Tracking Interacting Objects Optimally Using Integer Programming

ECCV'14

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Problem

- Tracking two classes of objects that interact

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  - One class of object (i.e., container) “contain” the other (i.e., containee)

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Cars-People

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People-Bags

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People-Bags

Players-Basketball

Conventional Tracking Method

- Track Objects Sequentially

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  - Track container first, and containee second

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Detection

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Sequential Tracking Using KSP [1]

[1] Multiple Object Tracking using K-Shortest Paths Optimization, J. Berclaz, F. Fleuret, E. Türetken, P. Fua. PAMI 2011

Our Tracking Method

- Tracking Objects Simultaneously

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  - Intuition: the evidence of one object may reveal the presence or absence of another

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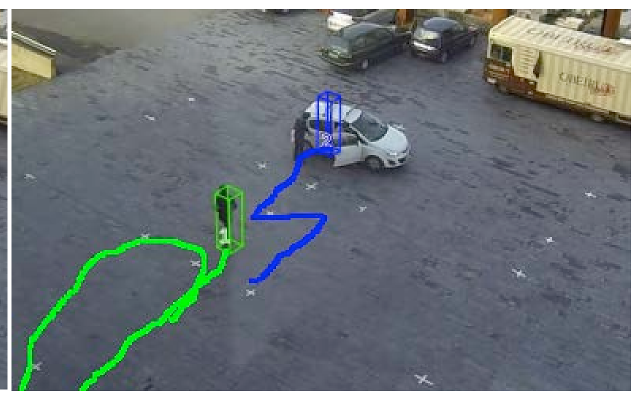
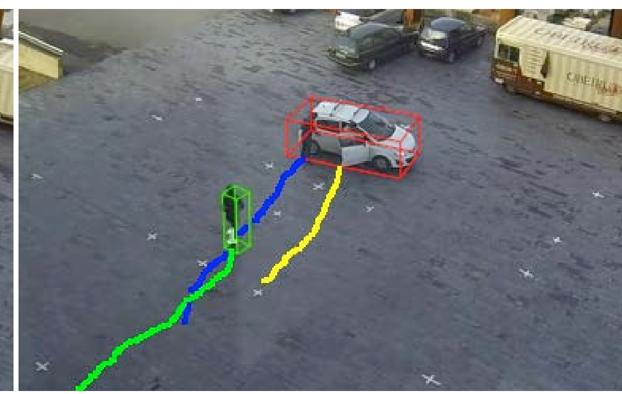


Sequential Tracking Using KSP



Ours

Our Tracking Method

- Even better…

Our Tracking Method

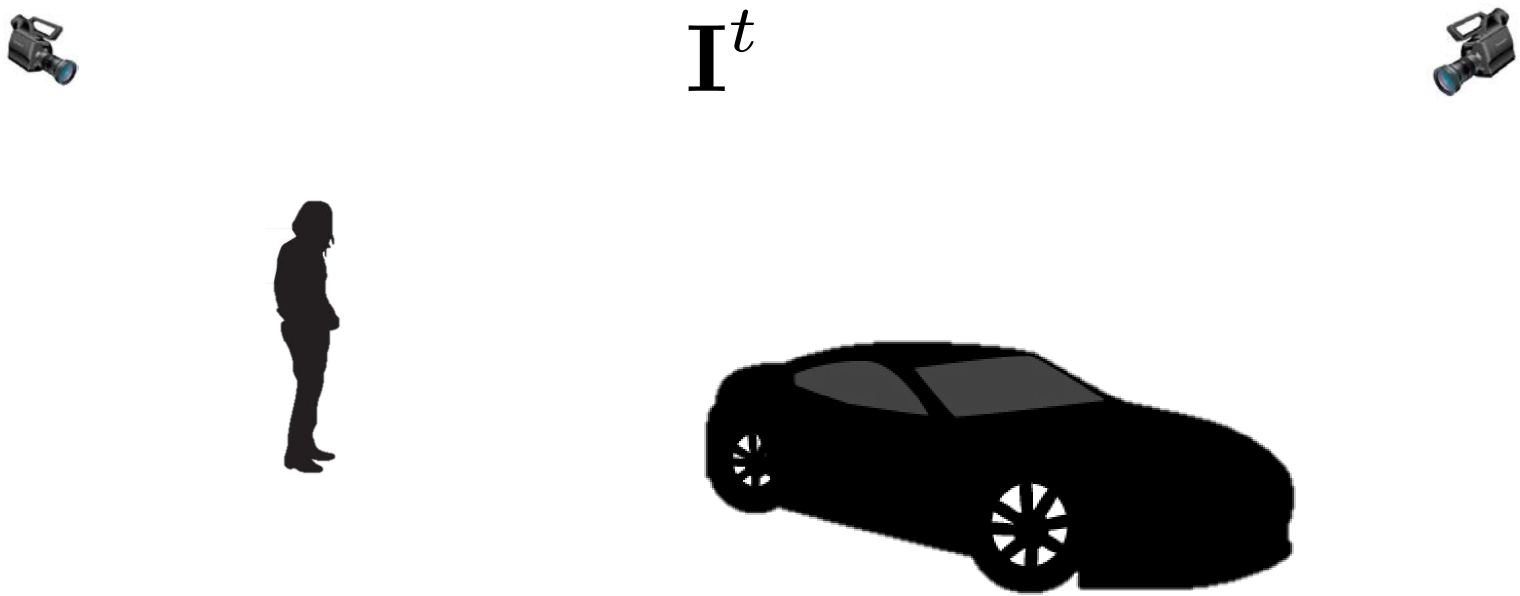
- Even better…
  - Track the Invisible!
    - Track the basketball when it is possessed by players

Our Tracking Method

- Even better…
  - Track the Invisible!
    - Track the basketball when it is possessed by players
  - Prior knowledge to improve tracking
    - At most one basketball ball in the match

Our Tracking Method

Detection: Probability Occupancy Map (POM)



