FG 2015, May 7, 2015

Multi-view Discriminant Analysis with Tensor Representation and Its Application to Cross-view Gait Recognition

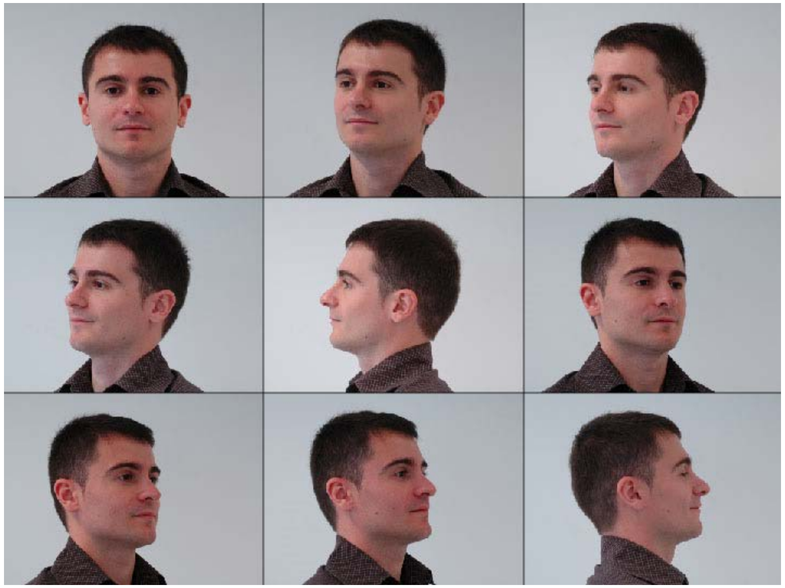
Yasushi Makihara[†], Al Mansur[†], Daigo Muramatsu[‡†], Zasim Uddin[†], Yasushi Yagi[†]

†Osaka University

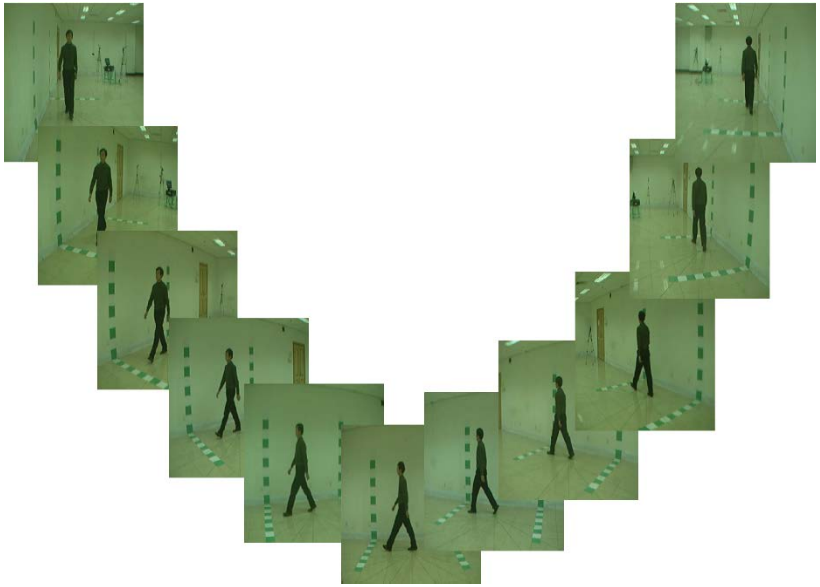
‡National Institute of Information and Communications Technology

Background

- Cross-view recognition



Face recognition [1]



Gait recognition [Yu et al. 2006]

[1] https://gtav.upc.edu/research-areas/face-database

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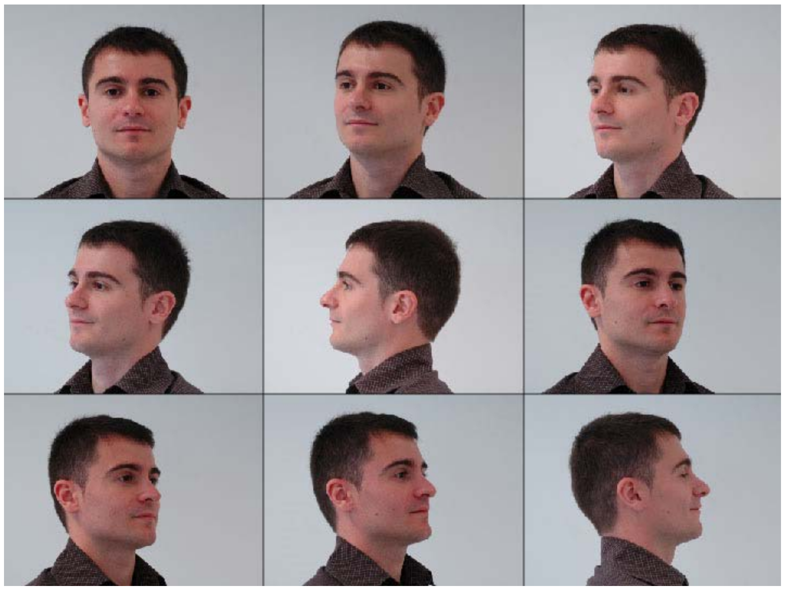

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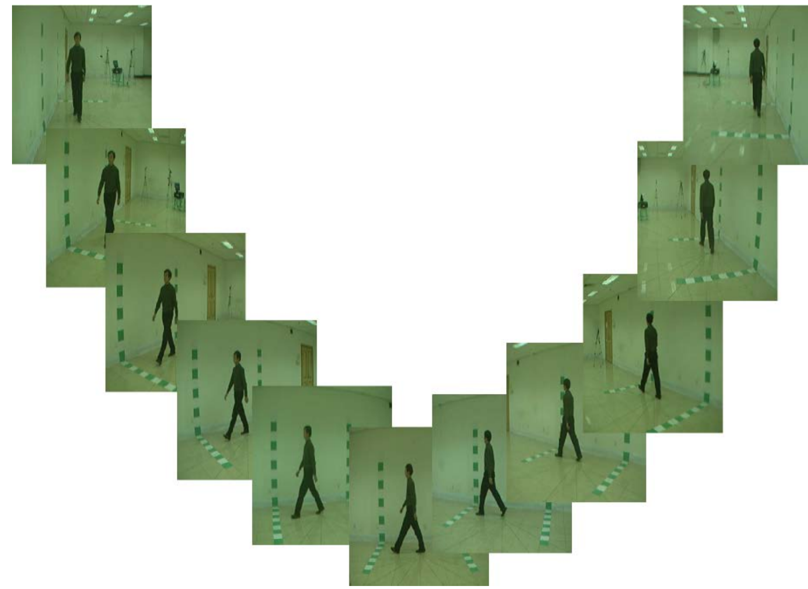

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Discriminant analysis
✓ Cross-view property
▪

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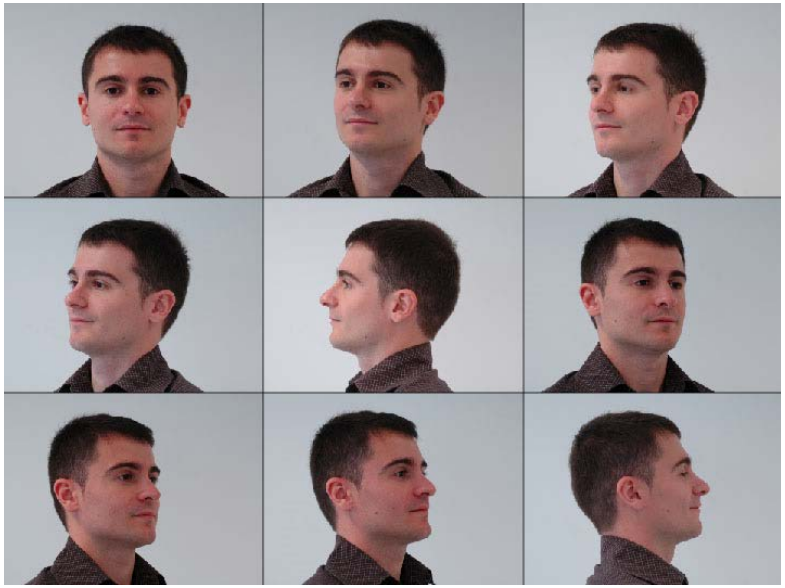

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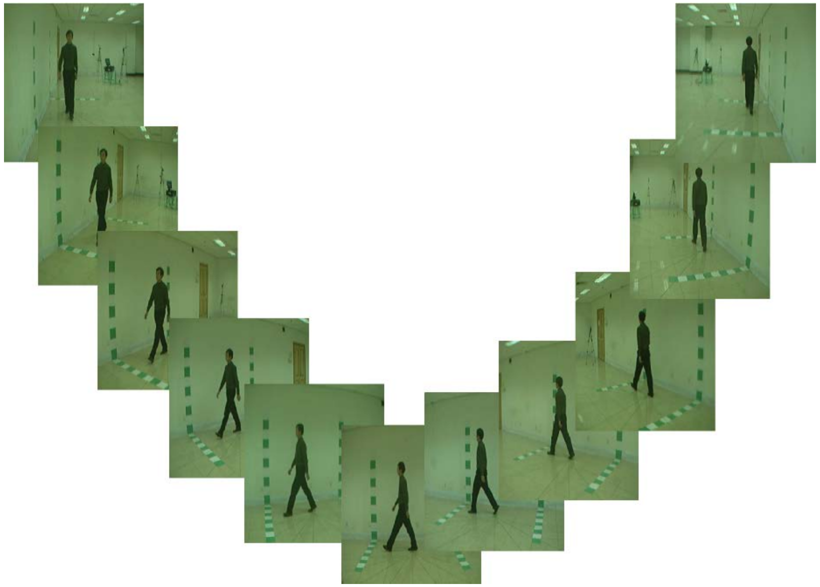

Gait recognition [Yu et al. 2006]

Discriminant analysis
- ✓ Cross-view property
- ✓ Small sample size (SSS) problem

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Related work

Method	Cross-view	SSS problem

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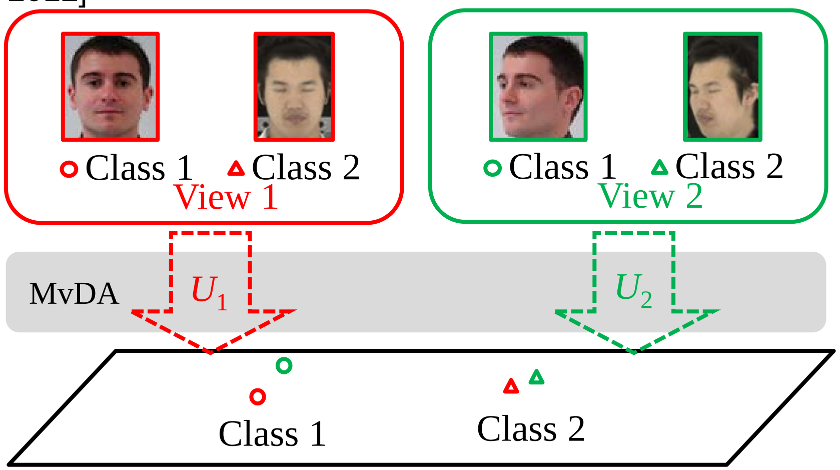
Multi-view discriminant analysis (MvDA) [Kan et al. 2012] 		

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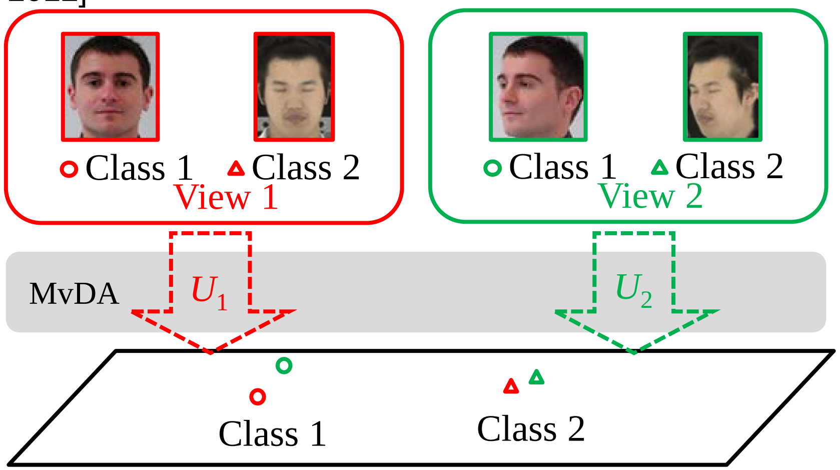
Multi-view discriminant analysis (MvDA) [Kan et al. 2012] 	✔	

