On the Quantitative Analysis of Craniofacial Asymmetry in 3D

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Research context: Craniofacial Dysmorphology

- Craniofacial geometry has been suggested as an index of early brain dysmorphogenesis in neuropsychiatric disorders
  - Down syndrome
  - Autism
  - Schizophrenia
  - Bipolar disorder
  - Fetal alcohol syndrome
  - Velocardiofacial syndrome
  - Cornelia de Large syndrome
  - ...
- Shape differences can be **<u>very subtle</u>**
  - Need for highly accuracy analysis

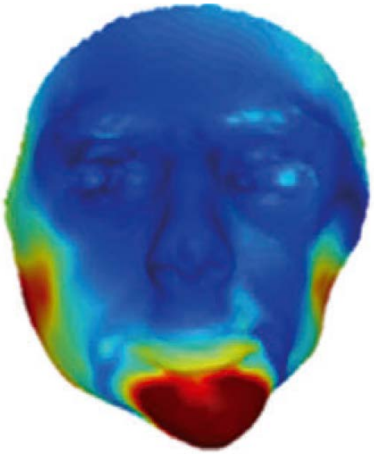
The interest in symmetry

- Wide application scope in computer vision
  - Face recognition
  - Gender classification
  - Landmark detection
- .. and beyond
  - Facial attractiveness, mate selection
  - Assessment of orthognatic surgery outcome
- Developmental dysmorphology
  - Autism spectrum disorders
  - Schizophrenia
  - Fetal alcohol syndrome
  - Differences can be very subtle

Measuring bilateral facial symmetry in 3D

What is our accuracy in asymmetry estimation?

- Shape-based definition
  - Difference between left and right hemispheres
  - Deviation from nearest symmetric shape
  - Estimation of the symmetry plane
    - Alignment of original and reflected shapes
- A synthetic motivation example
  - Traditional approaches perform poorly
  - Landmark- and surface-based discrepancies
- Quantitative evaluation
  - Point-wise 3D measurements of asymmetry
  - Comparison to ground-truth asymmetries
    - Synthetic patterns of asymmetry
    - Starting from a perfectly symmetric face



Claes et al (2011)
Journal of Anatomy

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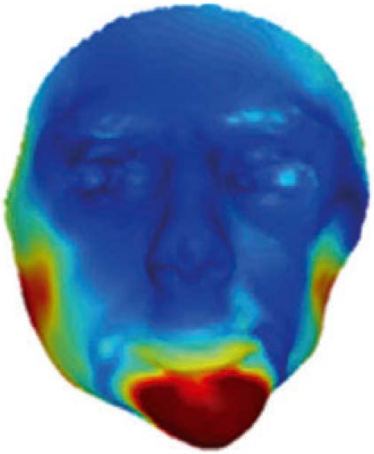

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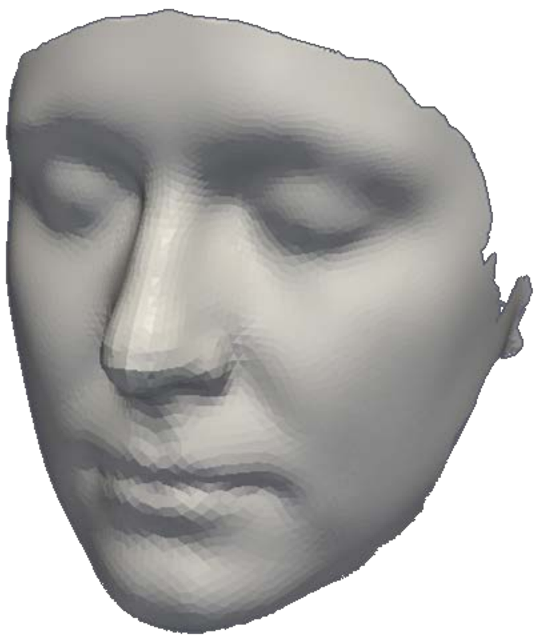
Bilateral symmetry: facial hemispheres



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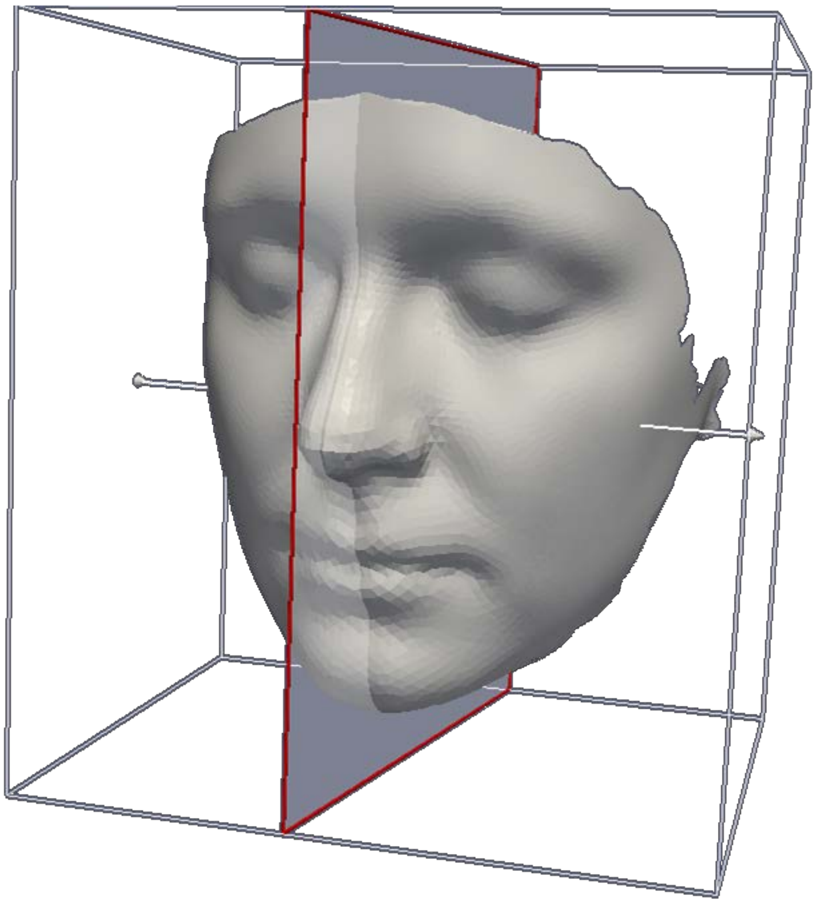

Plane of bilateral
symmetry

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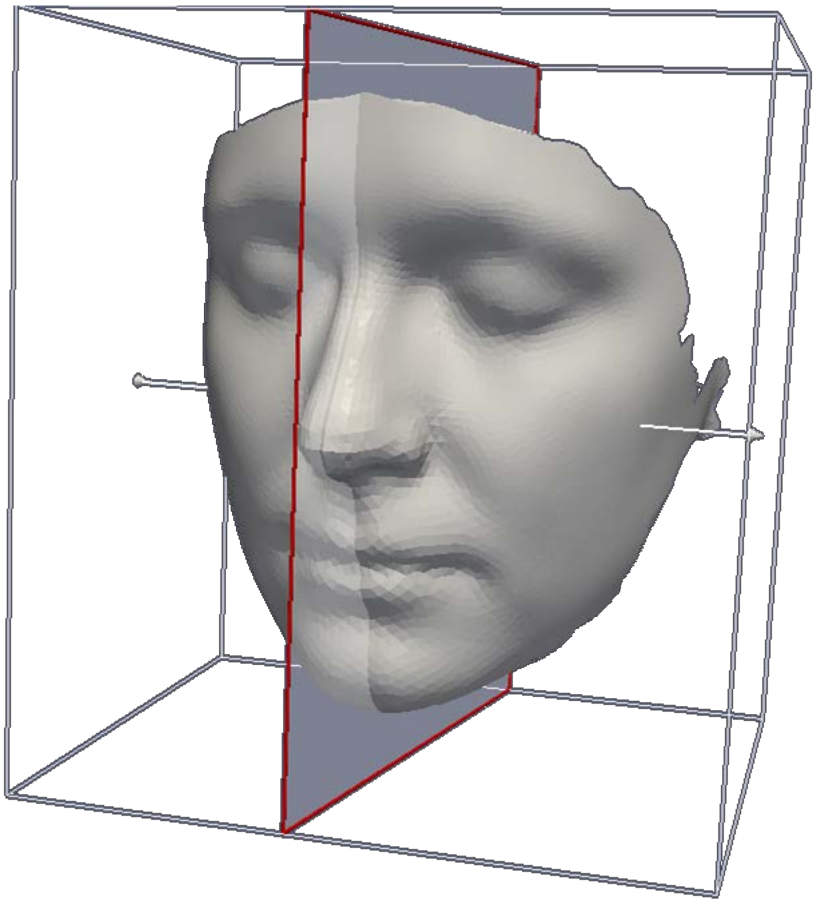

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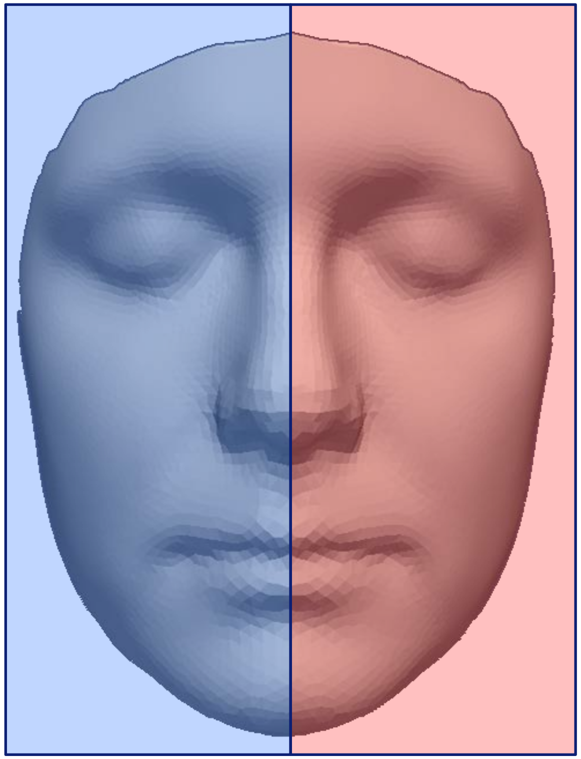
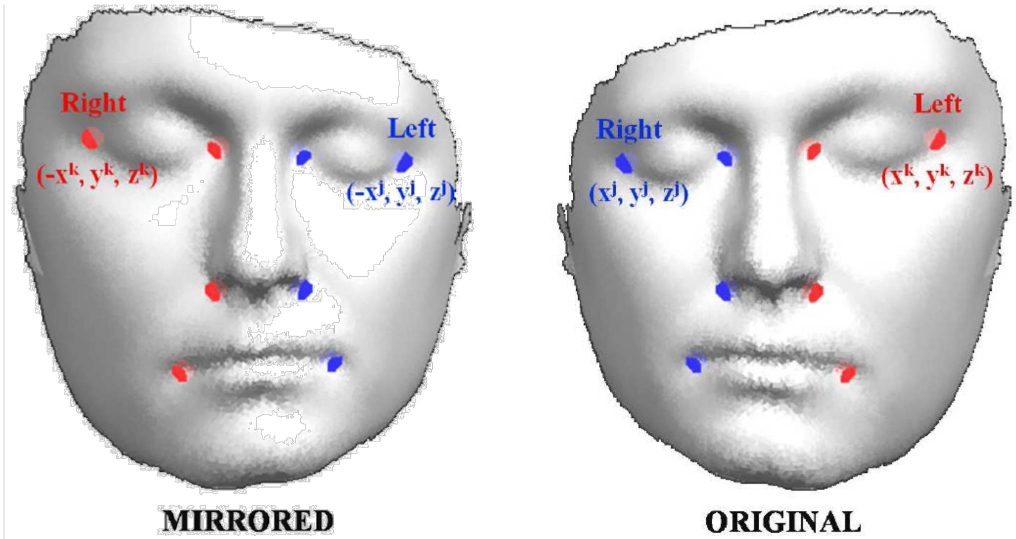