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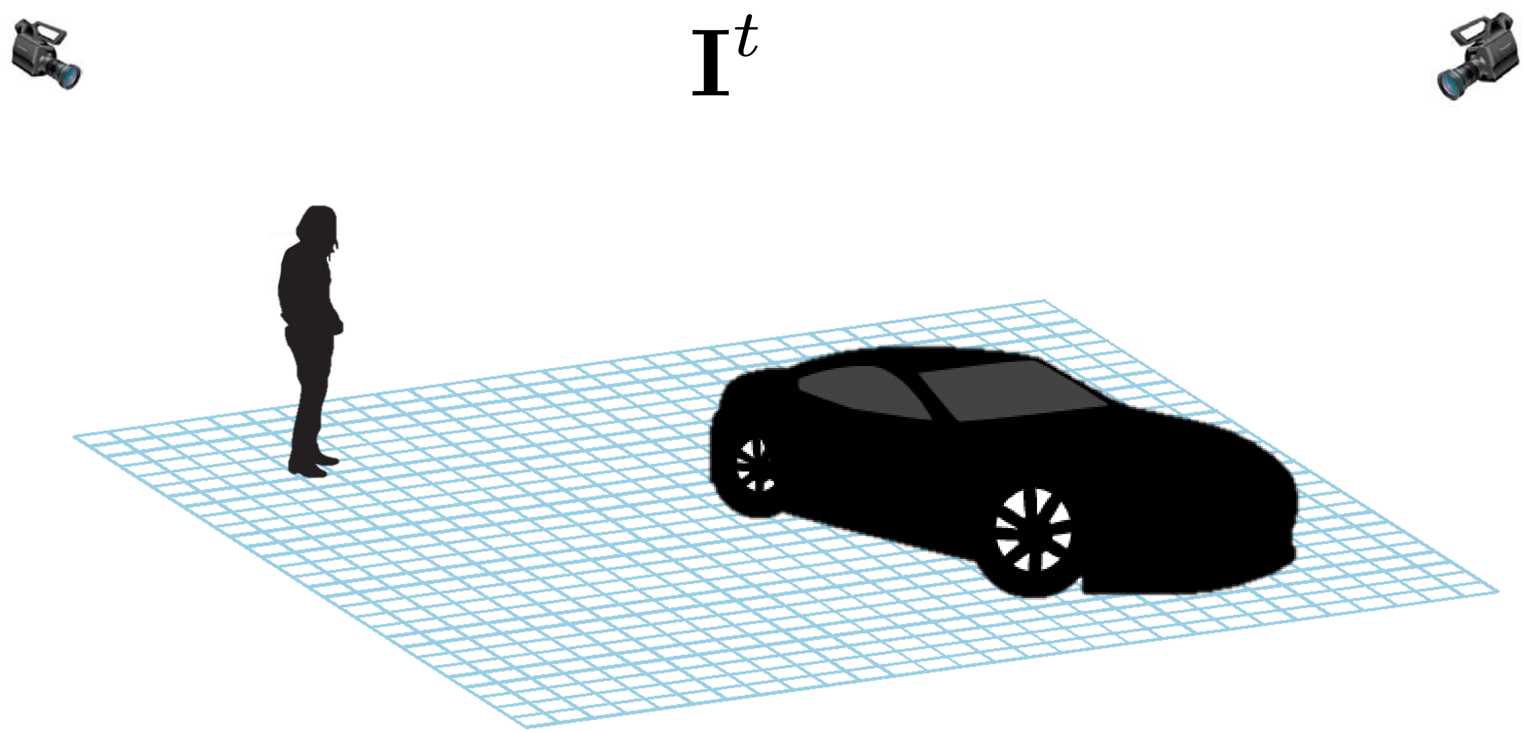

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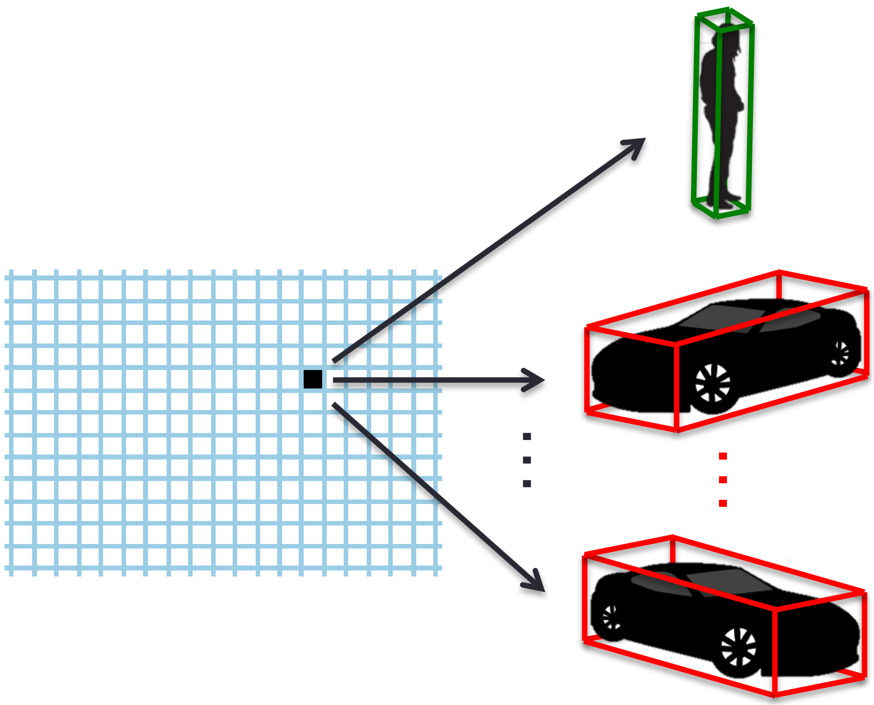


$X^t_{k_0} \in \{0, 1\}$

containe
e

$Y^t_{k_0} \in \{0, 1\}$

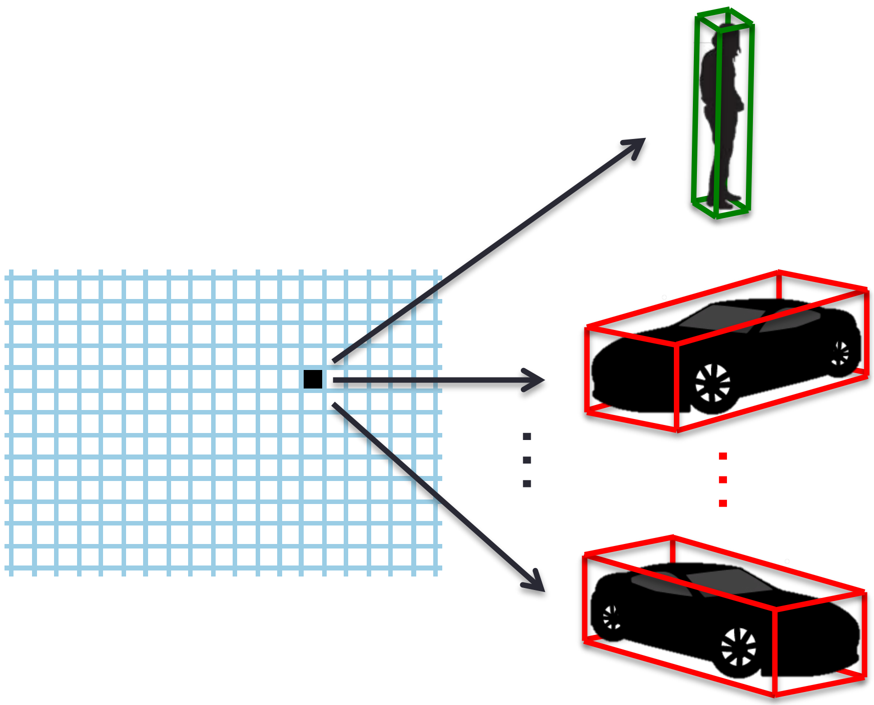
$\vdots$

$Y^t_{k_n} \in \{0, 1\}$

containe
r

k index $\underbrace{\text{location}}_{l(k)}$ and $\underbrace{\text{orientation}}_{o(k)}$!