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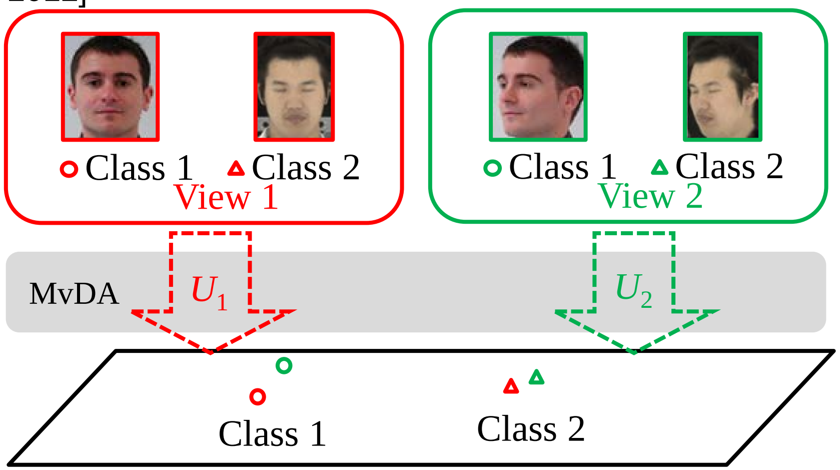
Multi-view discriminant analysis (MvDA) [Kan et al. 2012] 	✔	X

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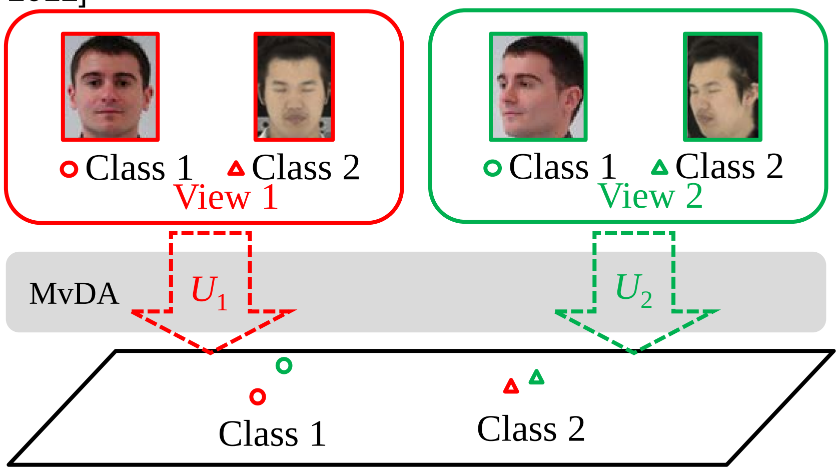
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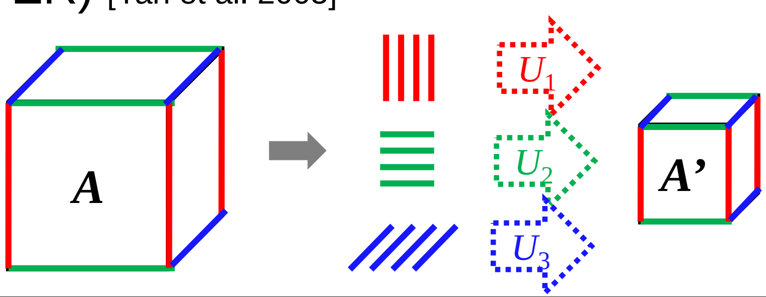
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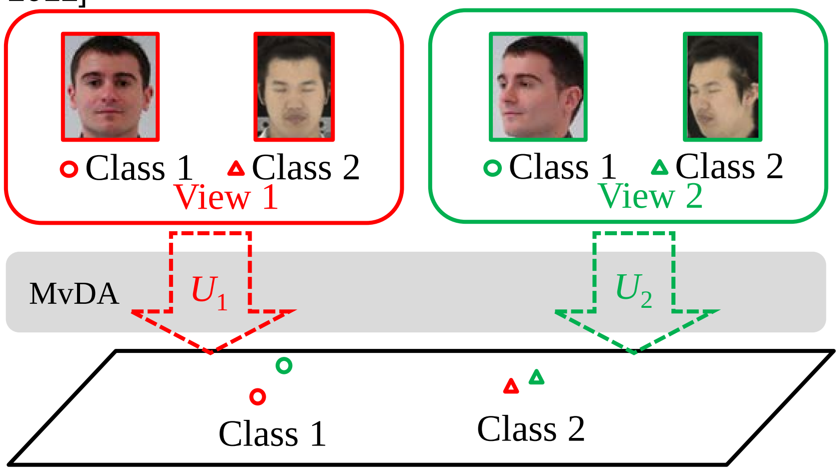
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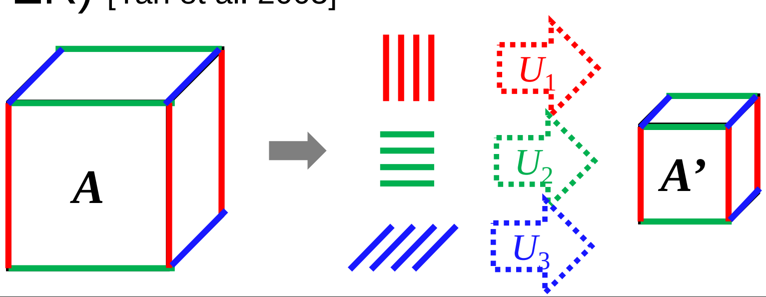
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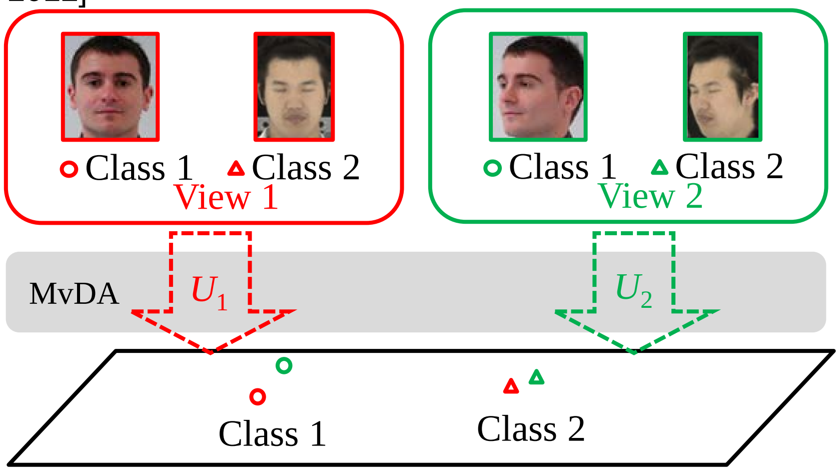
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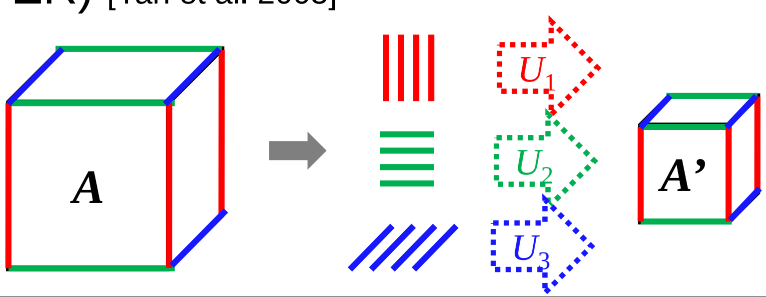
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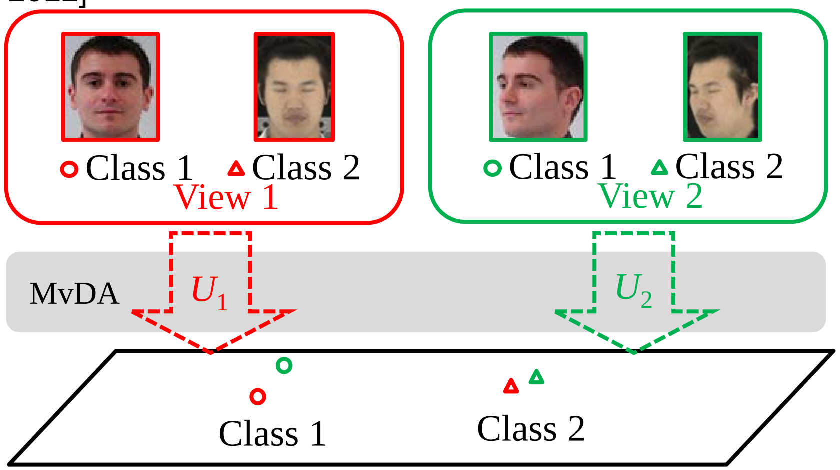
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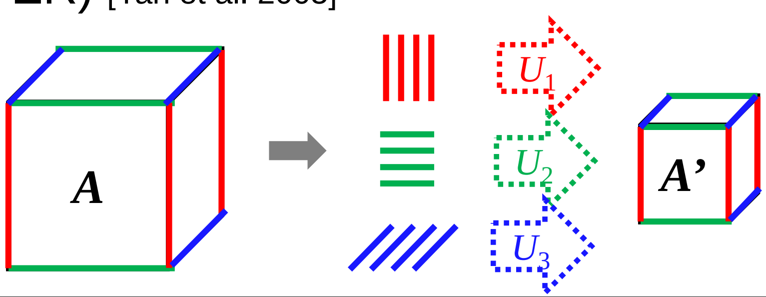
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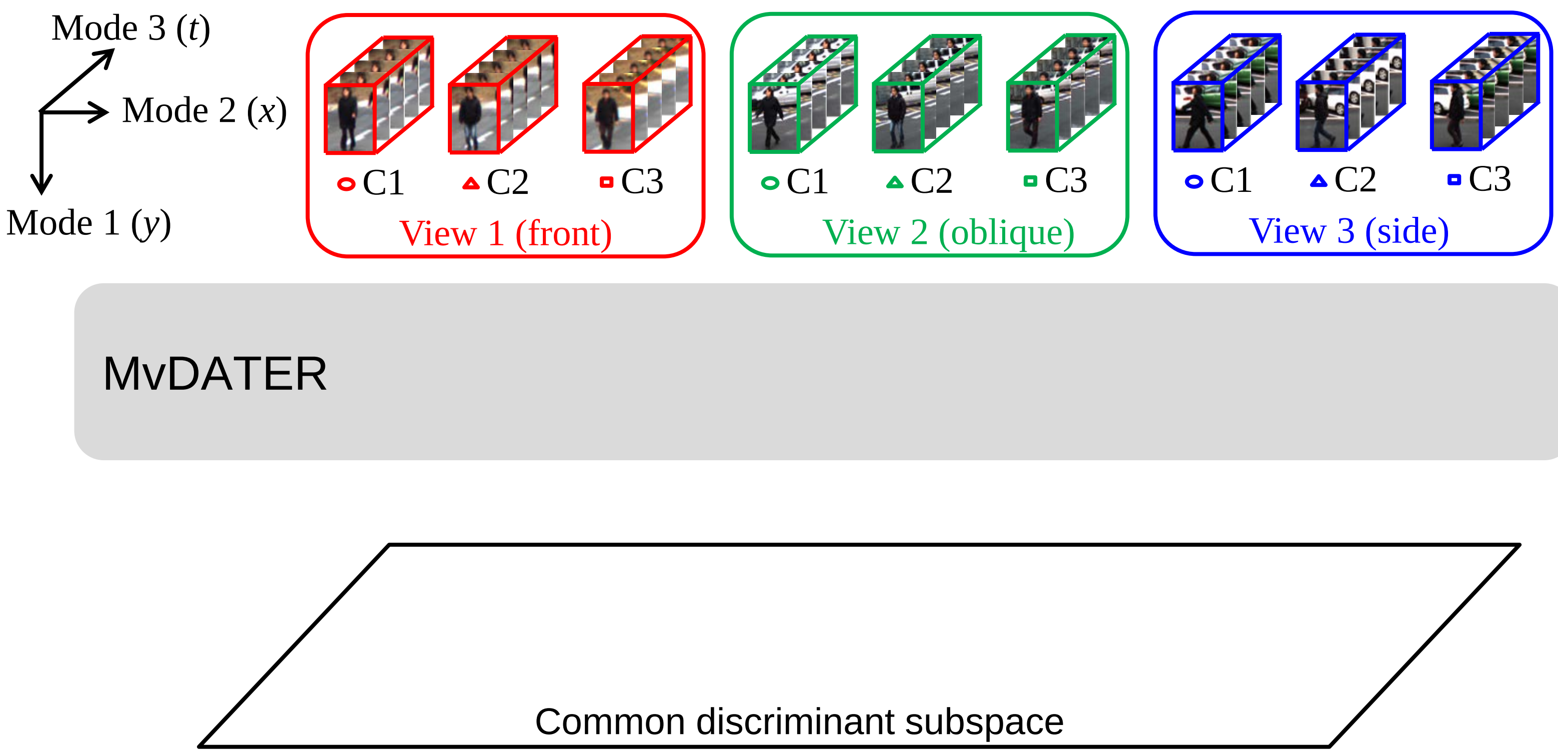
Objective

- **MvDATER**: **M**ulti-**v**iew **D**iscriminant **A**nalysis with **TE**nsor **R**epresentation



