

# Homotopska teorija tipov

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3.000.000 článkov

100.000 článkov/leto



3.000.000 člankov

100.000 člankov/leto

```
end.
```

```
Definition S1_decode (x:S1) : S1_code x -> (base = x).
```

```
Proof.
```

```
  revert x; refine (S1_ind (fun x => S1_code x -> base = x) looptothe _).
```

```
  apply path_forall; intros z; simpl in z.
```

```
  refine (transport_arrow _ _ _ @ _).
```

```
  refine (transport_paths_r loop _ @ _).
```

```
  rewrite transport_S1_code_loopV.
```

```
  destruct z as [[In] | | [In]]; simpl.
```

```
  by apply concat_pV_p.
```

```
  by apply concat_pV_p.
```

```
U:--- Circle.v      76% (191,0)  Git-master (Coq Script(1-) Holes Wrap)
```

```
1 subgoals, subgoal 1 (ID 150)
```

```
H : Univalence
```

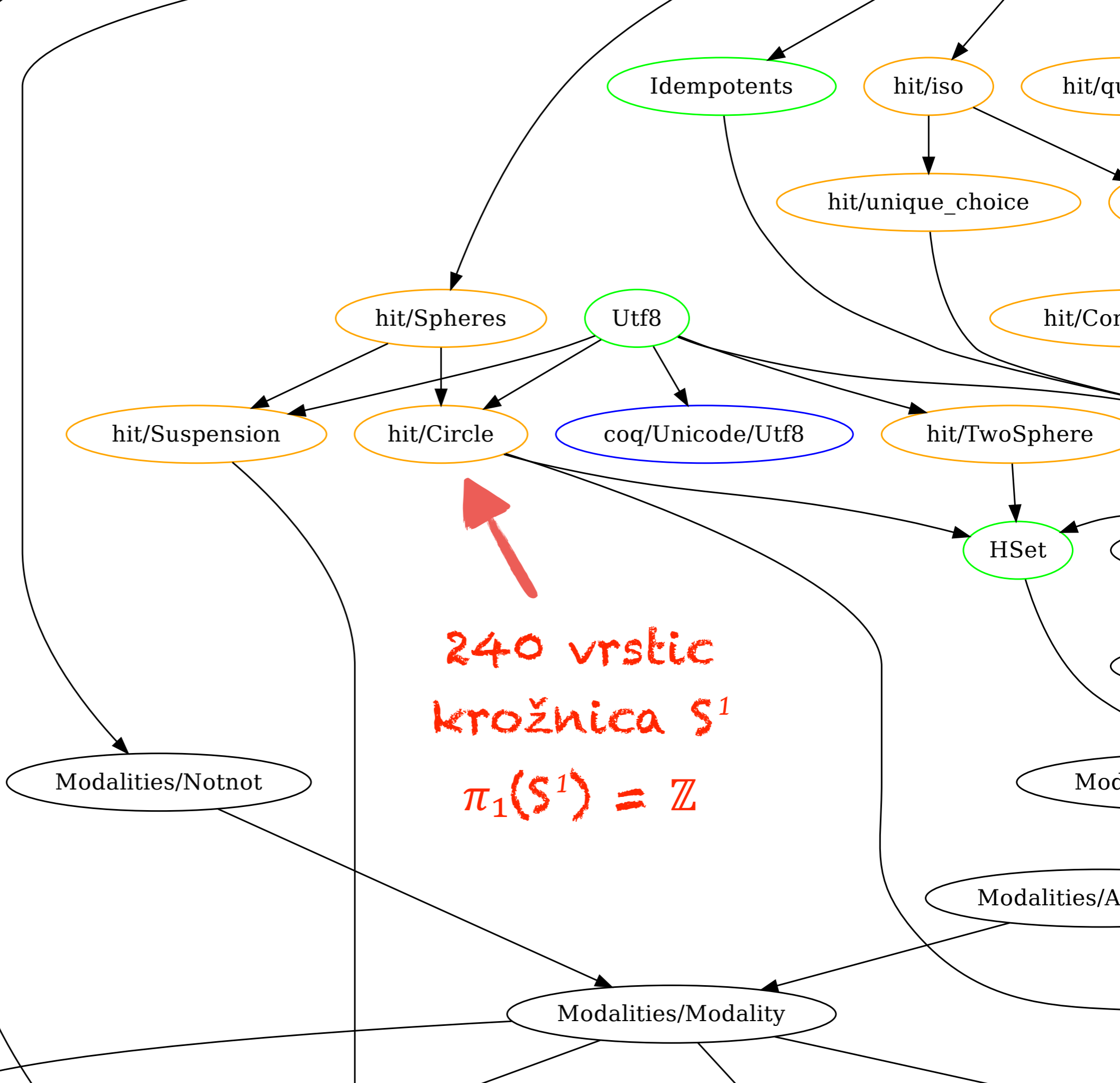
```
z : Int
```

```
=====
transport (paths base) loop (looptothe (transport S1_code loop^ z)) =
looptothe z
```

```
U:%%- *goals*      All (7,0)      (Coq Goals Wrap)
```

