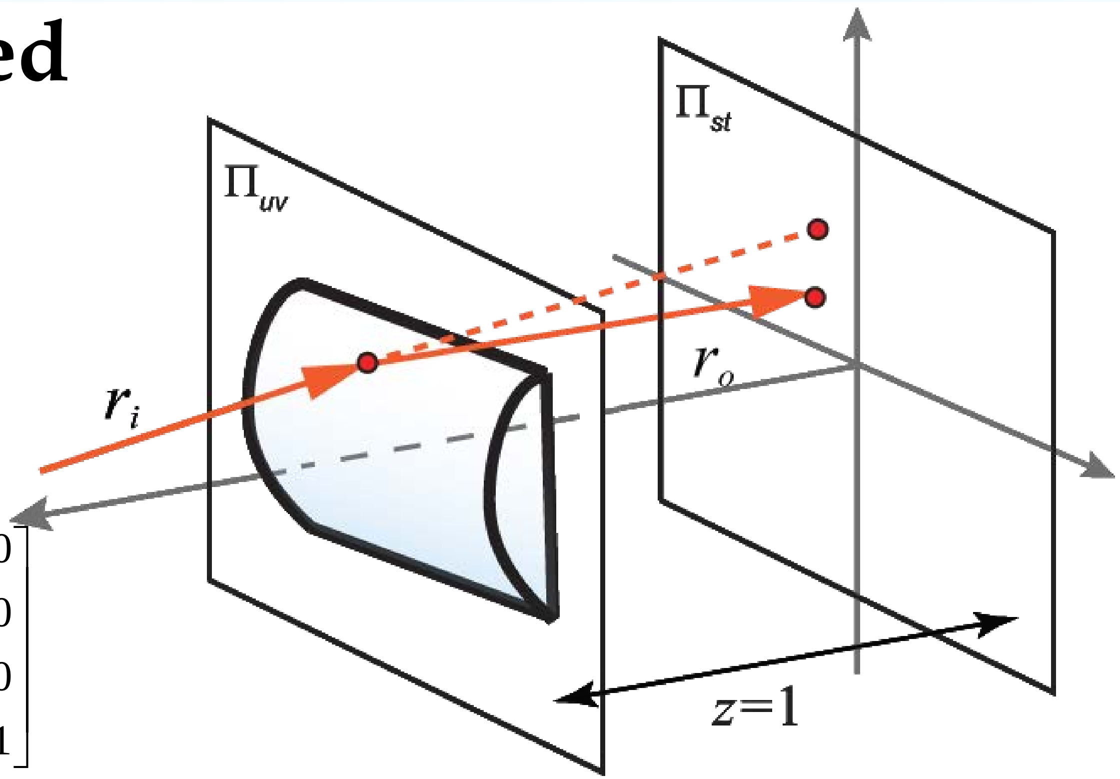
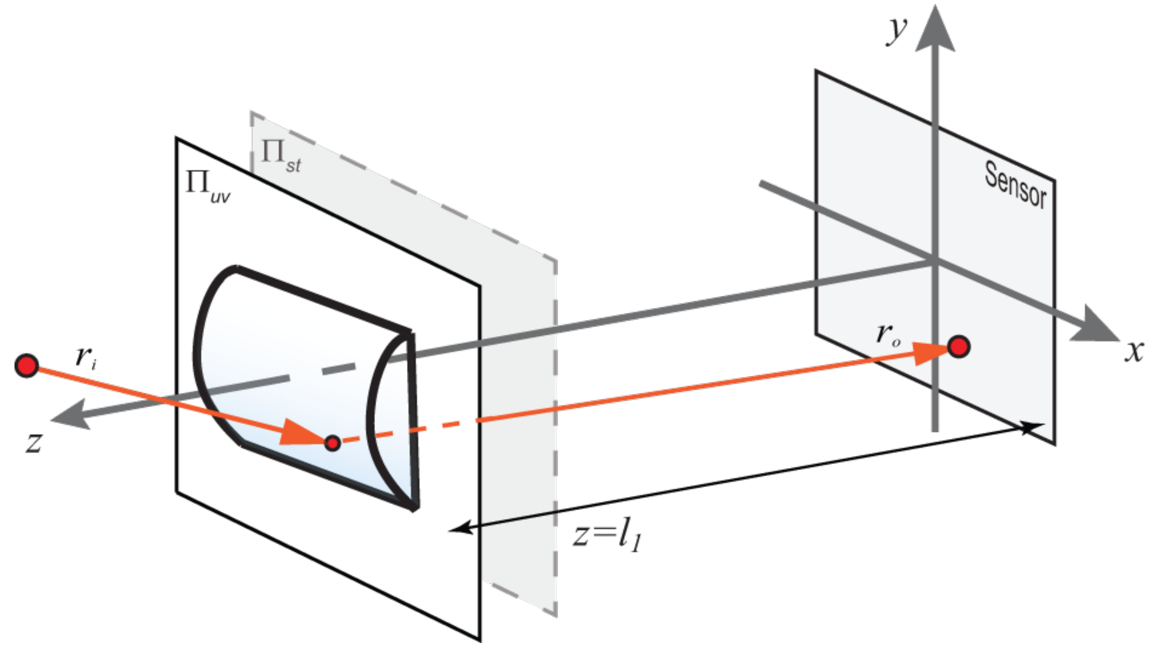


- **Non-Axis Aligned: Add Rotation R**

$$[u_o, v_o, s_o, t_o]^T = \mathbf{RCR}^T[u_i, v_i, s_i, t_i]^T$$

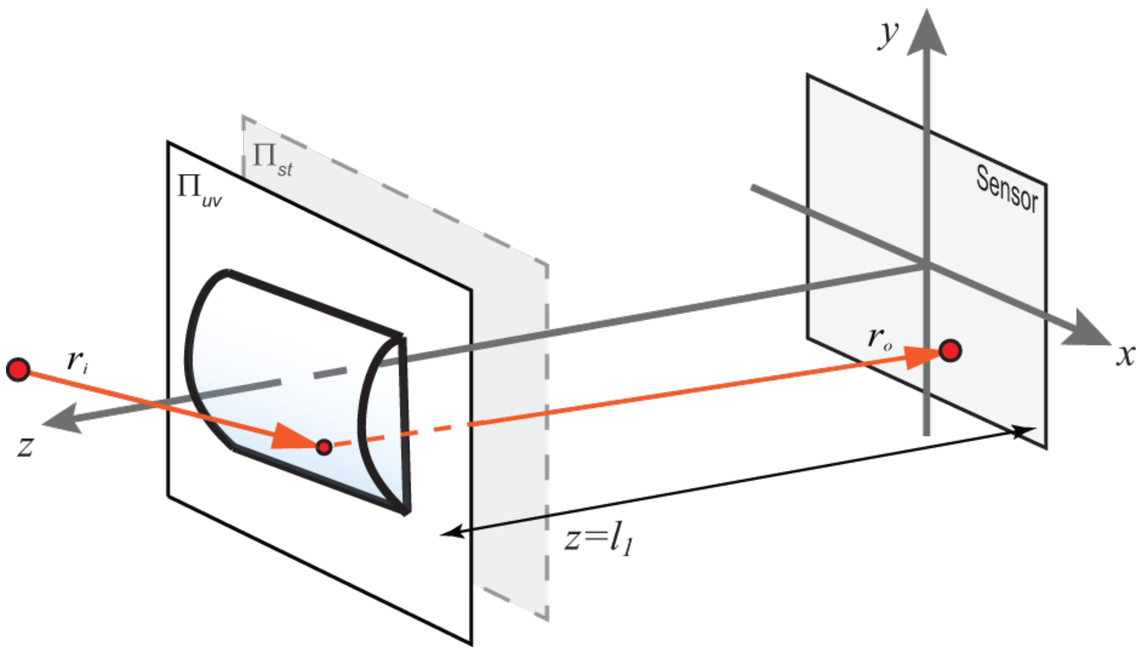
XSlit Lens Ray Transform

- Relay Two Orthogonal Cylindrical Lenses



XSlit Lens Ray Transform

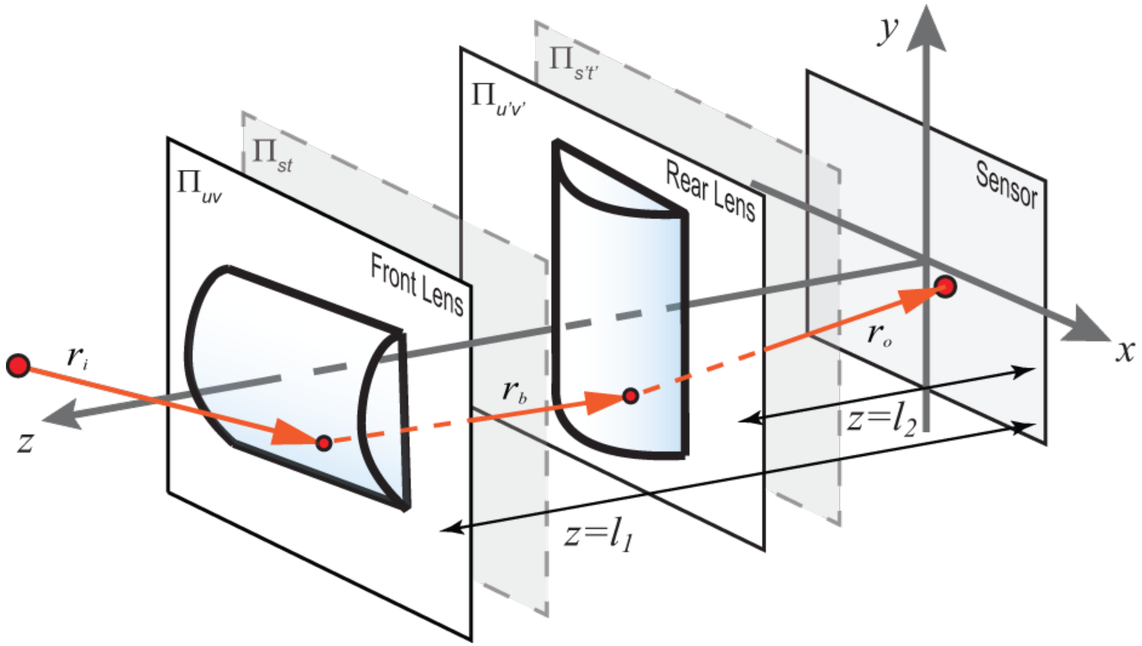
- Relay Two Orthogonal Cylindrical Lenses



$$horizontal : \mathbf{C}_h(f) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1/f & 0 & 1 \end{bmatrix}$$