Detection: Probability Occupancy Map (POM)



$$\rho^t_{k_0} = P(X^t_{k_0} = 1 \mid \mathbf{I}^t)$$

$$\beta^t_{k_0} = P(Y^t_{k_0} = 1 \mid \mathbf{I}^t)$$

$$\vdots$$

$$\beta^t_{k_n} = P(Y^t_{k_n} = 1 \mid \mathbf{I}^t)$$

Formulation: Objective

$$(\mathbf{x},\mathbf{y})^* = \operatorname*{argmax}_{(\mathbf{x},\mathbf{y})\in\mathcal{F}} P(\mathbf{X}=\mathbf{x},\ \mathbf{Y}=\mathbf{y} \mid \mathbf{I}) = \ldots$$

$$= \operatorname*{argmax}_{(\mathbf{x},\mathbf{y})\in\mathcal{F}} \sum_{t,k} \log\left(\frac{\rho_k^t}{1-\rho_k^t}\right) x_k^t + \log\left(\frac{\beta_k^t}{1-\beta_k^t}\right) y_k^t$$

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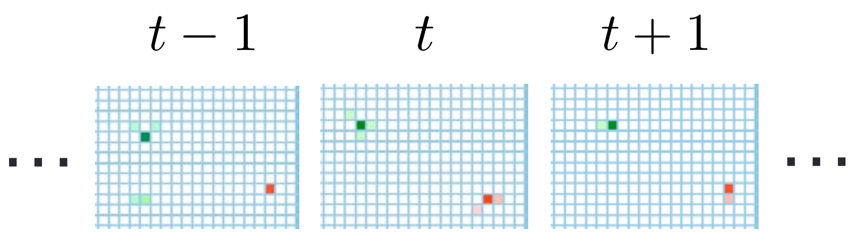

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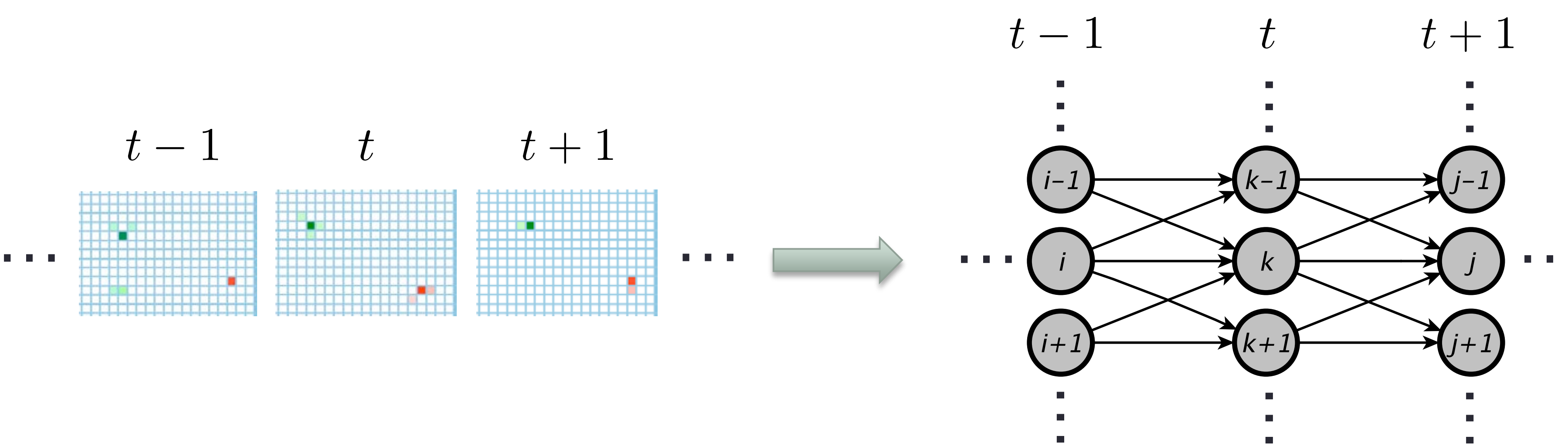

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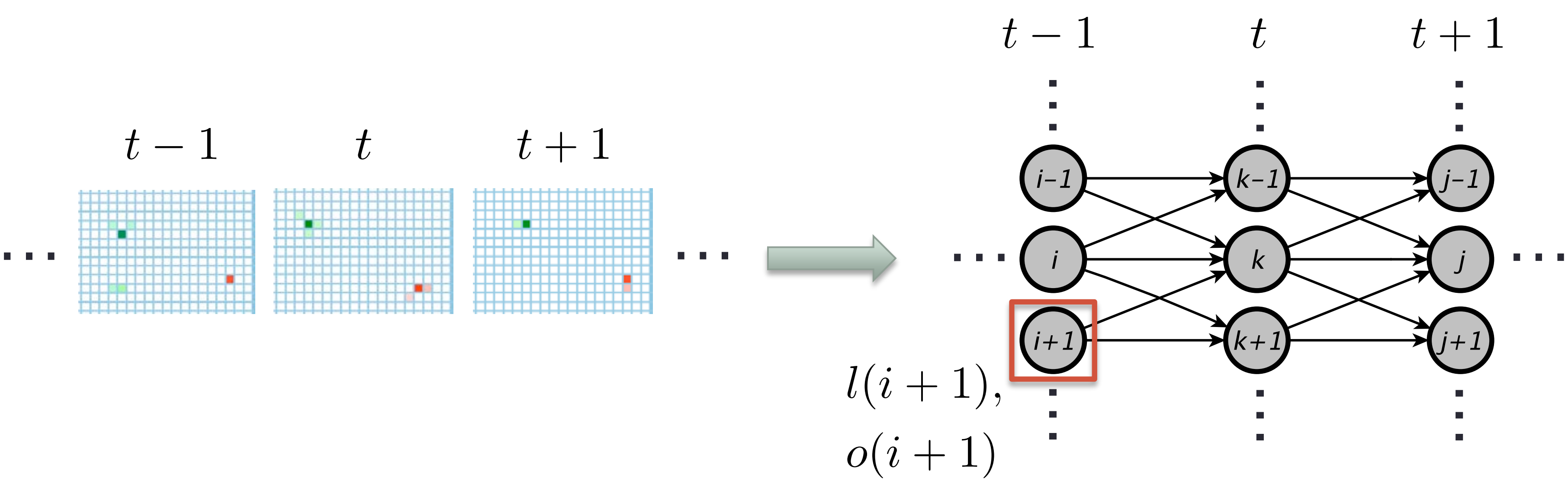