Bilateral symmetry computation

Original surface S

1. Obtained mirrored surface S_M
   - Arbitrary plane
2. Put S and S_M in correspondence
   - Rigid transformation T
   - Vertex index reflection
3. Compute asymmetry by point-wise difference between S and the mirrored-aligned S_M



$(\cdot)^{ref}$ Reflection operator: a function that swaps left and right vertex indices

$$(j)^{ref} = \bar{j}, \qquad (\bar{j})^{ref} = j$$

$$Asymmetry \sim S - T((SM)^{ref})$$

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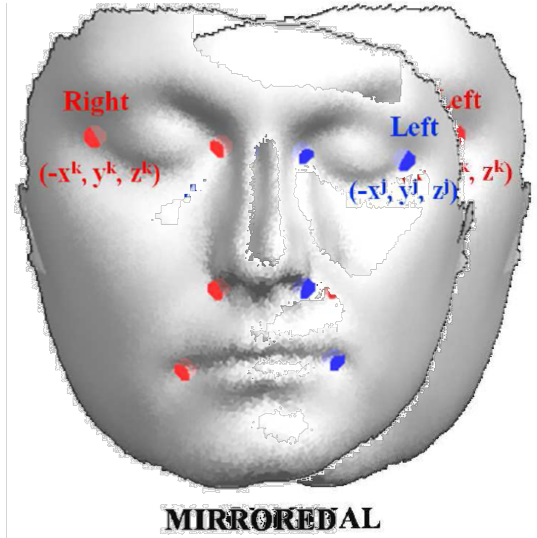

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A motivating example … against traditional Procrustes alignment to estimate T

- Let $\mathcal{S}_{symm}$ be a facial surface that is perfectly symmetric about the plane $x = 0$. Thus, symmetric paired

- Let us generate $\mathcal{S}_1$ by expanding the left side of $\mathcal{S}_{symm}$ linearly with respect to the coordinates of the x-axis. If we represent surface $\mathcal{S}_{symm}$ by a set of n_V vertices $\{\mathbf{v}_s^i\}_{i=1}^{n_V}$ with $\mathbf{v}_s^i = (x_s^i, y_s^i, z_s^i)^T$ then:

$$\forall(\mathbf{v}_s^i \in \mathcal{S}_{symm}, \mathbf{v}_1^i \in \mathcal{S}_1) :$$

$$\mathbf{v}_1^i = \begin{cases} \mathbf{v}_s^i + (\alpha\, x_s^i, 0, 0)^T & \text{if } x_s^i > 0 \\ \mathbf{v}_s^i & \text{otherwise} \end{cases}$$

where α is a constant that controls the degree of expansion.

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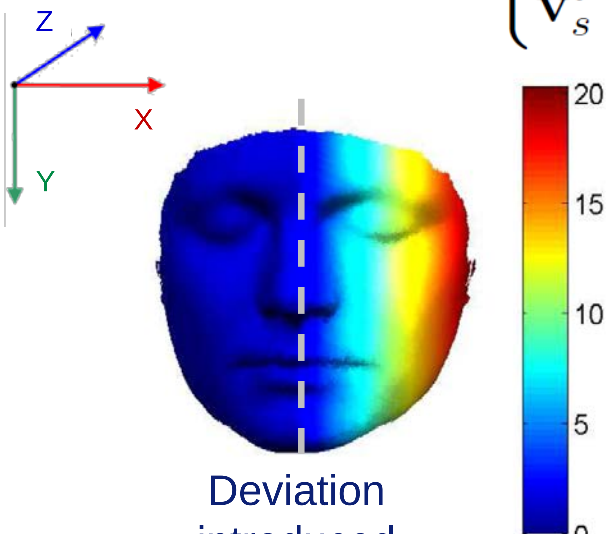

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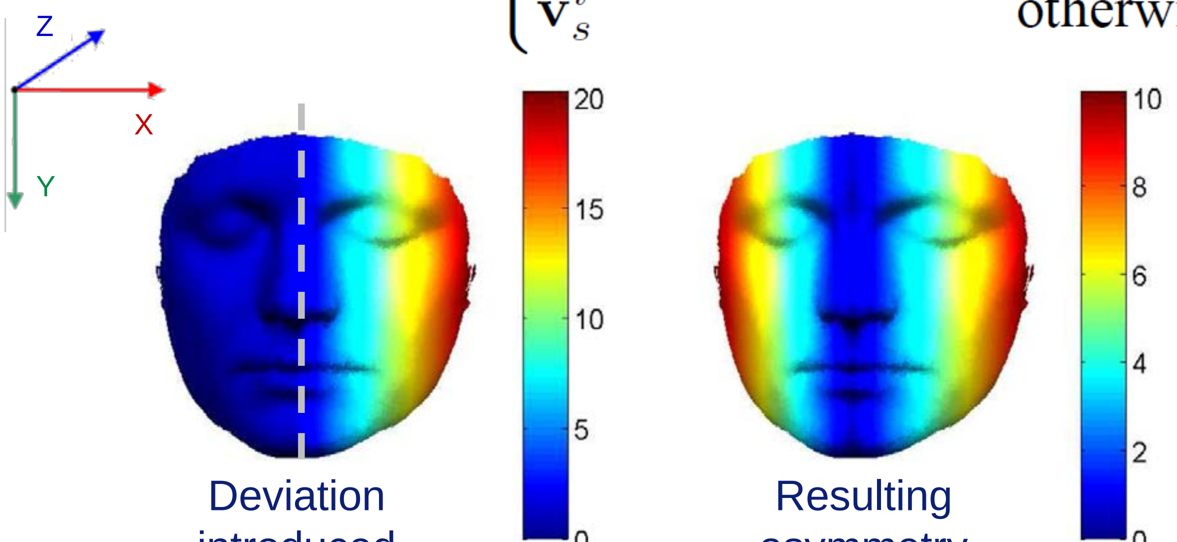

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Resulting asymmetry

Asymmetry patterns must be themselves symmetric

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  -
  -
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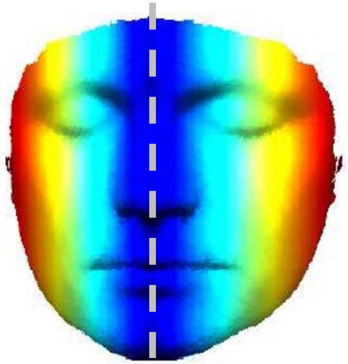

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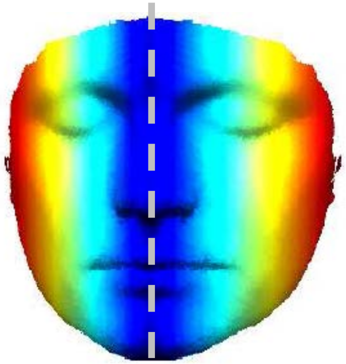

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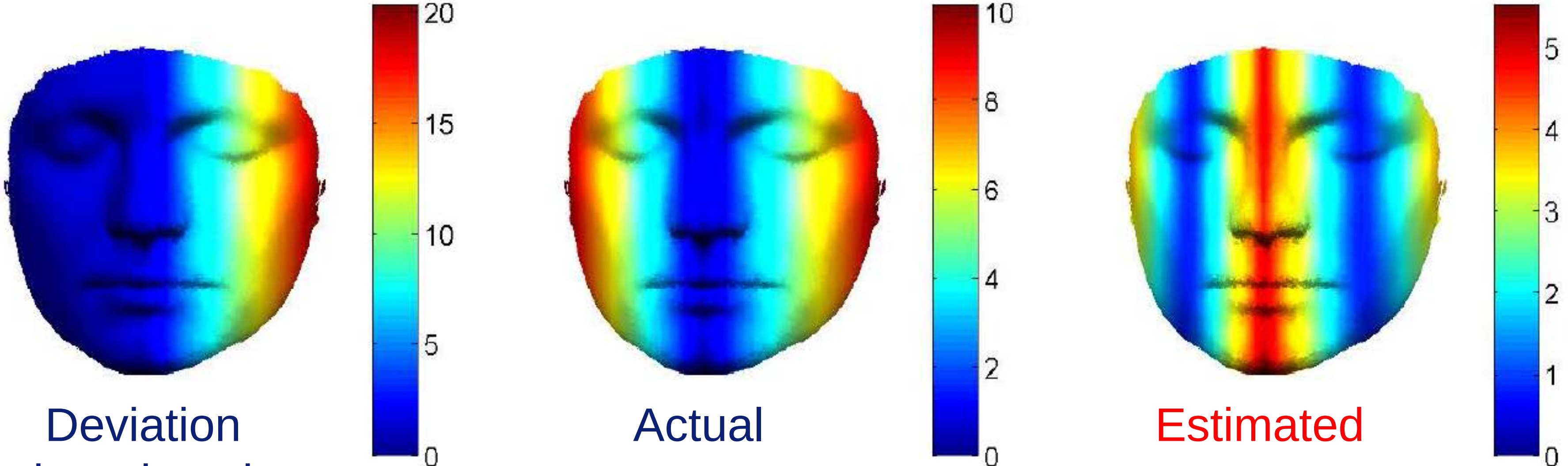