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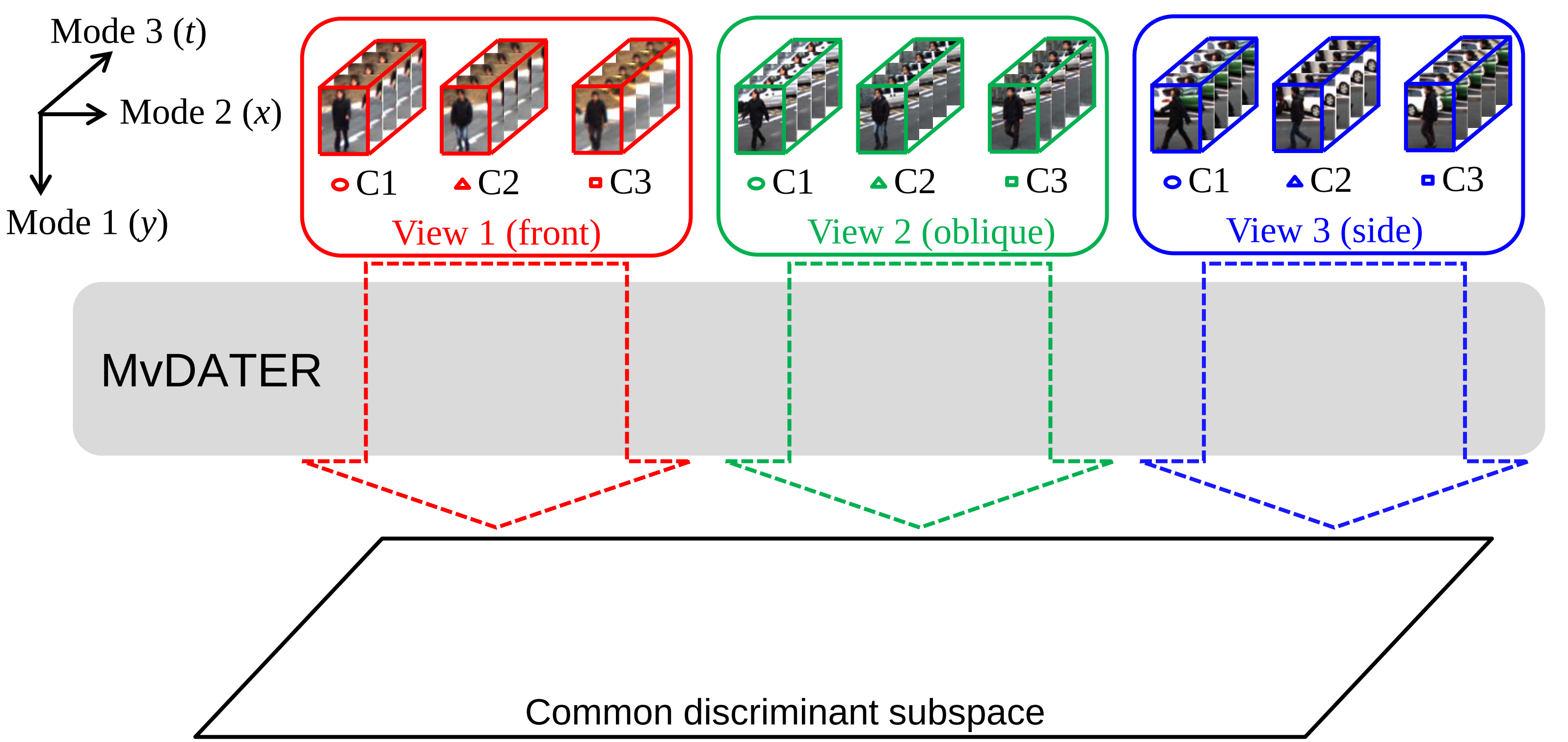

Multi-view projections

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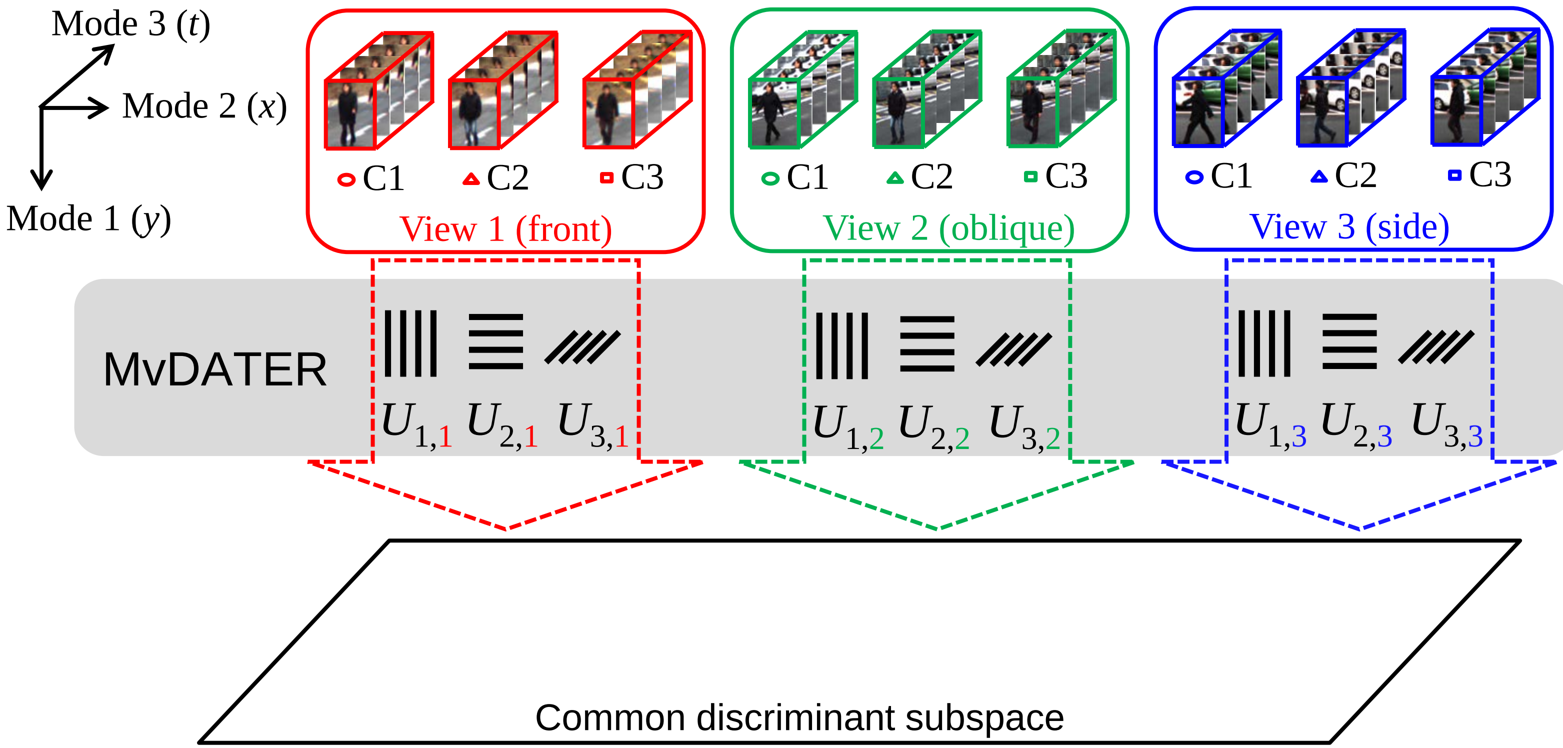

Multi-view projections + Multi-mode projections

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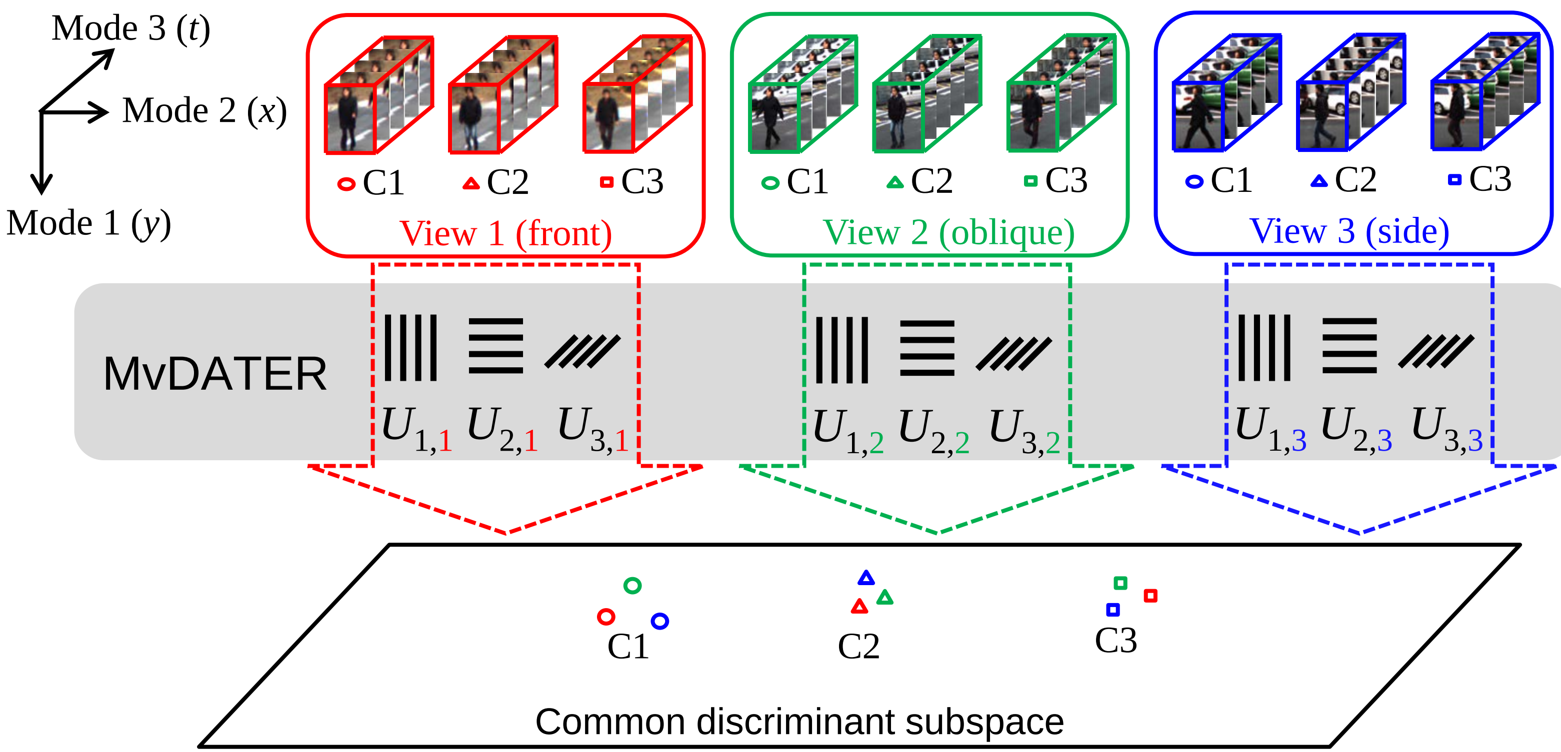

Multi-view projections + Multi-mode projections

Tensor representation (1)

- L-th order tensor object

$$\mathcal{A} \in \mathbf{R}^{M_1 \times \cdots \times M_L}$$

M_l: l-th mode dimension

$\mathcal{A}(m_1, \cdots, m_L)$: Tensor component

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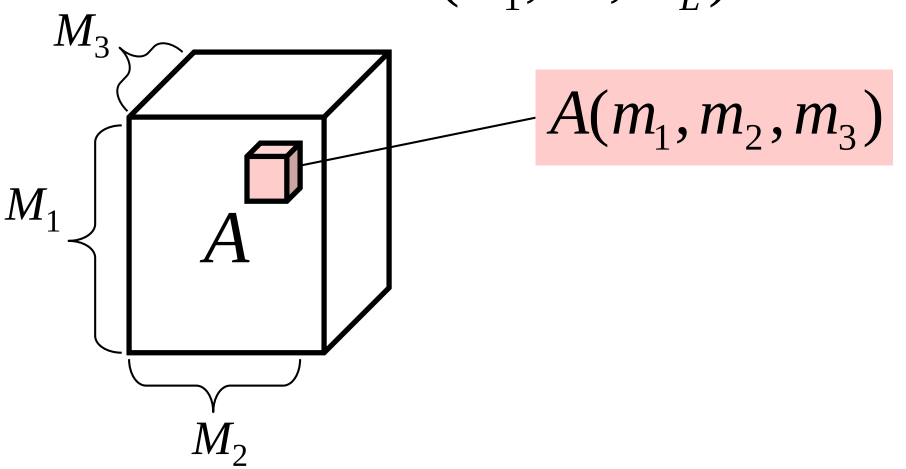

3rd order tensor object

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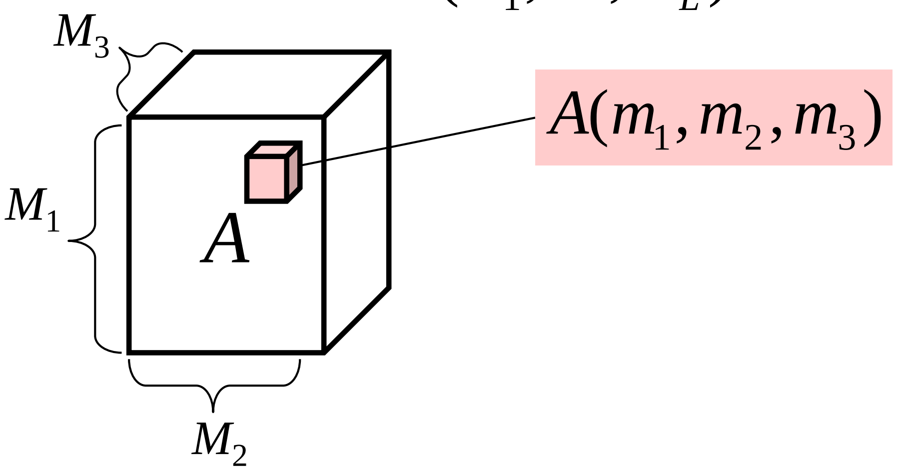