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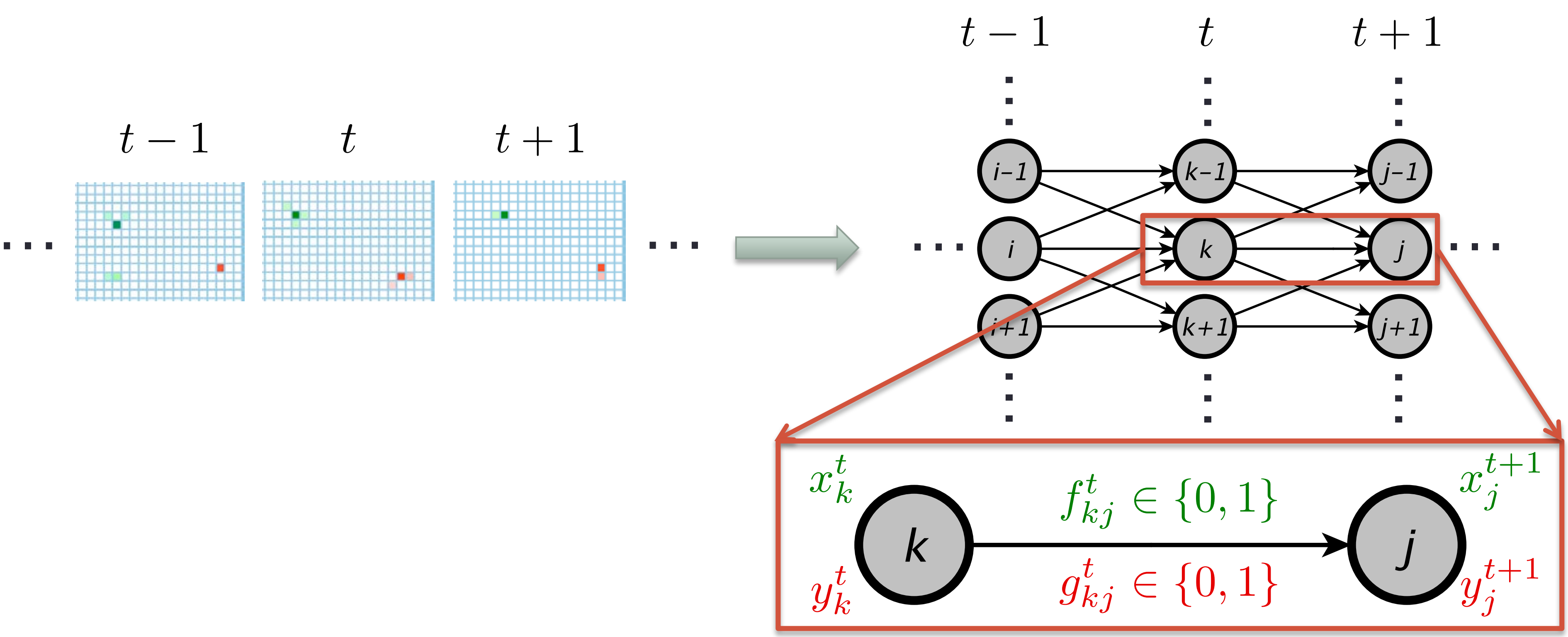

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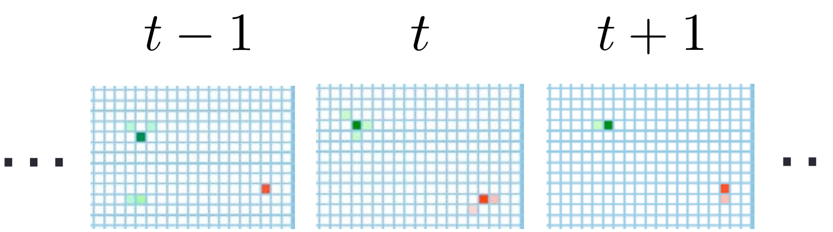

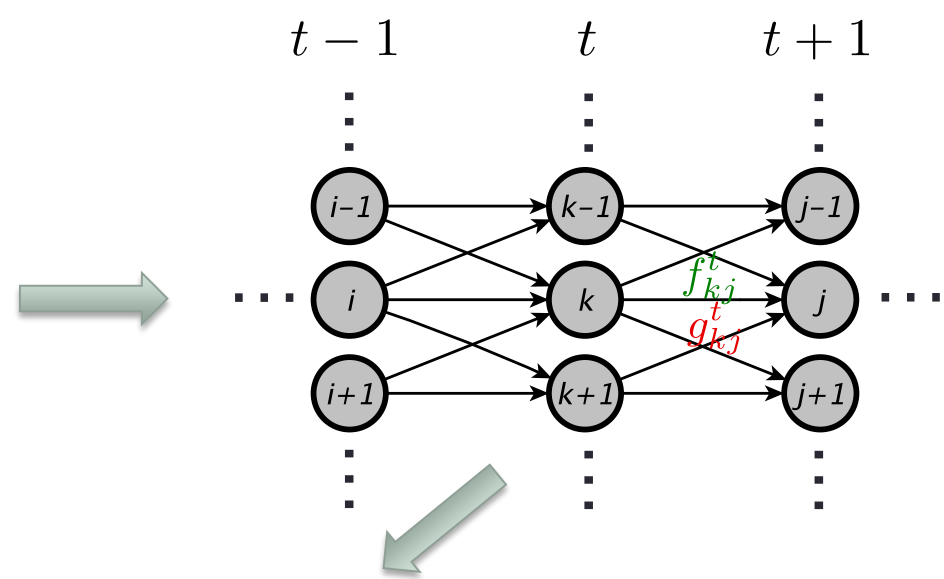

$$\operatorname*{argmax}_{(\mathbf{f},\mathbf{g})\in\mathcal{F}} \sum_{t} \sum_{j\in\mathcal{N}(k)} \sum_{k} \log\left(\frac{\rho_k^t}{1-\rho_k^t}\right) f_{kj}^t + \log\left(\frac{\beta_k^t}{1-\beta_k^t}\right) g_{kj}^t$$

