Reinforcement Learning in Multi-Agent Systems with Partial History Sharing

Jalal Arabneydi and Aditya Mahajan

Electrical and Computer Engineering Department, McGill University

Website: www.cim.mcgill.ca/ jarabney

Date: June 9th, 2015

Motivation

- We are interested in systems with multiple agents (decision makers) that wish to cooperate in order to accomplish a common task while
  - agents have different information (decentralized information)
  - agents do not know the complete model of the system i.e., they may only know the partial model or may not know the model at all.
- Multi-agent systems arise in various applications: Networked control systems, Robotics, Communication networks, Transportation networks, Sensor networks, Smart grids, Economics, etc.
- Advantages of multi-agent (decentralized) over single-agent (centralized) systems:
  - distributes computational resources and capacities.
  - provides robustness, maintainability, and flexibility.
  - implements the solution efficiently (physically and economically).

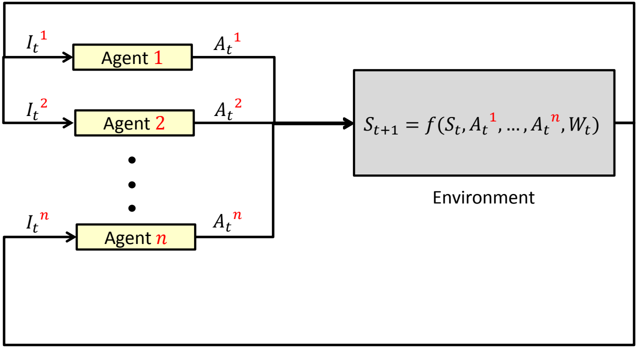
Challenges

- The **discrepancy in perspectives** makes establishing cooperation among agents conceptually challenging.

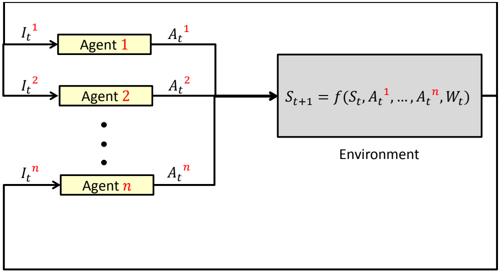
- In general, these problems belong to **NEXP complexity** class.
- Finding team-optimal solution is more challenging when agents have only **partial knowledge** or **no knowledge** of system model.

Problem Formulation

- Consider a system with finite-valued variables that consists of $n \in \mathbb{N}$ agents.
- State of system $S_t \in \mathcal{S}$ and action of agent i: $A^i_t \in \mathcal{A}^i$, where $t \in \mathbb{N}$ denotes time.
- Observation of agent i: $O^i_t = h^i(S_t, A^1_{t-1}, \dots, A^n_{t-1}, V^i_t)$
- Information of agent i: $I^i_t \subseteq \{O^1_{1:t}, \dots, O^n_{1:t}, A^1_{1:t-1}, \dots, A^n_{1:t-1}\}$.



Problem Formulation



- Control law of agent i: $A_t^i = g_t^i(I_t^i)$.
- Control strategy $\mathbf{g} := (g_1, g_2, \ldots)$, where $g_t := (g_t^1, \ldots, g_t^n)$.
- Reward given control strategy $\mathbf{g}$: $J(\mathbf{g}) = \mathbb{E}^{\mathbf{g}} \left[\sum_{t=1}^{\infty} \gamma^{t-1} r(S_t, A_t^1, \ldots, A_t^n) \right]$.
- Agents observe the immediate reward.
- Objective: Develop a (model-based or model-free) reinforcement learning algorithms that guarantees an ϵ-optimal strategy $\mathbf{g}^*$ i.e. $J^* - J(\mathbf{g}^*) \leq \epsilon$.

Partial History Sharing

Split $I_t^i = \{C_t, M_t^i\}$, where $C_t = \bigcap_i I_t^i$ is common information and $M_t^i = I_t^i \backslash C_t$ is local information.

Common Information	Local Information

(A1) Common information is nested: $C_{t+1} = \{C_t, Z_t\}$, where $Z_t := C_{t+1} \backslash C_t$ is common observation such that $C_{t+1} = Z_{1:t}$.

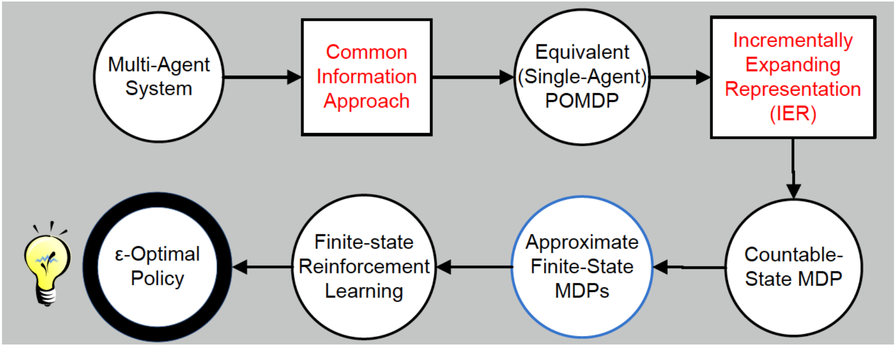
(A2) The update of local information $M_{t+1}^i \subseteq \{M_t^i, A_t^i, O_{t+1}^i\}$.