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Actual asymmetry

Estimated asymmetry

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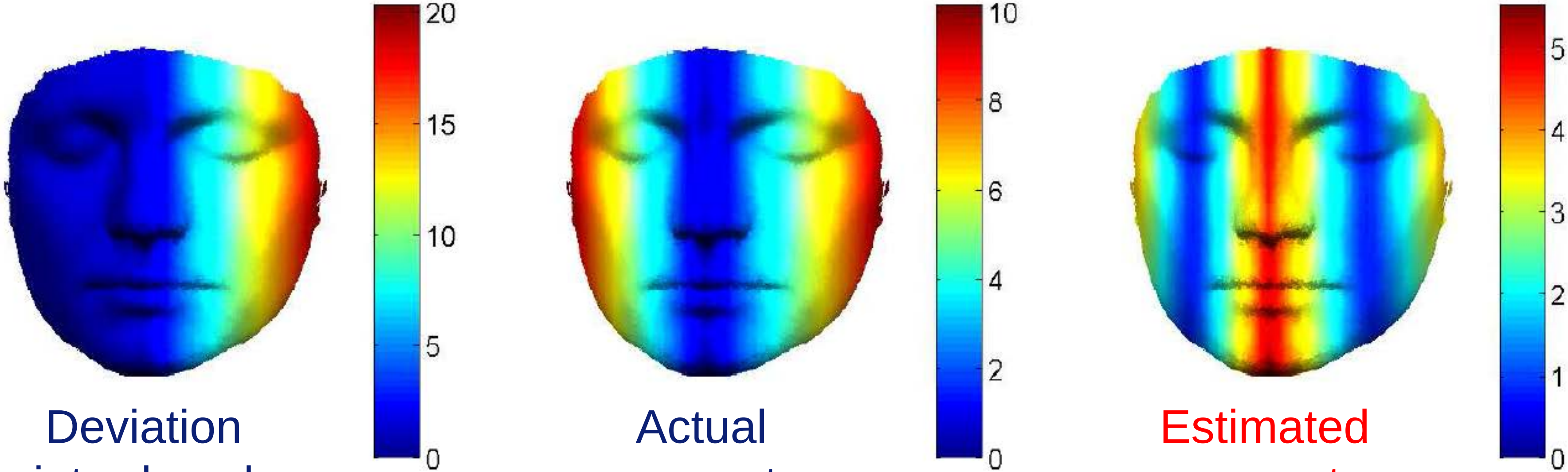

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Correlation = – 0.158

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Asymmetry estimation / Alignment methods

- LMS (Least mean of squares)

$$\arg\min_{\hat{T}} \sum_{\forall i} \|\mathbf{v}_S^i - \hat{T}(\mathbf{v}_M^{\bar{i}})\|^2$$

- CW-LMS (Confidence-weighted LMS)

$$\arg\min_{\hat{T}} \sum_{\forall i} w_i \|\mathbf{v}_S^i - \hat{T}(\mathbf{v}_M^{\bar{i}})\|^2$$

  - Gaussian distribution N(0, σ) for inliers
  - Uniform distribution λ^{-1} for outliers
  - Van Leemput et al. ÌEEE TMI, 20(8), 2001

$$e_i = \|\mathbf{v}_S^i - \hat{T}(\mathbf{v}_M^{\bar{i}})\|$$

$$\sigma = \sqrt{\frac{\sum_{\forall i} w_i e_i}{\sum_{\forall i} e_i}}$$

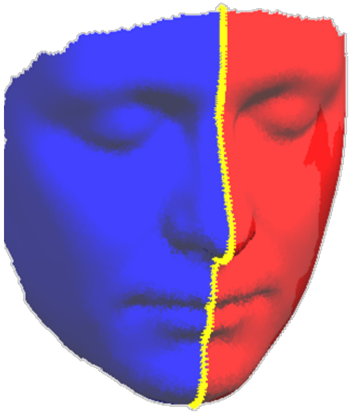
$$\lambda = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{1}{2}K^2\right)$$

- LMedS (Least median of squares)

$$\arg\min_{\hat{T}} \left(\text{median} \left\{ \|\mathbf{v}_S^i - \hat{T}(\mathbf{v}_M^{\bar{i}})\|^2 \right\}_{i=1}^{n_V} \right),$$

Asymmetry estimation / Alignment methods

- DW-LMedS (Distance-weighted LMedS)
  - Symmetry enforcement of the recovered pattern of asymmetry
  - Weights decrease with the distance to the mid-line d_{ML}
    - Facial asymmetry tends to grow as we depart from the mid-line



$$\arg\min_{\hat{T}} \left(\text{median} \{ \| e^i_{DW} \|^2 \}_{i=1}^{n_V} + \sum_{i=1}^{n_V} \frac{|e_i^2 - e_{\bar{i}}^2|}{n_V} \right)$$

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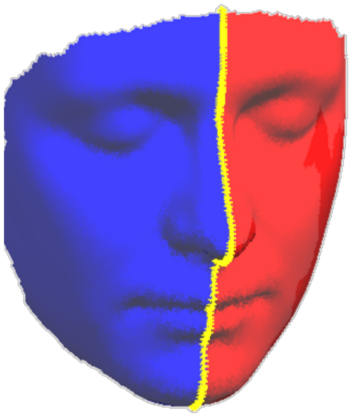

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$$e^i_{DW} = \exp\left(\frac{-1}{\tau} \frac{d_{ML}(\mathbf{v}_S^i)}{\max\left(d_{ML}(\mathbf{v}_S)\right)} \right) (\mathbf{v}_S^i - \hat{T}(\mathbf{v}_M^{\bar{i}}))$$

$$e_i = \|\mathbf{v}_S^i - \hat{T}(\mathbf{v}_M^{\bar{i}})\|, \qquad e_{\bar{i}} = \|\mathbf{v}_S^{\bar{i}} - \hat{T}(\mathbf{v}_M^i)\|$$

Quantitative evaluation with synthetic patterns of asymmetry

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Unknown asymmetry pattern

- 100 facial scans
- Healthy subjects to serve as controls in dysmorphology studies

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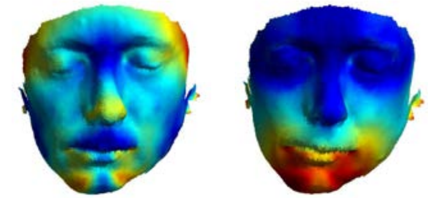


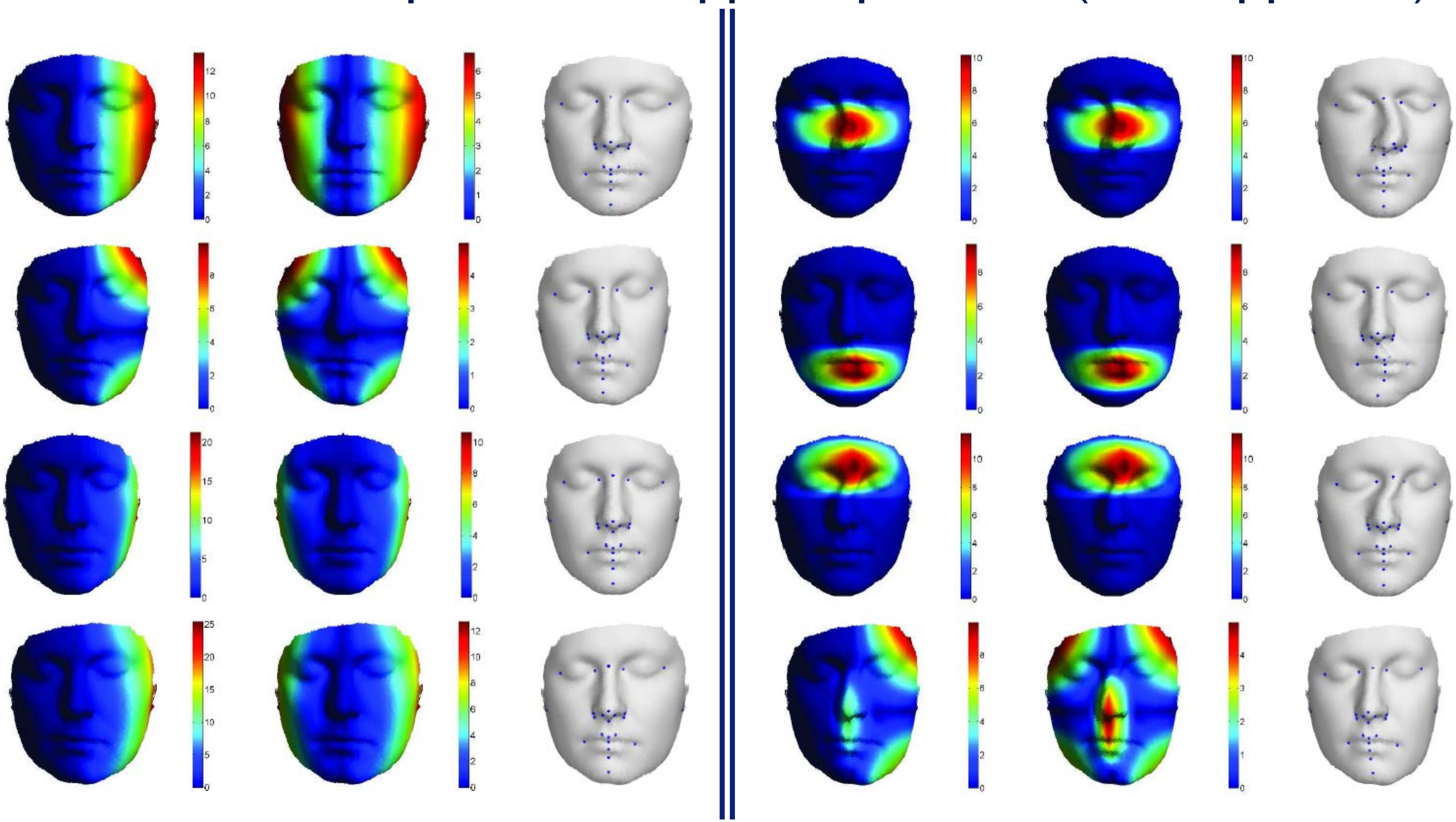
Deformation with synthetic asymmetry pattern

SYNTHESIZED

The resulting shape has a known asymmetry pattern (ground truth)

25 synthetic patterns applied applied to 100 “real” facial scans

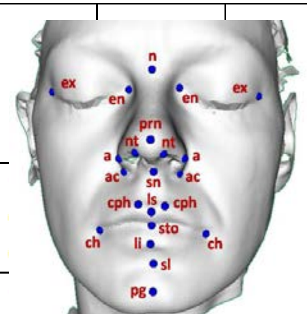
- A few examples of the applied patterns (see suppl mat)