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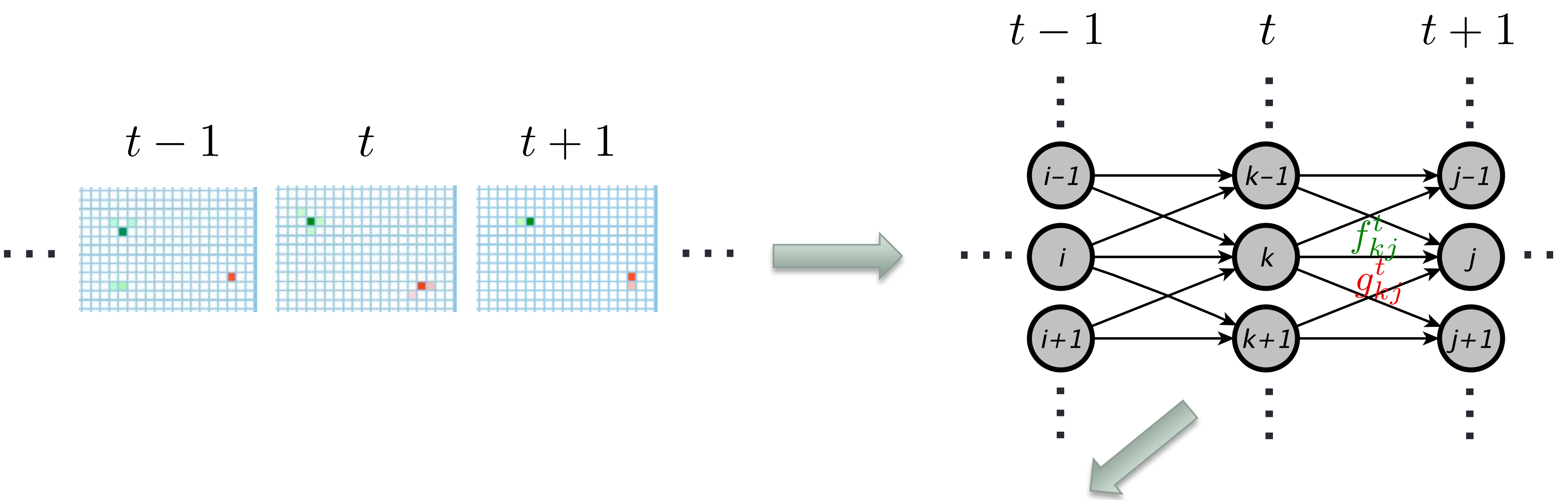

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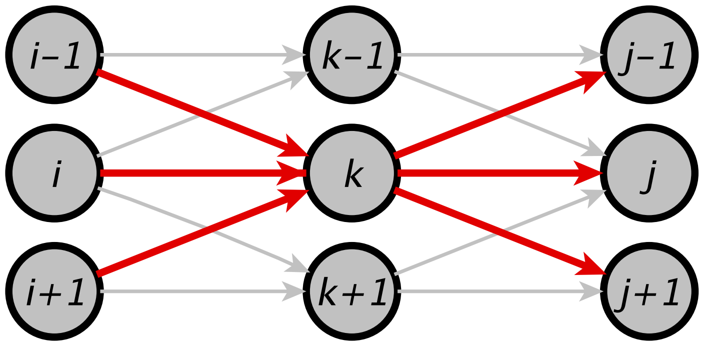
Formulation: Constraints

- Flow Conservation on Container Objects
- Spatial Exclusion on the ground
- ✓ Consistency of Interacting Flows
- ✓ Tracking the Invisible
- ✓ Additional Bound Constraint

Formulation: Constraints

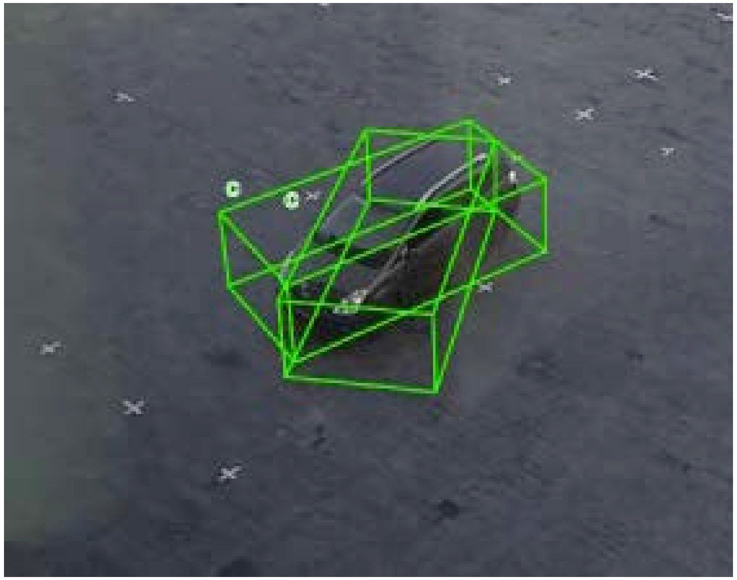
- Flow Conservation on Container Objects
  - Container Objects don't disappear



Formulation: Constraints

- Flow Conservation on Container Objects
- Spatial Exclusion on the ground
  - Objects don't overlap in the 3D space

✗



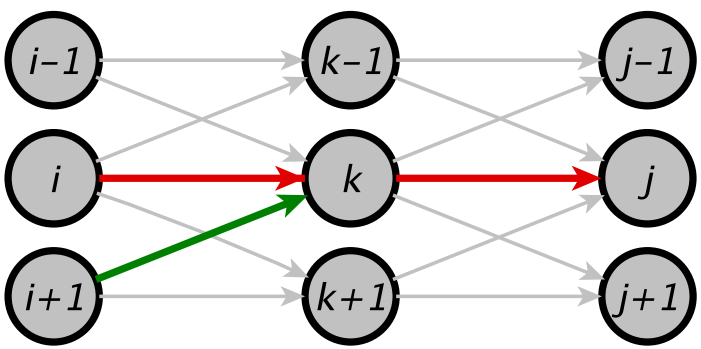
Formulation: Constraints

- Flow Conservation on Container Objects
- Spatial Exclusion on the ground
- Consistency of Interacting Flows
  - Models the interaction between objects