3rd order tensor object

- Inner product

$$\langle A, B \rangle = \sum_{m_1=1}^{M_1} \cdots \sum_{m_L=1}^{M_L} A(m_1, \cdots, m_L) B(m_1, \cdots, m_L)$$

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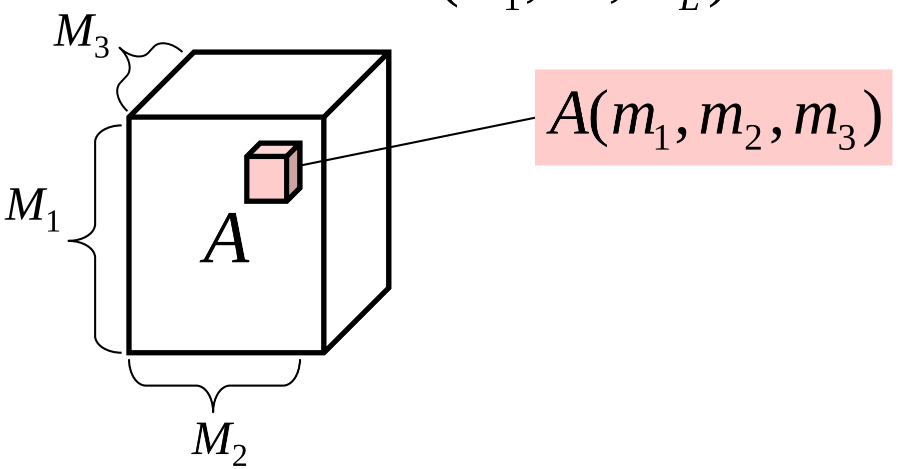

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$$\langle \mathcal{A}, \mathcal{B} \rangle = \sum_{m_1=1}^{M_1} \cdots \sum_{m_L=1}^{M_L} \mathcal{A}(m_1, \cdots, m_L) \mathcal{B}(m_1, \cdots, m_L)$$

- Frobenius norm

$$\|\mathcal{A}\|_F = \sqrt{\langle \mathcal{A}, \mathcal{A} \rangle}$$

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$$Y = X \times_l U_l$$

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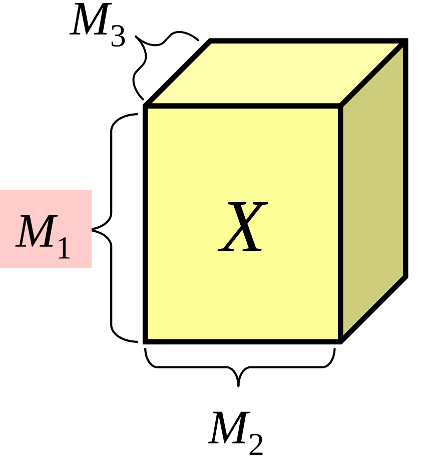

1-mode product for 3rd order tensor object

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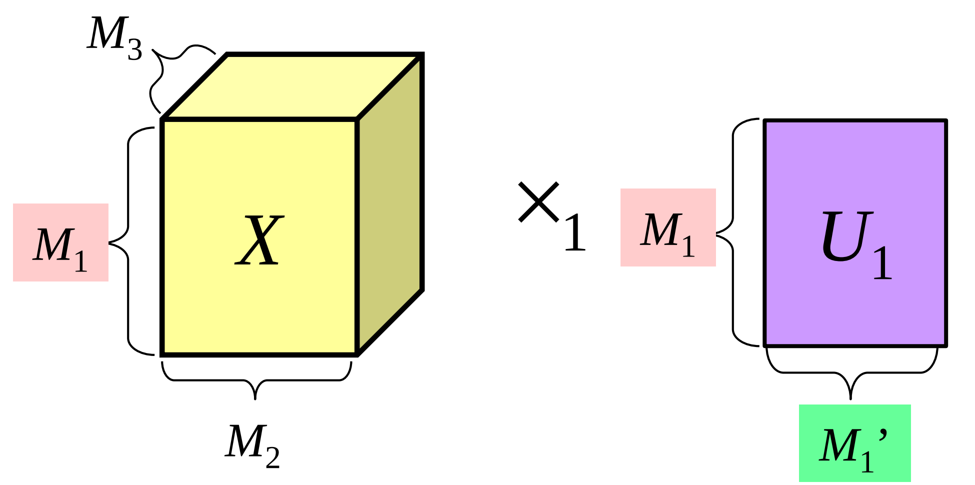

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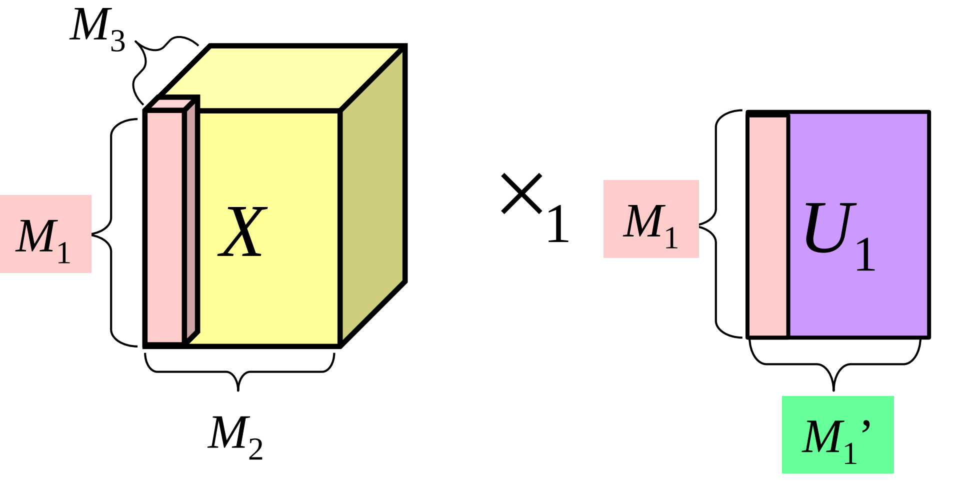

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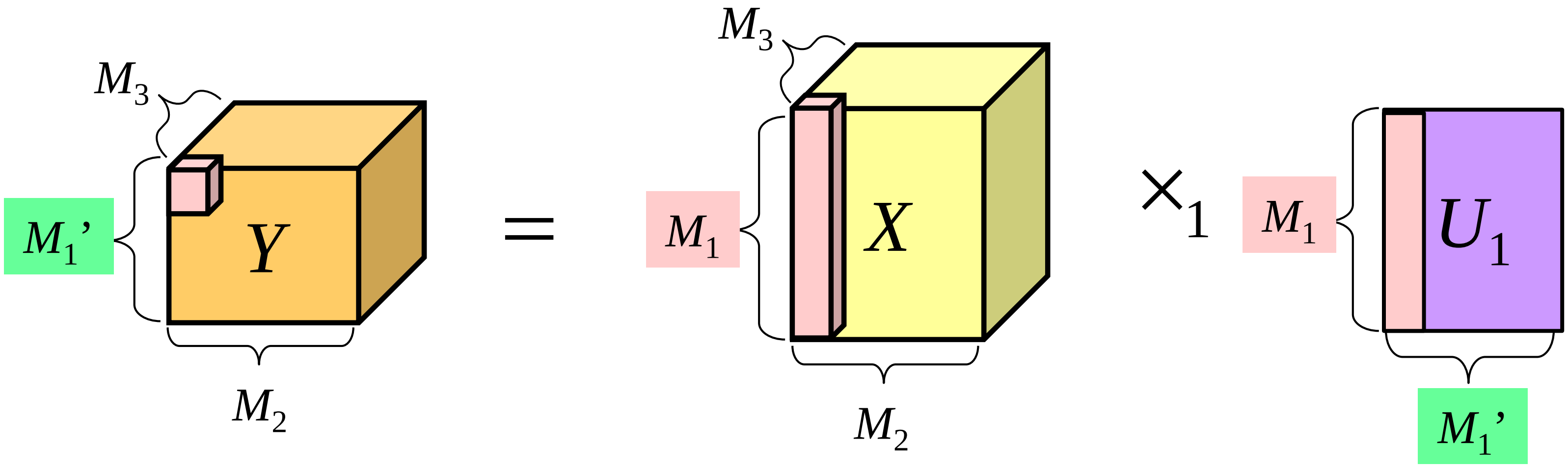

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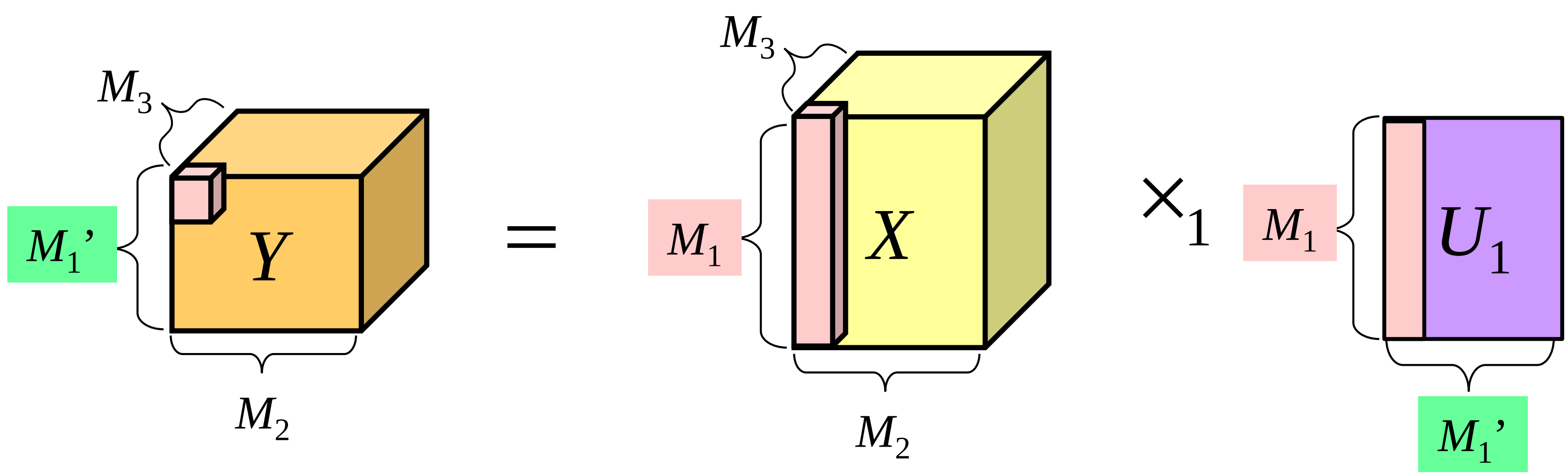


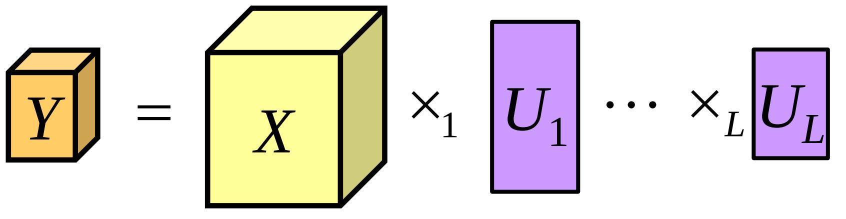
1-mode product for 3rd order tensor object

$M_l' < M_l$ → l-mode dimension reduction

Multi-view multi-mode projections

- Multi-mode projections into low dimensional space

$$Y = X \times_1 U_1 \cdots \times_L U_L$$



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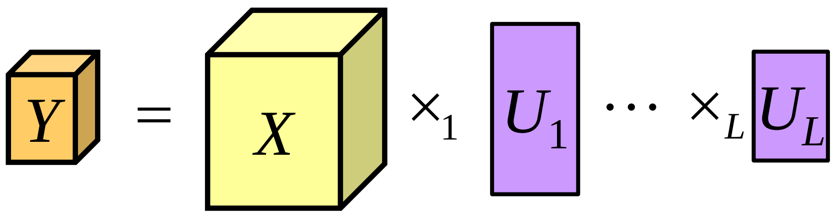

Extend to multiple views

$$\mathbf{U} = \{U_{l,j}\} \left(l = 1, \ldots, L,\; j = 1, \ldots, N_V \right)$$

Mode index

View index

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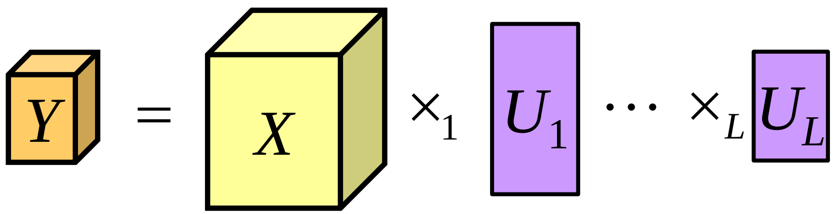