XSlit Lens Ray Transform

- **Relay Two Orthogonal Cylindrical Lenses**

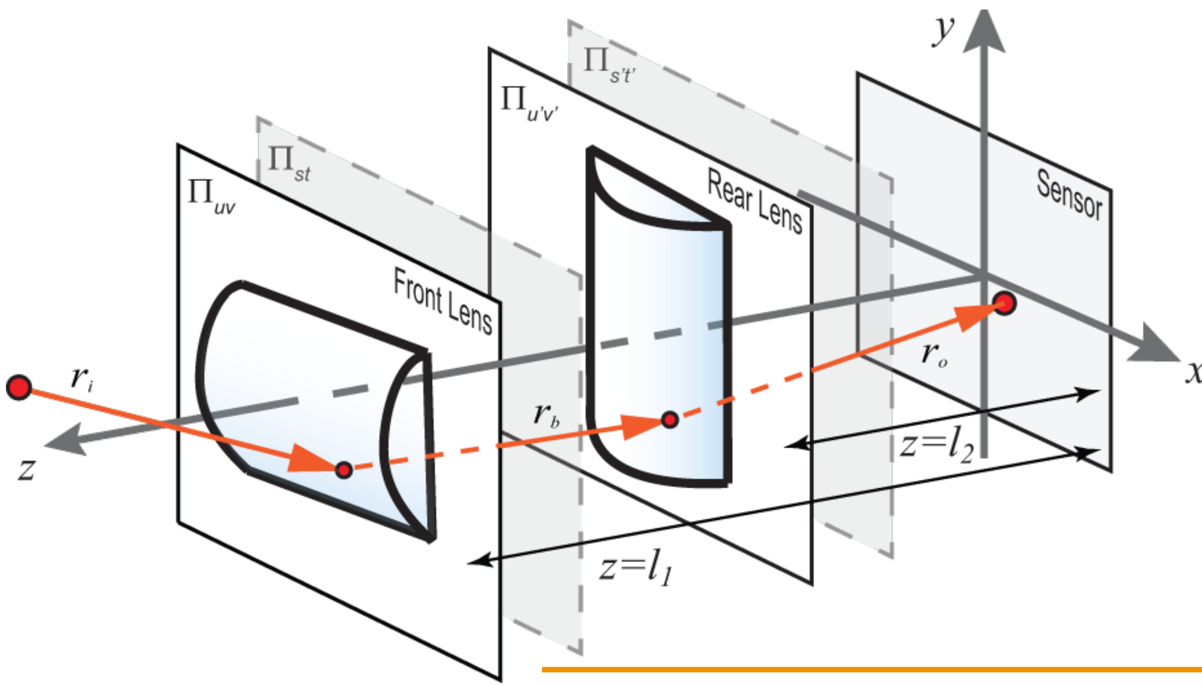


$$horizontal: \mathbf{C}_h(f) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1/f & 0 & 1 \end{bmatrix} \qquad vertical: \mathbf{C}_v(f) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1/f & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

XSlit Lens Operator

- XSlit Lens Operator
  - Concatenation of multiple linear operators

$$[u_o, v_o, s_o, t_o]^\top = \mathrm{L}(l)\mathrm{C}_v(f_2)\mathrm{L}^{-1}(l)\mathrm{C}_h(f_1)[u_i, v_i, s_i, t_i]^\top$$
$$= \mathrm{S}(f_1, f_2, l)[u_i, v_i, s_i, t_i]^\top$$



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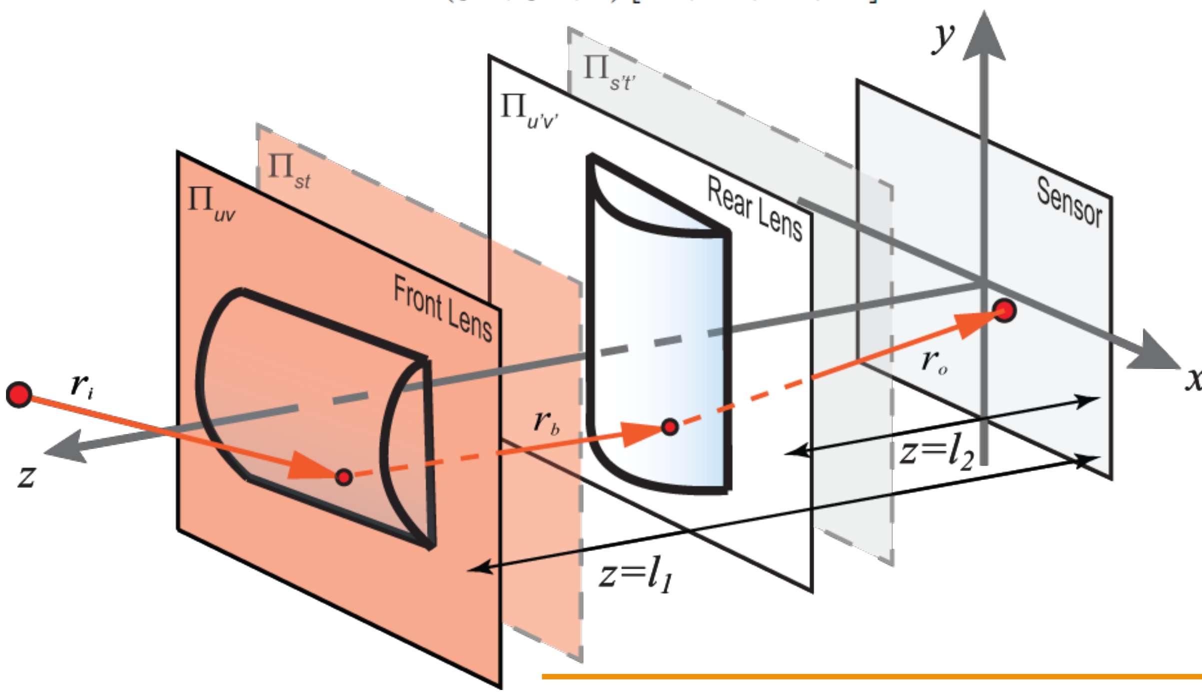

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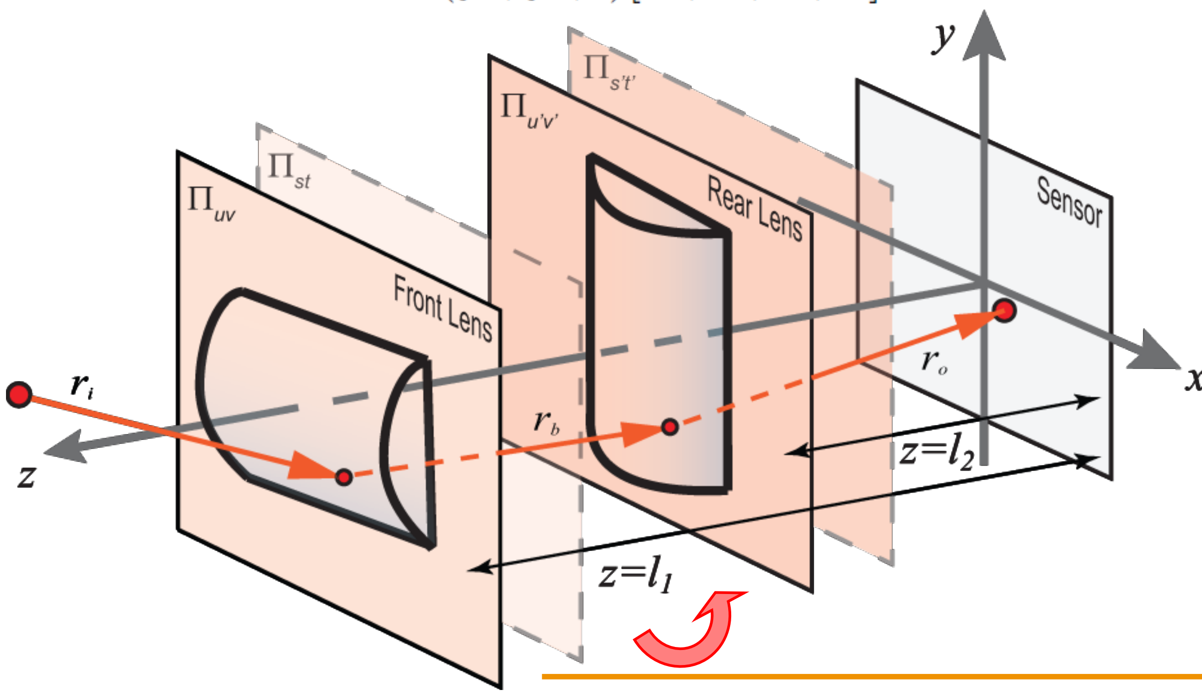

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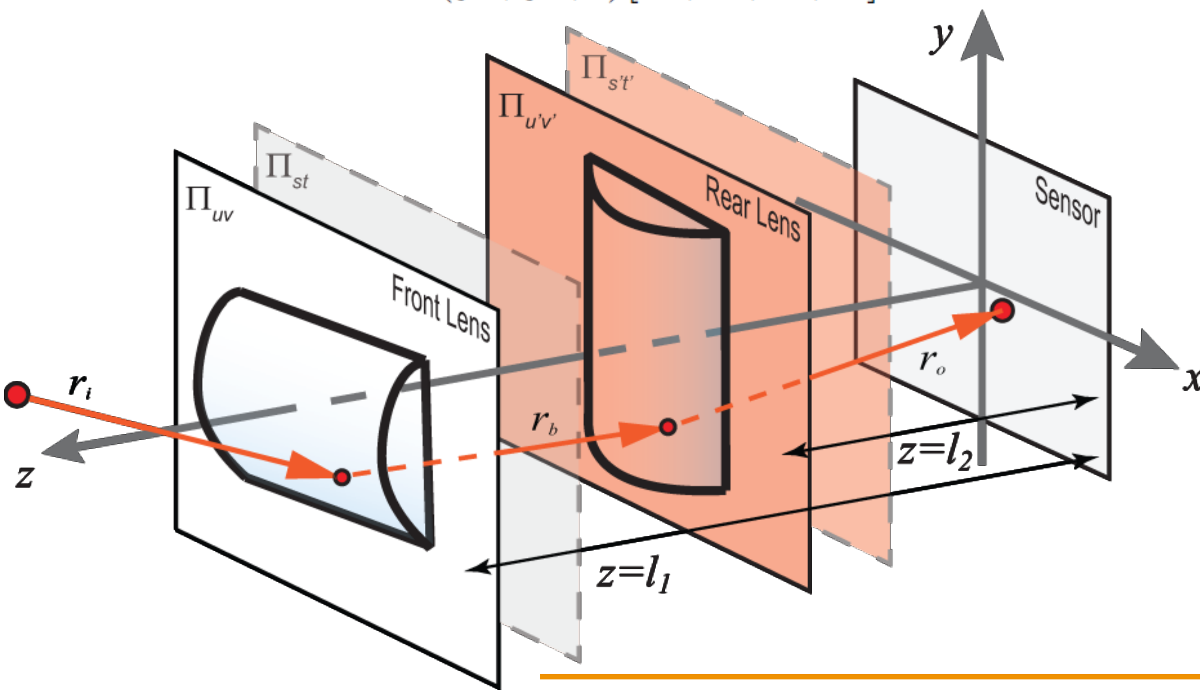