Formulation: Constraints

- Consistency of Interacting Flows

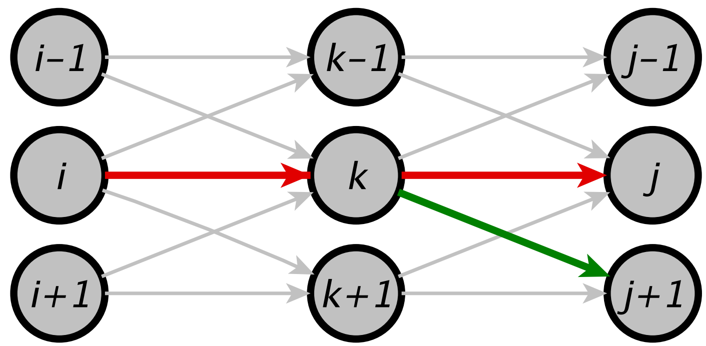
If a person disappears, he



Formulation: Constraints

- Consistency of Interacting Flows

If a person appears,



Formulation: Constraints

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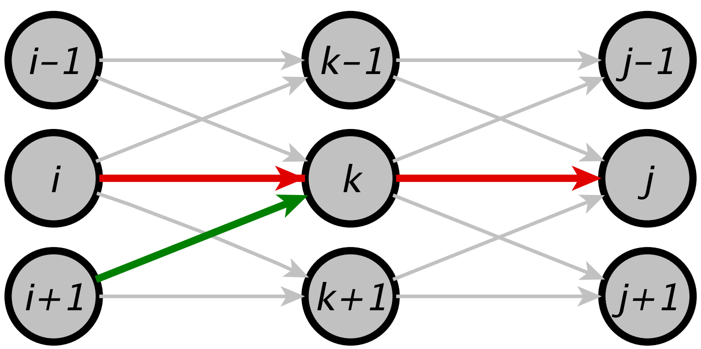
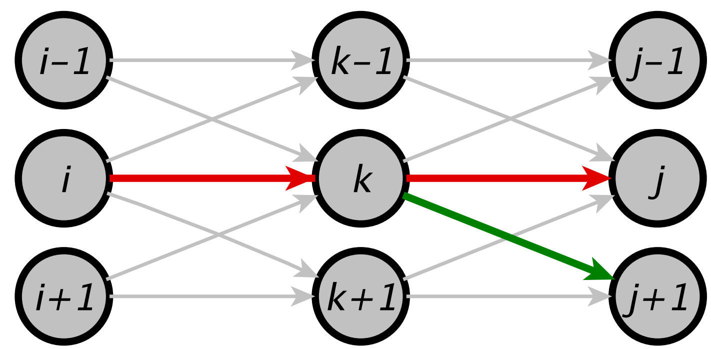


If a person appears,



$$-\sum_{\substack{m:l(k)=l(m),\\ i:m\in\mathcal{N}(i)}} g_{im}^{t-1} \le \sum_{i:k\in\mathcal{N}(i)} f_{ik}^{t-1} - \sum_{j\in\mathcal{N}(k)} f_{kj}^{t} \le \sum_{\substack{m:l(k)=l(m),\\ j\in\mathcal{N}(m)}} g_{mj}^{t}, \quad \forall t,k$$

Formulation: Constraints

- Flow Conservation on Container Objects
- Spatial Exclusion on the ground
- Consistency of Interacting Flows
- Tracking the Invisible
  - Track the invisible containee objects

Formulation: Constraints

- Tracking the Invisible – Counter Flow Variable h

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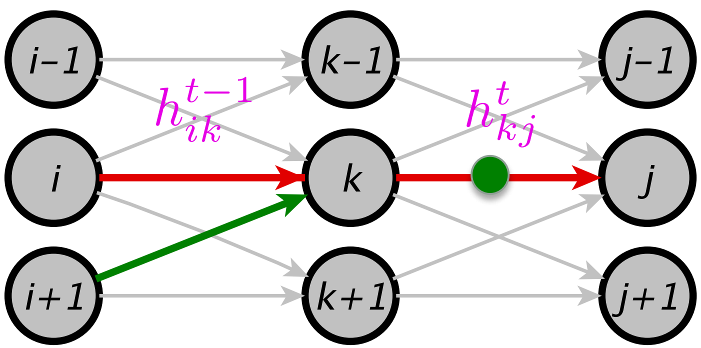
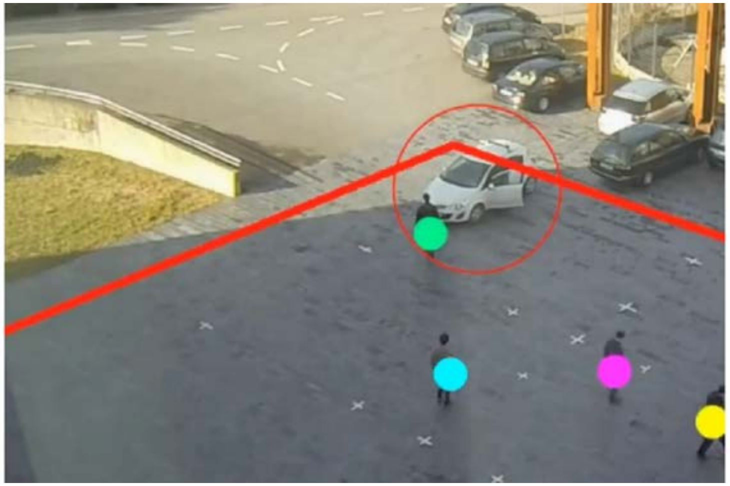
- Tracking the Invisible – Counter Flow Variable h

The person is in the car after he enters it.



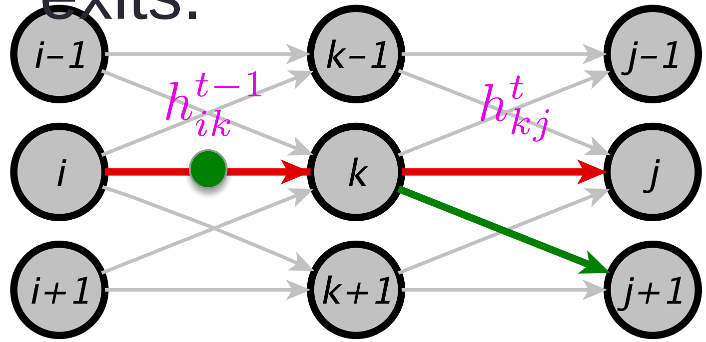
$$h_{kj}^{t} = h_{ik}^{t-1} + 1$$



Formulation: Constraints

- Tracking the Invisible – Counter Flow Variable h

The person was in the car before he exits.



$$h_{kj}^{t} = h_{ik}^{t-1} - 1$$

Formulation: Constraints

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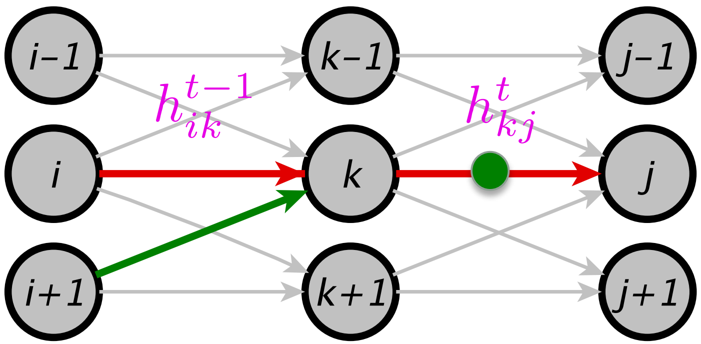