Results: average correlation coefficients

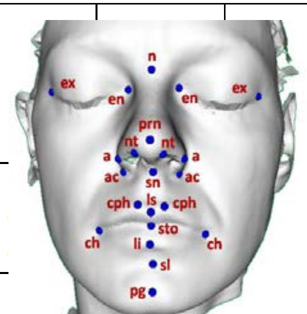
Asymm Patterns		Landmarks			Midline		Whole surface				Mixed	
		LMS	LMedS	DW-LMedS	LMS	LMedS	LMS	CW-LMS	LMedS	DW-LMedS	HM-LMedS	HM-DW-LMedS
Linear expansions of individual axes (with the nose tip at the origin)	1	0.92	0.95	0.94	1.00	1.00	−0.15	−0.17	−0.93	−0.92	0.82	0.87
	2	0.87	1.00	1.00	1.00	1.00	0.90	0.90	0.97	0.98	0.98	0.99
	3	0.73	0.99	1.00	1.00	1.00	0.70	0.77	0.95	0.95	0.96	0.96
Module expansions with respect to a given landmark	4	0.95	1.00	1.00	1.00	1.00	0.67	0.70	0.93	0.94	0.95	0.97
	5	0.95	0.99	1.00	1.00	1.00	0.48	0.50	0.87	0.86	0.93	0.94
	6	0.75	0.96	0.95	0.99	0.99	0.64	0.67	0.85	0.88	0.90	0.93
	7	0.93	0.98	0.98	0.99	0.99	0.62	0.64	0.84	0.88	0.92	0.94
Combinations of vertical shift (y axis) and expansion of x or z axis	8	0.54	0.99	1.00	1.00	1.00	0.61	0.65	0.94	0.95	0.95	0.96
	9	0.94	0.99	0.99	1.00	1.00	0.46	0.47	0.86	0.43	0.92	0.94
	10	0.89	0.94	0.95	1.00	1.00	0.51	0.51	−0.05	−0.05	0.88	0.90
Linear or rotational horizontal shifts of the upper or lower face	11	0.65	0.97	0.97	−0.13	1.00	0.84	0.92	1.00	1.00	1.00	1.00
	12	0.95	1.00	1.00	−0.33	1.00	0.65	0.71	1.00	1.00	1.00	1.00
	13	0.84	1.00	1.00	−0.01	1.00	0.92	0.97	1.00	1.00	1.00	1.00
	14	0.67	1.00	1.00	−0.14	1.00	0.60	0.64	1.00	1.00	1.00	1.00
Progressive horizontal shifts of specific regions (e.g. mouth, nose, eyes)	15	0.76	1.00	1.00	−0.18	1.00	0.88	0.94	1.00	1.00	1.00	1.00
	16	0.61	1.00	1.00	−0.04	1.00	0.92	0.97	1.00	1.00	1.00	1.00
	17	0.83	1.00	1.00	−0.47	1.00	0.76	0.84	1.00	1.00	1.00	1.00
Quadratic distortions limited both vertically and horizontally from specific landmarks	18	0.12	0.98	0.97	−0.25	1.00	0.82	0.90	1.00	1.00	1.00	1.00
	19	0.37	1.00	1.00	0.07	1.00	0.91	0.97	1.00	1.00	1.00	1.00
	20	−0.53	0.27	0.09	−0.44	−0.42	0.97	1.00	1.00	1.00	0.98	0.97
	21	0.87	0.85	0.96	0.83	0.60	0.53	0.56	0.78	0.65	0.93	0.94
	22	0.72	1.00	1.00	−0.51	−0.33	0.84	0.91	1.00	1.00	0.94	0.94
Combinations of quadratic distortion and axes expansion	23	0.70	0.62	0.75	0.27	0.28	0.92	0.92	0.97	0.97	0.99	0.99
	24	0.95	0.85	0.86	0.96	0.95	−0.07	−0.18	−0.77	−0.76	0.67	0.84
	25	−0.08	0.78	−0.38	0.92	0.92	0.27	0.29	0.89	0.88	0.90	0.91

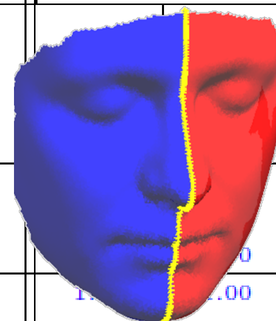
Results: average correlation coefficients



Asymm Patterns		Landmarks			Midline		Whole surface				Mixed	
					LMS	LMedS	LMS	CW-LMS	LMedS	DW-LMedS	HM-LMedS	HM-DW-LMedS
Linear expansions of individual axes (with the nose tip at the origin)	1	[illegible]	[illegible]	[illegible]	1.00	1.00	−0.15	−0.17	−0.93	−0.92	0.82	0.87
	2	[illegible]	[illegible]	[illegible]	1.00	1.00	0.90	0.90	0.97	0.98	0.98	0.99
	3	[illegible]	[illegible]	[illegible]	1.00	1.00	0.70	0.77	0.95	0.95	0.96	0.96
Module expansions with respect to a given landmark	4	[illegible]	[illegible]	[illegible]	1.00	1.00	0.67	0.70	0.93	0.94	0.95	0.97
	5	0.95	0.99	1.00	1.00	1.00	0.48	0.50	0.87	0.86	0.93	0.94
	6	0.75	0.96	0.95	0.99	0.99	0.64	0.67	0.85	0.88	0.90	0.93
	7	0.93	0.98	0.98	0.99	0.99	0.62	0.64	0.84	0.88	0.92	0.94
Combinations of vertical shift (y axis) and expansion of x or z axis	8	0.54	0.99	1.00	1.00	1.00	0.61	0.65	0.94	0.95	0.95	0.96
	9	0.94	0.99	0.99	1.00	1.00	0.46	0.47	0.86	0.43	0.92	0.94
	10	0.89	0.94	0.95	1.00	1.00	0.51	0.51	−0.05	−0.05	0.88	0.90
Linear or rotational horizontal shifts of the upper or lower face	11	0.65	0.97	0.97	−0.13	1.00	0.84	0.92	1.00	1.00	1.00	1.00
	12	0.95	1.00	1.00	−0.33	1.00	0.65	0.71	1.00	1.00	1.00	1.00
	13	0.84	1.00	1.00	−0.01	1.00	0.92	0.97	1.00	1.00	1.00	1.00
	14	0.67	1.00	1.00	−0.14	1.00	0.60	0.64	1.00	1.00	1.00	1.00
Progressive horizontal shifts of specific regions (e.g. mouth, nose, eyes)	15	0.76	1.00	1.00	−0.18	1.00	0.88	0.94	1.00	1.00	1.00	1.00
	16	0.61	1.00	1.00	−0.04	1.00	0.92	0.97	1.00	1.00	1.00	1.00
	17	0.83	1.00	1.00	−0.47	1.00	0.76	0.84	1.00	1.00	1.00	1.00
Quadratic distortions limited both vertically and horizontally from specific landmarks	18	0.12	0.98	0.97	−0.25	1.00	0.82	0.90	1.00	1.00	1.00	1.00
	19	0.37	1.00	1.00	0.07	1.00	0.91	0.97	1.00	1.00	1.00	1.00
	20	−0.53	0.27	0.09	−0.44	−0.42	0.97	1.00	1.00	1.00	0.98	0.97
	21	0.87	0.85	0.96	0.83	0.60	0.53	0.56	0.78	0.65	0.93	0.94
	22	0.72	1.00	1.00	−0.51	−0.33	0.84	0.91	1.00	1.00	0.94	0.94
Combinations of quadratic distortion and axes expansion	23	0.70	0.62	0.75	0.27	0.28	0.92	0.92	0.97	0.97	0.99	0.99
	24	0.95	0.85	0.86	0.96	0.95	−0.07	−0.18	−0.77	−0.76	0.67	0.84
	25	−0.08	0.78	−0.38	0.92	0.92	0.27	0.29	0.89	0.88	0.90	0.91

Results: average correlation coefficients





Asymm Patterns		Landmarks			Midline		Whole surface				Mixed	
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	LMS	CW-LMS	LMedS	DW-LMedS	HM-LMedS	HM-DW-LMedS
Linear expansions of individual axes (with the nose tip at the origin)	1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	−0.15	−0.17	−0.93	−0.92	0.82	0.87
	2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.90	0.90	0.97	0.98	0.98	0.99
	3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.70	0.77	0.95	0.95	0.96	0.96
Module expansions with respect to a given landmark	4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.67	0.70	0.93	0.94	0.95	0.97
	5	0.95	0.99	1.00	1.00	1.00	0.48	0.50	0.87	0.86	0.93	0.94
	6	0.75	0.96	0.95	0.99	0.99	0.64	0.67	0.85	0.88	0.90	0.93
	7	0.93	0.98	0.98	0.99	0.99	0.62	0.64	0.84	0.88	0.92	0.94
Combinations of vertical shift (y axis) and expansion of x or z axis	8	0.54	0.99	1.00	1.00	1.00	0.61	0.65	0.94	0.95	0.95	0.96
	9	0.94	0.99	0.99	1.00	1.00	0.46	0.47	0.86	0.43	0.92	0.94
	10	0.89	0.94	0.95	1.00	1.00	0.51	0.51	−0.05	−0.05	0.88	0.90
Linear or rotational horizontal shifts of the upper or lower face	11	0.65	0.97	0.97	−0.13	1.00	0.84	0.92	1.00	1.00	1.00	1.00
	12	0.95	1.00	1.00	−0.33	1.00	0.65	0.71	1.00	1.00	1.00	1.00
	13	0.84	1.00	1.00	−0.01	1.00	0.92	0.97	1.00	1.00	1.00	1.00
	14	0.67	1.00	1.00	−0.14	1.00	0.60	0.64	1.00	1.00	1.00	1.00
Progressive horizontal shifts of specific regions (e.g. mouth, nose, eyes)	15	0.76	1.00	1.00	−0.18	1.00	0.88	0.94	1.00	1.00	1.00	1.00
	16	0.61	1.00	1.00	−0.04	1.00	0.92	0.97	1.00	1.00	1.00	1.00
	17	0.83	1.00	1.00	−0.47	1.00	0.76	0.84	1.00	1.00	1.00	1.00
Quadratic distortions limited both vertically and horizontally from specific landmarks	18	0.12	0.98	0.97	−0.25	1.00	0.82	0.90	1.00	1.00	1.00	1.00
	19	0.37	1.00	1.00	0.07	1.00	0.91	0.97	1.00	1.00	1.00	1.00
	20	−0.53	0.27	0.09	−0.44	−0.42	0.97	1.00	1.00	1.00	0.98	0.97
	21	0.87	0.85	0.96	0.83	0.60	0.53	0.56	0.78	0.65	0.93	0.94
	22	0.72	1.00	1.00	−0.51	−0.33	0.84	0.91	1.00	1.00	0.94	0.94
Combinations of quadratic distortion and axes expansion	23	0.70	0.62	0.75	0.27	0.28	0.92	0.92	0.97	0.97	0.99	0.99
	24	0.95	0.85	0.86	0.96	0.95	−0.07	−0.18	−0.77	−0.76	0.67	0.84
	25	−0.08	0.78	−0.38	0.92	0.92	0.27	0.29	0.89	0.88	0.90	0.91