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Mode index

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$$Y_{ijk} = X_{ijk} \times_1 U_{1,j} \cdots \times_L U_{L,j}$$

i: Class index ($i = 1, \ldots, n_c$)

k: Sample index of class i from view j ($k = 1, \ldots, n_{ij}$)

Discriminant tensor criterion

- Ratio of between-class and within-class scatter

$$\mathbf{U}^* = \arg\max_{\mathbf{U}} \frac{\sum_{i=1}^{N_C} n_i \left\| \overline{Y}_i - \overline{Y} \right\|_F^2}{\sum_{i=1}^{N_C} \sum_{j=1}^{N_V} \sum_{k=1}^{n_{ij}} \left\| Y_{ijk} - \overline{Y}_i \right\|_F^2}$$

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Substitute projection matrices

$$\mathbf{U}^* = \arg\max_{\mathbf{U}} \frac{\sum_{i=1}^{N_C} n_i \left\| \sum_{r=1}^{N_V} w_{ir} \left(\bar{X}_{ir} \times_1 U_{1,r} \cdots \times_L U_{L,r} \right) - \sum_{q=1}^{N_C} w_q \sum_{r=1}^{N_V} w_{qr} \left(\bar{X}_{qr} \times_1 U_{1,r} \cdots \times_L U_{L,r} \right) \right\|_F^2}{\sum_{i=1}^{N_C}\sum_{j=1}^{N_V}\sum_{k=1}^{n_{ij}} \left\| X_{ijk} \times_1 U_{1,r} \cdots \times_L U_{L,r} - \sum_{r=1}^{N_V} w_{ir} \left(\bar{X}_{ir} \times_1 U_{1,r} \cdots \times_L U_{L,r} \right) \right\|_F^2}$$

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No closed-form solution due to higher order tensor structure

Discriminant tensor criterion

-

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l-mode discriminant analysis

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$$\mathbf{U}^* = \arg\max_{\mathbf{U}} \frac{\sum_{i=1}^{N_C} n_i \left\| \sum_{r=1}^{N_V} w_{ir} \left(\bar{X}_{ir} \times_1 U_{1,r} \cdots \times_L U_{L,r} \right) - \sum_{q=1}^{N_C} w_q \sum_{r=1}^{N_V} w_{qr} \left(\bar{X}_{qr} \times_1 U_{1,r} \cdots \times_L U_{L,r} \right) \right\|_F^2}{\sum_{i=1}^{N_C} \sum_{j=1}^{N_V} \sum_{k=1}^{n_{ij}} \left\| X_{ijk} \times_1 U_{1,r} \cdots \times_L U_{L,r} - \sum_{r=1}^{N_V} w_{ir} \left(\bar{X}_{ir} \times_1 U_{1,r} \cdots \times_L U_{L,r} \right) \right\|_F^2}$$

Focus only on *l*-th mode product

$$U_l^* = \arg\max_{U_l} \frac{\sum_{i=1}^{N_C} n_i \left\| \sum_{r=1}^{N_V} w_{ir} \left(\bar{X}_{ir} \times_l U_{l,r} \right) - \sum_{q=1}^{N_C} w_q \sum_{r=1}^{N_V} w_{qr} \left(\bar{X}_{qr} \times_l U_{l,r} \right) \right\|}{\sum_{i=1}^{N_C} \sum_{j=1}^{N_V} \sum_{k=1}^{n_{ij}} \left\| X_{ijk} \times_l U_{l,j} - \sum_{r=1}^{N_V} w_{ir} \left(\bar{X}_{ir} \times_l U_{l,r} \right) \right\|_F^2}$$

l-mode discriminant analysis

$$\mathbf{U}^* = \arg\max_{\mathbf{U}} \frac{\sum_{i=1}^{N_C} n_i \left\| \sum_{r=1}^{N_V} w_{ir} \left(\overline{X}_{ir} \times_1 U_{1,r} \cdots \times_L U_{L,r} \right) - \sum_{q=1}^{N_C} w_q \sum_{r=1}^{N_V} w_{qr} \left(\overline{X}_{qr} \times_1 U_{1,r} \cdots \times_L U_{L,r} \right) \right\|_F^2}{\sum_{i=1}^{N_C} \sum_{j=1}^{N_V} \sum_{k=1}^{n_{ij}} \left\| X_{ijk} \times_1 U_{1,r} \cdots \times_L U_{L,r} - \sum_{r=1}^{N_V} w_{ir} \left(\overline{X}_{ir} \times_1 U_{1,r} \cdots \times_L U_{L,r} \right) \right\|_F^2}$$

Focus only on l-th mode product

$$U_l^* = \arg\max_{U_l} \frac{\sum_{i=1}^{N_C} n_i \left\| \sum_{r=1}^{N_V} w_{ir} \left(\overline{X}_{ir} \times_l U_{l,r} \right) - \sum_{q=1}^{N_C} w_q \sum_{r=1}^{N_V} w_{qr} \left(\overline{X}_{qr} \times_l U_{l,r} \right) \right\|}{\sum_{i=1}^{N_C} \sum_{j=1}^{N_V} \sum_{k=1}^{n_{ij}} \left\| X_{ijk} \times_l U_{l,j} - \sum_{r=1}^{N_V} w_{ir} \left(\overline{X}_{ir} \times_l U_{l,r} \right) \right\|_F^2}$$

Rearrange (see proceeding for detailed derivation)

$$U_l^* = \arg\max_{U_l} \frac{Tr\left(\sum_{s=1}^{N_V} \sum_{r=1}^{N_V} U_{l,r}{}^T S_{B,rs}^{(l)} U_{l,s} \right)}{Tr\left(\sum_{s=1}^{N_V} \sum_{r=1}^{N_V} U_{l,r}{}^T S_{W,rs}^{(l)} U_{l,s} \right)}$$

l-mode between-class scatter matrix

l-mode within-class scatter matrix

l-mode discriminant analysis

$$U_l^* = \arg\max_{U_l} \frac{Tr\left(\sum_{s=1}^{N_V}\sum_{r=1}^{N_V} U_{l,r}{}^T S_{B,rs}^{(l)} U_{l,s}\right)}{Tr\left(\sum_{s=1}^{N_V}\sum_{r=1}^{N_V} U_{l,r}{}^T S_{W,rs}^{(l)} U_{l,s}\right)}$$

l-mode discriminant analysis

$$U_l^* = \arg\max_{U_l} \frac{Tr\left(\sum_{s=1}^{N_V}\sum_{r=1}^{N_V} U_{l,r}{}^T S_{B,rs}^{(l)} U_{l,s}\right)}{Tr\left(\sum_{s=1}^{N_V}\sum_{r=1}^{N_V} U_{l,r}{}^T S_{W,rs}^{(l)} U_{l,s}\right)}$$

⇩ Concatenate multi-view projection matrices

$$U_l = \begin{bmatrix} U_{l,1} \\ \vdots \\ U_{l,N_V} \end{bmatrix} \quad S_W^{(l)} = \begin{bmatrix} S_{W,11}^{(l)} & \cdots & S_{W,1N_V}^{(l)} \\ \vdots & \ddots & \vdots \\ S_{W,N_V1}^{(l)} & \cdots & S_{W,N_VN_V}^{(l)} \end{bmatrix} \quad S_B^{(l)} = \begin{bmatrix} S_{B,11}^{(l)} & \cdots & S_{B,1N_V}^{(l)} \\ \vdots & \ddots & \vdots \\ S_{B,N_V1}^{(l)} & \cdots & S_{B,N_VN_V}^{(l)} \end{bmatrix}$$

l-mode discriminant analysis

$$U_l^* = \arg\max_{U_l} \frac{Tr\left(\sum_{s=1}^{N_V}\sum_{r=1}^{N_V} U_{l,r}{}^T S_{B,rs}^{(l)} U_{l,s}\right)}{Tr\left(\sum_{s=1}^{N_V}\sum_{r=1}^{N_V} U_{l,r}{}^T S_{W,rs}^{(l)} U_{l,s}\right)}$$

Concatenate multi-view projection matrices

$$U_l^* = \arg\max_{U_l} \frac{Tr\left(U_l{}^T S_B^{(l)} U_l\right)}{Tr\left(U_l{}^T S_W^{(l)} U_l\right)}$$

$$U_l = \begin{bmatrix} U_{l,1} \\ \vdots \\ U_{l,N_V} \end{bmatrix} \quad S_W^{(l)} = \begin{bmatrix} S_{W,11}^{(l)} & \cdots & S_{W,1N_V}^{(l)} \\ \vdots & \ddots & \vdots \\ S_{W,N_V 1}^{(l)} & \cdots & S_{W,N_V N_V}^{(l)} \end{bmatrix} \quad S_B^{(l)} = \begin{bmatrix} S_{B,11}^{(l)} & \cdots & S_{B,1N_V}^{(l)} \\ \vdots & \ddots & \vdots \\ S_{B,N_V 1}^{(l)} & \cdots & S_{B,N_V N_V}^{(l)} \end{bmatrix}$$

l-mode discriminant analysis

$$U_l^* = \arg\max_{U_l} \frac{Tr\left(\sum_{s=1}^{N_V}\sum_{r=1}^{N_V} U_{l,r}{}^T S_{B,rs}^{(l)} U_{l,s}\right)}{Tr\left(\sum_{s=1}^{N_V}\sum_{r=1}^{N_V} U_{l,r}{}^T S_{W,rs}^{(l)} U_{l,s}\right)}$$

Concatenate multi-view projection matrices

$$U_l^* = \arg\max_{U_l} \frac{Tr\left(U_l{}^T S_B^{(l)} U_l\right)}{Tr\left(U_l{}^T S_W^{(l)} U_l\right)}$$

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Trace ratio: intractable

l-mode discriminant analysis

$$U_l^* = \arg\max_{U_l} \frac{Tr\left(\sum_{s=1}^{N_V}\sum_{r=1}^{N_V} U_{l,r}{}^T S_{B,rs}^{(l)} U_{l,s}\right)}{Tr\left(\sum_{s=1}^{N_V}\sum_{r=1}^{N_V} U_{l,r}{}^T S_{W,rs}^{(l)} U_{l,s}\right)}$$

Concatenate multi-view projection matrices

$$U_l^* = \arg\max_{U_l} \frac{Tr\left(U_l{}^T S_B^{(l)} U_l\right)}{Tr\left(U_l{}^T S_W^{(l)} U_l\right)}$$

$$U_l = \begin{bmatrix} U_{l,1} \\ \vdots \\ U_{l,N_V} \end{bmatrix} \quad S_W^{(l)} = \begin{bmatrix} S_{W,11}^{(l)} & \cdots & S_{W,1N_V}^{(l)} \\ \vdots & \ddots & \vdots \\ S_{W,N_V1}^{(l)} & \cdots & S_{W,N_VN_V}^{(l)} \end{bmatrix} \quad S_B^{(l)} = \begin{bmatrix} S_{B,11}^{(l)} & \cdots & S_{B,1N_V}^{(l)} \\ \vdots & \ddots & \vdots \\ S_{B,N_V1}^{(l)} & \cdots & S_{B,N_VN_V}^{(l)} \end{bmatrix}$$

Trace ratio: intractable

Relax to ratio trace

$$U_l^* = \arg\max_{U_l} Tr\left(\frac{U_l{}^T S_B^{(l)} U_l}{U_l{}^T S_W^{(l)} U_l}\right)$$

Ratio trace: tractable

l-mode discriminant analysis

$$U_l^* = \arg\max_{U_l} \frac{Tr\left(\sum_{s=1}^{N_V}\sum_{r=1}^{N_V} U_{l,r}{}^T S_{B,rs}^{(l)} U_{l,s}\right)}{Tr\left(\sum_{s=1}^{N_V}\sum_{r=1}^{N_V} U_{l,r}{}^T S_{W,rs}^{(l)} U_{l,s}\right)}$$

Concatenate multi-view projection matrices

$$U_l^* = \arg\max_{U_l} \frac{Tr\left(U_l{}^T S_B^{(l)} U_l\right)}{Tr\left(U_l{}^T S_W^{(l)} U_l\right)}$$

$$U_l = \begin{bmatrix} U_{l,1} \\ \vdots \\ U_{l,N_V} \end{bmatrix} \quad S_W^{(l)} = \begin{bmatrix} S_{W,11}^{(l)} & \cdots & S_{W,1N_V}^{(l)} \\ \vdots & \ddots & \vdots \\ S_{W,N_V 1}^{(l)} & \cdots & S_{W,N_V N_V}^{(l)} \end{bmatrix} \quad S_B^{(l)} = \begin{bmatrix} S_{B,11}^{(l)} & \cdots & S_{B,1N_V}^{(l)} \\ \vdots & \ddots & \vdots \\ S_{B,N_V 1}^{(l)} & \cdots & S_{B,N_V N_V}^{(l)} \end{bmatrix}$$