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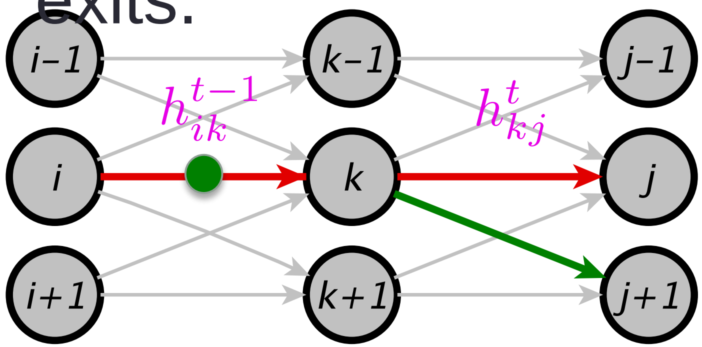

$$h_{kj}^{t} = h_{ik}^{t-1} - 1$$

$$\sum_{\substack{k:l=l(k)\\ j\in\mathcal{N}(k)}} h_{kj}^{t} = \sum_{\substack{k:l=l(k),\\ i:k\in\mathcal{N}(i)}} h_{ik}^{t-1} + \sum_{\substack{k:l=l(k),\\ i:k\in\mathcal{N}(i)}} f_{ik}^{t-1} - \sum_{\substack{k:l=l(k)\\ j\in\mathcal{N}(k)}} f_{kj}^{t}, \ \forall t, l$$

Formulation: Constraints

- Tracking the Invisible – Counter Flow Variable h

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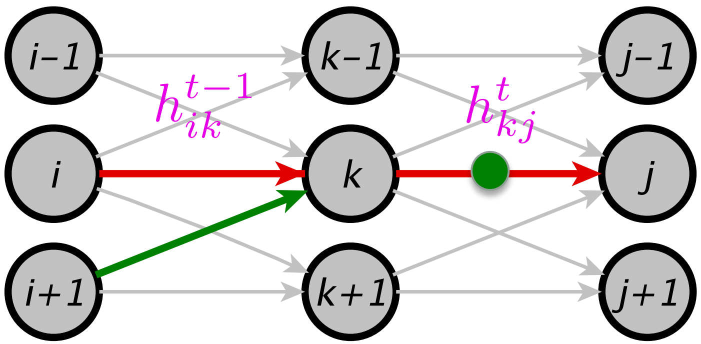

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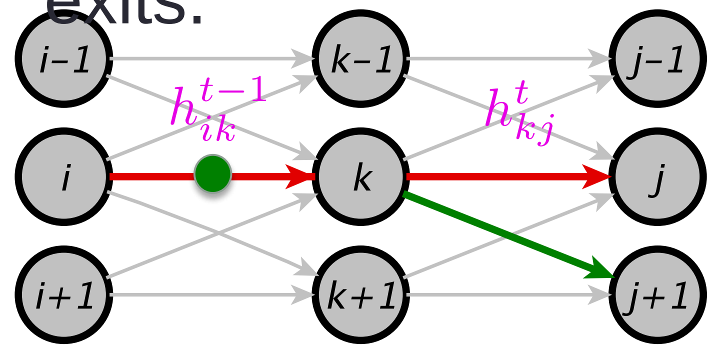

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Integers!

Formulation: Constraints

- Tracking the Invisible – Counter Flow Variable h

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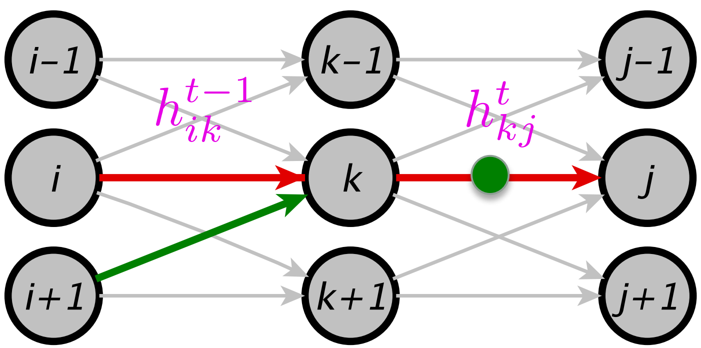

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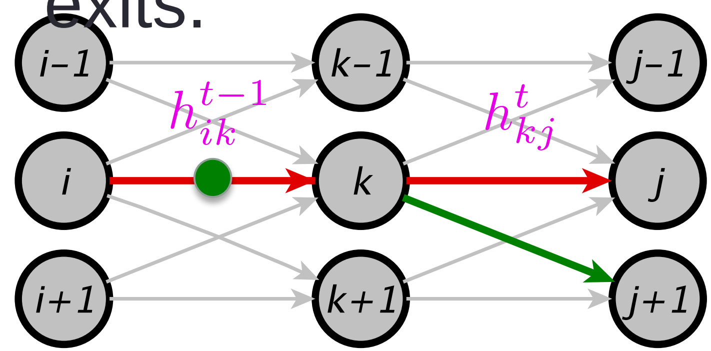

$$h_{kj}^{t} = h_{ik}^{t-1} - 1$$

Relax to fractional!

$$\sum_{\substack{k:l=l(k)\\ j\in\mathcal{N}(k)}} h_{kj}^{t} = \sum_{\substack{k:l=l(k),\\ i:k\in\mathcal{N}(i)}} h_{ik}^{t-1} + \sum_{\substack{k:l=l(k),\\ i:k\in\mathcal{N}(i)}} f_{ik}^{t-1} - \sum_{\substack{k:l=l(k)\\ j\in\mathcal{N}(k)}} f_{kj}^{t}, \quad \forall t, l$$

Integers!

Formulation: Constraints

- Flow Conservation on Container Objects
- Spatial Exclusion on the ground
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- Additional Bound Constraint

Formulation: Constraints

- Additional Bound Constraint
  - A Car carries limited number of people:

$$h_{kj}^{t} \leq c * g_{kj}^{t}, \quad \forall t, k, j$$

Formulation: Constraints

- Additional Bound Constraint
  - A Car carries limited number of people
  - At most one basketball in the match:

$$\sum_{\substack{v_k^t \in V(t),\\ j \in \mathcal{N}(k)}} h_{kj}^t + \sum_{\substack{v_k^t \in V(t),\\ j \in \mathcal{N}(k)}} f_{kj}^t \leq 1 \ , \ \forall t$$

Formulation: Constraints

- Additional Bound Constraint
  - A Car carries limited number of people
  - At most one basketball in the match
  - A moving car has at least one driver:

$$h_{kj}^{t} \geq g_{kj}^{t}, \ \forall t, k, j : j \in \mathcal{N}(k), l(k) \neq l(j)$$

Mixed Integer Programming

- Objective:

$$\underset{(\mathbf{f},\mathbf{g})\in\mathcal{F}}{\operatorname{argmax}} \sum_{t} \sum_{j\in\mathcal{N}(k)} \sum_{k} \log\left(\frac{\rho_k^t}{1-\rho_k^t}\right) f_{kj}^t + \log\left(\frac{\beta_k^t}{1-\beta_k^t}\right) g_{kj}^t$$