XSlit Lens Operator

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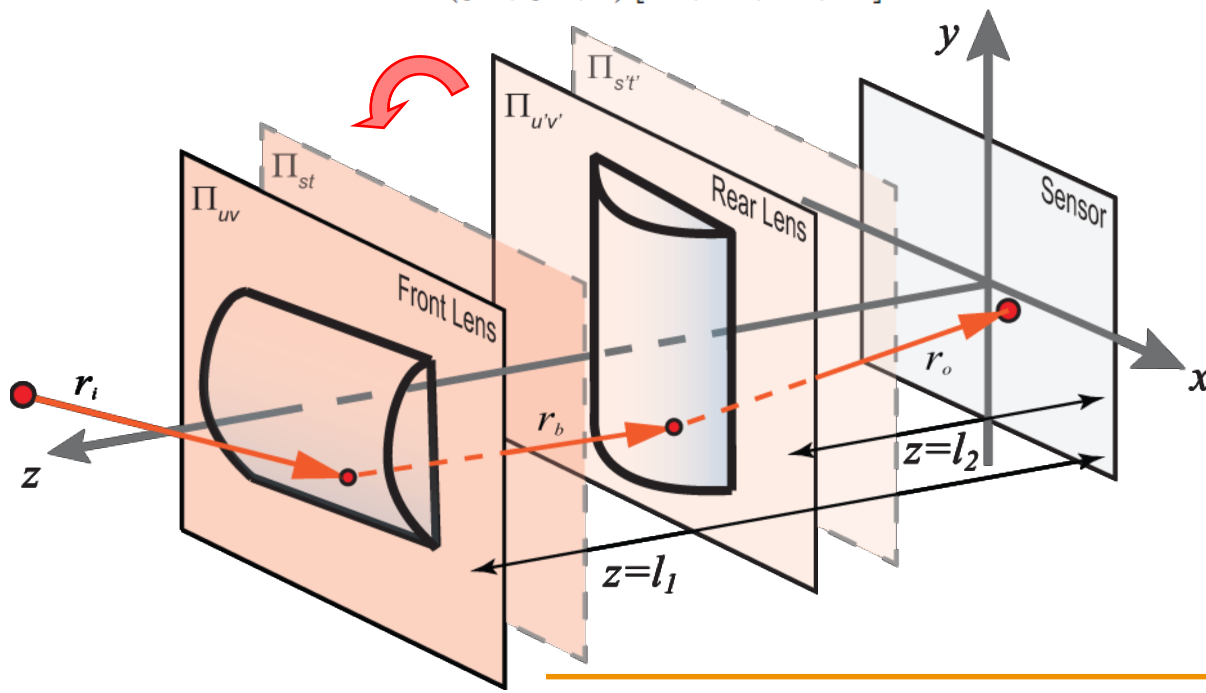

XSlit Lens Operator

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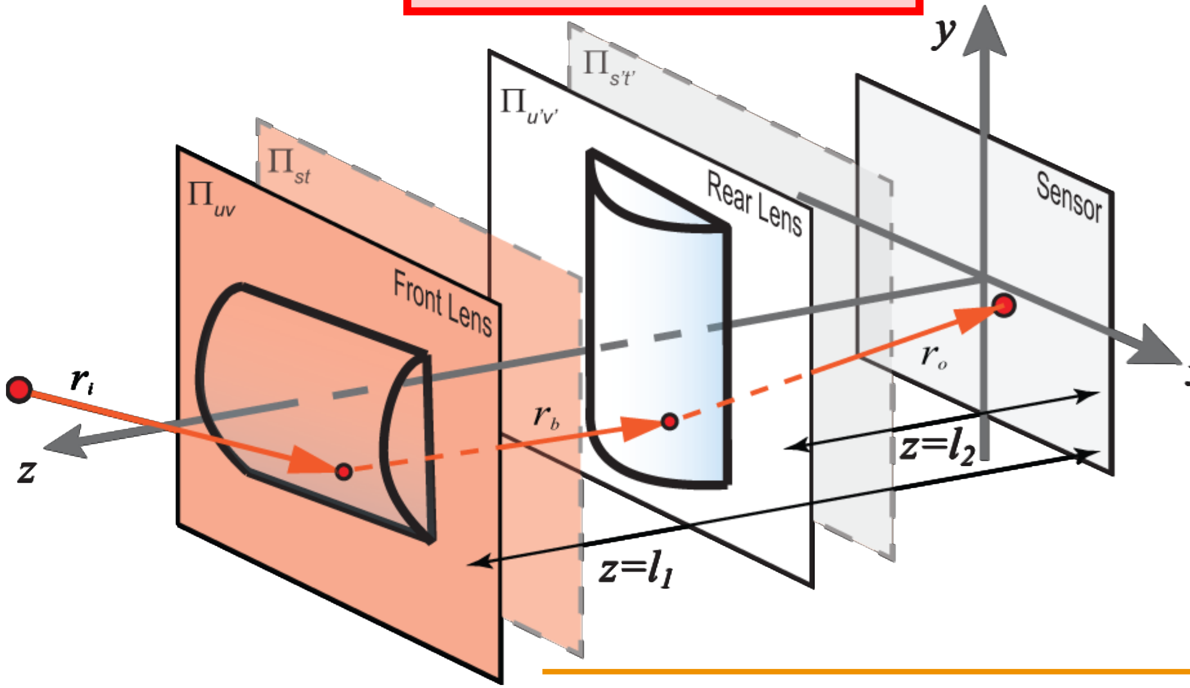
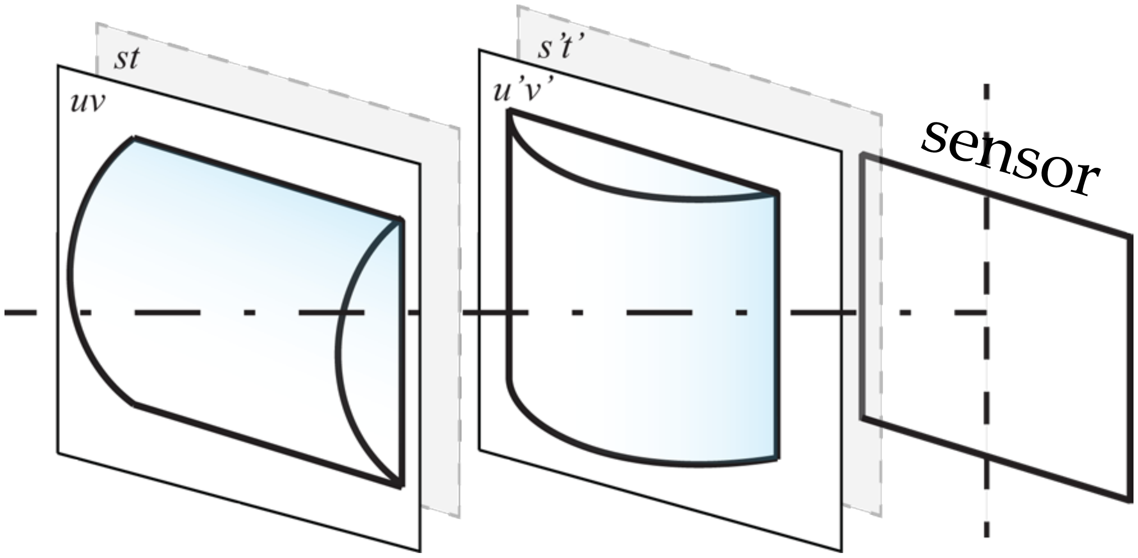
  - Concatenation of multiple linear operators

$$[u_o, v_o, s_o, t_o]^\top = \mathrm{L}(l)\mathrm{C}_v(f_2)\mathrm{L}^{-1}(l)\mathrm{C}_h(f_1)[u_i, v_i, s_i, t_i]^\top$$
$$= \mathrm{S}(f_1, f_2, l)[u_i, v_i, s_i, t_i]^\top$$



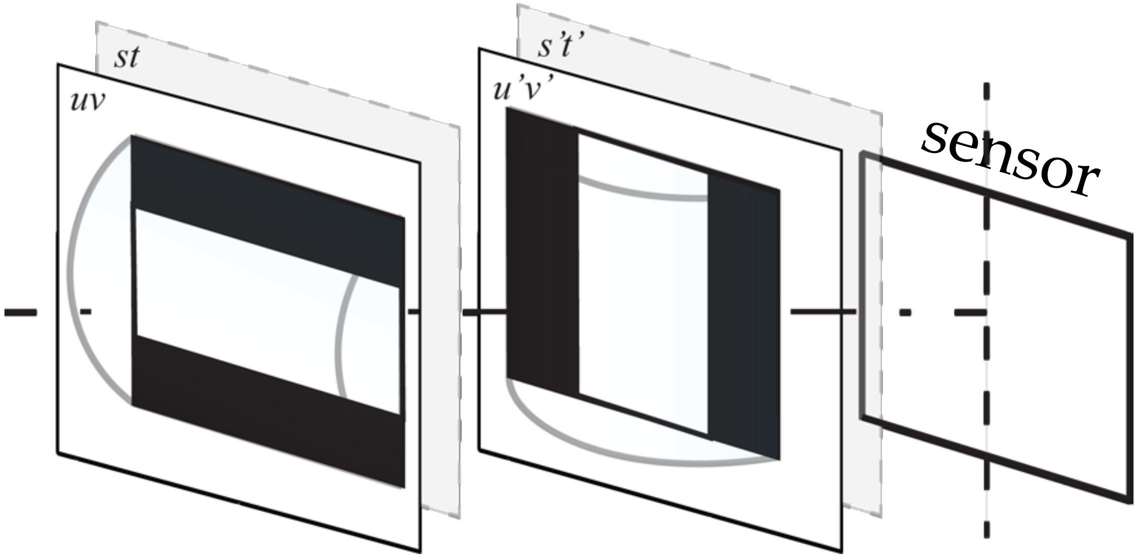
Aperture Operator



Aperture Operator

$$A_1(v) = \begin{cases} 1 & |v| \leq w_1/2 \\ 0 & else \end{cases}$$

$$A_2(u') = \begin{cases} 1 & |u'| \leq w_2/2 \\ 0 & else \end{cases}$$



$$[u_o, v_o, s_o, t_o]^{\mathrm{T}} = L(l)C_v(f_2)L^{-1}(l)C_h(f_1)[u_i, v_i, s_i, t_i]^{\mathrm{T}}$$
$$= S(f_1, f_2, l)[u_i, v_i, s_i, t_i]^{\mathrm{T}}$$

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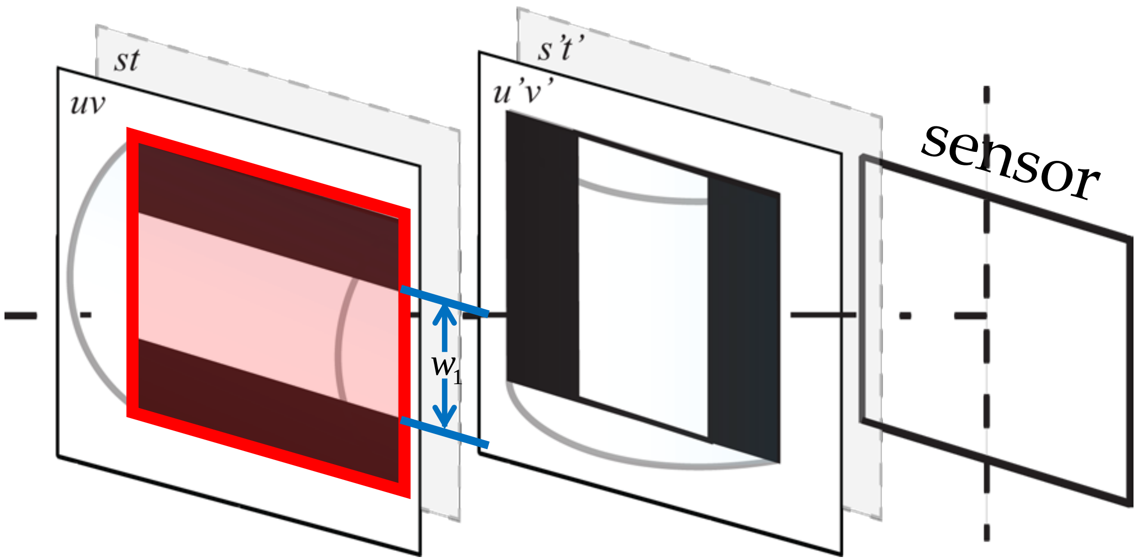

$$[u_o, v_o, s_o, t_o]^T = L(l)C_v(f_2)L^{-1}(l)C_h(f_1)[u_i, v_i, s_i, t_i]^T$$
$$= S(f_1, f_2, l)[u_i, v_i, s_i, t_i]^T$$