(A3) The size of Z_t and the size of $M_t^i, \forall i$, are uniformely bounded in time t.

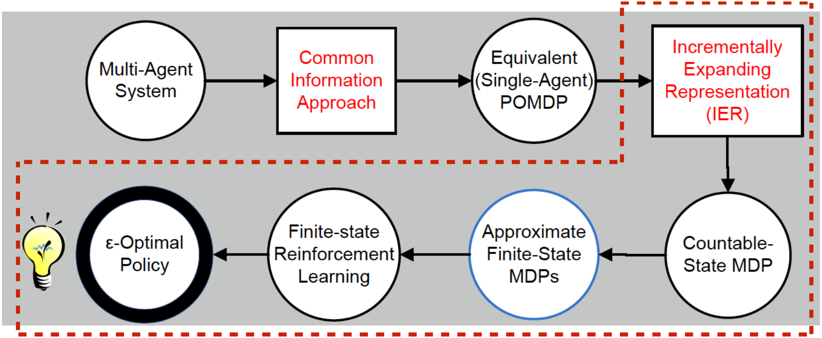
- (A1), (A2), and (A3) are **mild** conditions. Also, C_t is allowed to be empty set.
- A large class of multi-agent systems have partial history sharing such as: **delayed sharing**, **control sharing**, **mean-field sharing,** etc.

Our approach has two main steps:

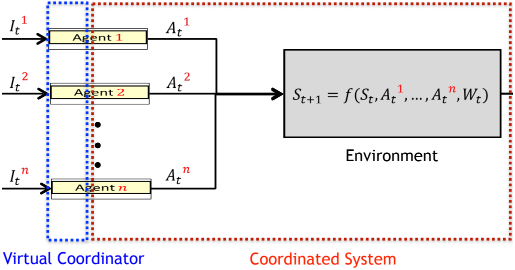
- Step 1) Common Information Approach
- Step 2) Approximate RL algorithm for centralized (single-agent) POMDPs



Salient Feature of the Approach



- This approach guarantees ϵ-optimality performance.
- It encompasses a large class of multi-agent systems.
- Various POMDP RL algorithms may be used in step 2 to obtain different approaches.
- The approach used in Step 2 is a novel POMDP RL algorithm.



Define partial function $\beta_t^i : \mathcal{M}^i \to \mathcal{A}^i$ as follows:

$$\beta_t^i(\cdot) := g_t^i(Z_{1:t}, \cdot) \quad \text{such that} \quad A_t^i = \beta_t^i(M_t^i).$$

Define coordinator's strategy as follows:

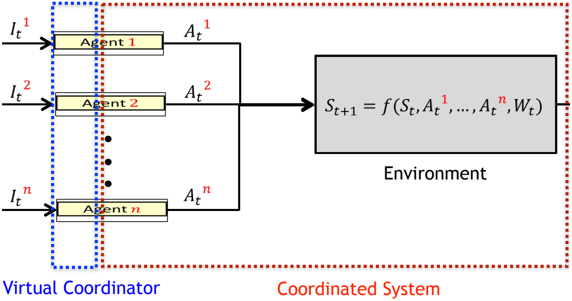
$$\psi_t(Z_{1:t}) := \mathbf{g}_t(Z_{1:t}, \cdot) \quad \text{such that} \quad (\beta_t^1, \dots, \beta_t^n) = \psi_t(Z_{1:t}).$$

Virtual coordinator observes C_t and prescribes $\beta_t =: (\beta_t^1, \dots, \beta_t^n) \in \mathcal{G}$.

An Equivalent Centralized POMDP

The total expected reward in coordinated system is as follows:

$$\hat{J}(\psi) = \mathbb{E}^{\psi}\left[\sum_{t=1}^{\infty} \gamma^{t-1} r_t(S_t, \beta_t^1(M_t^1), \dots, \beta_t^n(M_t^n))\right].$$



- $\Pi_t = \mathbb{P}(S_t, M_t^1, \ldots, M_t^n | Z_{1:t-1}, \beta_{1:t-1})$ is an information state.
- Let $\mathcal{R}$ be the reachable set of the obtained POMDP with action $\beta \in \mathcal{G}$ and observation $Z \in \mathcal{Z}$.
- Given fixed initial distribution π_1, reachable set $\mathcal{R}$ is at most countable.