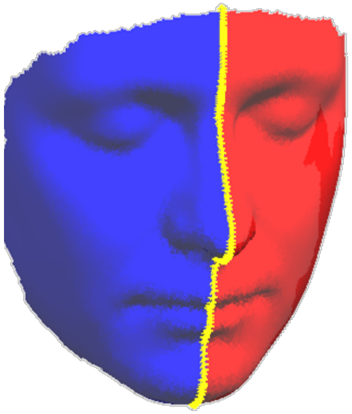
	Landmarks			Midline		Whole surface				Mixed	
	LMS	LMedS	DW-LMedS	LMS	LMedS	LMS	CW-LMS	LMedS	DW-LMedS	HM-LMedS	HM-DW-LMedS
1	0.92	0.95	0.94	1.00	1.00	−0.15	−0.17	−0.93	−0.92	0.82	0.87
2	0.87	1.00	1.00	1.00	1.00	0.90	0.90	0.97	0.98	0.98	0.99
3	0.73	0.99	1.00	1.00	1.00	0.70	0.77	0.95	0.95	0.96	0.96
4	0.95	1.00	1.00	1.00	1.00	0.67	0.70	0.93	0.94	0.95	0.97
5	0.95	0.99	1.00	1.00	1.00	0.48	0.50	0.87	0.86	0.93	0.94
6	0.75	0.96	0.95	0.99	0.99	0.64	0.67	0.85	0.88	0.90	0.93
7	0.93	0.98	0.98	0.99	0.99	0.62	0.64	0.84	0.88	0.92	0.94
8	0.54	0.99	1.00	1.00	1.00	0.61	0.65	0.94	0.95	0.95	0.96
9	0.94	0.99	0.99	1.00	1.00	0.46	0.47	0.86	0.43	0.92	0.94
10	0.89	0.94	0.95	1.00	1.00	0.51	0.51	−0.05	−0.05	0.88	0.90
11	0.65	0.97	0.97	−0.13	1.00	0.84	0.92	1.00	1.00	1.00	1.00
12	0.95	1.00	1.00	−0.33	1.00	0.65	0.71	1.00	1.00	1.00	1.00
13	0.84	1.00	1.00	−0.01	1.00	0.92	0.97	1.00	1.00	1.00	1.00
14	0.67	1.00	1.00	−0.14	1.00	0.60	0.64	1.00	1.00	1.00	1.00
15	0.76	1.00	1.00	−0.18	1.00	0.88	0.94	1.00	1.00	1.00	1.00
16	0.61	1.00	1.00	−0.04	1.00	0.92	0.97	1.00	1.00	1.00	1.00
17	0.83	1.00	1.00	−0.47	1.00	0.76	0.84	1.00	1.00	1.00	1.00
18	0.12	0.98	0.97	−0.25	1.00	0.82	0.90	1.00	1.00	1.00	1.00
19	0.37	1.00	1.00	0.07	1.00	0.91	0.97	1.00	1.00	1.00	1.00
20	−0.53	0.27	0.09	−0.44	−0.42	0.97	1.00	1.00	1.00	0.98	0.97
21	0.87	0.85	0.96	0.83	0.60	0.53	0.56	0.78	0.65	0.93	0.94
22	0.72	1.00	1.00	−0.51	−0.33	0.84	0.91	1.00	1.00	0.94	0.94
23	0.70	0.62	0.75	0.27	0.28	0.92	0.92	0.97	0.97	0.99	0.99
24	0.95	0.85	0.86	0.96	0.95	−0.07	−0.18	−0.77	−0.76	0.67	0.84
25	−0.08	0.78	−0.38	0.92	0.92	0.27	0.29	0.89	0.88	0.90	0.91

	Landmarks			Midline		Whole surface				Mixed	
	LMS	LMedS	DW-LMedS	LMS	LMedS	LMS	CW-LMS	LMedS	DW-LMedS	HM-LMedS	HM-DW-LMedS
1	0.92	0.95	0.94	1.00	1.00					0.82	0.87
2	0.87	1.00	1.00	1.00	1.00	0.90	0.90	0.97	0.98	0.98	0.99
3	0.73	0.99	1.00	1.00	1.00	0.70	0.77	0.95	0.95	0.96	0.96
4	0.95	1.00	1.00	1.00	1.00	0.67	0.70	0.93	0.94	0.95	0.97
5	0.95	0.99	1.00	1.00	1.00	0.48	0.50	0.87	0.86	0.93	0.94
6	0.75	0.96	0.95	0.99	0.99	0.64	0.67	0.85	0.88	0.90	0.93
7	0.93	0.98	0.98	0.99	0.99	0.62	0.64	0.84	0.88	0.92	0.94
8	0.54	0.99	1.00	1.00	1.00	0.61	0.65	0.94	0.95	0.95	0.96
9	0.94	0.99	0.99	1.00	1.00	0.46	0.47	0.86	0.43	0.92	0.94
10	0.89	0.94	0.95	1.00	1.00	0.51	0.51			0.88	0.90
11	0.65	0.97	0.97		1.00	0.84	0.92	1.00	1.00	1.00	1.00
12	0.95	1.00	1.00		1.00	0.65	0.71	1.00	1.00	1.00	1.00
13	0.84	1.00	1.00		1.00	0.92	0.97	1.00	1.00	1.00	1.00
14	0.67	1.00	1.00		1.00	0.60	0.64	1.00	1.00	1.00	1.00
15	0.76	1.00	1.00		1.00	0.88	0.94	1.00	1.00	1.00	1.00
16	0.61	1.00	1.00		1.00	0.92	0.97	1.00	1.00	1.00	1.00
17	0.83	1.00	1.00		1.00	0.76	0.84	1.00	1.00	1.00	1.00
18	0.12	0.98	0.97		1.00	0.82	0.90	1.00	1.00	1.00	1.00
19	0.37	1.00	1.00	0.07	1.00	0.91	0.97	1.00	1.00	1.00	1.00
20		0.27	0.09			0.97	1.00	1.00	1.00	0.98	0.97
21	0.87	0.85	0.96	0.83	0.60	0.53	0.56	0.78	0.65	0.93	0.94
22	0.72	1.00	1.00			0.84	0.91	1.00	1.00	0.94	0.94
23	0.70	0.62	0.75	0.27	0.28	0.92	0.92	0.97	0.97	0.99	0.99
24	0.95	0.85	0.86	0.96	0.95					0.67	0.84
25		0.78		0.92	0.92	0.27	0.29	0.89	0.88	0.90	0.91

	Landmarks			Midline		Whole surface				Mixed	
	LMS	LMedS	DW-LMedS	LMS	LMedS	LMS	CW-LMS	LMedS	DW-LMedS	HM-LMedS	HM-DW-LMedS
1	0.92	0.95	0.94	1.00	1.00					0.82	0.87
2	0.87	1.00	1.00	1.00	1.00	0.90	0.90	0.97	0.98	0.98	0.99
3	0.73	0.99	1.00	1.00	1.00	0.70	0.77	0.95	0.95	0.96	0.96
4	0.95	1.00	1.00	1.00	1.00	0.67	0.70	0.93	0.94	0.95	0.97
5	0.95	0.99	1.00	1.00	1.00			0.87	0.86	0.93	0.94
6	0.75	0.96	0.95	0.99	0.99	0.64	0.67	0.85	0.88	0.90	0.93
7	0.93	0.98	0.98	0.99	0.99	0.62	0.64	0.84	0.88	0.92	0.94
8	0.54	0.99	1.00	1.00	1.00	0.61	0.65	0.94	0.95	0.95	0.96
9	0.94	0.99	0.99	1.00	1.00			0.86		0.92	0.94
10	0.89	0.94	0.95	1.00	1.00	0.51	0.51			0.88	0.90
11	0.65	0.97	0.97		1.00	0.84	0.92	1.00	1.00	1.00	1.00
12	0.95	1.00	1.00		1.00	0.65	0.71	1.00	1.00	1.00	1.00
13	0.84	1.00	1.00		1.00	0.92	0.97	1.00	1.00	1.00	1.00
14	0.67	1.00	1.00		1.00	0.60	0.64	1.00	1.00	1.00	1.00
15	0.76	1.00	1.00		1.00	0.88	0.94	1.00	1.00	1.00	1.00
16	0.61	1.00	1.00		1.00	0.92	0.97	1.00	1.00	1.00	1.00
17	0.83	1.00	1.00		1.00	0.76	0.84	1.00	1.00	1.00	1.00
18		0.98	0.97		1.00	0.82	0.90	1.00	1.00	1.00	1.00
19		1.00	1.00		1.00	0.91	0.97	1.00	1.00	1.00	1.00
20						0.97	1.00	1.00	1.00	0.98	0.97
21	0.87	0.85	0.96	0.83	0.60	0.53	0.56	0.78	0.65	0.93	0.94
22	0.72	1.00	1.00			0.84	0.91	1.00	1.00	0.94	0.94
23	0.70	0.62	0.75			0.92	0.92	0.97	0.97	0.99	0.99
24	0.95	0.85	0.86	0.96	0.95					0.67	0.84
25		0.78		0.92	0.92			0.89	0.88	0.90	0.91