Trace ratio: intractable

Relax to ratio trace

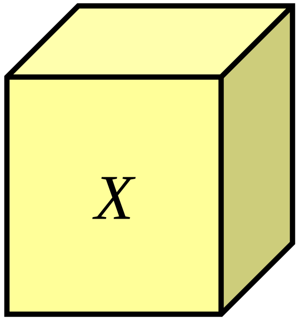
$$U_l^* = \arg\max_{U_l} Tr\left(\frac{U_l{}^T S_B^{(l)} U_l}{U_l{}^T S_W^{(l)} U_l}\right)$$

Ratio trace: tractable

Generalized eigenvalue problem

$$S_B^{(l)} U_l = S_W^{(l)} U_l \Lambda$$

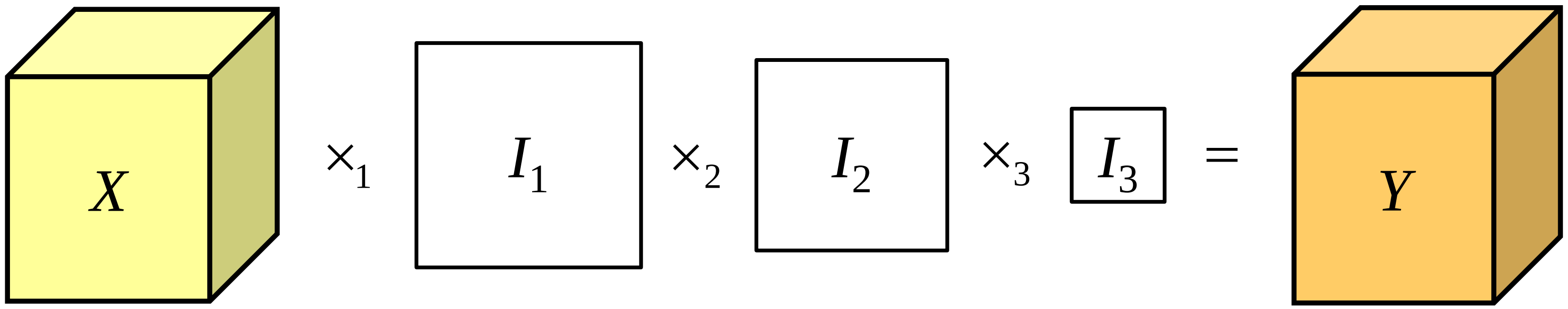
Iterative solution



3rd order tensor object

Iterative solution

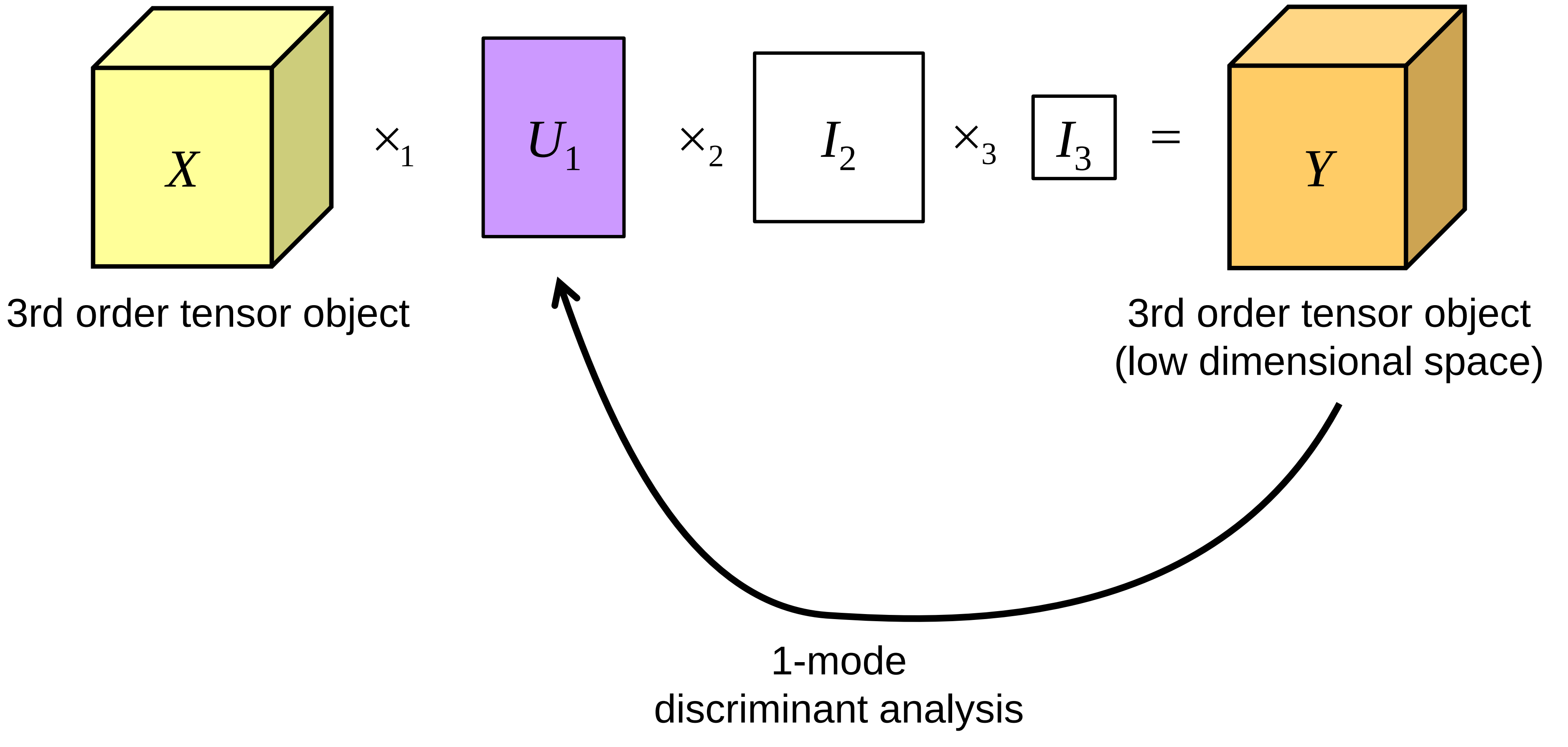
Initialize projection matrices as identity matrices



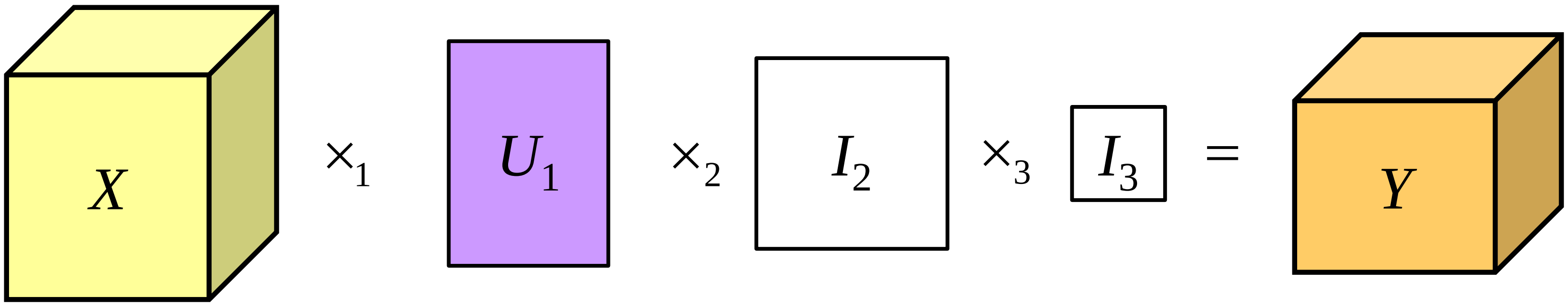
3rd order tensor object

3rd order tensor object
(low dimensional space)

Iterative solution



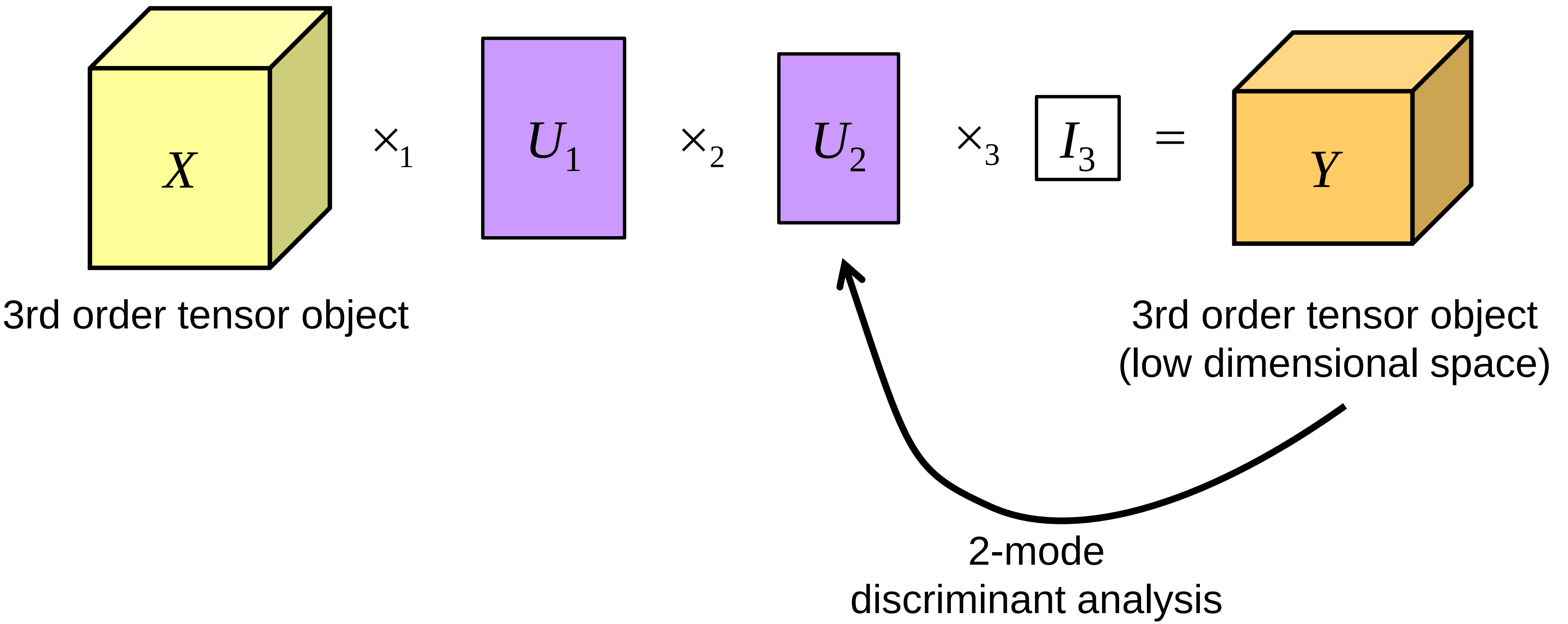
Iterative solution



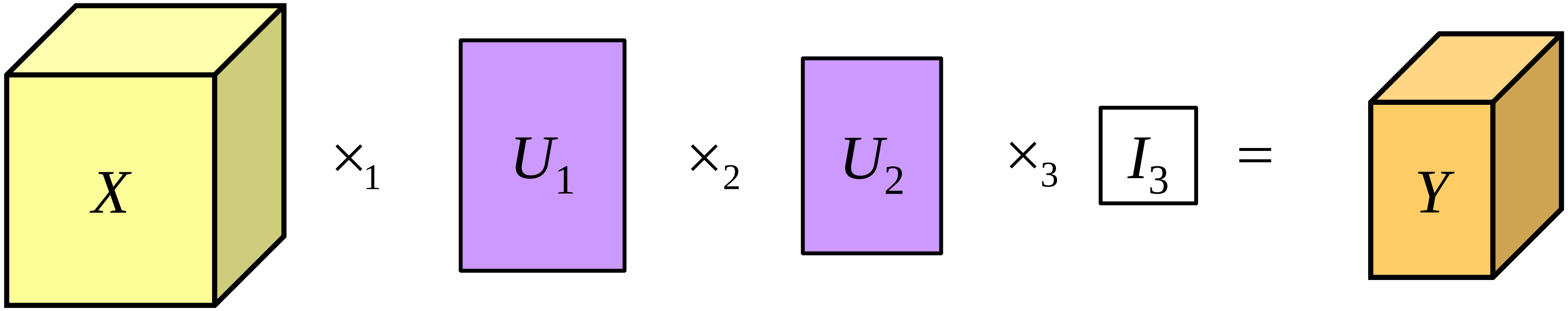
3rd order tensor object

3rd order tensor object (low dimensional space)