Aperture Operator

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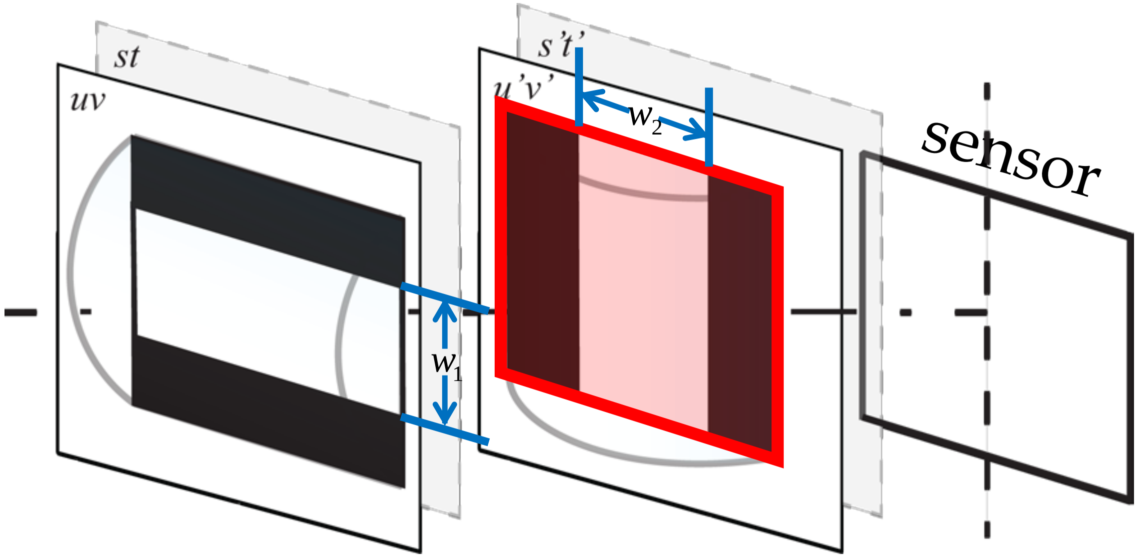

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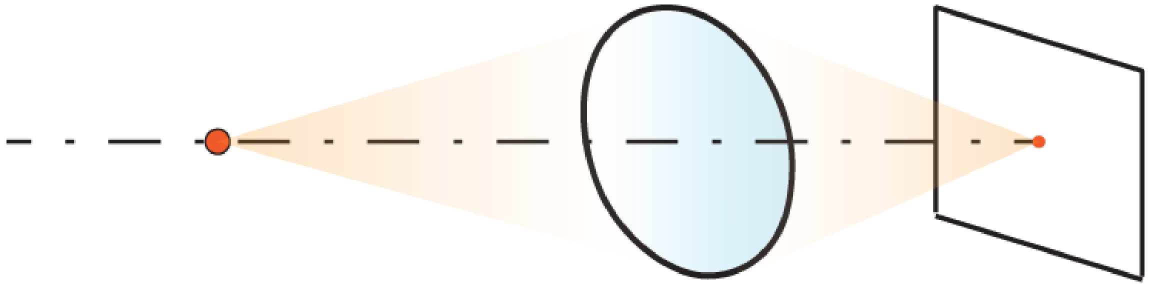
Point Spread Function (under Rectangular Aperture)



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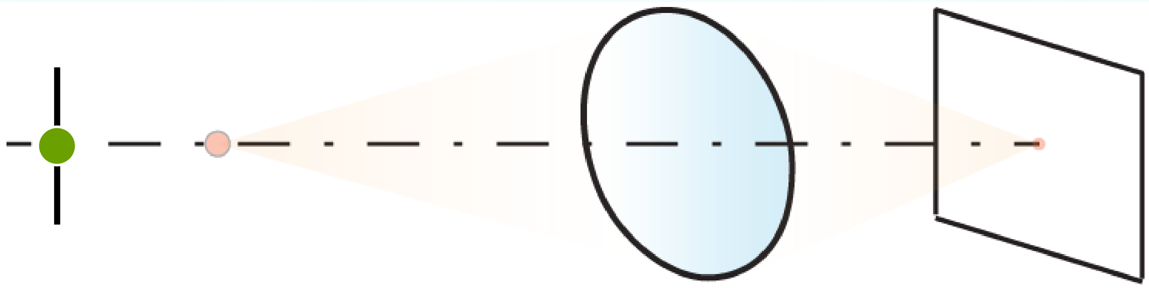

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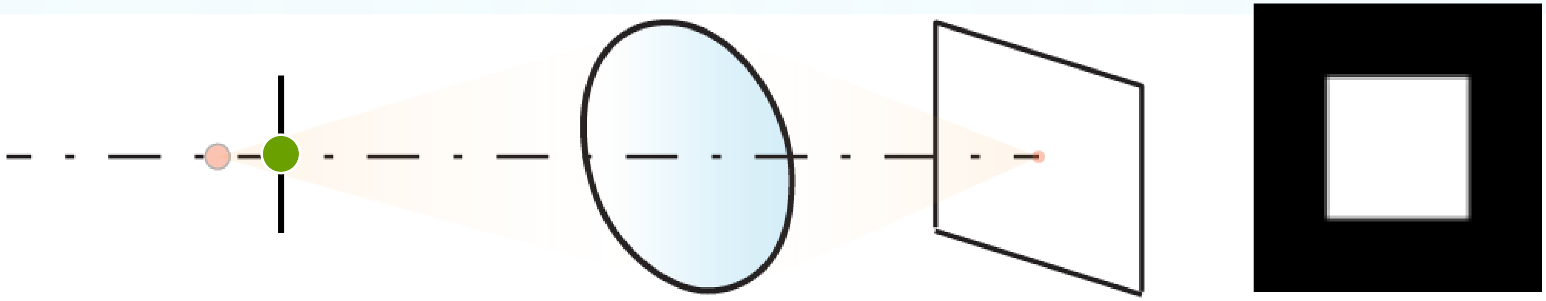

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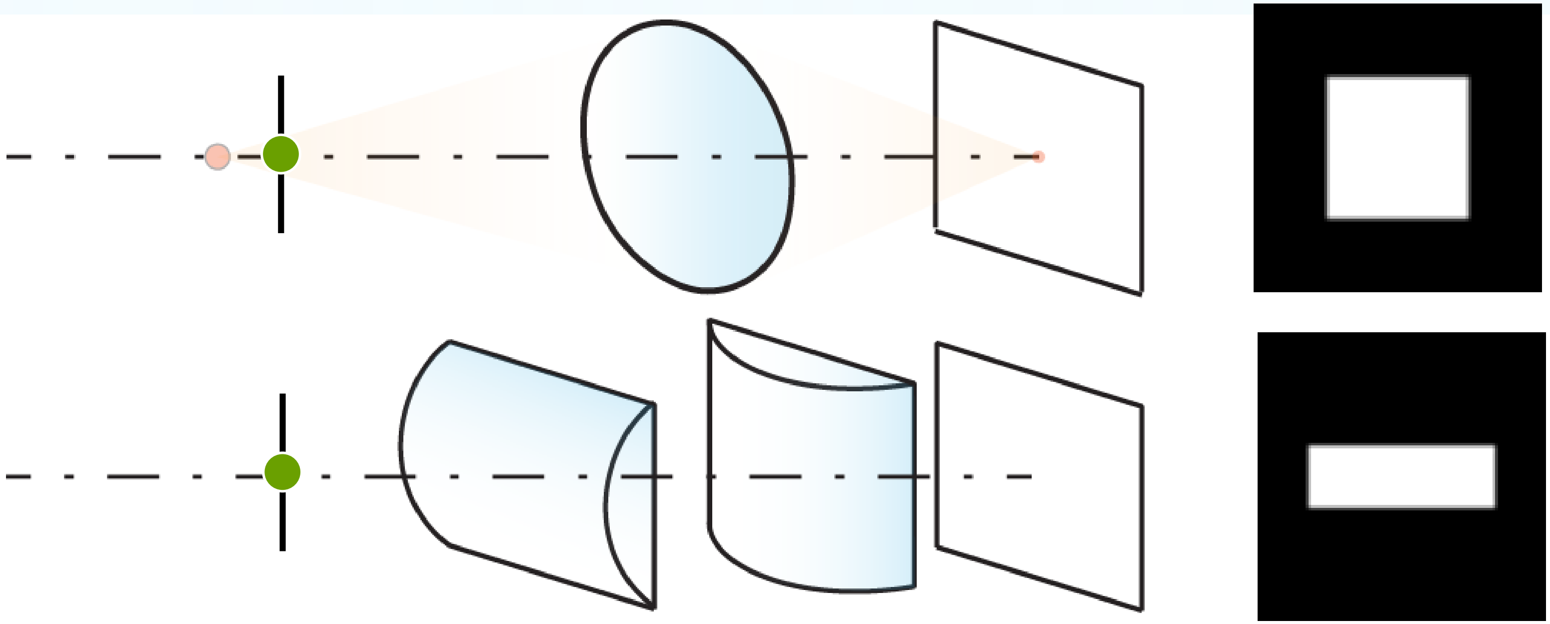