	Landmarks			Midline		Whole surface				Mixed	
	LMS	LMedS	DW-LMedS	LMS	LMedS	LMS	CW-LMS	LMedS	DW-LMedS	HM-LMedS	HM-DW-LMedS
1	0.92	0.95	0.94	1.00	1.00					0.82	0.87
2	0.87	1.00	1.00	1.00	1.00	0.90	0.90	0.97	0.98	0.98	0.99
3	0.73	0.99	1.00	1.00	1.00	0.70	0.77	0.95	0.95	0.96	0.96
4	0.95	1.00	1.00	1.00	1.00	0.67	0.70	0.93	0.94	0.95	0.97
5	0.95	0.99	1.00	1.00	1.00			0.87	0.86	0.93	0.94
6	0.75	0.96	0.95	0.99	0.99	0.64	0.67	0.85	0.88	0.90	0.93
7	0.93	0.98	0.98	0.99	0.99	0.62	0.64	0.84	0.88	0.92	0.94
8	0.54	0.99	1.00	1.00	1.00	0.61	0.65	0.94	0.95	0.95	0.96
9	0.94	0.99	0.99	1.00	1.00			0.86		0.92	0.94
10	0.89	0.94	0.95	1.00	1.00	0.51	0.51			0.88	0.90
11	0.65	0.97	0.97		1.00	0.84	0.92	1.00	1.00	1.00	1.00
12	0.95	1.00	1.00		1.00	0.65	0.71	1.00	1.00	1.00	1.00
13	0.84	1.00	1.00		1.00	0.92	0.97	1.00	1.00	1.00	1.00
14	0.67	1.00	1.00		1.00	0.60	0.64	1.00	1.00	1.00	1.00
15	0.76	1.00	1.00		1.00	0.88	0.94	1.00	1.00	1.00	1.00
16	0.61	1.00	1.00		1.00	0.92	0.97	1.00	1.00	1.00	1.00
17	0.83	1.00	1.00		1.00	0.76	0.84	1.00	1.00	1.00	1.00
18		0.98	0.97		1.00	0.82	0.90	1.00	1.00	1.00	1.00
19		1.00	1.00		1.00	0.91	0.97	1.00	1.00	1.00	1.00
20						0.97	1.00	1.00	1.00	0.98	0.97
21	0.87	0.85	0.96	0.83	0.60	0.53	0.56	0.78	0.65	0.93	0.94
22	0.72	1.00	1.00			0.84	0.91	1.00	1.00	0.94	0.94
23	0.70	0.62	0.75			0.92	0.92	0.97	0.97	0.99	0.99
24	0.95	0.85	0.86	0.96	0.95					0.67	0.84
25		0.78		0.92	0.92			0.89	0.88	0.90	0.91

Hybrid methods



- Hemispheres + midline (HM-LMedS & HM-DW.LMedS)
  - The midline provides complementary performance to surface-based methods
  - Large imbalance between the number of surface and midline points

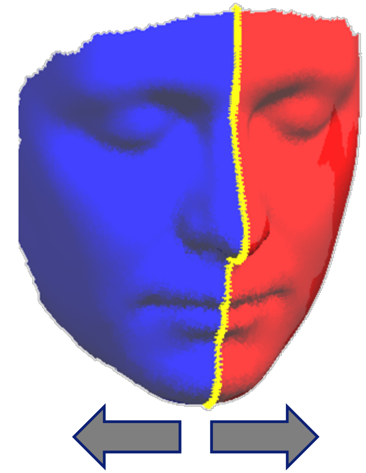
$$\arg\min_{\hat{T}} \left(\text{median} \left\{ \| e_{DW}^{i} \|^{2} \right\}_{\forall i \notin \mathcal{I}_{ML}} + \right.$$

$$\left. + \frac{1}{2} \text{median} \left\{ e_{i}^{2} \right\}_{\forall i \in \mathcal{I}_{ML}} + \sum_{i=1}^{n_V} \frac{|e_{i}^{2} - e_{\bar{i}}^{2}|}{n_V} \right)$$

$$e_{DW}^{i} = \exp\left(\frac{-1}{\tau} \frac{d_{ML}(\mathbf{v}_{S}^{i})}{\max\left(d_{ML}(\mathbf{v}_{S})\right)} \right) \left(\mathbf{v}_{S}^{i} - \hat{T}(\mathbf{v}_{M}^{\bar{i}}) \right)$$

$$e_{i} = \|\mathbf{v}_{S}^{i} - \hat{T}(\mathbf{v}_{M}^{\bar{i}})\|, \qquad e_{\bar{i}} = \|\mathbf{v}_{S}^{\bar{i}} - \hat{T}(\mathbf{v}_{M}^{i})\| \qquad \mathcal{I}_{ML} = \left\{ i \in [1; n_V] \,|\, \bar{i} = i \right\}$$

Hybrid methods



- Hemispheres + midline (HM-LMedS & HM-DW.LMedS)
  - The midline provides complementary performance to surface-based methods
  - Large imbalance between the number of surface and midline points

$$\arg\min_{\hat{T}}\left(\operatorname{median}\{\| e^i_{DW} \|^2\}_{\forall i \notin \mathcal{I}_{ML}} + \right.$$
$$\left. + \frac{1}{2}\operatorname{median}\{e_i^2\}_{\forall i \in \mathcal{I}_{ML}} + \sum_{i=1}^{n_V} \frac{|e_i^2 - e_{\bar{i}}^2|}{n_V} \right)$$

$$e^i_{DW} = \exp\left(\frac{-1}{\tau}\frac{d_{ML}(\mathbf{v}^i_S)}{\max\left(d_{ML}(\mathbf{v}_S)\right)}\right)(\mathbf{v}^i_S - \hat{T}(\mathbf{v}^{\bar{i}}_M))$$

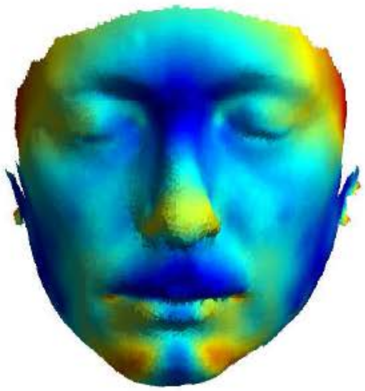
$$e_i = \|\mathbf{v}^i_S - \hat{T}(\mathbf{v}^{\bar{i}}_M)\|, \qquad e_{\bar{i}} = \|\mathbf{v}^{\bar{i}}_S - \hat{T}(\mathbf{v}^i_M)\| \qquad \mathcal{I}_{ML} = \{i \in [1; n_V] \,|\, \bar{i} = i\}$$

Conclusions

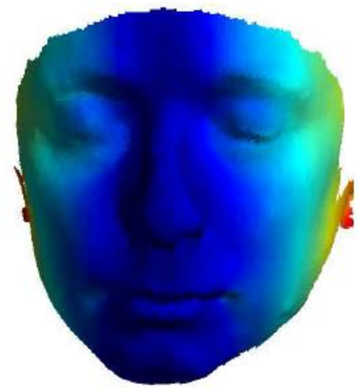
- We tested different alignment methods for the estimation of point-wise 3D surface asymmetry
  - Important differences between methods
  - Widespread least-squares cost functions performed the worst
  - Landmark-based methods were generally better than methods using the whole surface
- A hybrid approach combining surface and mid-line terms achieved the best performance
  - Exponential decaying of weights as points depart from the midline
  - As a byproduct we get more robustness to the exact definition of face boundaries