Iterative solution



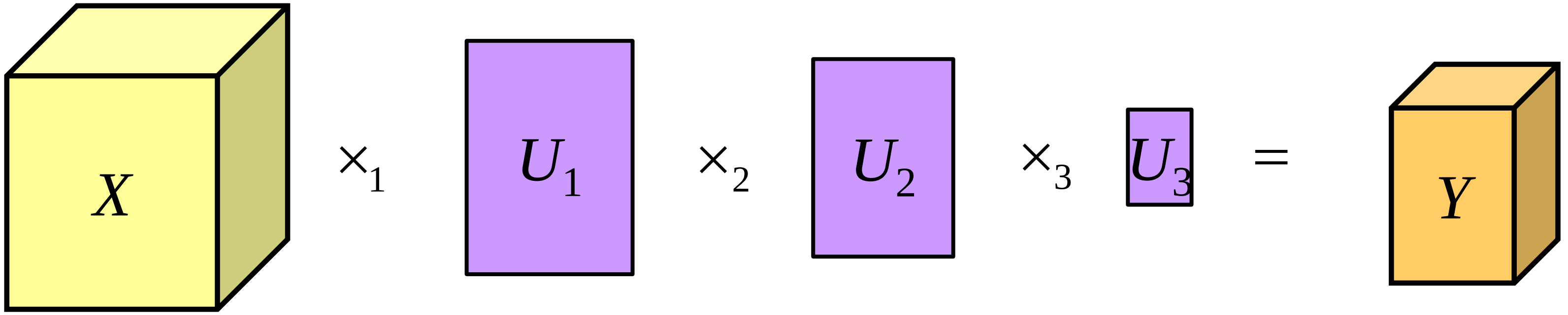
Iterative solution



3rd order tensor object

3rd order tensor object
(low dimensional space)

Iterative solution



3rd order tensor object

3rd order tensor object
(low dimensional space)

Analysis of algorithms

- Assumption
  - Equal dimension in each mode $M_l = M \ \forall l$

Algorithms	LDA	MvDA	DATER	MvDATER
Dimensionality				
#Effective training samples				
Complexity				
Cross-view discrimination capability				
Robustness to SSS problem				
Computational efficiency				

Analysis of algorithms

- Assumption
  - Equal dimension in each mode $M_l = M \ \forall l$

Algorithms	LDA	MvDA	DATER	MvDATER
Dimensionality	M^L			
#Effective training samples	n			
Complexity	$O((M^L)^3)$			
Cross-view discrimination capability	★			
Robustness to SSS problem	★			
Computational efficiency	★			

Analysis of algorithms

- Assumption
  - Equal dimension in each mode $M_l = M \ \forall l$

Algorithms	LDA	MvDA	DATER	MvDATER
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Computational efficiency	★			

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Cross-view discrimination capability	★			
Robustness to SSS problem	★			
Computational efficiency	★			

Analysis of algorithms

- Assumption
  - Equal dimension in each mode $M_l = M \ \forall l$

Algorithms	LDA	MvDA	DATER	MvDATER
Dimensionality	M^L			
#Effective training samples	n			
Complexity	$O((M^L)^3)$			
Cross-view discrimination capability	★			
Robustness to SSS problem	★			
Computational efficiency	★			

Analysis of algorithms

- Assumption
  - Equal dimension in each mode $M_l = M \; \forall l$

Algorithms	LDA	MvDA	DATER	MvDATER
Dimensionality	M^L	$N_V M^L$		
#Effective training samples	n	n/N_V		
Complexity	$O((M^L)^3)$	$O((N_V M^L)^3)$		
Cross-view discrimination capability	★	★★★		
Robustness to SSS problem	★	☆		
Computational efficiency	★	☆		

Analysis of algorithms

- Assumption
  - Equal dimension in each mode $M_l = M \ \forall l$

Algorithms	LDA	MvDA	DATER	MvDATER
Dimensionality	M^L	$N_V M^L$		
#Effective training samples	n	n/N_V		
Complexity	$O((M^L)^3)$	$O((N_V M^L)^3)$		
Cross-view discrimination capability	★	★★★		
Robustness to SSS problem	★	☆		
Computational efficiency	★	☆		

Analysis of algorithms

- Assumption
  - Equal dimension in each mode $M_l = M \ \forall l$

Algorithms	LDA	MvDA	DATER	MvDATER
Dimensionality	M^L	$N_V M^L$		
#Effective training samples	n	n/N_V		
Complexity	$O((M^L)^3)$	$O((N_V M^L)^3)$		
Cross-view discrimination capability	★	★★★		
Robustness to SSS problem	★	☆		
Computational efficiency	★	☆		

Analysis of algorithms

- Assumption
  - Equal dimension in each mode $M_l = M \;\forall l$

Algorithms	LDA	MvDA	DATER	MvDATER
Dimensionality	M^L	$N_V M^L$	M	
#Effective training samples	n	n/N_V	nM^{L-1}	
Complexity	$O((M^L)^3)$	$O((N_V M^L)^3)$	$O(N_{iter} L M^3)$	
Cross-view discrimination capability	★	★★★	☆	
Robustness to SSS problem	★	☆	★★★	
Computational efficiency	★	☆	★★★	

Analysis of algorithms

- Assumption
  - Equal dimension in each mode $M_l = M \ \forall l$

Algorithms	LDA	MvDA	DATER	MvDATER
Dimensionality	M^L	$N_V M^L$	M	
#Effective training samples	n	n/N_V	nM^{L-1}	
Complexity	$O((M^L)^3)$	$O((N_V M^L)^3)$	$O(N_{iter} L M^3)$	
Cross-view discrimination capability	★	★★★	☆	
Robustness to SSS problem	★	☆	★★★	
Computational efficiency	★	☆	★★★	

Analysis of algorithms

- Assumption
  - Equal dimension in each mode $M_l = M \ \forall l$

Algorithms	LDA	MvDA	DATER	MvDATER
Dimensionality	M^L	$N_V M^L$	M	
#Effective training samples	n	n/N_V	nM^{L-1}	
Complexity	$O((M^L)^3)$	$O((N_V M^L)^3)$	$O(N_{iter} L M^3)$	
Cross-view discrimination capability	★	★★★	☆	
Robustness to SSS problem	★	☆	★★★	
Computational efficiency	★	☆	★★★	

Analysis of algorithms

- Assumption
  - Equal dimension in each mode $M_l = M \ \forall l$

Algorithms	LDA	MvDA	DATER	MvDATER
Dimensionality	M^L	$N_V M^L$	M	$N_V M$
#Effective training samples	n	n/N_V	nM^{L-1}	nM^{L-1}/N_V
Complexity	$O((M^L)^3)$	$O((N_V M^L)^3)$	$O(N_{iter} L M^3)$	$O(N_{iter} L (N_V M)^3)$
Cross-view discrimination capability	★	★★★	☆	★★☆
Robustness to SSS problem	★	☆	★★★	★★☆
Computational efficiency	★	☆	★★★	★★☆

Analysis of algorithms

- Assumption
  - Equal dimension in each mode $M_l = M \ \forall l$

Algorithms	LDA	MvDA	DATER	MvDATER
Dimensionality	M^L	$N_V M^L$	M	$N_V M$
#Effective training samples	n	n/N_V	nM^{L-1}	nM^{L-1}/N_V
Complexity	$O((M^L)^3)$	$O((N_V M^L)^3)$	$O(N_{iter} L M^3)$	$O(N_{iter} L (N_V M)^3)$
Cross-view discrimination capability	★	★★★	☆	★★☆
Robustness to SSS problem	★	☆	★★★	★★☆
Computational efficiency	★	☆	★★★	★★☆

Application to cross-view gait recognition: Setup

- Data set

CASIA Gait Database B [Yu et al. 2006]
(CASIA)

The OU-ISIR Gait Database
Large Population data set [Iwama et al. 2012]
(OU-LP)

Application to cross-view gait recognition: Setup

- Data set

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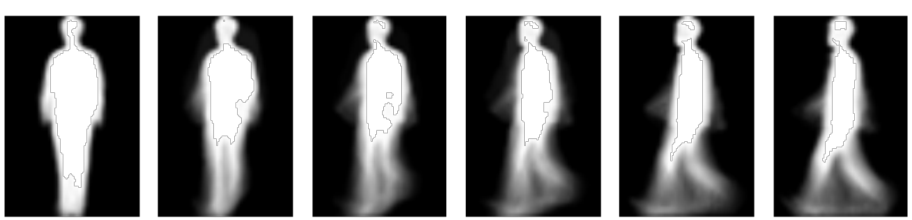
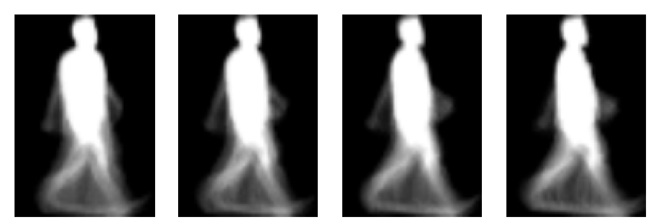
- Gait feature
  - Gait energy image (GEI) [Han and Bhanu 2006]



0° 18° 36° 54° 72° 90°

CASIA



55° 65° 75° 85°

OU-LP

Application to cross-view gait recognition: Setup

- Data set

CASIA Gait Database B [Yu et al. 2006]
(CASIA)

The OU-ISIR Gait Database
Large Population data set [Iwama et al. 2012]
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- Gait feature
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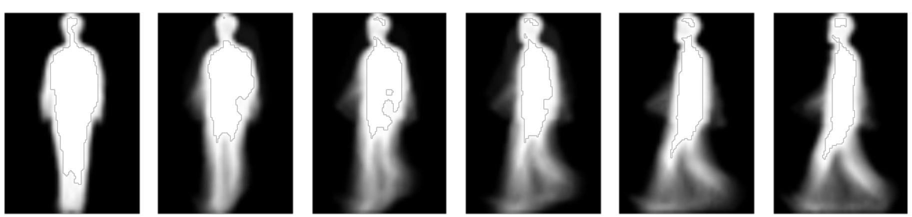
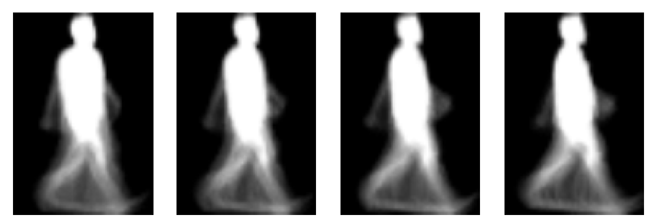


0° 18° 36° 54° 72° 90°

CASIA

Gallery



55° 65° 75° 85°

OU-LP

Gallery

Application to cross-view gait recognition: Setup

- Data set

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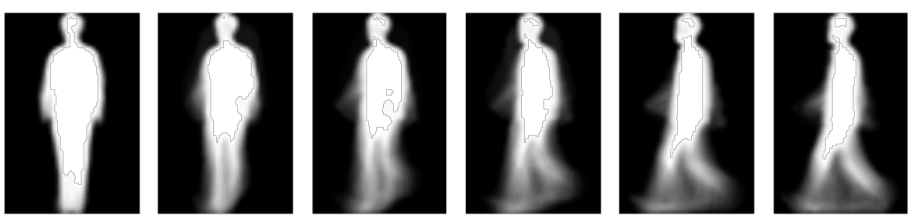
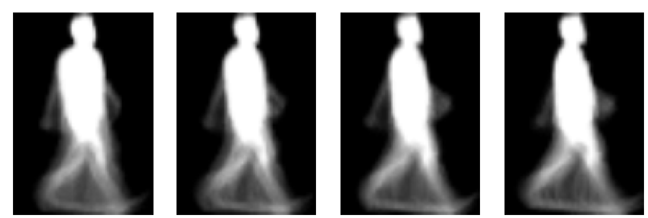