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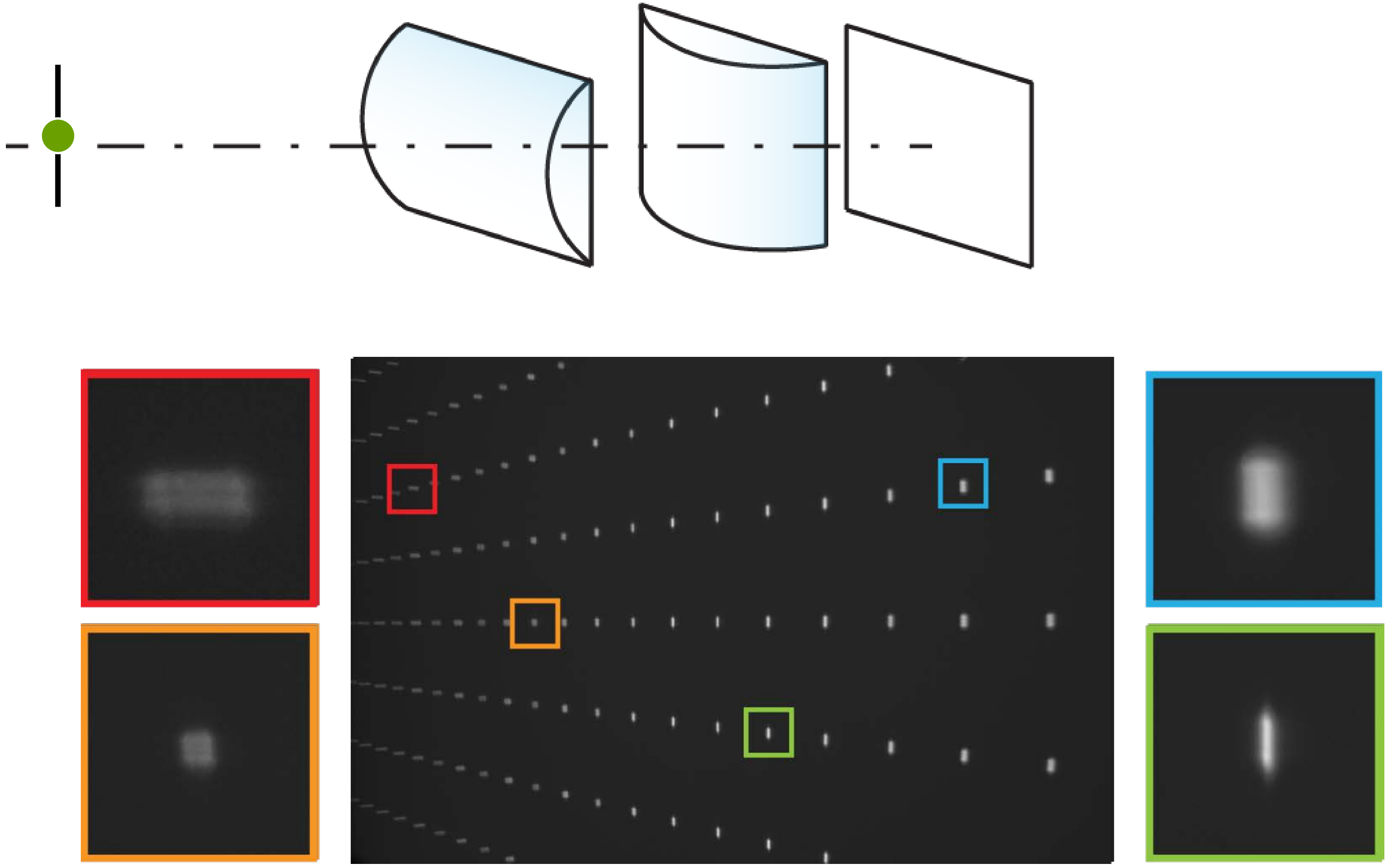
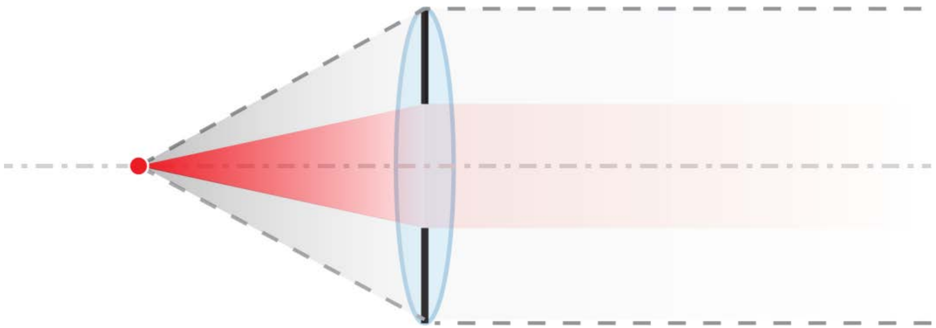


Light Throughput Comparisons



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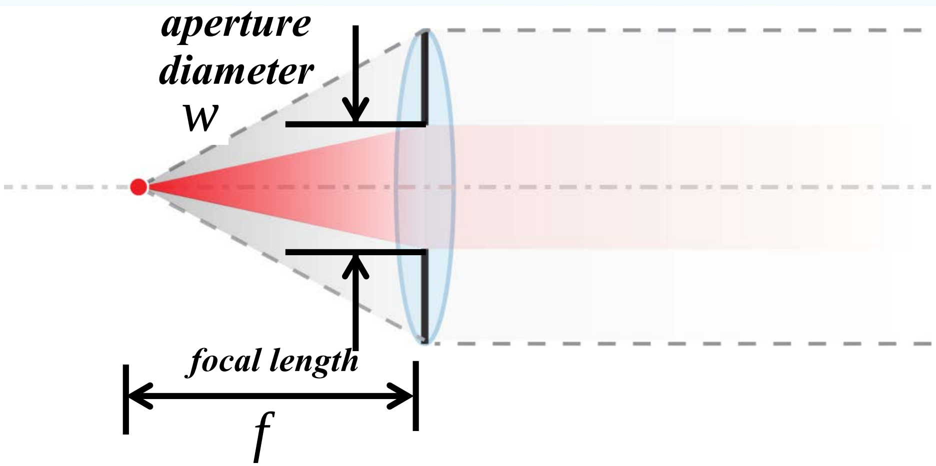

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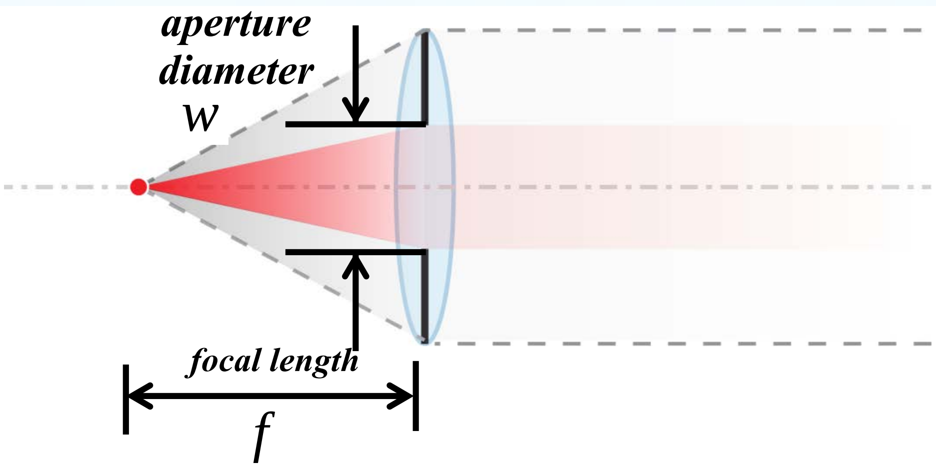

F-number: $Np = \frac{f}{w}$

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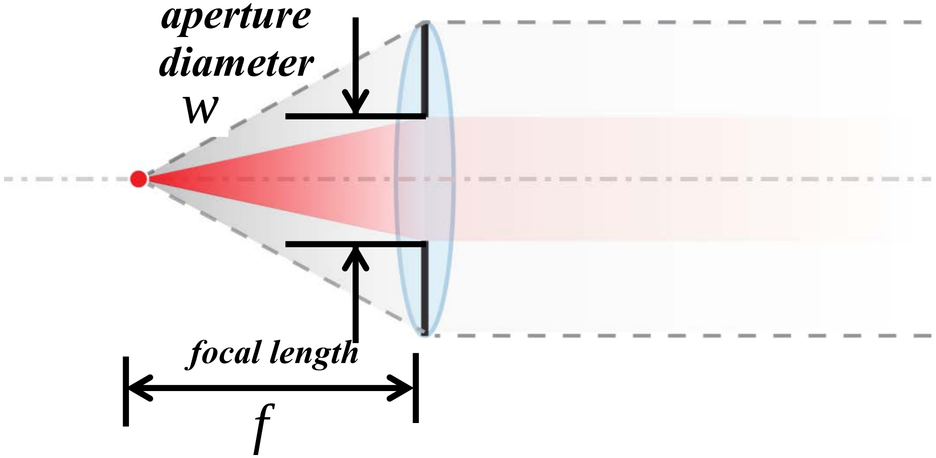

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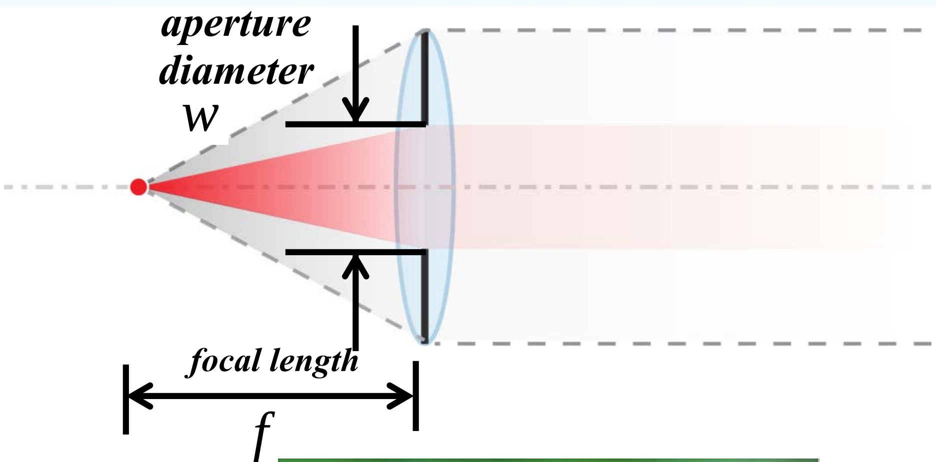

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Throughput Equivalent Spherical Lens (TESL)

- **Pinhole Lens: f-number N_p**

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B – source irradiance; $\cos^4\alpha$ – cosine-fourth law of illumination falloff.