  - Subject to
    - Flow Conservation on Container Objects
    - Spatial Exclusion on the ground
    - Consistency of Interacting Flows
    - Tracking the Invisible
    - Additional Bound Constraint

- Solved using Gurobi

Quantitative Results

- 6 baselines, 3 scenario, 15K+ frames

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Quantitative Results

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SSP | KSP–fixed | KSP–free | KSP–sequential | OURS–LP | OURS–MIP

Objects can appear anywhere

SSP: Globally-Optimal Greedy Algorithms for Tracking a Variable Number of Objects,
H. Pirsiavash, D. Ramanan, C. Fowlkes. CVPR 2011

KSP: Multiple Object Tracking using K-Shortest Paths Optimization,
J. Berclaz, F. Fleuret, E. Türetken, P. Fua. T-PAMI 2011

Quantitative Results

- 6 baselines, 3 scenario, 15K+ frames

Objects can appear on the boundary

Quantitative Results

- 6 baselines, 3 scenario, 15K+ frames

Track Car first, People second

Quantitative Results

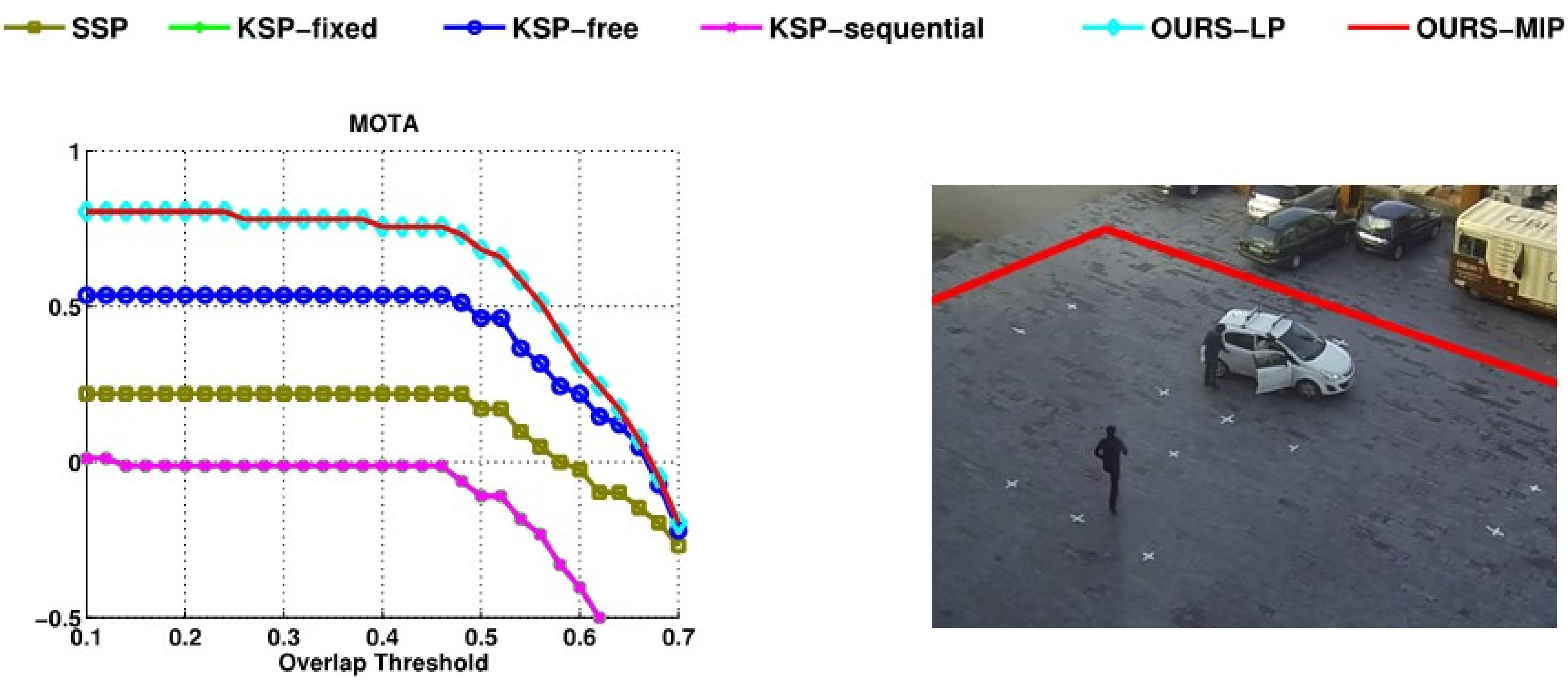
- 6 baselines, 3 scenario, 15K+ frames

SSP KSP–fixed KSP–free KSP–sequential **OURS–LP** OURS–MIP

Relax integrality constraint

Quantitative Results

- 6 baselines, 3 scenario, 15K+ frames



People-Car

Quantitative Results

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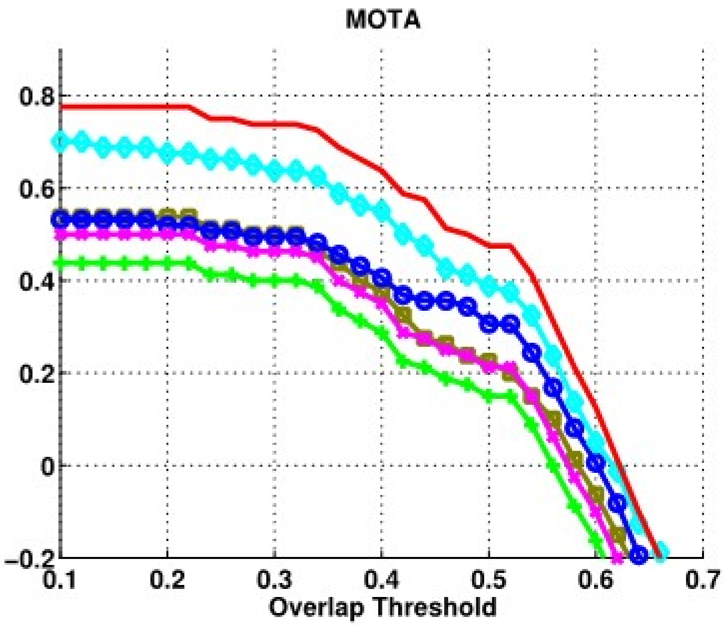

PETS2006

Conclusion

- Tracking interacting objects using network flow, achieved by f and g .

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- Tracking the invisible, achieved by counter flow variable h .

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Conclusion

- Tracking interacting objects using network flow, achieved by f and g .
- Tracking the invisible, achieved by counter flow variable h .
- Solve MIP to obtain the global optimum
- Future Work:
  - More scenarios: tracking crowd of people
  - More classes of objects
  - Re-Identification

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http://cvlab.epfl.ch/TrackInteractObj