0° 18° 36° 54° 72° 90°

Probe CASIA Gallery

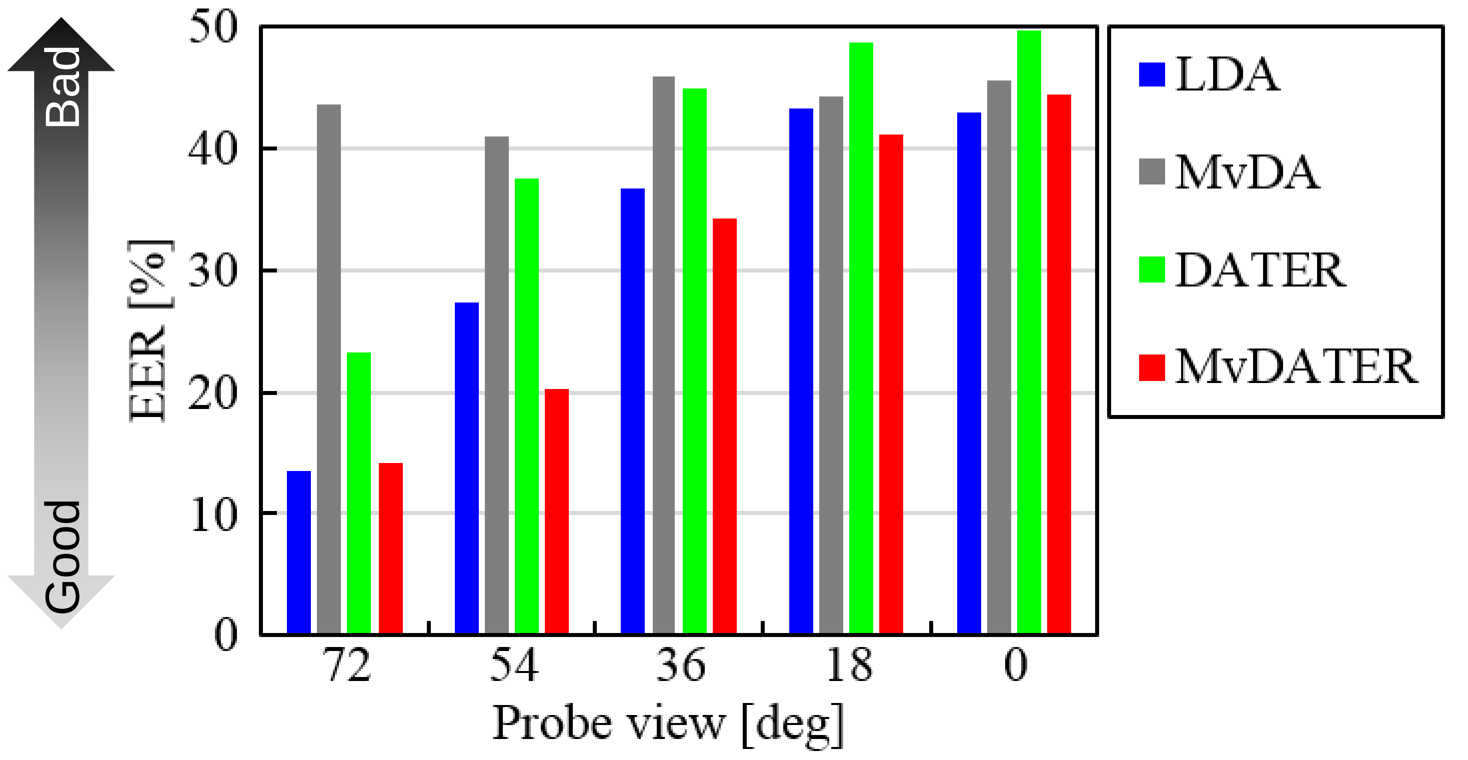


55° 65° 75° 85°

Probe OU-LP Gallery

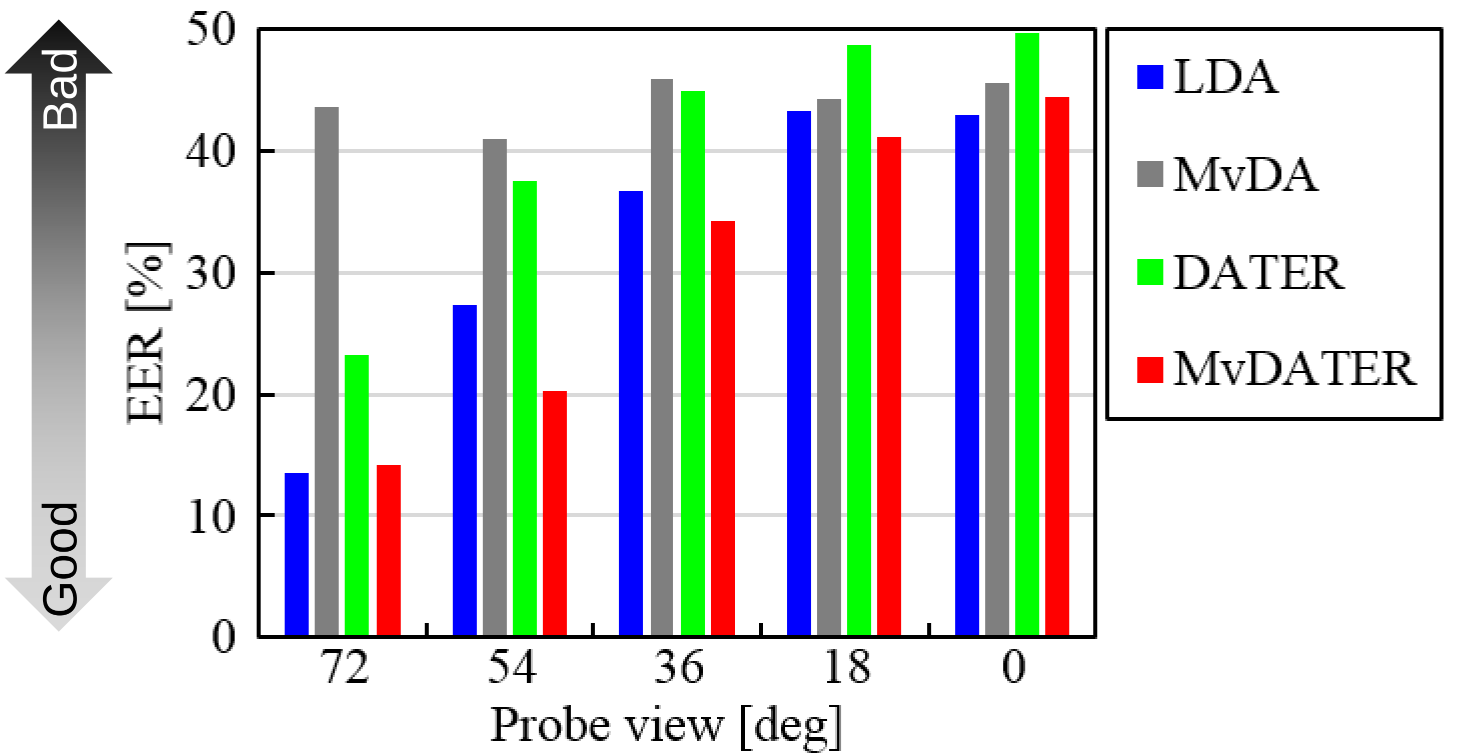
Application to cross-view gait recognition: Result for CASIA

- Single sample per training subject
  - #Training subjects: 61
  - #Test subjects: 61



Application to cross-view gait recognition: Result for CASIA

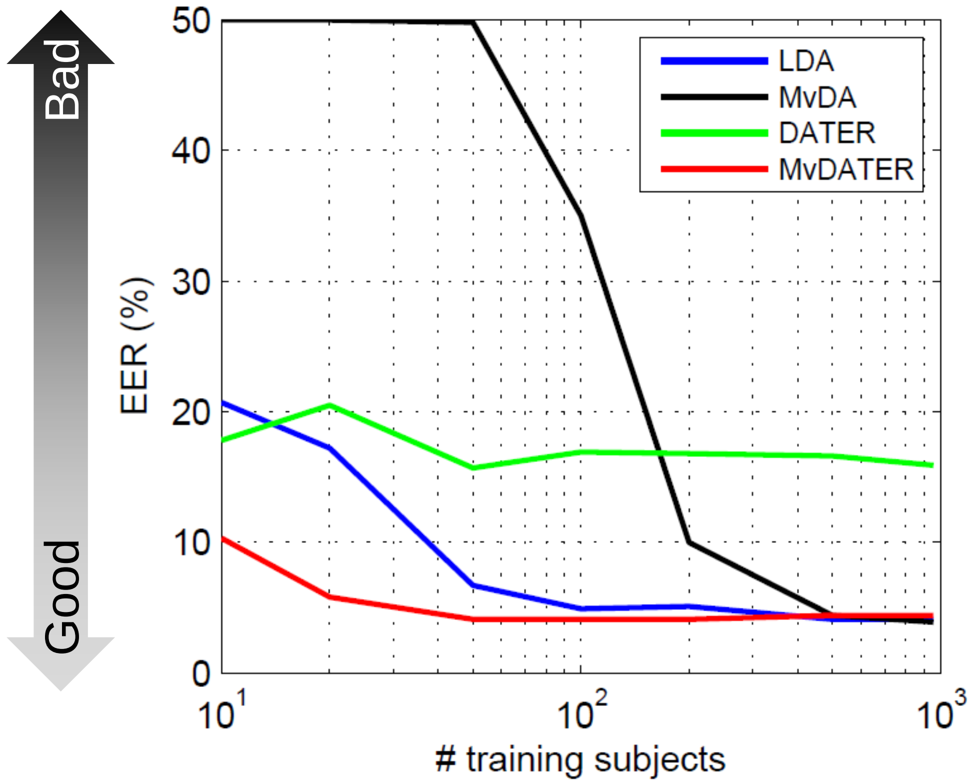
- Single sample per training subject
  - #Training subjects: 61
  - #Test subjects: 61



Proposed method yields the best accuracy for the most of cases

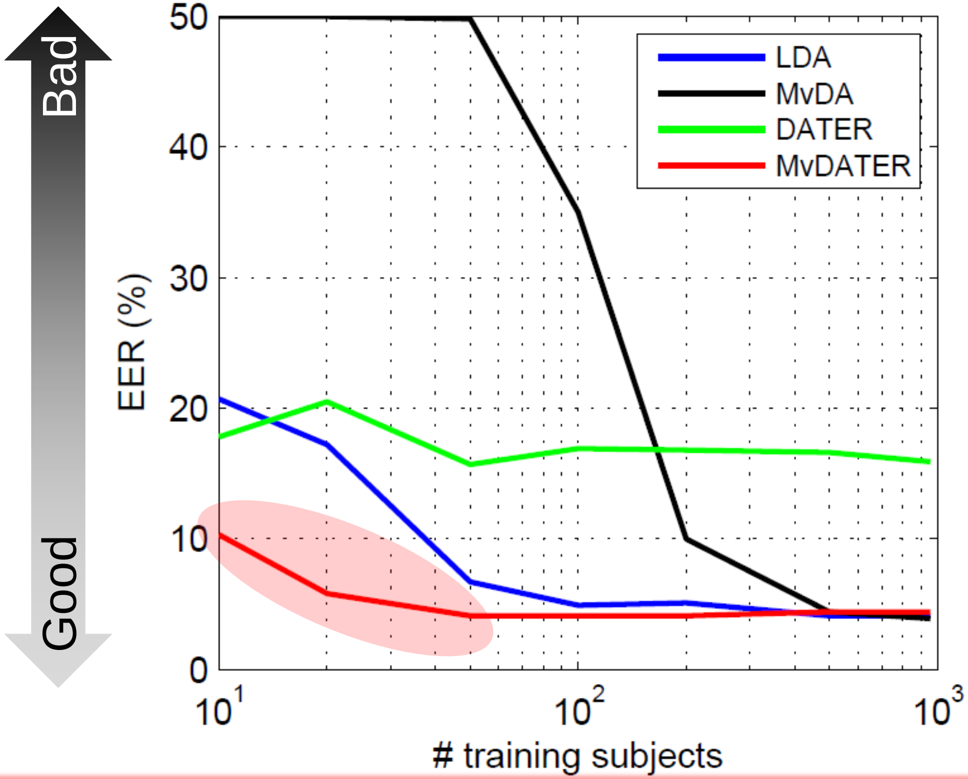
Application to cross-view gait recognition: Result for OULP (Probe: 75 deg)

- Sensitivity analysis of #training subjects
  - #Training subjects: 10 to 956
  - #Test subjects: 956



Application to cross-view gait recognition: Result for OULP (Probe: 75 deg)

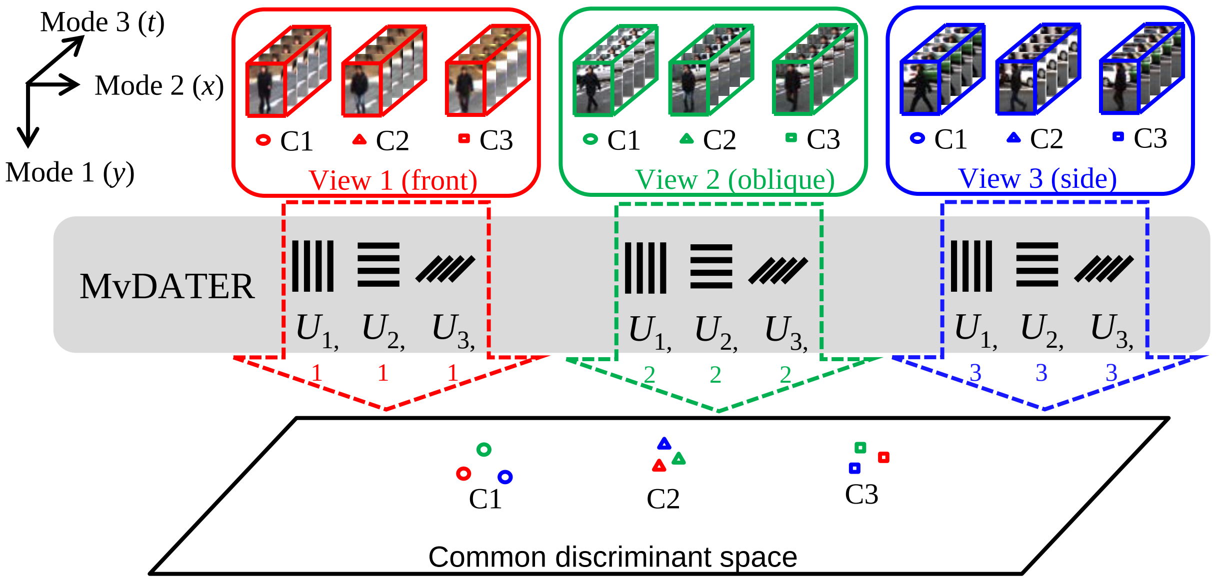
- Sensitivity analysis of #training subjects
  - #Training subjects: 10 to 956
  - #Test subjects: 956



Proposed method suppresses accuracy drop for small sample size

Summary

- **MvDATER**: **M**ulti-**v**iew **D**iscriminant **A**nalysis with **TE**nsor **R**epresentation



- Future work
  - Comparison with other cross-view gait recognition
  - Validation in various cross-view recognition