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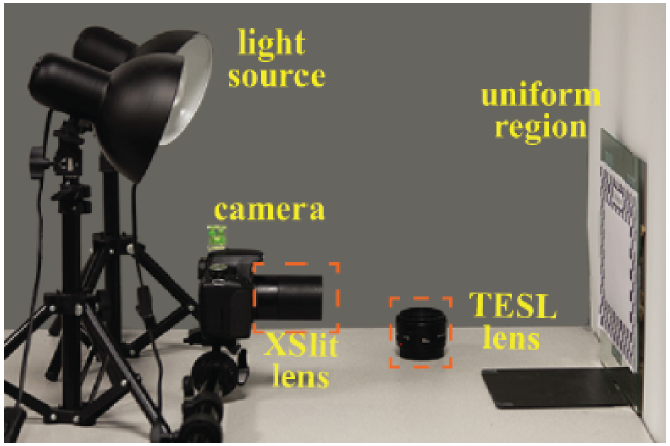

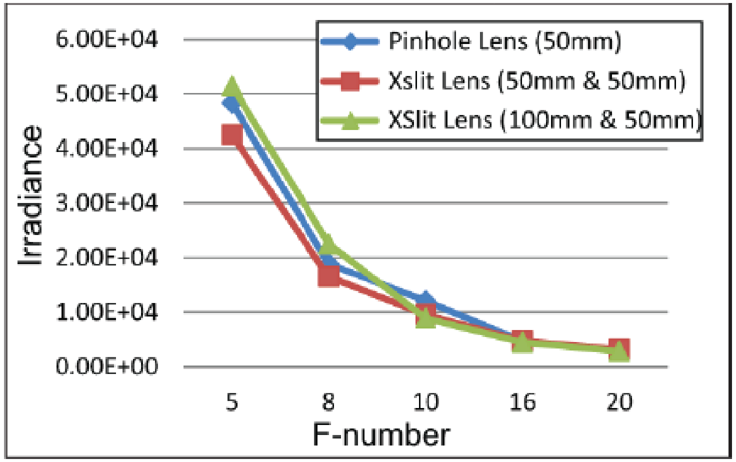

Finally…
Defocus Blur Comparisons

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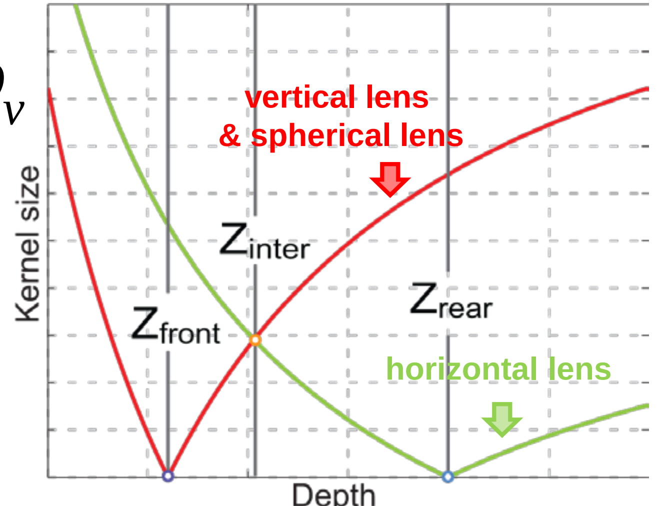

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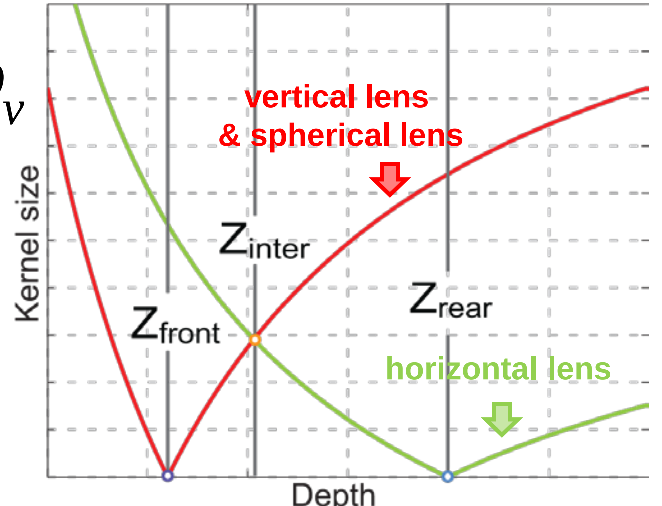

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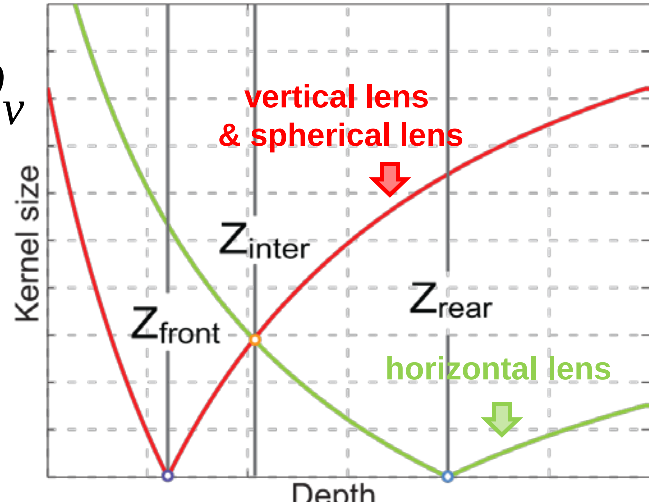
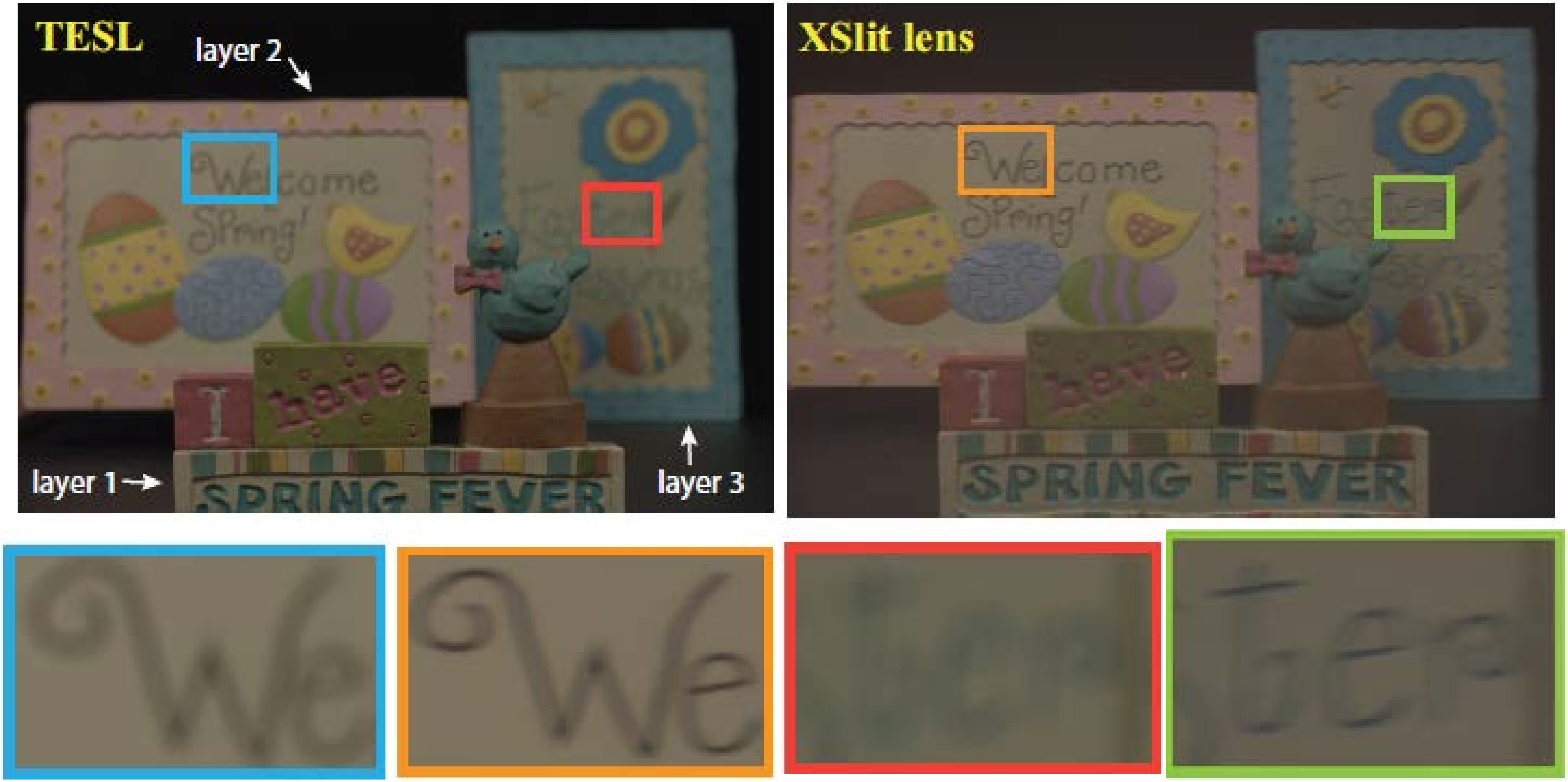
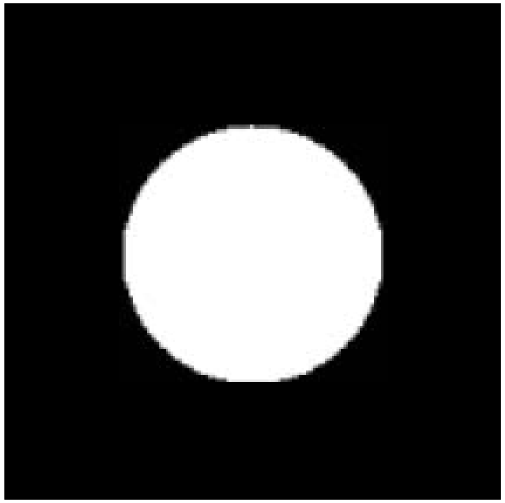


Experiments



Coded Aperture

- Coded Aperture Imaging
  - Produce invertible PSFs for depth estimation and extended Depth-of-Field (DoF)



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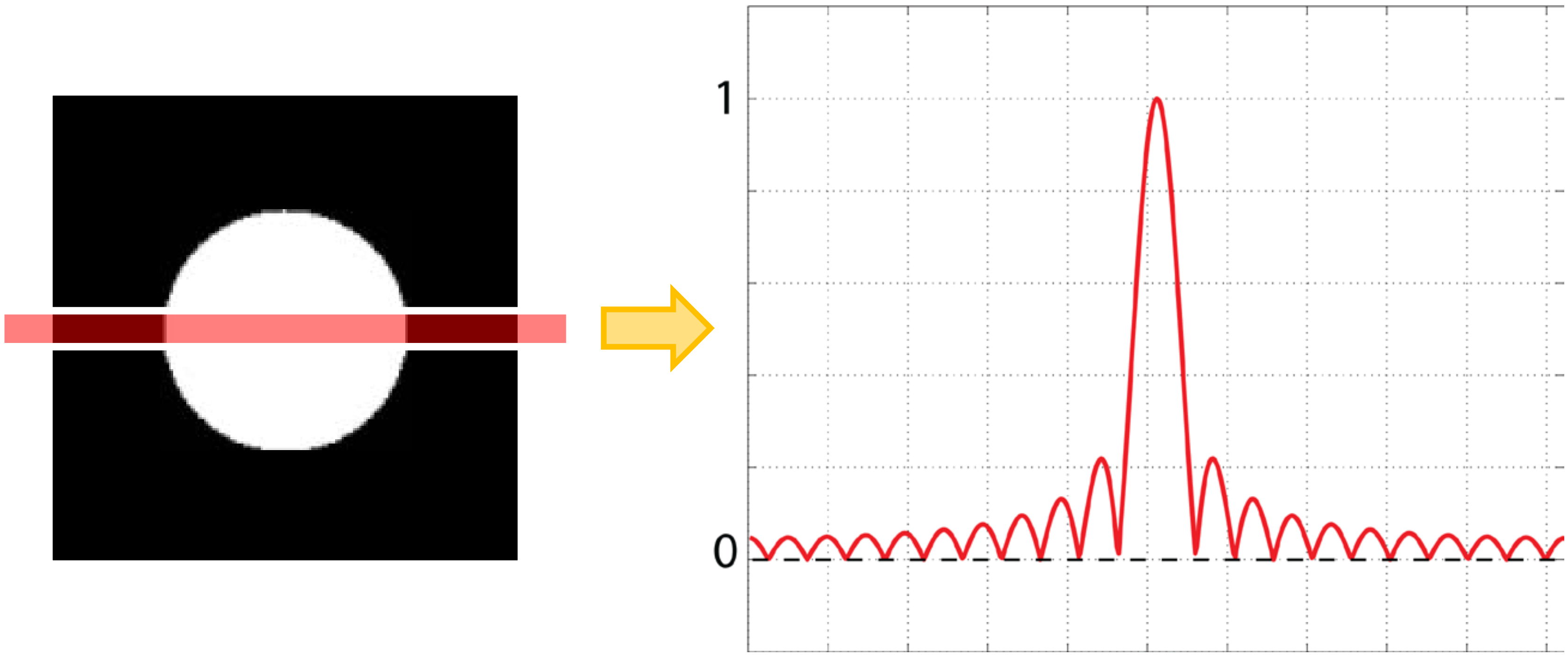