Tip	Logika	Množice	Homotopija
<b><math>A</math></b>	izjava	množica	prostor
<b><math>a : A</math></b>	dokaz	element	točka
<b><math>B(x)</math></b>	hipotetični dokaz	družina množic	vlaknenje
<b><math>Id_A</math></b>	enakost	$\{ (x,x) \mid x \in A \}$	prostor poti $A^{[0,1]}$





# Homotopy Type Theory

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# Homotopy Type Theory

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600 strani  
20+ avtorjev  
prosto dostopna

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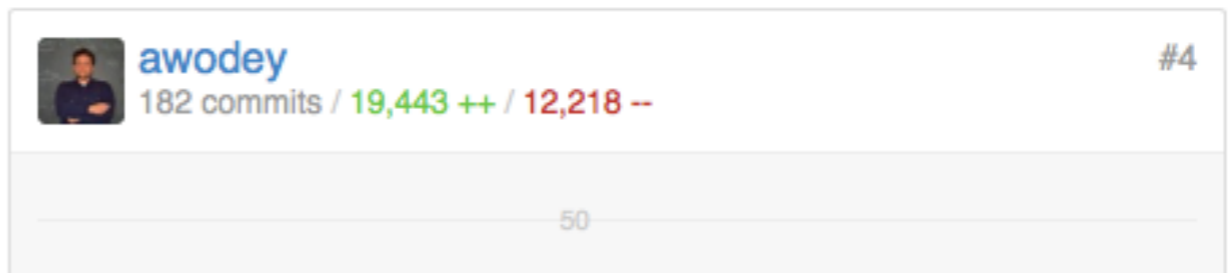
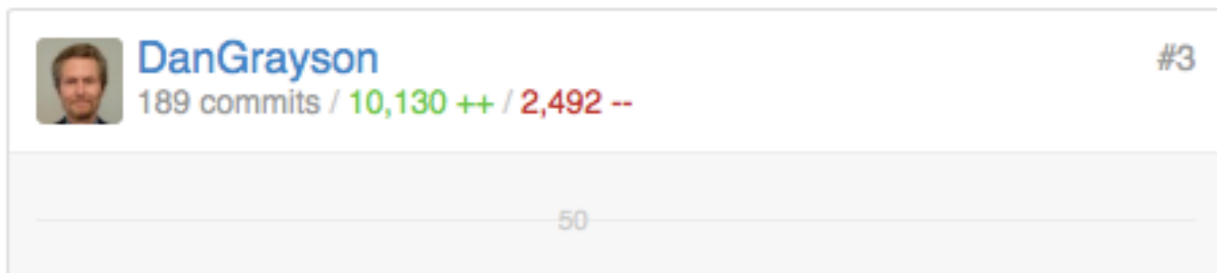
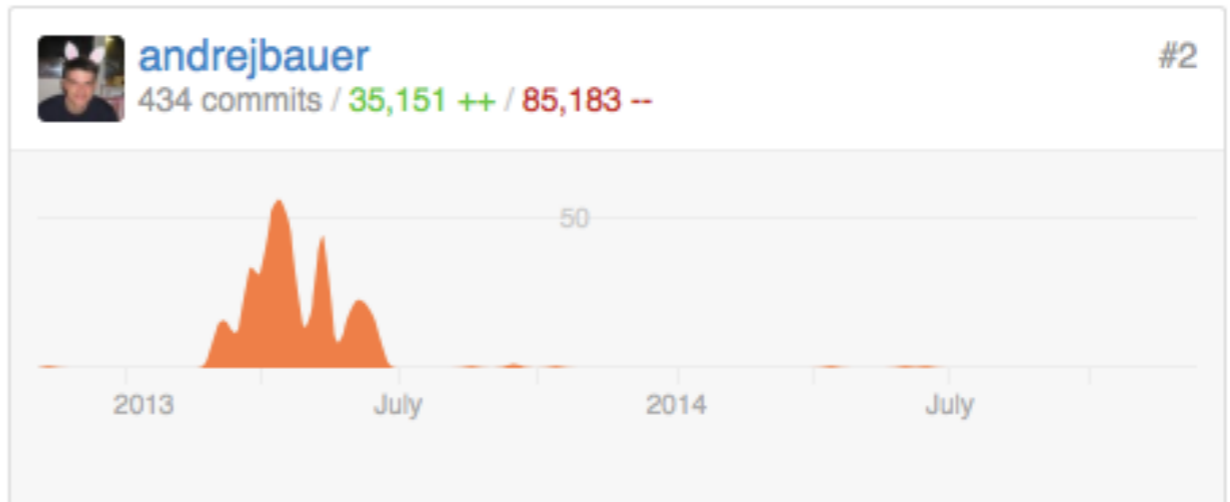
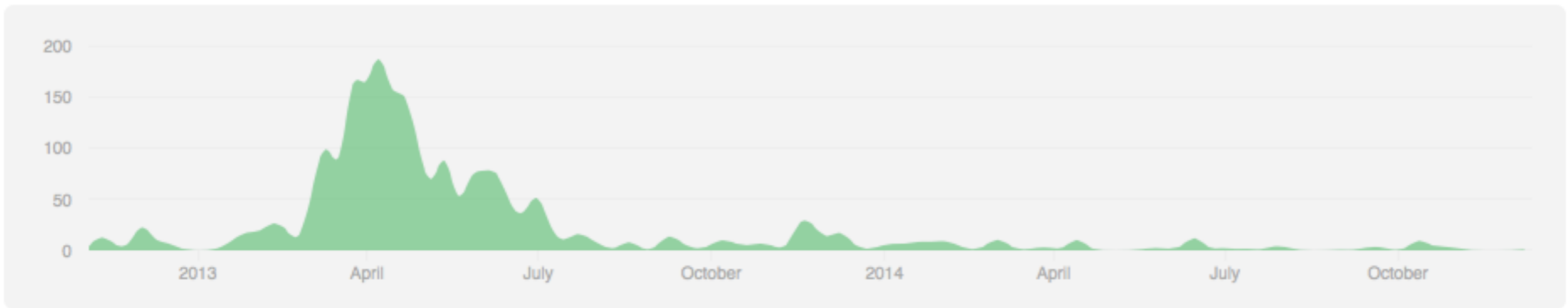
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Contributors Traffic Commits Code frequency Punch card Network Members