Step 2: An Approximate POMDP RL Algorithm

Incrementally Expanding Representation (IER)

IER is a 3-tuple $\langle \{\mathcal{X}\}_{N=1}^{\infty}, \tilde{f}, B\} \rangle$ such that

- $\{\mathcal{X}\}_{N=1}^{\infty}$ is a sequence of finite sets such that $\mathcal{X}_1 \subsetneq \mathcal{X}_2 \subsetneq \dots \mathcal{X}_N \subsetneq \dots$, and $\mathcal{X}_1$ is singleton say $\mathcal{X}_1 = \{x^*\}$. Let $\mathcal{X} = \lim_{N \to \infty} \mathcal{X}_N$.
- For any β and z, and $x \in \mathcal{X}_N$, we have that $\tilde{f}(x, \beta, z) \in \mathcal{X}_{N+1}$.
- B is surjective function that maps $\mathcal{X}$ to the reachable set s.t. $\Pi_t = B(X_t)$.

Lemma 1

For every multi-agent system with partial history sharing information structure, there exists at least one IER such that $\mathcal{X}$ and $\tilde{f}$ do not depend on unknowns .

Note that B may depend on unknowns.

Step 2: An Approximate POMDP RL Algorithm

- Construct countable-state MDP Δ with state space $\mathcal{X}$, action space $\mathcal{G}$, dynamics $\tilde{f}$, and reward $\tilde{r}(B(X_t), \beta_t) := \hat{r}_t(\Pi_t, \beta_t)$.
- Approximate Δ by finite-state MDPs $\{\Delta_N\}_{N=1}^{\infty}$ where state space is $\mathcal{X}_N$, action space $\mathcal{G}$, dynamics $\tilde{f}$, and reward $\tilde{r}(B(X_t), \beta_t)$.
- Apply a generic finite-state RL algorithm ζ to learn optimal strategy of Δ_N. We assume ζ converges to an optimal strategy of Δ_N.
- Translate the strategies in Δ_N to strategies in the original multi-agent system.

Main Theorem

Let J^* be the optimal performance (reward) of the original MAS system and $\tilde{J}$ be the performance under the learned strategy. Then,

$$J - \tilde{J} \leq \epsilon_N,$$

where $\epsilon_N = \frac{2\gamma^{\tau_N}}{1-\gamma}(r_{max} - r_{min}) \leq \frac{2\gamma^N}{1-\gamma}(r_{max} - r_{min})$ and τ_N is a model dependent parameter that $\tau_N \geq N$. Note that error goes to zero **exponentially** in N.

Multi-Agent RL Algorithm

(1) Given $\epsilon > 0$, choose N such that $\frac{2\gamma^N}{1-\gamma}(r_{max} - r_{min}) \leq \epsilon$. Then, construct Δ_N; particularly, state space $\mathcal{X}_N$ and dynamics $\tilde{f}$.

(2) At iteration k, ζ chooses prescriptions $\beta_k = (\beta_k^1, \ldots, \beta_k^n)$. (Agents have access to a common random generator to explore consistently). Agent i takes action a_k^i based on prescription β_k^i and local information m_k^i:

$$a_k^i = \beta_k^i(m_k^i), \forall i.$$

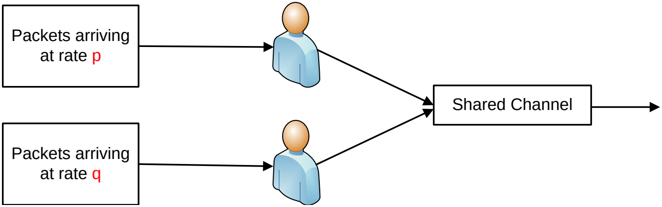
(3) Based on taken actions, system incurs reward r_k, evolves, and generates common observation z_k that is observable to every agent. Agents consistently compute next state as follows

$$x_{k+1} = \tilde{f}(x_k, \beta_k, z_k) \in \mathcal{X}_N.$$

(4) ζ learns (updates) the coordinated strategy according to observed reward r_k by performing prescriptions β_k at state x_k and transiting to state x_{k+1}.

(5) $k \leftarrow k + 1$, and got step 2 until termination.