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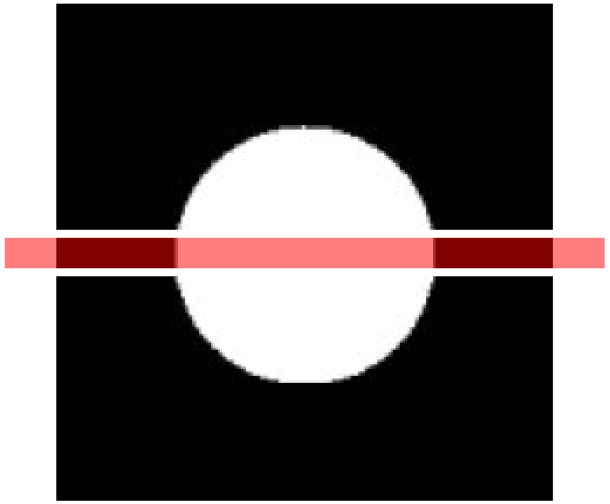

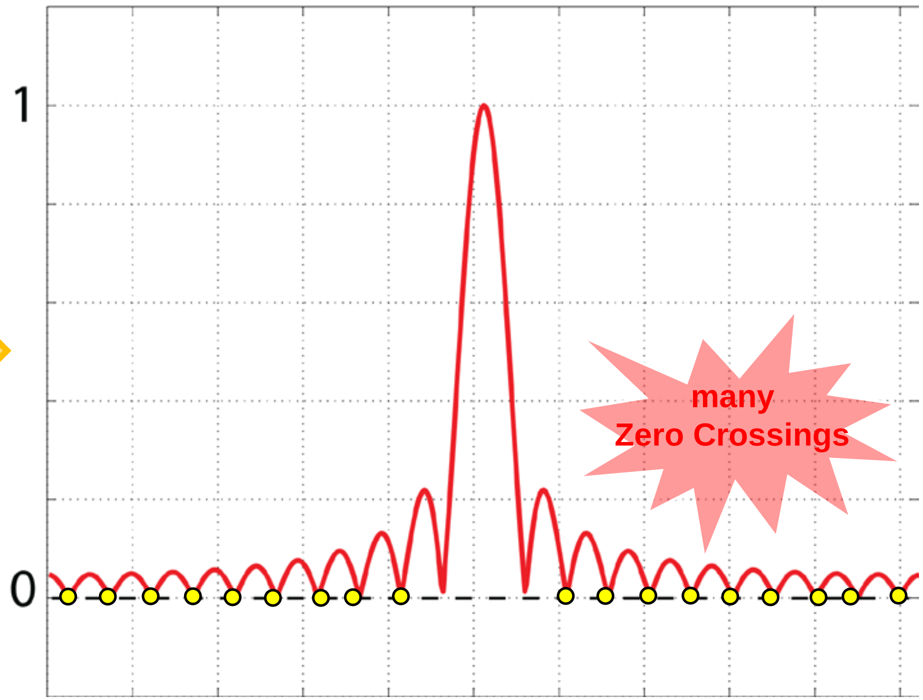

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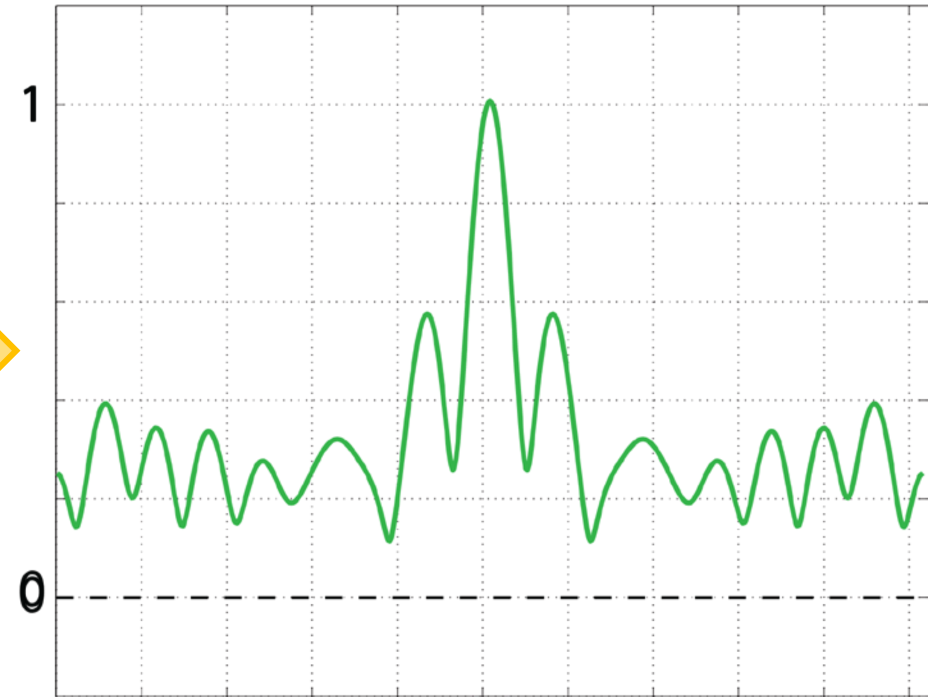


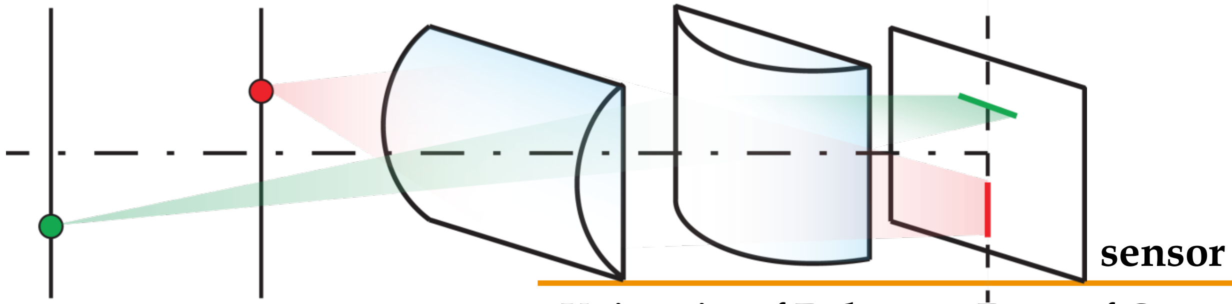
Coded Pattern Design

- Code Design Dilemma
  - Better depth discrepancy: Code with zero crossings [Levin ’07]
  - Robust deconvolution: Code that is broadband [Veeraraghavan ‘07]
  - One possible solution: Capture twice, each with a different coded pattern [Zhou ’09]

- Our Solution: Dual Aperture Coding

XSlit Coded Imaging

- **XSlit defocus is formed by convolving two orthogonal 1D kernels**



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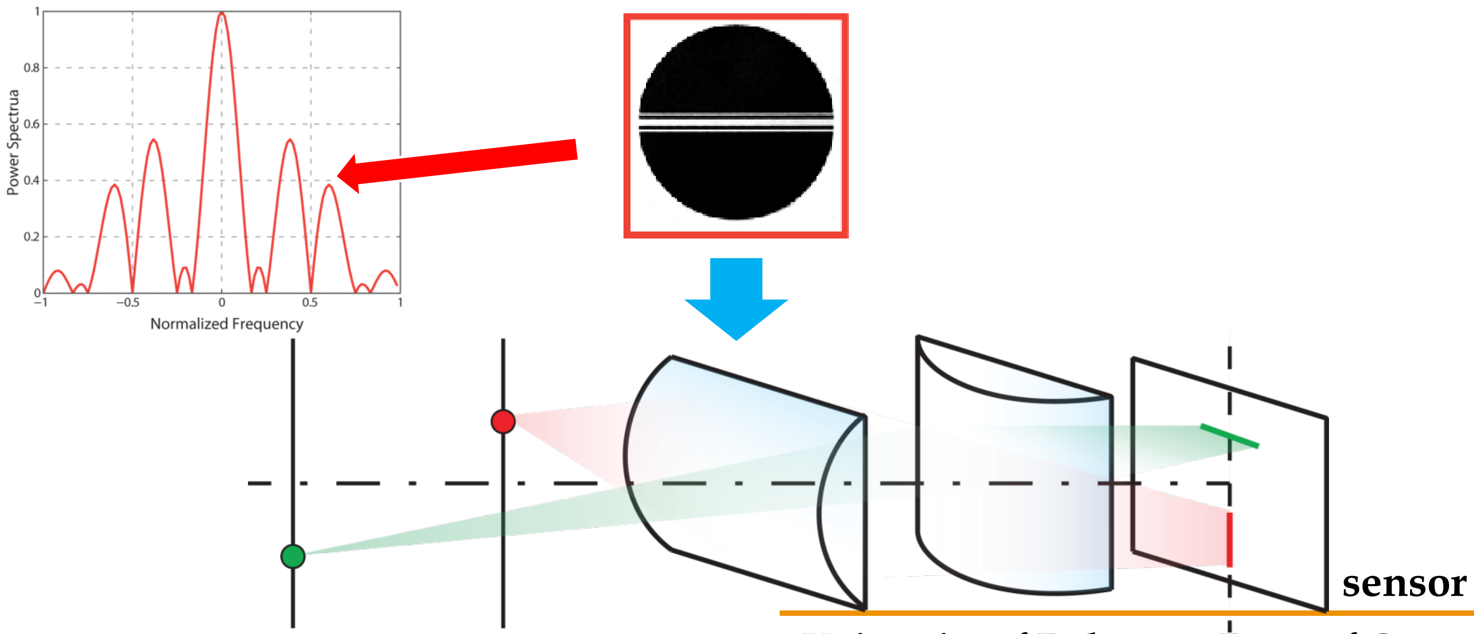
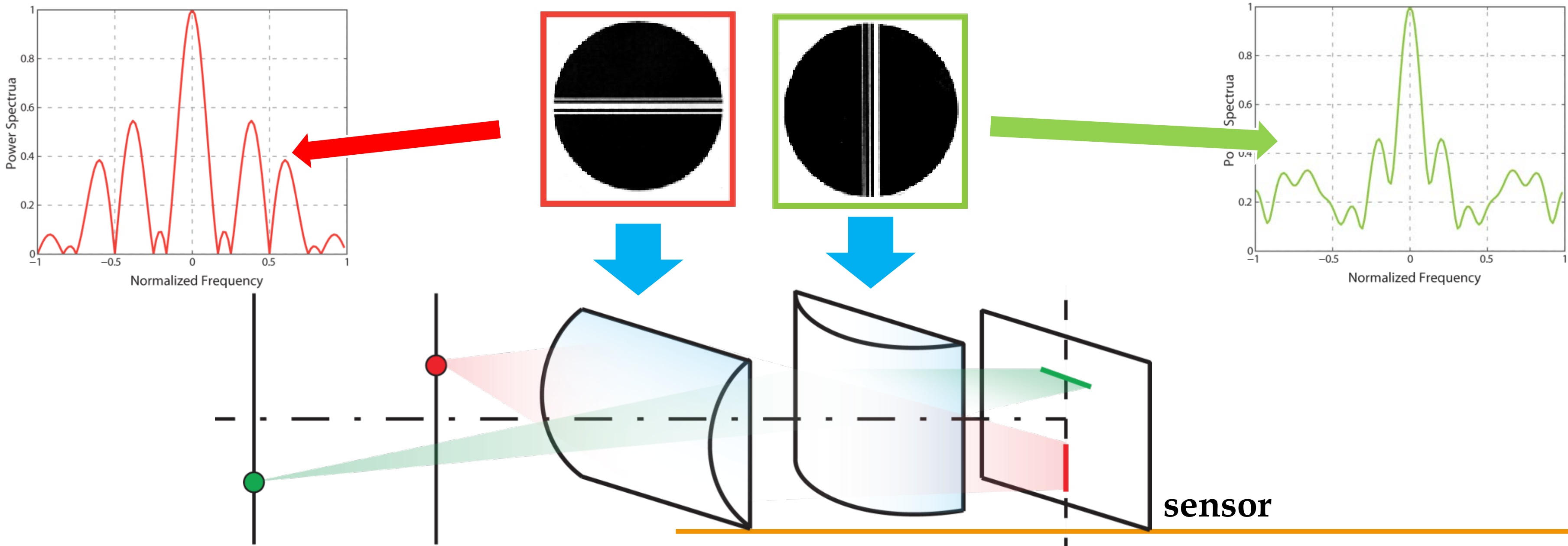