A qualitative example estimating the unknown asymmetry of a real scan

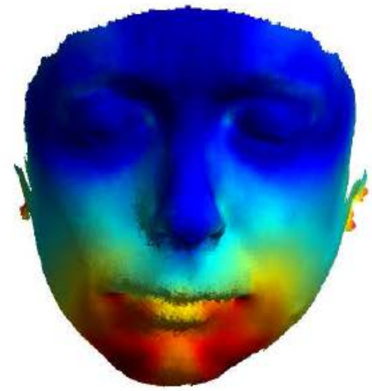
Landmark-based LMS



Midline-based LMS

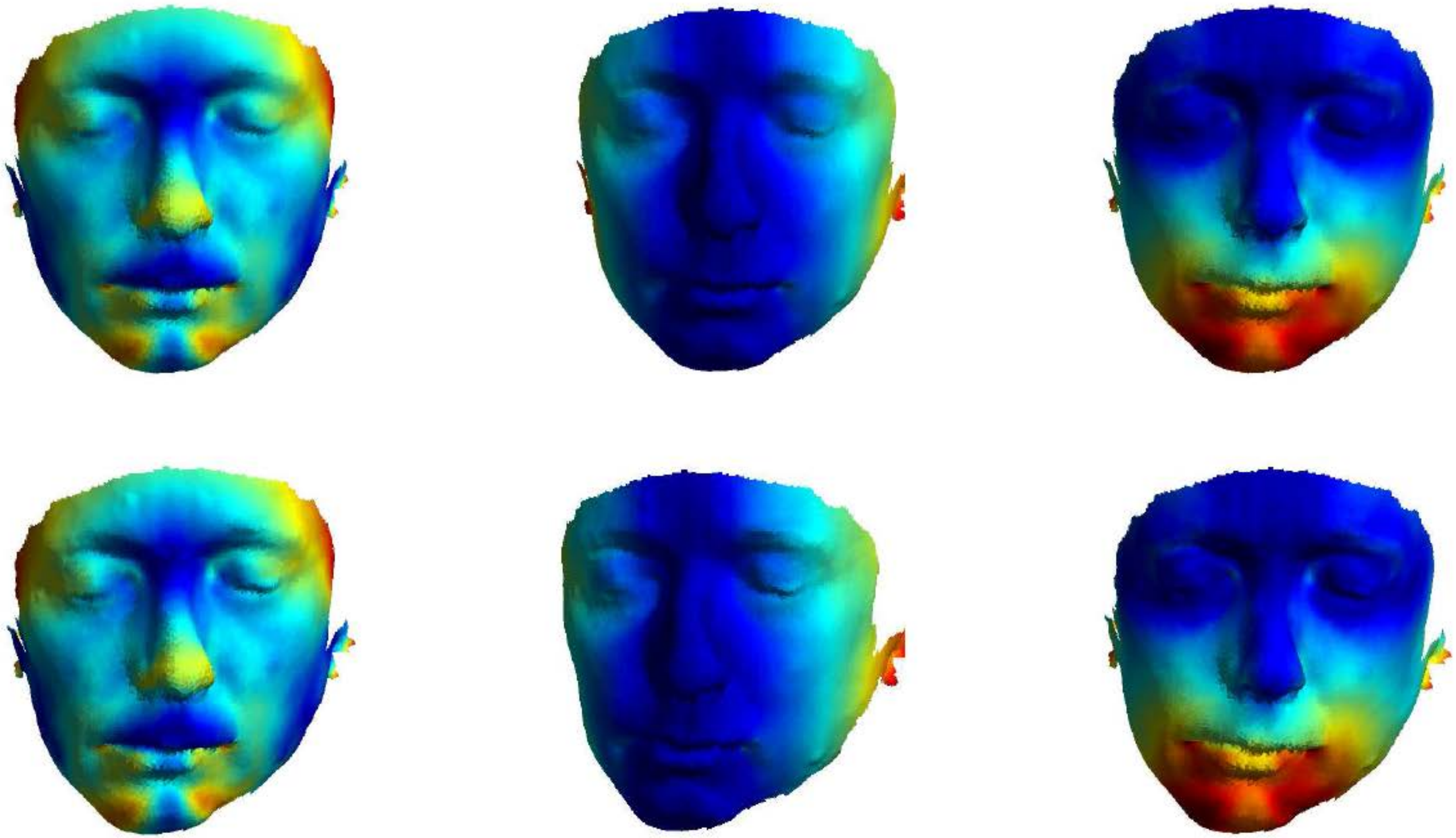


DW-HM-LMdeS



A qualitative example estimating the unknown asymmetry of a real scan

Landmark-based LMS | Midline-based LMS | DW-HM-LMdeS



Asymmetry patterns magnified by 2.0 : 1

Thank you for your attention

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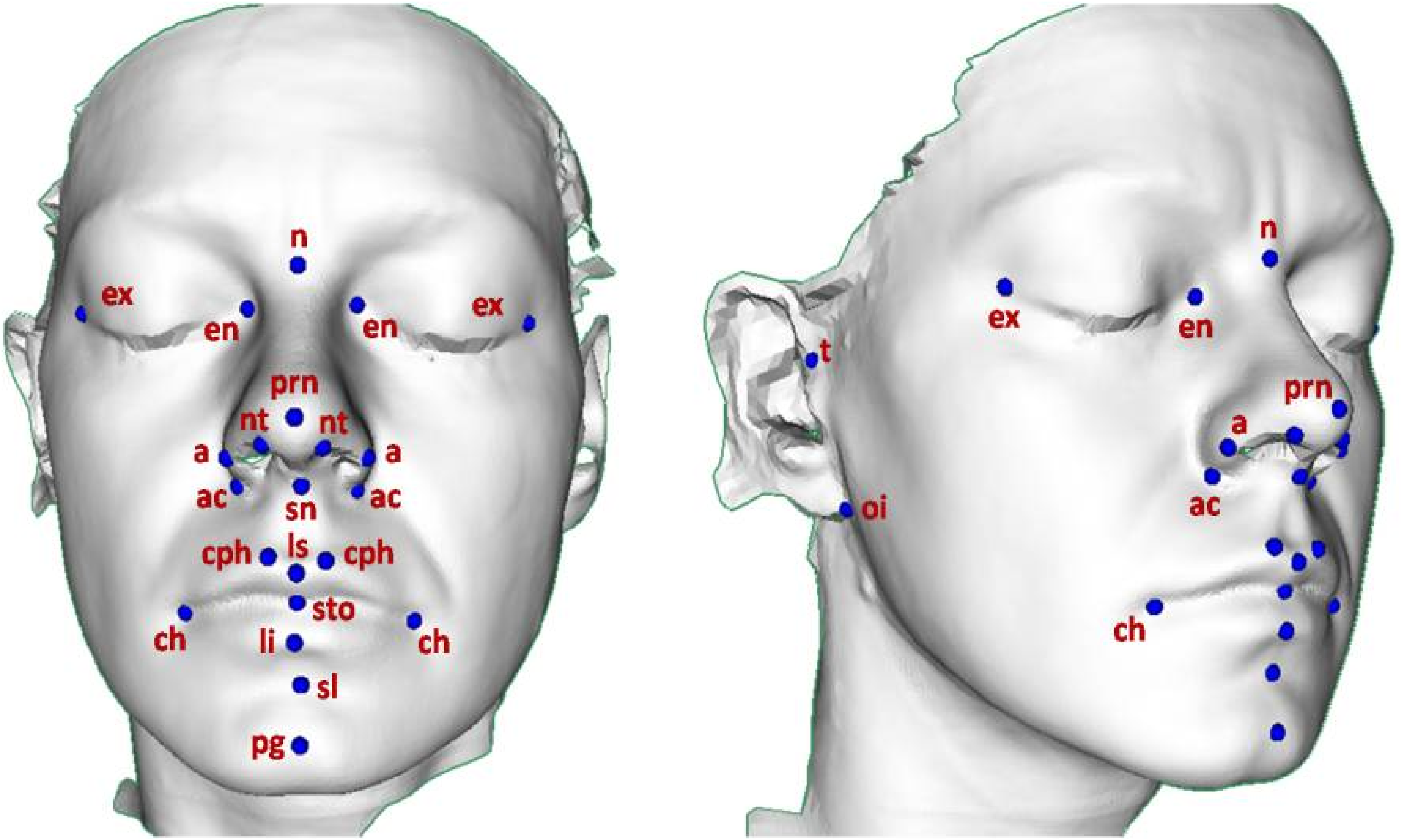
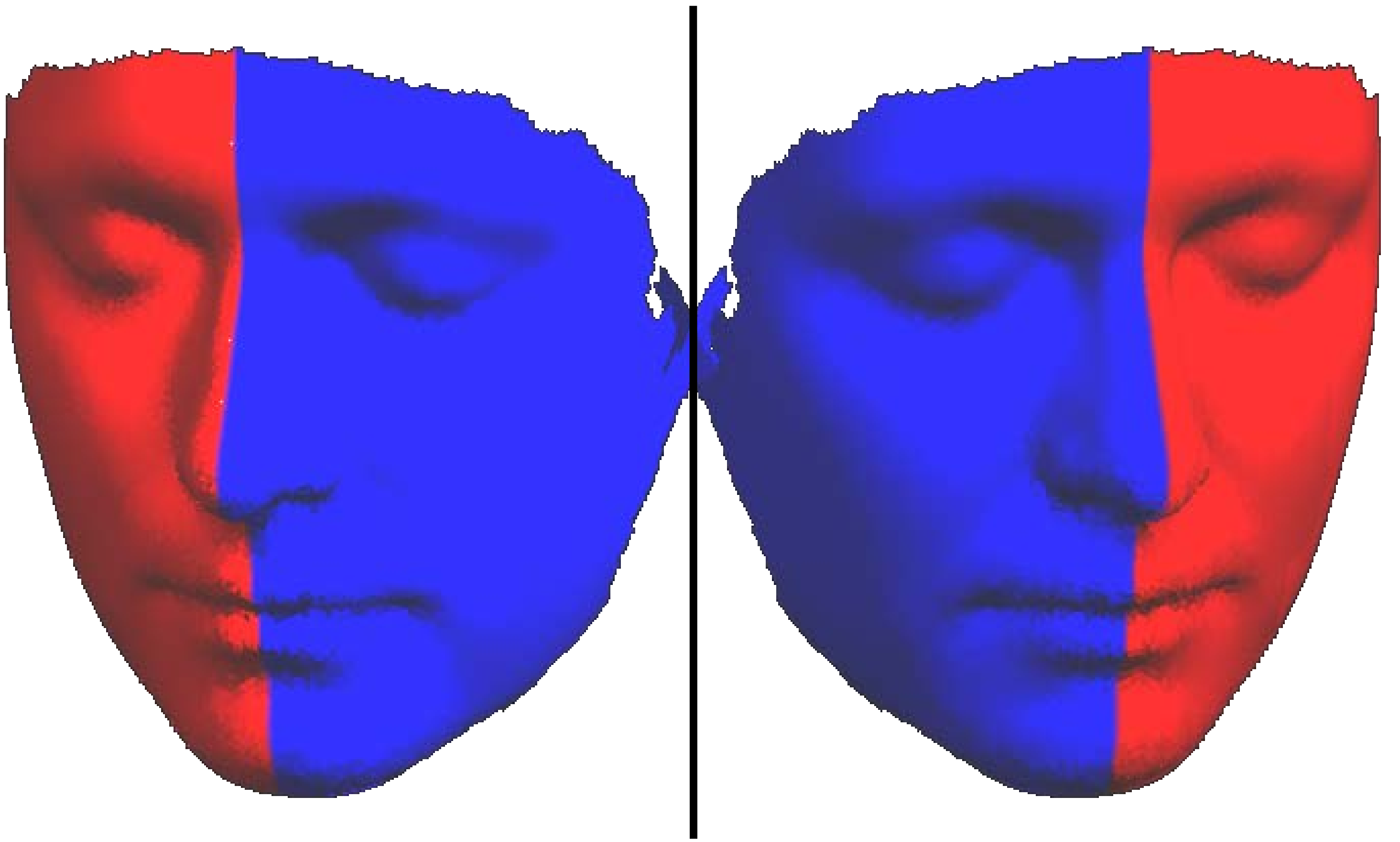
Craniofacial landmarks