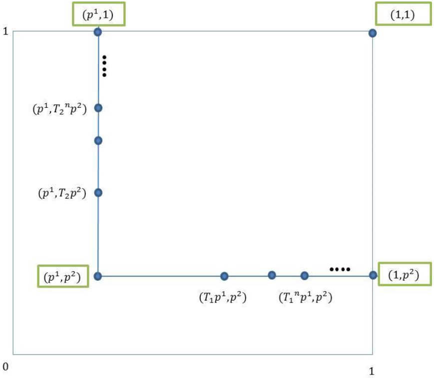
Example: Multi-Access Broadcasting Channel (MABC)



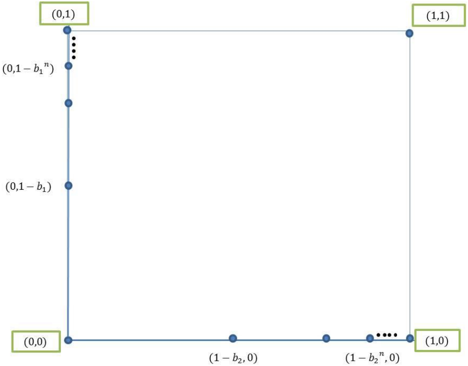
- $S_t = (S_t^1, S_t^2) \in \{0,1\}^2, \quad A_t^i \in \{0,1\}.$
- Packets arrive at user i according to independent Bernoulli process with rate $p^i \in (0,1)$ tat are unknown.
- Each user transmits if it has a packet i.e. $A_t^i \le S_t^i$.
- Information at each agent $I_t^i = \{S_t^i, A_{1:t-1}^1, A_{1:t-1}^2\}$.
- The objective is to maximize the throughput; hence, reward function $r(S_t, A_t^1, A_t^2) = A_t^1 + A_t^2 - 2A_t^1 A_t^2$.

Example: Multi-Access Broadcasting Channel (MABC)

Reachable set



Countable state Space



Example: Multi-Access Broadcasting Channel (MABC)

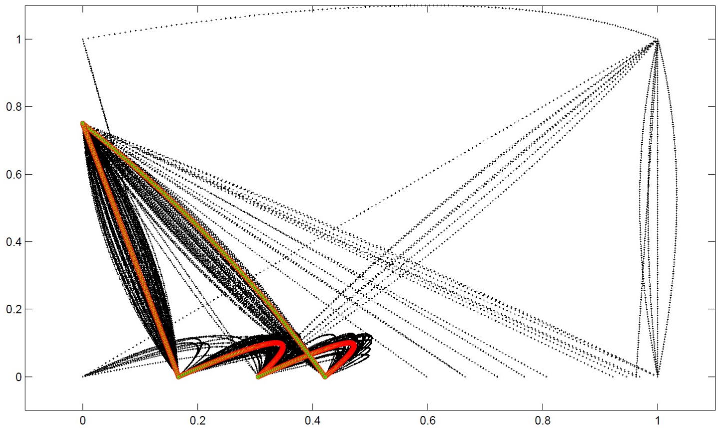
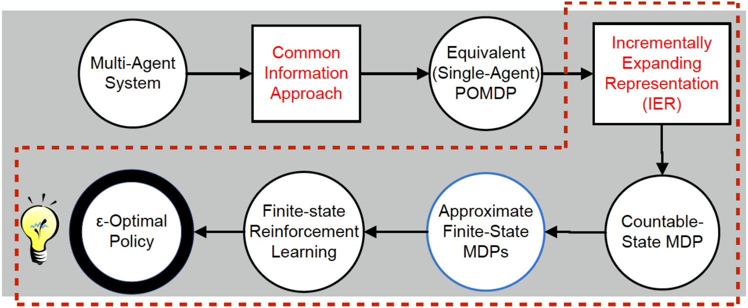


Figure: This figures shows the learning process of MDP Δ_N in a few snapshots. Numerical values: $b_1 = 0.25, b2 = 0.83, N = 50, \gamma = 0.99, p^1 = 0.3, p^2 = 0.6$.

Summary

- Given $\epsilon > 0$, we presented a (model-based or model-free) RL algorithm that guarantees ϵ-optimality for a large class of multi-agent systems with partial history sharing.
- Our approach has two main steps: Common Information Approach + POMDP RL.
- We provided a novel approach for approximate solution of POMDPs (model known and unknown model).
- We developed a multi-agent Q-learning algorithm for MABC problem that converges to optimal policy.
- The obtained error bound is conservative and in practice, the actual error is less.



Thank You

Poster number: T47

"Reinforcement Learning in Decentralized Stochastic Control Systems with Partial History Sharing", Accepted in IEEE American Control Conference (ACC), Jul., 2015.

Only for (online) learning purposes

Main Property

When state x_t steps out of Δ_N, it will come back to $\mathcal{X}_\mathcal{N}$ after a finite time.

This property is required for a model to have every pair of state and action in Δ_N visited infinitely often. In the literature, different versions of this property have been considered.

- There exists an oracle that provides the agent with exact information about the current state, upon request; however, using the oracle is expensive and reserved for the learning phase.
- In sensor networks, where the communication is sensing is cheap but communication is expensive.
- Agents have access to "reset" or "off-line" simulation.
- Δ is such that after a finite time, the state will come back to Δ_N, so it is better to wait until the state comes back.