XSlit Coded Imaging

- XSlit defocus is formed by convolving two orthogonal 1D kernels
  - Vertical aperture: Broadband for invertibility
  - Horizontal aperture: high depth discrepancy

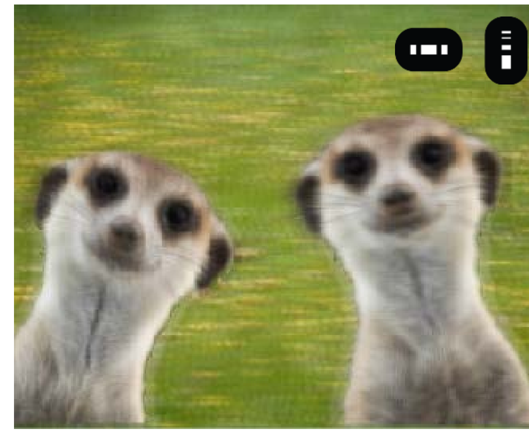


Synthetic Result

Input Shape Image

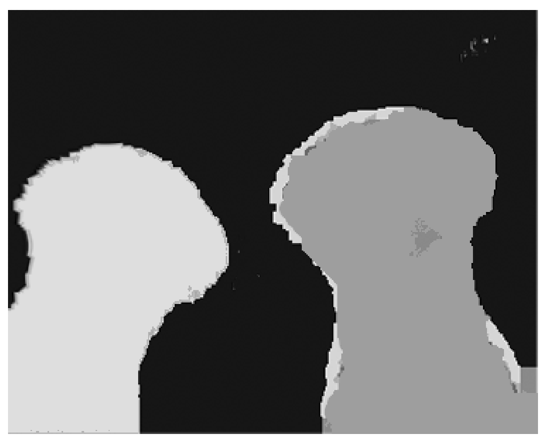
TESL Coded Aperture

XSlit Coded Aperture

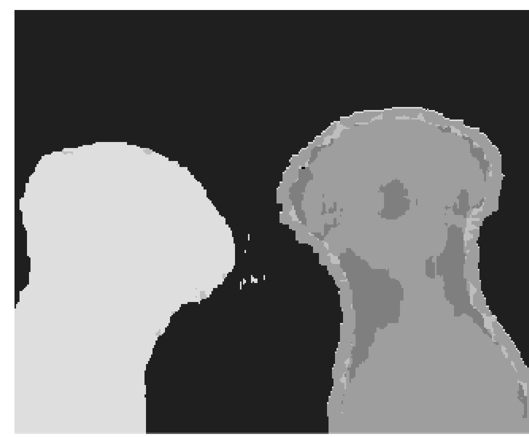


Ground Truth Depth Map

TESL Depth Map



Our Depth Map



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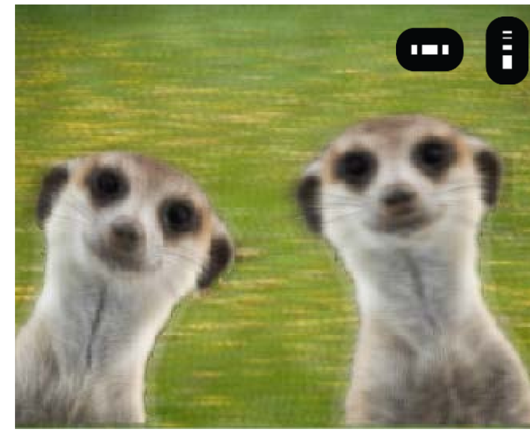


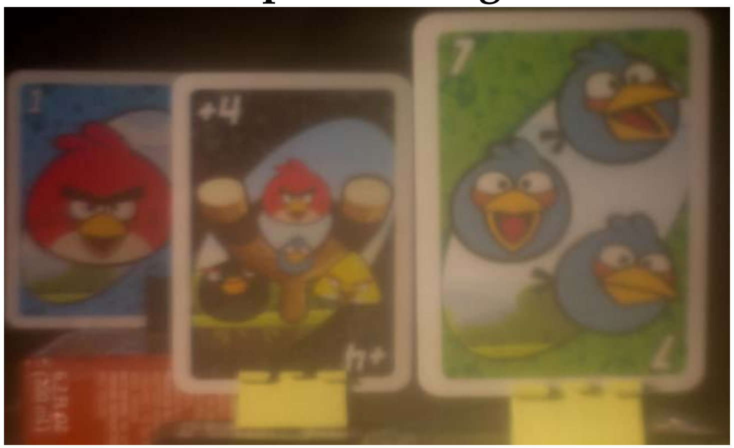
Ground Truth Depth Map

TESL Deconvolution Result

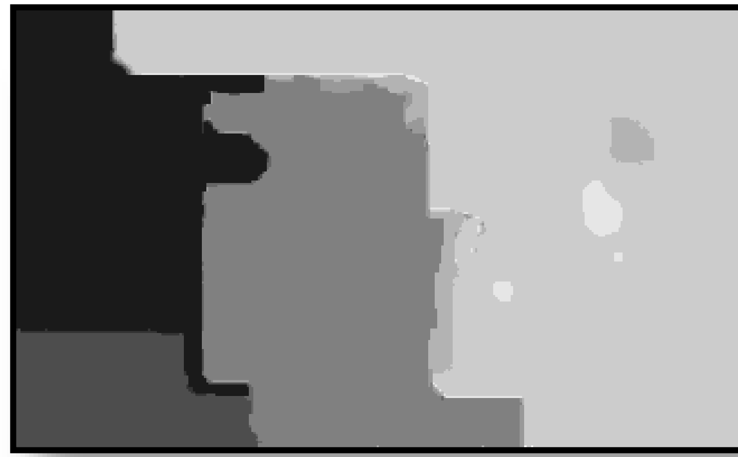
Our Deconvolution Result

Real Result

Captured Image



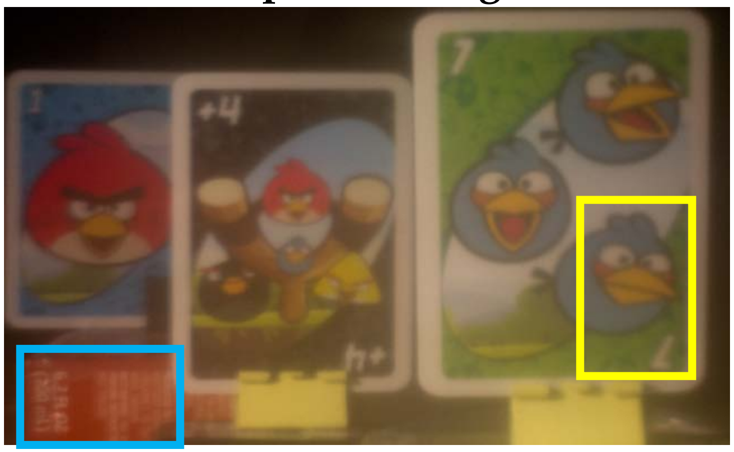
Recovered Depth Map



Deblurred Result

Real Result

Captured Image



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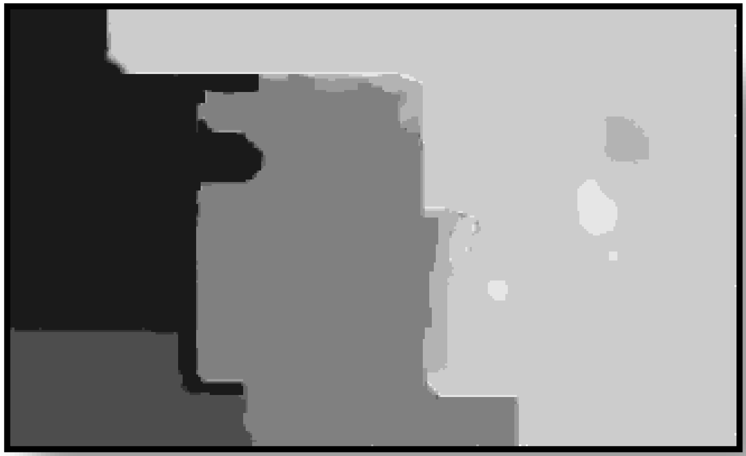

Deblurred Result

Conclusions

- Non-centric cameras can be useful (even with lenses)!
- A ray-geometry framework that enables
  - Lens transform analysis
  - Aperture analysis
  - Defocus and light efficiency analysis
- XSlit Coded Aperture Imaging
  - Address the dilemma on code pattern designs

Future Work

- Sensor Front:
  - Alternative XSlit lens designs
  - Alternative LF-camera designs
- Algorithm Front:
  - Use ray geometry to model Seidel aberration [Tang and Kutulakos '13]
  - Exploit the shape of the blur kernels
  - Other XSlit imaging properties
- Scene-aware Coded Aperture

Acknowledgement

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Danke schön!

Thank You !