Manual annotations from: R. Hennessy et al. Biol Psychiat **51 (2002) 507–514**

Comparison of hemispheres by reflection

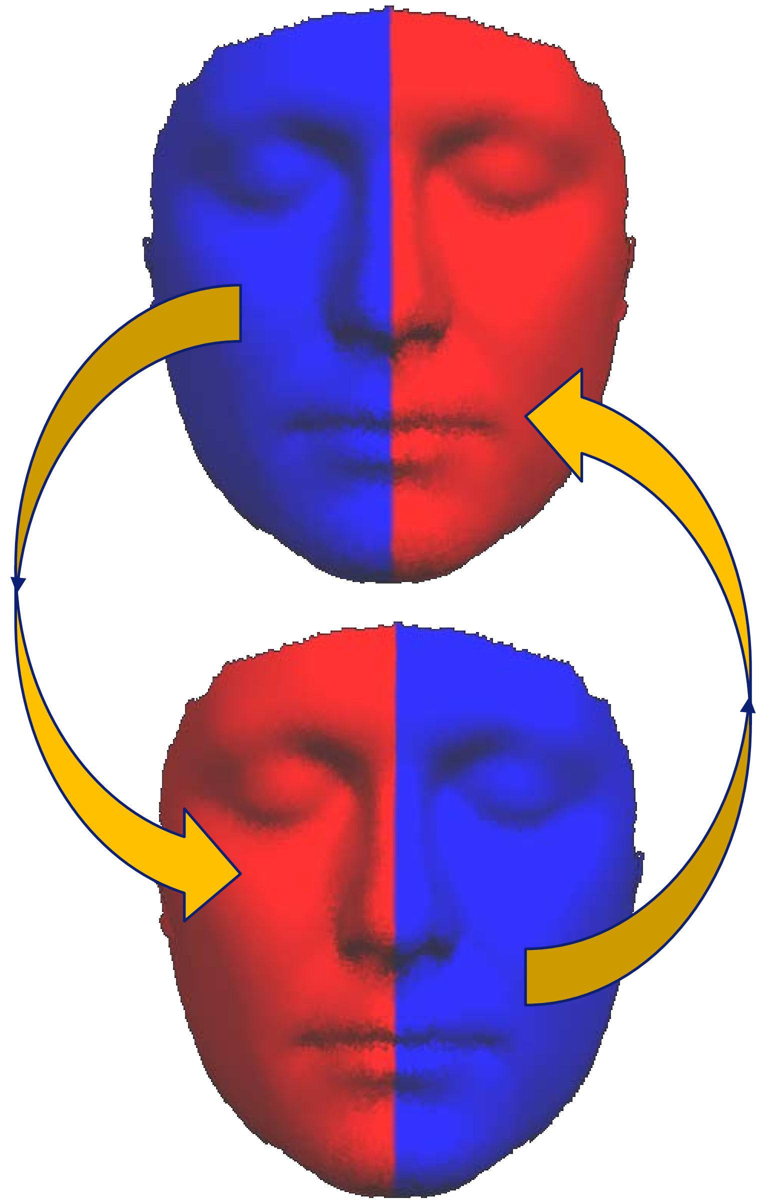
- Reflection into an arbitrary plane



We can register the mirrored surface with the original

Comparison of hemispheres by reflection

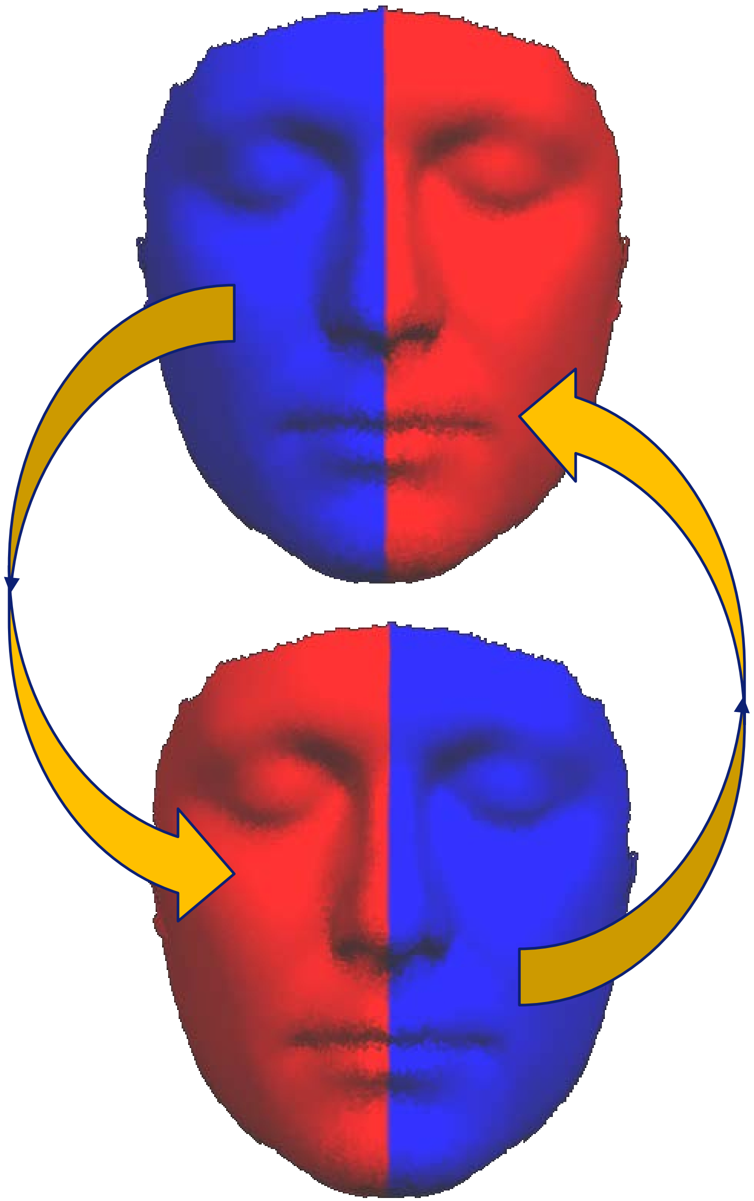
- Reflection into an arbitrary plane
- Alignment of the original and mirrored surfaces
- Straight-forward comparison
  - Left w/ right
  - Right w/ left



Comparison of hemispheres by reflection

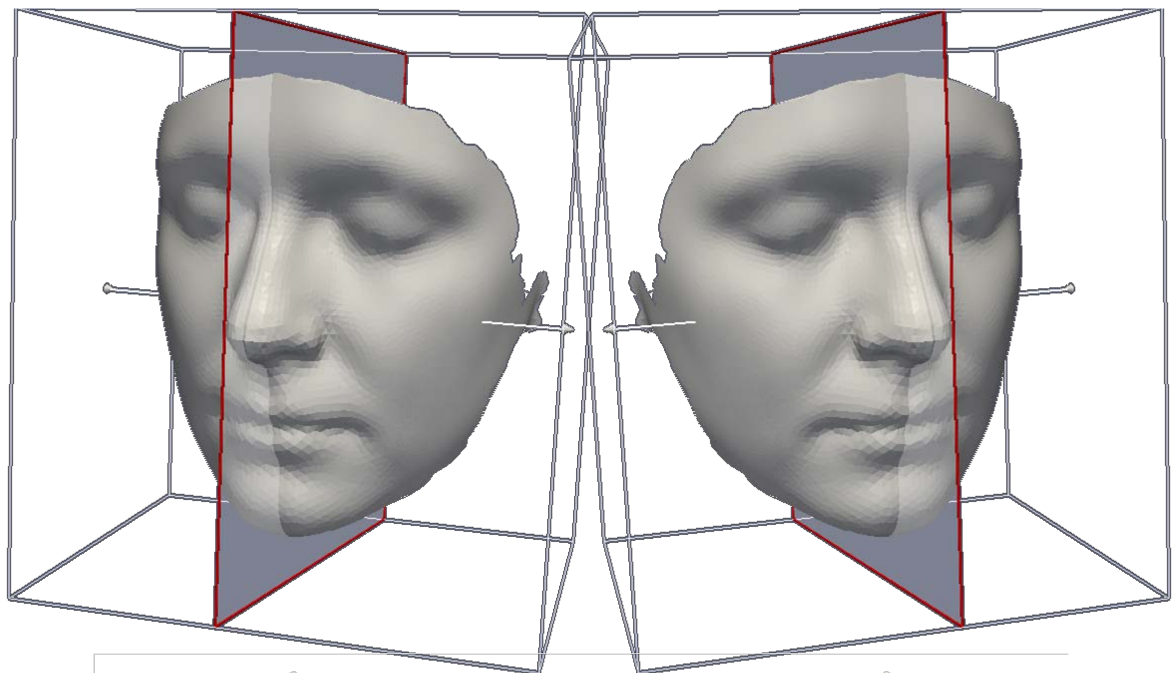
- Reflection into an arbitrary plane
- Alignment of the original and mirrored surfaces
- Straight-forward comparison
  - Left w/ right
  - Right w/ left

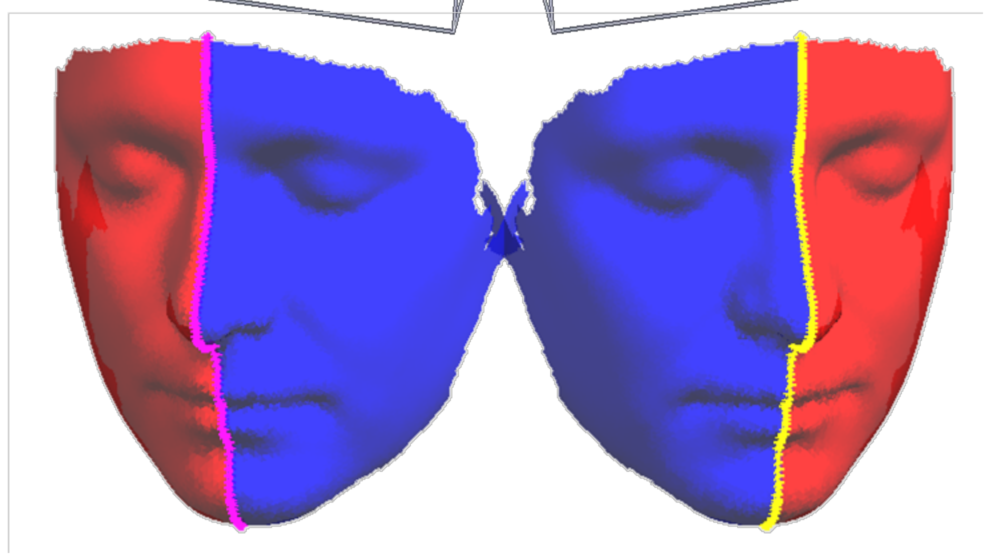
How do we align both surfaces ?



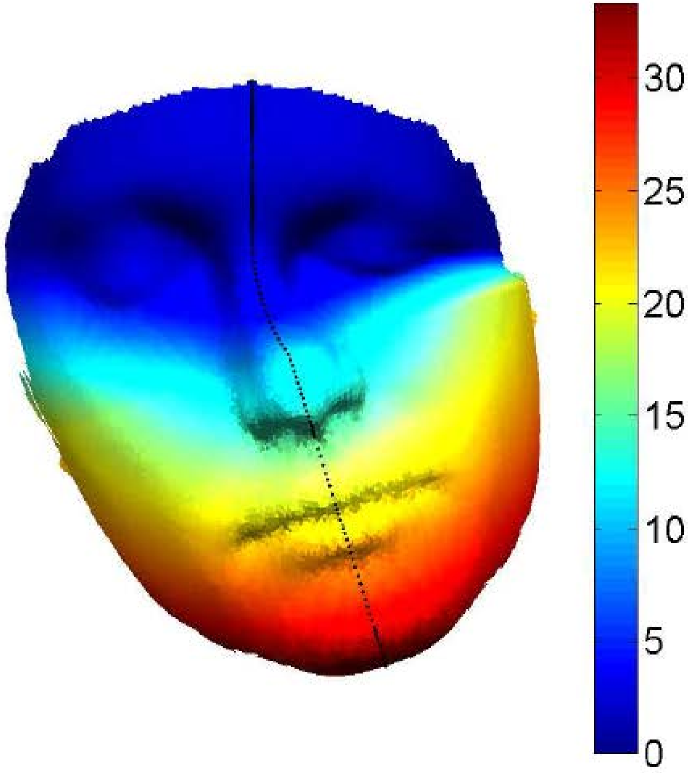
Alignment of the mid-sagittal plane

- The mid-sagittal plane of the original and mirrored surfaces should be the same
- Hence, making both mid-sagittal planes coincide should align the surfaces





Complex example 1



Complex example 1