Nov 4, 2012 – Dec 11, 2014

Contributions: Commits

Contributions to master, excluding merge commits



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New Issue

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<input type="checkbox"/>	<b>31 Open</b> ✓ 434 Closed	Author	Labels	Milestones	Assignee	Sort
<input type="checkbox"/>	<b>Proofs of Lems. 7.2.9 and 7.3.1</b> <span>math question</span> #744 opened 5 days ago by kristinas					0
<input type="checkbox"/>	<b>Think about the beginning of chapter 2</b> <span>feature request</span> <span>Fix in 2nd edition</span> #740 opened on Nov 5 by mikeshulman <span>Second edition</span>					3
<input type="checkbox"/>	<b>Is univalence really needed to prove <math>\pi_1(S^1)=Z</math>?</b> #736 opened on Oct 25 by martinescardo					6
<input type="checkbox"/>	<b>Motivation of definition of surjection</b> #735 opened on Oct 24 by fpvandoorn					15
<input type="checkbox"/>	<b>Explain more category theory</b> <span>feature request</span> <span>Fix in 2nd edition</span> #733 opened on Oct 21 by mikeshulman <span>Second edition</span>					1
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<input type="checkbox"/>	<b>Definition of concatenation</b> #729 opened on Oct 20 by mikeshulman					0
<input type="checkbox"/>	<b>Tell readers that Introduction is not essential for book understanding.</b> #727 opened on Oct 17 by andrejbauer					7

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2012-11-06

*From the Introduction:*

*Homotopy type theory is a new branch of mathematics that combines aspects of several different fields in a surprising way. It is based on a recently discovered connection between homotopy theory and type theory. It touches on topics as seemingly distant as the homotopy groups of spheres, the algorithms for type checking, and the definition of weak  $n$ -groupoids.*

Homotopy type theory brings new ideas into the very foundation of mathematics. On the one hand, there is Voevodsky's subtle and beautiful univalence axiom. The univalence axiom implies, in particular, that isomorphic structures can be identified, a principle that mathematicians have been happily using on workdays, despite its incompatibility with the "official" doctrines of conventional foundations. On the other hand, we have higher inductive types, which provide direct, logical descriptions of some of the basic spaces and constructions of homotopy theory: spheres, cylinders, truncations, localizations, etc. Both ideas are impossible to capture directly in classical set-theoretic foundations, but when combined in homotopy type theory, they permit an entirely new kind of "logic of homotopy types".

This suggests a new conception of foundations of mathematics, with intrinsic homotopical content, an "invariant" conception of the objects of mathematics — and convenient machine implementations, which can serve as a practical aid to the working mathematician. This is the *Univalent Foundations* program.

The present book is intended as a first systematic exposition of the basics of univalent foundations, and a collection of examples of this new style of reasoning — but without requiring the reader to know or learn any formal logic, or to use any computer proof assistant. We believe that univalent foundations will eventually become a viable alternative to set theory as the "implicit foundation" for the informalized mathematics done by most mathematicians.

*Get a free copy of the book at [HomotopyTypeTheory.org](http://HomotopyTypeTheory.org).*

Homotopy Type Theory

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